Measurement of the top quark mass in the lepton+jets channel using the Ideogram

Method

A measurement of the top quark mass using events with one charged lepton, missing transverse energy, and jets in the final state, collected by the D0 detector from $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV at the Fermilab Tevatron collider, is presented. A constrained fit is used to fully reconstruct the kinematics of the events. For every event a top quark mass likelihood is calculated taking into account all possible jet assignments and the probability that an event is signal or background.
Lifetime-based identification of $b$ jets is employed to enhance the separation between $t\bar{t}$ signal and background from other physics processes and to improve the assignment of the observed jets to the quarks in the $t\bar{t}$ hypothesis. We extract a multiplicative jet energy scale factor JES in situ, greatly reducing the systematic effect related to the jet energy measurement. In a data sample with an integrated luminosity of 425 pb$^{-1}$, we observe 230 candidate events, with an estimated background of 123 events, and measure $m_t = 173.7\pm4.4 \text{ (stat + JES)}^{+2.0}_{-1.5} \text{ (syst)}$ GeV. This result represents the first application of the Ideogram technique to the measurement of the top quark mass in lepton+jets events.

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I. INTRODUCTION

The top quark is by far the heaviest known fermion. Its discovery in 1995 [1] confirmed the structure of the standard model, while its strikingly large mass compared to other fermions highlights remaining open questions related to the large range of quark and lepton masses and the precise mechanism of electroweak symmetry breaking that explains fermion masses in the theory. Within the framework of the standard model, the top quark mass is related to the Higgs boson mass and the $W$ boson mass through radiative corrections. A precise measurement of the top quark mass helps to constrain the standard model and to predict the mass of the Higgs boson [2]. At the same time it provides a challenge to the standard model with increased precision and distinguishes possible extensions of it.

At the Fermilab Tevatron Collider, which collides protons and antiprotons with a center-of-mass energy of 1.96 TeV, the top quark is predominantly produced in $t\bar{t}$ pairs through $q\bar{q}$ annihilation ($\approx 85\%$) and gluon-gluon fusion. In the framework of the standard model, the top quark decays almost exclusively to a $b$ quark and a $W$ boson. Thus the final state topology of a $t\bar{t}$ event is determined by the decay modes of the two $W$ bosons. The analysis presented in this paper uses the lepton+jets ($\ell$+jets) channel, where one $W$ boson decays hadronically and the other $W$ boson decays leptonically to a muon or an electron and the corresponding (anti)neutrino. Tau leptons are not explicitly reconstructed in the analysis. Throughout this paper, charge conjugate modes are implicitly included.

The $\ell$+jets topology combines a sizable branching fraction with a striking signature of the isolated energetic lepton and large missing transverse energy from the escaping neutrino. The background from $W$+jets and QCD multijet events is manageable. This means that the $\ell$+jets channel is particularly suited for studies of top quark properties, and it has provided the most precise measurements of the top quark mass to date [3–7].

Nonetheless, serious challenges exist. A direct measurement of the top quark mass requires that the kinematics of the event are fully reconstructed, including the momentum of the neutrino. The signal events need to be separated from backgrounds in a manner that does not bias the mass measurement. Furthermore, with four jets in the final state, the assignment of jets to the original top quark decay products gives a twelve-fold ambiguity. Finally, a proper calibration of the jet energy scale is crucial. In previous measurements, this was the dominant systematic uncertainty.

Early measurements of the top quark mass [3] (and some recent analyses [4]) used a constrained fit to reconstruct the kinematics of the event, choosing one jet assignment based on the quality of the fit. A distribution of some event variable strongly correlated to the top mass, typically the fitted top mass, was plotted for data events. This distribution was then compared to distributions based on Monte Carlo simulation generated for different top quark masses to determine the value of the top quark mass that best agrees with the data. In the case of the D0 analysis [3], a multi-variate discriminant that separates signal from background was also used in a two-dimensional likelihood fit to the Monte Carlo reference distributions. However, in these analyses, only a certain amount of information per event is used in the final fit.

The D0 Matrix Element analysis [5] demonstrated for the first time that the statistical precision of the measurement can be greatly enhanced by constructing event-by-event likelihoods that reflect the full ambiguity of the events. A dramatic improvement was achieved, albeit at the cost of computationally intensive methods.

The analysis presented in this paper uses the Ideogram technique. This method is based on a constrained kinematic fit and strives to obtain a similar improvement in statistical precision as the Matrix Element analysis with minimal additional computation. The constrained fit is used to determine the kinematics of the events and to improve their reconstruction beyond the detector resolution. A top quark mass likelihood is derived for every event including all possible assignments of jets to quarks in the $t\bar{t}$ hypothesis, and taking into account the possibility that the event is background. The top quark mass is extracted through a combined likelihood fit including all events. This approach is very similar to the Ideogram technique used by the DELPHI experiment to measure the $W$ boson mass at the CERN LEP collider [8]. Also there the different possible jet permutations lead to an ambiguity in the mass fit which is reflected in the event likelihood as the sum of Gaussian resolution functions. The similarity with the ideogram plots used by the Particle Data Group [9] to visualize a set of measurements is what gave the method its name. This is the first time the method is used to determine the top quark mass in
the $\ell$+jets channel. Recently, it has also been applied to the all-hadronic decay channel [10].

The free parameters in the fit are the top quark mass, the $t\bar{t}$ signal fraction in the sample, and an overall jet energy scale (JES) factor. Including the JES factor as a free parameter in the fit greatly reduces the systematic uncertainty related to the jet energy scale calibration [4, 7]. We employ the tagging of $b$ jets, i.e., $b$ taggging, to enhance the separation between signal and background from other physics processes. The $b$ tags also help to better distinguish between correct and wrong jet assignments in the likelihood. Events with and without $b$ tags are included in the overall likelihood fit.

This paper is organized as follows: Sections II and III describe the D0 Run II detector and the event reconstruction, respectively. Sections IV to VI describe the data and simulation samples used and outline the event selection. In Sec. VII, the sample composition is estimated using topological and $b$ tagging information. Section VIII describes in detail the calculation of the Ideogram likelihood and the Monte Carlo calibration procedure. The method is applied to data in Sec. IX and the systematic uncertainties are discussed in Sec. X. Section XI presents a cross-check of the JES calibration, followed by the conclusion in Sec. XII.

II. THE D0 DETECTOR

The Run II of the Fermilab Tevatron collider started in 2001 after substantial detector upgrades following the first Tevatron collider run in 1992-1996. The D0 Run II detector [11] consists of a magnetic central tracking system, composed of a silicon micro-strip tracker (SMT) and a central fiber tracker (CFT), both located within a 2 T superconducting solenoidal magnet. The SMT has approximately 800,000 individual strips, with typical pitch of 50 – 80 $\mu$m, and a design optimized for tracking and vertexing capabilities at pseudorapidities of $|\eta| < 2.5$. The system has a six-barrel longitudinal structure, each with a set of four layers arranged axially around the beam pipe, and interspersed with 16 radial disks. The CFT has eight thin coaxial barrels, each supporting two doublets of overlapping scintillating fibers of 0.835 mm diameter, one doublet being parallel to the beam axis, and the other alternating by $\pm 3^{\circ}$ relative to the axis. Light signals are transferred via clear fibers to solid-state photon counters (VLPCs) that have $\approx 80\%$ quantum efficiency.

Central and forward preshower detectors located just outside of the superconducting coil (in front of the calorimetry) are constructed of several layers of extruded triangular scintillator strips that are read out using wavelength-shifting fibers and VLPCs. The next layer of detection involves three liquid-argon/uranium calorimeters: a central section (CC) covering approximately $|\eta| < 1.1$, and two end calorimeters (EC) that extend coverage to $|\eta| \approx 4.2$, all housed in separate cryostats [12]. The calorimeter consists of an electromagnetic (EM) section followed by fine and coarse hadronic sections with modules assembled in a projective geometry to the interaction region. In addition to the preshower detectors, scintillators between the CC and EC cryostats provide sampling of developing showers for $1.1 < |\eta| < 1.4$.

