Measurement of the forward-backward asymmetry in the distribution of leptons in $t\bar{t}$ events in the lepton+jets channel

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We present measurements of the forward-backward asymmetry in the angular distribution of leptons from decays of top quarks and antiquarks produced in proton-antiproton collisions. We consider the final state containing a lepton and at least three jets. The entire sample of data collected by the D0 experiment during Run II of the Fermilab Tevatron Collider, corresponding to 9.7 fb$^{-1}$ of integrated luminosity, is used. The asymmetry measured for reconstructed leptons is $A_{FB}^l = (2.9 \pm 2.1 \text{(stat.)} +1.5_{-1.6} \text{(syst.)})\%$. When corrected for efficiency and resolution effects within the lepton rapidity coverage of $|y| < 1.5$, the asymmetry is found to be $A_{FB}^l = (4.2 \pm 2.3 \text{(stat.)} +1.1_{-1.4} \text{(syst.)})\%$. Combination with the asymmetry measured in the dilepton final state yields $A_{FB}^{ll} = (4.2 \pm 2.0 \text{(stat.)} \pm 1.4 \text{(syst.)})\%$. We examine the dependence of $A_{FB}^l$ on the transverse momentum and rapidity of the lepton. The results are in agreement with predictions from the next-to-leading-order QCD generator MC@NLO, which predicts an asymmetry of $A_{FB}^l = 2.0\%$ for $|y| < 1.5$.

I. INTRODUCTION

Within the standard model of particle physics (SM), top quarks are usually produced via quantum chromodynamic (QCD) interactions in quark-antiquark pairs. The process $p\bar{p} \rightarrow t\bar{t}(X)$ is predicted to produce more events for which the rapidity of the top quark, $y_t$, is greater than the rapidity of the top antiquark, $y_{\bar{t}}$, than events for which $y_{\bar{t}}$ is less than $y_t$. The rapidity $y$ is defined as $y(\theta, \beta) = \frac{1}{2} \ln \left( (1 + \beta \cos \theta) / (1 - \beta \cos \theta) \right)$, where $\theta$ is the polar angle and $\beta$ is the ratio of a particle’s momentum to its energy. The angle $\theta = 0$ corresponds to the direction of the incoming proton. This predicted forward-backward asymmetry is mostly due to contributions at order $\alpha_s^3$, where $\alpha_s$ is the QCD coupling constant [1]. There are also smaller contributions to the forward-backward asymmetry from electroweak (EW) interactions [2]. Forward-backward asymmetries in $p\bar{p} \rightarrow t\bar{t}(X)$ production previously measured at the Fermilab Tevatron Collider [3,4] were found to be somewhat higher than the SM predictions [5,6]. With a mass of approximately 173 GeV [8], the top quark is the most massive known elementary particle, which raises the possibility that the asymmetry is enhanced by effects beyond the SM. Hence, the previously measured asymmetries led to studies of possible causes not only within, but also beyond the SM [9].

Top quarks decay almost exclusively into a $b$ quark and a $W$ boson, and $W$ bosons decay either hadronically to a quark and an antiquark or leptonically to a lepton and a neutrino. Thus, $t\bar{t}$ events are usually classified based on the number of leptons from the decays of the $W$ bosons into the dilepton, lepton-jets ($l+\text{jets}$), and all-jets channels. The $t\bar{t}$ production asymmetry was first measured by the D0 Collaboration [10] in the $l+\text{jets}$ channel. The result of Ref. [10] was superseded by that of Ref. [3], where a dataset corresponding to an integrated luminosity of 5.4 fb$^{-1}$ was used to measure an inclusive asymmetry of $(20^{+6}_{-5})\%$. The CDF Collaboration measured this asymmetry in the $l+\text{jets}$ channel with 9.4 fb$^{-1}$ of integrated luminosity.
luminosity, finding $A_{FB} = (16.4 \pm 4.5)\%$ \cite{5}. These measured values can be compared to SM predictions, for example, $A_{FB} = (8.8 \pm 0.9)\%$ \cite{6}.

The forward-backward asymmetry in the production of $t\bar{t}$ pairs leads to a forward-backward asymmetry $A_{FB}^t$ in the angular distribution of the leptons produced in the $t\bar{t}$ decay \cite{11}. The asymmetry $A_{FB}^t$ was first measured by D0 \cite{3} as a cross-check of the asymmetry of the $t\bar{t}$ pair and to demonstrate that the observed tension with the SM should not be attributed to biases introduced by the algorithm used to reconstruct the $t\bar{t}$ system or to the corrections for the detector acceptance and resolution effects. In SM $p\bar{p} \rightarrow t\bar{t}(X)$ production, the polarization of top quarks is negligible and the leptons are produced isotropically (in the appropriate reference frames) leading to an $A_{FB}^t$ that is smaller than $A_{FB}$. The D0 Collaboration measured $A_{FB}^t = (15.2 \pm 4.0)\%$ in the $l+\text{jets}$ channel for $|y_l| < 1.5$, where $y_l$ is the rapidity of the lepton from top quark decay \cite{3} and $A_{FB}^t = (5.8 \pm 5.3)\%$ in the dilepton channel for $|y_l| < 2$ \cite{12}. The CDF Collaboration measured this asymmetry in the $l+\text{jets}$ channel and found values extrapolated to the full acceptance of $A_{FB}^t = (9.4^{+3.2}_{-2.5})\%$ \cite{13}. The corresponding SM predictions range from 2.0\% to 3.8\% \cite{6,11,15}. The higher predictions include electroweak corrections, which increase $A_{FB}^t$ by less than a percent (absolute). The dominant uncertainty on these predictions is from the renormalization and factorization scales, and is evaluated to be up to 1.0\% \cite{6,11,15}. The results of the previous measurements could be taken as an indication of effects beyond the SM that lead to the production of polarized top quarks \cite{10}. Motivated by the desire to further investigate this tension and by the potential sensitivity of $A_{FB}^t$ to new physics, we pursue this analysis in greater detail and with a larger dataset.

Measuring the leptonic asymmetry rather than the $t\bar{t}$ asymmetry has additional benefits. The measurements of the $t\bar{t}$ asymmetry require full reconstruction of the $t\