A muon system [13] resides beyond the calorimetry and consists of a layer of tracking detectors and scintillation trigger counters before 1.8 T iron toroids, followed by two similar layers after the toroids. Tracking for $|\eta| < 1$ relies on 10 cm wide drift tubes [12], while 1 cm mini-drift tubes are used for $1 < |\eta| < 2$.

Trigger and data acquisition systems are designed to accommodate the high luminosities of Run II. Based on preliminary information from tracking, calorimetry, and muon systems, the output of the first level of the trigger is used to limit the rate for accepted events to approximately 2 kHz. At the next trigger stage, with more refined information, the rate is reduced further to about 1 kHz. These first two levels of triggering rely mainly on hardware and firmware. The third and final level of the trigger, with access to all of the event information, uses software algorithms and a computing farm, and reduces the output rate to about 50 Hz, which is written to tape.

III. EVENT RECONSTRUCTION

This section summarizes the offline event reconstruction. We use a right-handed Cartesian coordinate system with the $z$ axis defined by the direction of the proton beam, the $y$ axis pointing vertically upwards and the $x$ axis pointing out from the center of the accelerator ring. The origin is at the center of the detector. The polar angle $\theta$ is defined with respect to the positive $z$ axis and $\phi$ is the azimuthal angle from the $x$ axis in the transverse $xy$ plane. The pseudorapidity $\eta$ is defined as $\eta \equiv -\ln(\tan(\theta/2))$.

A. Tracks and event vertex

Tracks are reconstructed from the hit information in the SMT and CFT. A Kalman filter [14] is used to fit track candidates found by a road-based algorithm or a technique searching for clusters of track parameters formed by tracker hits. Using a vertex search procedure [15], a list of reconstructed primary vertices is returned. The primary event vertex for the $t\bar{t}$ reconstruction is chosen from this list based on the $p_T$ spectrum of the tracks associated with a given vertex. Only vertices with at least three tracks associated with them are considered.

B. Electrons

We reconstruct electrons using information from the calorimeter and the central tracker. Clusters of EM calorimeter cells (EM clusters) are built with a simple
The jet cone are further corrected for the momentum carried by the muon and the associated neutrino. Since the method to extract the top quark mass is calibrated with respect to the Monte Carlo simulation, it is important to determine the relative jet energy scale $S$ between data and the Monte Carlo simulation,

$$ S = \frac{\langle p_T^{\text{jet}} - p_T^{\gamma} \rangle_{\text{data}}}{\langle p_T^{\text{jet}} - p_T^{\gamma} \rangle_{\text{MC}}} . $$

$S$ is parameterized as a function of photon $p_T$ for several bins in $(p_T, \eta)$ space and is found to be flat within its uncertainties. No corrections from this source are therefore applied. Effects of a potential $p_T$ dependence are taken into account as a systematic uncertainty. For the overall jet energy scale, a uniform factor, $JES$, is introduced as a free parameter in the analysis. This factor is fitted in situ, simultaneously with the top quark mass in data by using information from the invariant mass of the hadronically decaying $W$ bosons. For every event, this mass is constrained in the kinematic fit to be equal to the known value of the $W$ boson mass [9]. The $\chi^2$ of the kinematic fit reflects the compatibility of the reconstructed jet energies with this constraint. The likelihood is sensitive to the $JES$ parameter through the $\chi^2$. The overall fit will give the maximum likelihood for the value of $JES$ which optimizes, on average, the compatibility between the reconstructed and fitted jet energies.

Apart from the $W$ boson mass information, no constraint on the overall energy scale is used in the top quark mass fit. The jet energy scale measured in situ is consistent with the result obtained from photon+jet studies (Sec. XI).

The analysis is calibrated such that in pseudo-experiments with Monte Carlo events the average fitted $JES$ value is equal to one. A fitted value $JES < 1$ means that the jet energies in the sample considered are underestimated with respect to the reference Monte Carlo scale described above ($JES < 1$ is equivalent to $S < 0$ when fitting the data sample).

### E. Missing transverse energy

We identify neutrinos indirectly from the energy imbalance in the event. The imbalance is reconstructed from the vector sum of the transverse energies in the calorimeter cells and the reconstructed muons. Energies from the cells in the coarse hadronic portion of the calorimeter are only added if associated with a reconstructed jet. The missing transverse energy, $E_T$, is corrected for the energy scale calibration of jets and electrons.

### F. $b$-jet identification

We identify $b$ jets using a lifetime tagging algorithm (secondary vertex tagger, SVT) based on the explicit re-
construction of a secondary vertex from the decay of a $b$-flavored hadron [17]. We call $dca$ the distance of closest approach between a track and the beam line, with $\sigma(dca)$ being the uncertainty on $dca$. After the reconstruction of the primary event vertex, we consider tracks with $dca/\sigma(dca) > 3.5$ for the reconstruction of additional (secondary) vertices. For a reconstructed secondary vertex, the transverse decay length $L_{xy}$ with respect to the primary event vertex is computed. A jet is tagged as a $b$ jet if a secondary vertex is reconstructed within $\Delta R < 0.5$ of the jet with $L_{xy}/\sigma(L_{xy}) > 7.0$, where $\sigma(L_{xy})$ is the uncertainty on $L_{xy}$. The $b$-jet tagging rate $\epsilon_b$ is measured in data using information from an independent $b$-tagging analysis that looks for the presence of a muon in the jet cone.

Light quark jets can also be tagged when a fake secondary vertex is reconstructed due to track mis-measurements and random overlaps of tracks. This light jet tagging rate $\epsilon_l$ is estimated from the rate of secondary vertices with $L_{xy}/\sigma(L_{xy}) < -7.0$ in a data sample with predominantly light quark jets. Negative values of $L_{xy}$ occur if the secondary vertex is on the opposite side of the event vertex with respect to the jet and are a sign of mis-measurement and resolution effects. Misreconstructed vertices with negative and positive values of $L_{xy}$ are expected to occur at the same rate. Corrections for the contamination with heavy flavor and the presence of long lived particles are applied as determined from Monte Carlo simulation. The $b$-jet and light-jet tagging rates are measured in data and are parametrized as a function of jet transverse momentum and pseudorapidity [15].

IV. DATA SAMPLES

This paper describes the analysis of data collected between April 2002 and August 2004, corresponding to an integrated luminosity of approximately 425 pb$^{-1}$. For this analysis, the data sample was selected by triggering on a lepton and at least one additional jet in the events. The specific trigger requirements are described in more detail in Ref. [15].

The event selection requires an isolated lepton of transverse momentum $p_T > 20$ GeV, with a pseudorapidity $|\eta| < 1.1$ for electrons and $|\eta| < 2$ for muons. Missing transverse energy $E_T > 20$ GeV is required as well as four or more jets with $p_T > 20$ GeV and $|\eta| < 2.5$. A $\Delta\phi$ cut between $E_T$ and lepton momentum is imposed to exclude events where the transverse energy imbalance is caused by a poor measurement of the lepton energy. The position of the event vertex along the beam direction has to be within 60 cm of the center of the detector. We select 246 candidate events.

A QCD multijet background sample is also extracted from data by reversing the final lepton quality requirement. Leaving all other event selection cuts unchanged, the candidate isolated muon must fail to be isolated from activity in the tracker or calorimeter (Sec. III C) in the muon+jets channel. Similarly, in the electron+jets channel the candidate electron must not be matched to a track or fail to pass the electron likelihood cut (Sec. III B).

V. SIMULATION

Monte Carlo event generators are used to create large samples of simulated signal and background events. These samples are used for the calibration of the central mass value and the estimate of the uncertainty. We use ALPGEN 1.3 [18] to generate signal and $W+$jets background events. The underlying event and hadronization is simulated using PYTHIA 6.2 [19]. Signal $t\bar{t}$ events are generated at nine mass points with masses ranging from 150 GeV to 200 GeV. The factorization and renormalization scales are set to $Q = m_t$ for the $t\bar{t}$ simulation and $Q^2 = M_W^2 + \sum(p_T^{jet})^2$ for $W+$jets. All events are passed through a full GEANT-based [20] D0 detector simulation and reconstructed with the same software as the collider data. Events are accepted according to the probability that a simulated event would pass the trigger requirements. This probability is typically between 0.9 and 1.0. The same object and event selections as for the data samples are applied. The simulation chain is tuned to reproduce resolutions of reconstructed objects seen in the collider data.

VI. KINEMATIC FIT AND FINAL EVENT SELECTION

The kinematics of the events, including the undetected neutrino from the $W$ boson decay, are reconstructed using the same kinematic constrained fit that was developed for the Run I analysis [3]. The resolutions of muons, electrons and jets were updated for Run II [7, 21, 22].

In events with more than four jets, only the four jets with highest $p_T$ are considered as possible candidates to be a light quark or $b$ quark in the $t\bar{t}$ hypothesis used in the constrained fit.

All twelve possible assignments of jets to quarks are considered. As a starting point for the kinematic fit, the unmeasured component of the neutrino momentum parallel to the beam, $p_T^\nu$, is chosen such that the two top quarks are assigned equal mass. This yields a quadratic equation for $p_T^\nu$. We use both solutions as input to the fit yielding twenty-four fit results per event. Depending on the event kinematics and resolution effects, the discriminant of the quadratic equation may be negative, in which case the discriminant is forced to be zero. Thus one or two solutions are always obtained. If only one solution is available, we include the same fit result twice in the likelihood.

For the kinematic fit, we relate the reconstructed jet energy to the unfragmented parton energy. To this end, a jet-parton energy mapping is applied, which is the same in data and MC simulation. The corrections depend on
the flavor (b quark or light quark) of the parent quark and therefore depend on the jet-to-parton assignment used. To derive the mapping functions, we use MC events where the jets are unambiguously matched to the partons of the $t\bar{t}$ decay and compare the jet energy to the MC generated parton energy. The jet-parton mapping functions contain the JES parameter as a uniform multiplicative factor.

The kinematic fit is performed by minimizing a $\chi^2$ subject to the kinematic constraints: $m(t \to \ell\nu b) = m(\bar{t} \to q\bar{q}\bar{b})$, $m(\ell\nu) = M_W$, and $m(q\bar{q}) = M_W$. We use $M_W = 80.4$ GeV [9]. The minimization algorithm uses the method of Lagrange multipliers; the nonlinear constraint equations are solved using an iterative technique. From the fit for each jet/neutrino solution $i$, we extract the mass $m_i$, the estimated uncertainty on the fitted mass $\sigma_i$, and the goodness of fit $\chi^2_i$. The fit is repeated for different values of $JES$. The JES parameter is varied in steps of 3% in an interval of $\pm 15\%$ around unity. Only jet combinations for which the fit converges at all values of JES are used. This requirement is needed to prevent discontinuities as a function of JES in the event likelihood. The fitted mass $m_i(JES)$, estimated uncertainty $\sigma_i(JES)$, and goodness of fit $\chi^2_i(JES)$ all depend on the JES parameter. In the following this dependence is not shown explicitly, to improve readability.

The final selection requirement is that at least one jet/neutrino solution yields $\chi^2 < 10$ for the kinematic fit with $JES = 1$. This cut reduces the number of events from 120 to 116 in the electron+jets channel and from 126 to 114 in the muon+jets channel. Most of the events removed by this cut are background events or badly reconstructed $t\bar{t}$ events that do not satisfy the $t\bar{t}$ fit hypothesis and do not carry useful information about the top quark mass. The algorithmic efficiency of the kinematic fit is excellent, as listed in Table I.

| Table I: The numbers of events and efficiencies for the electron+jets (e) and muon+jets (μ) channel having at least one jet combination for which the fit converges at $JES = 1$, without and with the requirement on the maximum value of the $\chi^2$. Each column shows the relative efficiency with respect to the previous column. |

<table>
<thead>
<tr>
<th>$JES$</th>
<th>Before fit converges</th>
<th>$\chi^2 &lt; 10$</th>
<th>All $JES^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>tt</td>
<td>9452</td>
<td>100.0%</td>
<td>97.7%</td>
</tr>
<tr>
<td>μ</td>
<td>9265</td>
<td>99.8%</td>
<td>94.4%</td>
</tr>
<tr>
<td>W+jets</td>
<td>5163</td>
<td>100.0%</td>
<td>94.2%</td>
</tr>
<tr>
<td>μ</td>
<td>5520</td>
<td>99.7%</td>
<td>89.9%</td>
</tr>
<tr>
<td>data</td>
<td>120</td>
<td>100.0%</td>
<td>97.0%</td>
</tr>
<tr>
<td>μ</td>
<td>126</td>
<td>100.0%</td>
<td>91.0%</td>
</tr>
</tbody>
</table>

* for all values of $JES$ in the fit range $0.85 < JES < 1.15$

VII. SAMPLE COMPOSITION

In order to obtain a good separation between $t\bar{t}$ signal and background events (mainly $W$+jets), a likelihood discriminant based on the ‘low-bias’ topological discriminant $D_{LB}$, developed in Run I [3], is used. The $D_{LB}$ discriminant was designed to have minimal correlation with the top mass and is based on the following four topological variables: $E_T$, aplanarity, $H_{T2}$, and $K_{Tmin}$. Aplanarity is defined as the smallest eigenvalue of the normalized laboratory-frame momentum tensor of the jets and the W boson. $H_{T2} \equiv \frac{H_T}{H_T^2}$ measures the event centrality, where $H_{T2}$ is the scalar sum of the transverse momenta of the jets excluding the leading jet and $H_T$ is the sum of the magnitudes of the momentum components parallel to the beamline of the jets, isolated lepton and neutrino. In this case the neutrino momentum parallel to the beam is estimated requiring that the mass computed from the measured lepton momentum, $E_T$ and unknown neutrino momentum parallel to the beam is equal to the W boson mass. If more than one solution is found, the one smallest in absolute value is used. The variable $K_{Tmin} = \frac{\Delta R_{min}^{lep}}{E_T}$ is a measure of the jet separation normalized by the transverse energy of the reconstructed W boson. $\Delta R_{min}^{lep}$ is the smallest distance in $\eta - \phi$ space between any two of the four leading jets. $E_T^{lep}$ is the smaller of the two jet $E_T$s. The transverse energy of the W boson is defined as $E_T^{lep} \equiv \sqrt{p_T^{lep} + E_T}$. These four variables are combined in a single discriminant variable $D_{LB}$ using the likelihood ratio procedure described in Ref. [3].

For the analysis presented here, the low-bias discriminant $D_{LB} (= x_1)$ was combined with a new variable called “$p_T$-fraction” and the number of b tags to build a combined discriminant $D$. The $p_T$-fraction, defined as $x_2 = \langle \sum \text{tracks in jets $p_T$} / \langle \sum \text{all tracks $p_T$} \rangle$, is the $p_T$-weighted fraction of all tracks in the event that point to an energy deposit defining a jet (with jet $p_T > 20$ GeV with $|\eta| < 2.5$). Only those tracks were considered that have a distance of closest approach of less than 1 cm along the beam direction with respect to at least one of the primary vertices in the event. In order to be included in the $p_T$ sum over tracks in a jet, a track was required to be within $\Delta R < 0.5$ from the jet axis. This variable distinguishes clean events with nicely collimated jets from events with broader jets and significant underlying hadronic activity. Finally, $x_3$ is the number of b tags. For each variable $x_i$, we use Monte Carlo simulation to determine the probability density functions $s_i$, for $t\bar{t}$ signal, and $b_i$, for $W$+jets background. To a good approximation, these three variables $x_i$ are uncorrelated, and the combined likelihood discriminant is derived as

$$D = \frac{\Pi_i s_i(x_i) / b_i(x_i)}{\Pi_i s_i(x_i) / b_i(x_i) + 1},$$

thus combining event topology with a tracking-based jet shape and $b$ tag information. This combined likelihood
discriminant offers a much better discrimination between $t\bar{t}$ and background than does the low-bias topological variable $D_{l\bar{b}b}$ by itself, while maintaining its low level of correlation with the fitted top quark mass (and therefore with the jet energy scale).

Figure 1 shows the distribution of the combined discriminant $D$ obtained in the electron+jets and muon+jets channels. The distribution observed in data is compared to a model consisting of simulated $t\bar{t}$ and $W$+jets events and the QCD multijets sample obtained from data (Sec. IV). A likelihood fit is performed to determine the estimated fraction of $t\bar{t}$ events. The fit results are shown in Table II. In the fit, the ratio between the number of QCD and $W$+jets events was kept fixed at a value based on the estimate used in Refs. [7, 23].

<table>
<thead>
<tr>
<th>TABLE II: Composition of the 425 pb$^{-1}$ data sample as determined by the likelihood fit.</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron+jets</td>
</tr>
<tr>
<td>$t\bar{t}$</td>
</tr>
<tr>
<td>$W$+jets</td>
</tr>
<tr>
<td>QCD multijet</td>
</tr>
<tr>
<td>Total observed</td>
</tr>
</tbody>
</table>

VIII. THE IDEOGRAM METHOD

To maximize the statistical information on the top quark mass extracted from the event sample, a likelihood to observe the event is calculated for each event as a function of the assumed top quark mass $m_{t}$, the jet energy scale parameter $JES$, and the fraction of $t\bar{t}$ events in the event sample, $f_{t\bar{t}}$. The likelihood is composed of two terms, describing the hypotheses that the event is $t\bar{t}$ signal or background:

$$\mathcal{L}_{\text{evt}}(x; m_{t}, JES, f_{t\bar{t}}) = f_{t\bar{t}} \cdot P_{\text{sgn}}(x; m_{t}, JES) + (1 - f_{t\bar{t}}) \cdot P_{\text{bkg}}(x; JES).$$

(4)

Here, $x$ denotes the full set of observables that characterizes the event, $f_{t\bar{t}}$ is the signal fraction of the event sample, and $P_{\text{sgn}}$ and $P_{\text{bkg}}$ are the probabilities for $t\bar{t}$ and $W$+jets production, respectively. The contribution from QCD multijet events is comparatively small and expected to have a fitted mass shape very similar to that of $W$+jets events. Therefore no explicit QCD multijet term is included in the likelihood. The event observables $x$ can be divided into two groups. One set is chosen to provide good separation between signal and background events while minimizing the correlation with the mass information in the event. These variables (topological variables and $b$ tagging) are used to construct a low-bias combined discriminant $D$, as described in Sec. VII. The other event information used is the mass information $x_{\text{fit}}$ from the constrained kinematic fit, which provides the sensitivity to the top quark mass and jet energy scale.

To good approximation $D$ is uncorrelated with $x_{\text{fit}}$, and with the jet energy scale. Thus the probabilities $P_{\text{sgn}}$ and $P_{\text{bkg}}$ can be written as the product of a probability to observe a value $D$ and a probability to observe $x_{\text{fit}}$, as

$$P_{\text{sgn}}(x; m_{t}, JES) = P_{\text{sgn}}^{D}(D) \cdot P_{\text{sgn}}(x_{\text{fit}}; m_{t}, JES)$$

(5)

and

$$P_{\text{bkg}}(x; JES) = P_{\text{bkg}}^{D}(D) \cdot P_{\text{bkg}}(x_{\text{fit}}; JES)$$

(6)

where $D$ is calculated for a $JES$ parameter equal to 1. The normalized probability distributions of the discriminant $D$ for signal $P_{\text{sgn}}^{D}(D)$ and background $P_{\text{bkg}}^{D}(D)$ are assumed to be independent of $JES$ and are obtained from Monte Carlo simulation as discussed in Sec. VII. They correspond to parameterized versions of the Monte Carlo templates shown in Fig. 1. The reconstruction of the signal and background probabilities for the mass information $x_{\text{fit}}$ is explained in Sec. VIII A. The mass information in the event $x_{\text{fit}}$ consists of all fitted masses $m_{i}(JES)$, estimated uncertainties $\sigma_{i}(JES)$, and goodness-of-fit $\chi^{2}_{i}(JES)$ obtained from the kinematic fit.

A. Calculation of signal and background probability

The signal and background probabilities are calculated as a sum over all twenty-four possible jet/neutrino solutions. Without $b$ tagging, the relative probability for each of the solutions $i$ to be correct depends only on the $\chi^{2}_i$ for the corresponding fit and is proportional to $\exp(-\frac{1}{2}\chi^{2}_i)$. To further improve the separation between correct and incorrect jet assignments, $b$ tagging is used. If one or more jets in the event are $b$ tagged, an additional relative weight $w_{\text{btag},i}$ is assigned, representing the probability that the observed $b$ tags are compatible with the jet assignment assumed for that particular jet permutation:

$$w_{\text{btag},i} = \prod_{j=1}^{\text{nj}_{\text{jet}}} p_{i,j}^{D},$$

(7)

where $p_{i,j}^{D}$ can either be $\varepsilon_{t,j}$, $(1-\varepsilon_{t})$, $\varepsilon_{b,j}$, or $(1-\varepsilon_{b})$, depending on the assumed flavor of the jet (light or $b$) and whether or not that particular jet is tagged. For this purpose the jets from the hadronic $W$ boson decay are always assumed to be light quark ($u, d, s$) jets. In the calibration of the analysis (see Sec. VIII E), however, the fraction of $W \rightarrow c\bar{s}$ decays and the higher tagging rate for $c$ quark jets are taken into account. The tagging rates for light and $b$ quark jets $\varepsilon_{t}$ and $\varepsilon_{b}$ are used as parameterized functions of jet $p_{T}$ and $\eta$. The jet $p_{T}$ is based on the reconstructed jet energy for $JES = 1$, consistent with the jet energy scale for which the tagging rate functions
are derived from data [17]. Thus, the weight assigned to each jet combination becomes

\[ w_i = \exp(-\frac{1}{2} \chi^2_i) \cdot w_{\text{btag},i}. \]  

(8)

The mass-dependent signal probability in Eq. 5 is calculated as

\[ P_{\text{sgn}}(x_{\text{fit}}; m_t, \text{JES}) = \sum_{i=1}^{24} w_i \left[ f^{\text{correct}}_{\text{fit}} \cdot \int_0^m G(m_i, m', \sigma_i) \cdot \mathbf{B W}(m', m_i) dm' + (1 - f^{\text{correct}}_{\text{fit}}) \cdot S^{\text{tag}}_{\text{wrong}}(m_i, m_i) \right]. \]  

(9)

The signal term consists of two parts: one part describes the compatibility of the solution with a certain value of the top quark mass, assuming that it is the correct solution. It takes into account the estimated mass resolution \( \sigma_i \) for each jet permutation. The second part of the signal term describes the expected shape of the mass spectrum for the “wrong” jet assignments, which also depends on the top quark mass. The “correct” solution part is given by a convolution of a Gaussian resolution function \( G(m_i, m', \sigma_i) \) and a relativistic Breit-Wigner \( \mathbf{B W}(m', m_i) \). The Gaussian function describes the experimental resolution. The relativistic Breit-Wigner represents the expected distribution of the average invariant mass \( m' \) of the top and anti-top quark in the event for a given top quark mass \( m_t \). The width of the Breit-Wigner is set to the standard model value of the top decay width [9]. The “wrong” permutation signal shape \( S^{\text{tag}}_{\text{wrong}}(m_i, m_i) \) is obtained from MC simulation using a procedure described in Sec. VIIIIB. These two terms are assigned relative weights depending on the fraction \( f^{\text{correct}}_{\text{fit}} \), which represents the relative probability that the weight is assigned to the correct jet permutation. For well-reconstructed events with exactly 4 jets, this probability is approximately 39% if b tagging is not used. For 4-jet events with 0, 1, or \( \geq 2 \) tagged jets, the values \( f^0_{\text{correct}}=0.45, f^1_{\text{correct}}=0.55, \) and \( f^2_{\text{correct}}=0.65 \) are used. For 5-jet events smaller fractions are used: 0.15, 0.30, and 0.40 for events with 0, 1, or \( \geq 2 \) tagged jets respectively. Ensemble tests (see Sec. VIIIIE) confirm that these values result in a pull width for the mass close to unity for the different tagging multiplicities.

The background term in Eq. 6 is calculated as:

\[ P_{\text{bkg}}(x_{\text{fit}}; \text{JES}) = \sum_{i=1}^{24} w_i \cdot \mathbf{BG}(m_i), \]  

(10)

where the background shape \( \mathbf{BG}(m) \) is the shape of the fitted mass spectrum for \( W+\text{jets} \) events. To obtain \( \mathbf{BG}(m) \), the kinematic fit (with JES equal to unity) is applied to simulated \( W+\text{jets} \) events and the fitted masses \( m_i \) for all possible jet/neutrino solutions \( i \) are plotted. All entries are weighted according to the permutation weight \( w_i \) defined in Eq. 8. The shapes of \( \mathbf{BG}(m) \) used in the analysis are shown in Fig. 2.

The Breit-Wigner and “wrong” permutation signal shape are normalized to unity within the integration interval of \( m_{\text{min}} = 100 \text{ GeV} \) to \( m_{\text{max}} = 300 \text{ GeV} \). This interval is chosen to be large enough so as not to bias the mass in the region of interest.

The normalization of the background shape \( \mathbf{BG}(m) \) is chosen such that the fitted signal fraction \( f_{\text{top}} \) reproduces the true \( tt \) fraction in ensemble tests (Sec. VIIIIE) containing \( tt \) and \( W+\text{jets} \) events. The mass fit tends to underestimate \( f_{\text{top}} \), due to the presence of \( tt \) events that are misreconstructed or affected by energetic gluon radiation and resemble \( W+\text{jets} \) events in the fact that their topology does not conform to the \( tt \) hypothesis in the kinematic fit. A constant normalization factor of 1.15 is found to reduce the offset in \( f_{\text{top}} \) to less than 1% both in the electron+jets and the muon+jets channel. The jet energy scale parameter is varied before performing
FIG. 2: Histograms showing the background shape from a weighted sum (see text) of all twenty-four masses from each event from the $W+$jets background sample (points with error bars), for the electron+jets channel (left) and muon+jets (right). The histograms show the shapes that are used in the likelihood. To reduce statistical fluctuations, the shapes are calculated as the average value in a sliding window of $\pm 5$ GeV around each fitted mass.

the constrained fit by scaling all jet energies by a constant factor. The event likelihoods are recalculated for each different value of the JES parameter. Since the constrained fit uses a $W$ boson mass constraint, the $\chi^2$ in the fit will be best when the invariant mass of the jets from the hadronically decaying $W$ boson is closest (on average) to the known $W$ boson mass. Additional sensitivity to the jet energy scale comes from the shape of the fitted mass distribution in background events. For the proper jet energy scale the spectrum will agree best with the background shape included in the background term in the likelihood.

B. Determination of the wrong-permutation signal shape

The convolution of Gaussian detector resolution and a Breit-Wigner, used in the signal term of the likelihood, implicitly assumes that the correct jet assignment is chosen. To describe the contribution from wrong jet assignments, a separate term is added to the signal part of the likelihood. To obtain the fitted mass spectrum of the wrong permutation signal, samples of parton-matched $t\bar{t}$ events are used in which all quarks are matched to jets. The fitted mass spectrum is plotted including all jet permutations except the correct solution (excluding both neutrino solutions corresponding to the correct jet permutation). Each entry is weighted according to the permutation weight assigned in the Ideogram likelihood. Samples of different generated top quark masses are used. For each mass, the weighted sum of wrong solutions is fitted with a double Gaussian. The fitted parameters for correct solutions and for the wrong permutation signal show a linear behavior as a function of the top quark mass. The fitted parameters are given in Table III. Since the permutation weights change when $b$ tagging is included, this exercise is repeated for events with 0 tags, 1 tag, and 2 or more tags.

The linear fits are used to construct a 2-dimensional wrong-permutation signal shape as a function of the fit mass and generated top quark mass $S_{\text{wrong}}^{\text{tag}}(m_t, m_t)$. For each value of the generated top quark mass, the shape as a function of fitted mass is described as the sum of two Gaussians. The resulting parameterizations are displayed as the wrong-permutation shapes in Fig. 3 and 4. Also shown are the shapes of the correct jet assignments, determined in a similar fashion from parton-matched events using a single Gaussian. A linear dependence of the parameters is found as a function of generated top quark mass. The sum of the correct solutions and wrong solutions is compared to a weighted histogram of all fitted masses in $t\bar{t}$ simulation. The parametrized functions give an adequate description of the overall (wrong + correct) signal shape. In Fig. 3, the corresponding distributions are shown for events with 0, 1, or 2 tags. It is clearly visible how the fraction of the weight given to the correct solution improves when including $b$ tag information in the permutation weights. In Fig. 4, the nine distributions are shown for generated top quark masses ranging from 150 GeV to 200 GeV.

C. Determination of JES offset correction

The likelihood fit relies on the invariant mass of the hadronically decaying $W$ boson in the $t\bar{t}$ events to set the jet energy scale. It is designed to give an unbiased fit of the JES parameter in well-reconstructed $t\bar{t}$ events when the correct jet assignment is used. However, in a significant fraction of the events, the jets that are presumed to originate from the $W$ boson may not really come from a $W$ boson. Such cases include events other than $t\bar{t}$, as well as $t\bar{t}$ events that are mis-reconstructed. In the presence of such events we can expect an offset in the fitted JES parameter. The slope of the JES cali-
TABLE III: Parameters used to describe the background shapes (arbitrary normalization). For each case, the shape is described by the sum of two Gaussians \( \mathcal{G}(m_t) = a \cdot \exp \left[-(\mu - m_{\text{fit}})^2/2\sigma^2\right] \), where the three parameters \( a, \mu, \) and \( \sigma \) evolve linearly as a function of the generated top quark mass \( m_t \) as \( p_0 + p_1 \cdot (m_t - 175 \text{ GeV}) \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Gaussian 1</th>
<th>Gaussian 2</th>
<th>Gaussian 1</th>
<th>Gaussian 2</th>
<th>Gaussian 1</th>
<th>Gaussian 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>284.9</td>
<td>-1.722</td>
<td>51.72</td>
<td>-0.4199</td>
<td>267.3</td>
<td>-1.0700</td>
</tr>
<tr>
<td>( \mu )</td>
<td>161.7</td>
<td>0.7383</td>
<td>223.1</td>
<td>1.242</td>
<td>162.6</td>
<td>1.400</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>23.55</td>
<td>0.2392</td>
<td>22.94</td>
<td>-0.2528</td>
<td>23.27</td>
<td>-0.4551</td>
</tr>
</tbody>
</table>

The JES o\- set and slope turn out to be independent of the generated top quark mass (see Fig. 5). Therefore we apply a straightforward mass-independent correction. A normalization factor \( f_{\text{JES}}(m_t, f_{\text{top}}) = \exp(a \cdot \text{JES}) \) is introduced which corrects for the offset without changing the statistical uncertainty estimated from the likelihood (in case the final sample likelihood is Gaussian):

\[
L_{\text{corr}}(m_t, \text{JES}, f_{\text{top}}) = f_{\text{JES}}(m_t, f_{\text{top}}) \cdot L_{\text{evt}}(m_t, \text{JES}, f_{\text{top}}). \tag{11}
\]

Since background events on average cause a larger bias than signal events, \( a \) is defined to be dependent on the measured signal fraction \( f_{\text{top}} \): \( a = 2.83 + 0.56(1 - f_{\text{top}}) \). The value of the correction constant is tuned using MC simulation to give an unbiased measurement of the JES at the reference scale \( \text{JES} = 1 \). As shown in Table IV, the application of this offset correction removes the JES offset, but it further reduces the JES calibration slope.

TABLE IV: The JES calibration slope and offset for different event samples are shown. The offset increases and the calibration slope becomes smaller when mis-reconstructed signal events or background events are added. The offset correction at the likelihood level (see text) fixes the JES offset but further reduces the JES calibration slope.

<table>
<thead>
<tr>
<th>JES slope</th>
<th>JES offset</th>
<th>( \delta m_t ) (*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>parton-matched ( t\bar{t} ) only</td>
<td>0.96</td>
<td>+0.026</td>
</tr>
<tr>
<td>( t\bar{t} ) only</td>
<td>0.88</td>
<td>+0.050</td>
</tr>
<tr>
<td>all events</td>
<td>0.80</td>
<td>+0.076</td>
</tr>
<tr>
<td>all, 50% offset correction</td>
<td>0.72</td>
<td>+0.036</td>
</tr>
<tr>
<td>all, 100% offset correction</td>
<td>0.63</td>
<td>+0.000</td>
</tr>
</tbody>
</table>

(*) expected mass uncertainty after full calibration.
FIG. 4: Same as Fig. 3, for different values of the generated top quark mass, combining all events irrespective of the number of $b$ tags.

the corrections. For illustrative purposes we also include a 50% offset correction in the table, where $0.5 \cdot a$ is used instead of $a$.

The correction described above ensures that the fit is well-behaved and that, for values of the JES parameter near 1, the fit results will stay well within the range for which the $(JES, m_t)$ likelihood is calculated. It does not, however, provide a full calibration of the analysis, which is described in Sec. VIII E.

D. Combined likelihood fit

Since each event is independent, the combined likelihood for the entire sample is calculated as the product of the single event likelihood curves:

$$L_{\text{samp}}(m_t, JES, f_{\text{top}}) = \prod_j L_{\text{corr}}^{\text{evt}}(m_t, JES, f_{\text{top}}). \quad (12)$$

This likelihood is maximized with respect to the top quark mass $m_t$, the jet energy scale parameter $JES$, and the estimated fraction of signal in the sample $f_{\text{top}}$.

E. Calibration using Monte Carlo simulation

The analysis is calibrated using Monte Carlo simulation. Both the bias on the measured mass and the correctness of the estimated statistical uncertainty are studied using ensemble tests, in which many simulated experiments (pseudoexperiments) are created, each matching the size of the observed data sample. Thousands of pseudoexperiments are constructed, combining $tt$ and $W+\text{jets}$ events from MC simulation. The event fractions for $tt$ and $W+\text{jets}$ are allowed to fluctuate according to binomial statistics around the estimated fractions in the actual data sample. The fractions used are those listed in Table II. In the pseudoexperiments, the QCD multijets contribution is replaced by $W+\text{jets}$ events. This deviation in QCD multijet fraction is treated as a systematic uncertainty (see Sec. X). The total sample size is fixed to the observed number of events in data (116 in electron+jets and 114 in muon+jets). To make optimal use of the available MC statistics, standard resampling techniques are used, allowing for the multiple use of MC events when constructing the pseudoexperiments [24]. For every pseudoexperiment the mass is fitted and the
FIG. 5: The mean fitted JES and pull width as a function of the generated top quark mass $M_{\text{gen}}$ for a “true” JES of 0.97 (left), 1.00 (middle), and 1.03 (right), for the lepton+jets channel ($e + \mu$ combined). The fitted JES is stable as a function of generated top quark mass.

FIG. 6: The difference between the mean fitted mass $M_{\text{fit}}$ and the generated top quark mass $M_{\text{gen}}$ as a function of the generated top quark mass for a “true” JES of 0.97 (left), 1.00 (middle), and 1.03 (right), for the lepton+jets channel ($e + \mu$ combined). At a generated mass of 175 GeV, the mass bias changes by 1 GeV when the true JES is varied by ±3%.
deviation of this mass from the mean of all pseudoexperiment masses is divided by the fitted uncertainty. This quantity is referred to as the “pull.” The pull distribution for all pseudoexperiments is fitted with a Gaussian to extract the width, which we call the “pull width.” The corresponding pull and pull width for the fitted JES are also determined.

Figure 5 shows how the mean fitted JES and its pull width behave as a function of the top quark mass for different values of the true jet energy scale. The fitted JES parameter is independent of the top quark mass over the full range considered. The plots also show that the fitted JES changes linearly as a function of the true JES with a slope of 0.63 (see discussion in Sec. VIII C). Figure 6 shows the change in the fitted top quark mass and the width of the pull as a function of the generated top quark mass for different values of the true JES. Using these plots a full two-dimensional calibration is performed, describing the fitted JES and top quark mass as a function of the “true” JES and top quark mass generated in the MC simulation. The estimated statistical uncertainties are corrected for the width of the pull and error propagation is used to take into account the effect of the two-dimensional calibration including the correlation between the JES parameter and the offset in measured mass.

**F. Alternative JES fitting strategies**

Including the uniform JES parameter as a free parameter in the fit reduces the systematic uncertainty due to the jet energy scale, at the cost of a larger statistical uncertainty. As a comparison, in Fig. 7 the expected statistical uncertainties on the top quark mass are shown for three different fitting scenarios. When fixing the JES parameter in the fit to 1, the statistical uncertainty is smallest: 2.59 GeV at a generated top quark mass of 175 GeV. When allowing the JES parameter to float freely in the fit, without correcting for the JES slope (=0.63) in the calibration, part of the sensitivity to the overall JES scale is reduced and absorbed in the statistical uncertainty, leading to an expected statistical uncertainty of 3.34 GeV at a top quark mass of 175 GeV. Fully calibrating the analysis as a function of fitted mass and JES (the default approach), allows an unbiased top quark mass measurement for any value of the “true” JES, at the cost of a larger statistical uncertainty: 4.01 GeV at a generated top quark mass of 175 GeV.

In order to be consistent with the approach used by the Matrix Element analysis [7], thus facilitating a combination of results, and to minimize the dependence on the external JES constraint from jet+photon studies, the third scenario is presented here as the main analysis result, applying the full calibration as a function of fitted top quark mass and JES. Results using the other two JES fitting strategies are quoted as a cross-check in Sec. XI.

**IX. RESULTS WITH DATA**

The overall likelihood curves obtained for data are shown in Fig. 8. The 2D likelihoods show the actual likelihood values in bins of 1 GeV in mass and 3% in JES. The jagged appearance of the ellipses is caused by the large bin size in the JES direction. To extract the mass and statistical error, a Gaussian fit is applied to the three bins closest to the minimum in the one-dimensional negative log likelihood curves. The fitted values are corrected according to the calibration derived in Sec. VIII E. The measured top quark mass is:

\[ m_t = 173.7 \pm 4.4 \text{ (stat + JES) GeV} \]

with

\[ JES = 0.989 \pm 0.029 \text{ (stat).} \]

All uncertainties shown are statistical. The fitted \( t\bar{t} \) signal fraction is \( f_{\text{top}} = 0.453 \pm 0.032 \). If the JES parameter is kept fixed to 1 in the fit, the estimated statistical uncertainty is 2.93 GeV. Hence the 4.43 GeV (stat+JES) uncertainty of the 2D fit can be interpreted as a combination of an intrinsic mass uncertainty of 2.93 GeV (stat) and an additional uncertainty of 3.32 GeV (JES) due to fitting the JES parameter. As shown in Fig. 9, the observed statistical uncertainties are slightly larger than the average uncertainties expected from Monte Carlo ensemble tests, but they fall well within the distribution. The fitted JES of 0.989±0.029 is in good agreement with the reference scale 1 (or \( S=0 \)), corresponding to the hypothesis that after all jet corrections the JES in data and MC are the same.

One can also compare the in-situ fitted JES parameter with the scale obtained in jet+photon studies. When correcting all jets in MC events for the jet-pT dependent difference between data and MC, \( S \), and redoing the ensemble tests in MC simulation, the mean fitted JES is 0.962±0.023, where the uncertainties correspond to the combined statistical and systematic bounds from the jet+photon studies. This is consistent with the value of 0.989 ± 0.029 measured in situ.

**X. SYSTEMATIC UNCERTAINTIES**

The calibration of the analysis relies on Monte Carlo simulation. Therefore any discrepancy between the Monte Carlo simulation and the data may lead to a bias and thus to a systematic shift in the measured top quark mass. In this section we describe the aspects of the simulation which may not accurately represent the data and evaluate the possible effect on the mass measurement. To determine the impact of each uncertainty, we perform ensemble tests using a pool of simulated events that are modified according to the uncertainty in question. The shift in the mean fitted ensemble mass compared to the default value gives the size of the corresponding systematic uncertainty.
FIG. 7: The expected statistical uncertainty from ensemble tests is shown as a function of the generated top quark mass for three scenarios: with the JES parameter fixed to 1 (left), allowing the JES parameter to float freely in the fit, but only calibrating the mass fit for a true JES = 1 (middle), allowing the JES parameter to float freely in the fit and applying the full calibration as a function of true top quark mass and true JES (right). In each plot the width of the band indicates the estimated uncertainty.

FIG. 8: Overall likelihood curves for the events observed in data, in the electron+jets channel (left), muon+jets (middle), and both channels combined (right). The top plots show the full 2-dimensional likelihood as a function of the jet energy scale parameter ("JES") and top quark mass. Each contour, \( n \), corresponds to a difference in likelihood of \( \Delta \ln(L) = -n^2/2 \) with respect to the maximum likelihood. The fitted value of the JES parameter as a function of the top quark mass is plotted as the gray line superimposed on the 2D likelihoods. The bottom plots show the likelihood as a function of the top quark mass along the gray line from the upper plots. The fitted values from these distributions have to be corrected for the calibration from MC simulation to obtain the final results.
The total systematic uncertainty on the top quark mass measurement is obtained by adding all contributions in quadrature. The following sources of systematics are considered (also see Table V):

- **Jet energy scale $p_T$ dependence**: The inclusion of a uniform jet energy scale parameter JES as a free parameter in the mass fitting and calibration procedure ensures that a relative difference in overall jet energy scale between data and Monte Carlo is corrected for. The corresponding uncertainty is included in the quoted statistical (stat + JES) uncertainty. Any residual discrepancy between data and Monte Carlo jet energy calibration that cannot be described by a uniform scale factor may lead to an additional systematic error on the top quark mass. The largest additional effect is expected from the uncertainty in the jet-$p_T$ dependence [7]. The size of the impact of a possible jet-$p_T$ dependent shape is estimated by scaling the energies of all jets in the MC with a factor $(1 + 0.02 \frac{p_T^{\text{jet}} - 100 \text{ GeV}}{100 \text{ GeV}})$, where $p_T^{\text{jet}}$ is the default reconstructed jet $p_T$. The value of 0.02 is suggested by the jet+photon studies. The mass obtained with the modified pseudo-experiments is compared to the default result and the shift of 0.45 GeV is quoted as a systematic uncertainty.

- **Jet reconstruction efficiency and resolution**: In addition to uncertainties on the reconstructed jet energies, differences between data and the Monte Carlo simulation in the jet reconstruction efficiency and jet energy resolution may lead to a mass bias. Both efficiency and resolution are varied as a function of jet $p_T$ and rapidity within estimated uncertainties. No significant effect is observed, with an estimated statistical precision of 0.15 GeV. For both effects combined, a systematic uncertainty of 0.22 GeV is quoted.

- **$b$-jet energy response**: Uncertainties in the simulation of the ratio between the calorimeter response to hadronic showers and electromagnetic showers ($h/e$ ratio) may lead to additional differences in the $b$/light jet energy scale ratio between data and simulation. The possible size of the effect is studied in simulation, combining the uncertainty in the modeling of $h/e$ calorimeter response ratio with the difference in particle content between light quark and $b$ jets. An estimated uncertainty of 1.4% on the $b$/light jet energy scale ratio is found. Ensemble tests show that this corresponds to a systematic uncertainty on the top quark mass of 1.15 GeV.

- **$b$-tagging**: The $b$ tagging rates for $b$ jets, $c$ jets, and light-quark jets are varied within the uncertainties known from the data, and the resulting uncertainty from the fit, differences between data and Monte Carlo in the ratio of $b$-jet and light-jet energy scale could still affect the measurement. One possible source for such differences could be the description of $b$-jet fragmentation in the simulation. To estimate the uncertainty from this source we used samples of simulated $t\bar{t}$ events with different fragmentation models for $b$ jets. The default Bowler [25] scheme with $r_b=1.0$ is replaced with $r_b=0.69$ or with Peterson [26] fragmentation with $\varepsilon_b=0.00191$. These parameter values were obtained by tuning PYTHIA simulation to LEP data [27–29]. The size of the variation in $r_b$ corresponds to a larger shift in mean scaled energy $\langle x_B \rangle$ of $b$ hadrons than the uncertainties reported in [27–30]. The comparison between the Bowler and Peterson scheme addresses the uncertainty on the shape of the $x_B$ distribution. Ensemble tests are repeated using events from each of the three simulations. The absolute values of the deviations in top quark mass results with respect to the standard sample are added in quadrature and quoted as a symmetric uncertainty of 1.3 GeV.
TABLE V: Summary of systematic uncertainties.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Size of the effect (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet energy scale ($p_T$ dependence)</td>
<td>0.45</td>
</tr>
<tr>
<td>Jet ID efficiency and resolution</td>
<td>0.22</td>
</tr>
<tr>
<td>$b$ fragmentation</td>
<td>1.30</td>
</tr>
<tr>
<td>$b$ response (h/e)</td>
<td>1.15</td>
</tr>
<tr>
<td>$b$ tagging</td>
<td>0.29</td>
</tr>
<tr>
<td>Trigger uncertainty</td>
<td>+0.61 – 0.28</td>
</tr>
<tr>
<td>Signal modeling</td>
<td>0.73</td>
</tr>
<tr>
<td>Signal fraction (stat+sys)</td>
<td>0.12</td>
</tr>
<tr>
<td>Background modeling</td>
<td>0.20</td>
</tr>
<tr>
<td>Multijet background</td>
<td>0.28</td>
</tr>
<tr>
<td>MC calibration</td>
<td>0.25</td>
</tr>
<tr>
<td>PDF uncertainty</td>
<td>0.023</td>
</tr>
<tr>
<td>Total systematic uncertainty</td>
<td>+2.10 – 2.04</td>
</tr>
</tbody>
</table>

Variations are propagated to the final mass results. Uncertainties in the heavy flavor composition of the background are also considered. The combined effect is 0.29 GeV.

• **Trigger:** The trigger efficiencies in the Monte Carlo simulation are varied by their uncertainties estimated from data. The resulting variations in fitted mass are summed in quadrature, leading to a combined trigger uncertainty of +0.61 – 0.28 GeV.

• **Signal modeling:** The main uncertainty in the modeling of $t\bar{t}$ events is related to the radiation of gluons in the production or decay of the $t\bar{t}$ system. A difference in the description of hard gluon radiation could affect the transverse momentum spectrum of the $t\bar{t}$ system or, for example, change the rate of confusion between jets from the hadronically decaying $W$ boson and initial state gluons, which could affect the reconstructed top quark mass. To assess the uncertainty related to the modeling of high energy gluons, the difference is studied between the default signal simulation and a dedicated $t\bar{t}$+jet simulation in which an energetic parton is produced in addition to the $t\bar{t}$ system in the production process simulated by alpgen. It is estimated that in the class of events that pass the full event selection, the fraction of simulated events with such an energetic gluon disagrees with the data by less than 35%. Pseudoexperiments are made with the usual sample composition, but replacing the default $t\bar{t}$ events with the events from the dedicated $t\bar{t}$+jet simulation. 35% of the observed shift in the fitted mass corresponds to 0.73 GeV, which is assigned as a systematic uncertainty.

• **Signal fraction:** Since the $t\bar{t}$ fraction $f_{top}$ is fitted together with the top quark mass and the JES parameter, the mass measurement is affected by the uncertainty on the signal fraction in the data sample. We estimate two sources of systematic uncertainty: a variation of the signal fraction in the ensemble test used to calibrate the method and the effect of a possible systematic offset in the fitted signal fraction with respect to the true signal fraction internally in the mass fit.

We take the 7% relative statistical uncertainty of the signal fraction found by combining the $\mu$+jets and $e$+jets numbers shown in Table II. We add in quadrature an estimated relative systematic uncertainty of 11% estimated from the cross section measurements [17]. New ensemble tests for the calibration procedure, are performed with the mean of the Poisson distribution (for the signal fraction) shifted by (11±7)%.

Following this procedure the combined fit will still correctly fit the different signal fraction and compensate for the effect. This does not take into account the effect of a possible systematic discrepancy between the data and the Monte Carlo model of signal and background, which could lead to a systematic bias in the fitted signal fraction. To evaluate this additional systematic, the mass fit is forced to systematically overestimate or underestimate the $t\bar{t}$ fraction by 11% (with respect to the value preferred by the likelihood fit), and the shift in fitted mass is quoted as a systematic uncertainty. The combined uncertainty, adding the above two contributions in quadrature, is 0.12 GeV.

• **Background modeling:** The sensitivity of the measurement to the choice of background model is studied by comparing two enlarged pseudoexperiments in which the background simulation is changed. One sample is based on the standard $W$+jets simulation using a factorization scale of $Q^2 = M_W^2 + \sum_j (p_T^j)^2$ while in the other pseudoexperiment a sample of $W$+jets events is used that are generated with a different factorization scale of $Q'^2 = (p_T^j)^2$. The observed difference in fitted
mass is 0.20 GeV which is assigned as a systematic uncertainty.

- **QCD multijet background**: In the calibration procedure, the $W+\text{jets}$ simulation is used to model the small multijet background in the selected data sample. To study the systematic uncertainty due to this approximation, we selected a dedicated multijet-enriched sample of events from data by inverting the lepton isolation cut in the event selection. The calibration of the method is carried out with pseudoexperiments in which these events are used to model the multijet background, according to the fractions given in Table II. The observed shift is 0.28 GeV, which is quoted as a systematic uncertainty.

- **MC calibration**: The statistical uncertainty on the calibration curves shown in Fig. 6 is propagated through the analysis and yields a systematic uncertainty on the result of 0.25 GeV.

- **Uncertainty due to the parton distribution functions (PDF)**: The Ideogram analysis measures the top quark mass directly from the invariant mass of the $t\bar{t}$ decay products without making specific assumptions regarding the production process. Nevertheless, the calibration of the analysis relies on Monte Carlo simulation in which a certain PDF set was used (CTEQ5L [31]). It is conceivable that a different choice of PDFs would lead to a slightly different calibration. To study the systematic uncertainty on the top quark mass due to the precise PDF description, several PDF uncertainties are considered. PDF variations provided with the next-to-leading-order PDF set CTEQ6M [32] are compared to the default CTEQ6M. The difference between CTEQ5L and MRST leading order PDFs is taken as a separate contribution. Also the effect of a variation in $\alpha_s$ is evaluated. In all cases a large pseudoexperiment composed of events generated with CTEQ5L is reweighted so that distributions corresponding to the desired PDF set are obtained. The difference between weighted and unweighted pseudoexperiments is then quoted as systematic uncertainty, and all individual uncertainties are added in quadrature. The resulting combined uncertainty is found to be very small: $\pm 0.02$ GeV.

**XI. CROSS-CHECK USING AN EXTERNAL JES CONSTRAINT**

As a cross-check, the analysis is repeated using the two alternative JES fitting strategies discussed in Sec. VIII F. Fixing the JES parameter in the fit and relying fully on the external JES constraint from jet+photon studies, the top quark mass is measured to be:

$$m_t = 175.8 \pm 2.9 \text{ (stat)}^{+2.1}_{-2.7} \text{ (JES)} \text{ GeV},$$

quoting only the statistical uncertainty (stat) and the systematic uncertainty due to the jet energy scale (JES).

In the other alternative approach, the JES parameter is allowed to float freely in the fit but no calibration of the JES slope is applied. Again, the external JES constraint from jet+photon studies is required to set the jet energy scale and the remaining JES systematics. Effectively this approach combines in-situ with external JES information, leading to the following result:

$$m_t = 173.9 \pm 3.6 \text{ (stat)}^{+1.3}_{-1.0} \text{ (JES)} \text{ GeV}.$$  

Comparing the last (most precise) cross-check with the main result, one can conclude that omitting the external JES constraint and relying fully on the in-situ information changes the central result only by 0.2 GeV. The 2 GeV difference between the first cross-check and the main result correlates very well with the 1.1% difference in JES value between the default Monte Carlo scale and in-situ JES measurements. This difference is fully covered by the quoted uncertainties.

**XII. CONCLUSION**

The Ideogram method has been used for the first time to measure the top quark mass in $t\bar{t}$ events with the $\ell+\text{jets}$ topology. This technique employs a kinematic fit to extract mass information from the events, while improving the statistical sensitivity by constructing an analytic likelihood for every event taking into account all jet permutations and the possibility that the event is background. Lifetime-based identification of $b$ jets is employed to enhance the separation between $t\bar{t}$ signal and background and to improve the assignment of the observed jets to the partons in the $t\bar{t}$ hypothesis. To reduce the systematic uncertainty due to the jet energy scale calibration, an overall scale factor $JES$ for the energy of the reconstructed jets is a free parameter in the fit determined simultaneously with the top quark mass and the signal fraction.

From a D0 Run II data sample of approximately 425 pb$^{-1}$, 116 events are selected in the electron+jets channel and 114 in the muon+jets channel. The top quark mass is measured to be

$$m_t = 173.7 \pm 4.4 \text{ (stat + JES)}^{+2.1}_{-2.0} \text{ (syst)} \text{ GeV}$$

with a fitted JES scaling factor:

$$JES = 0.989 \pm 0.029 \text{ (stat only)},$$

which is consistent with the reference jet energy scale ($\pm 1.0$) and with the results from the jet+photon calibration ($\approx 0.962^{+0.023}_{-0.024}$). The mass result is in good agreement with the Matrix Element measurement using the
same data set [7] and with other recent top quark mass measurements [4, 33].

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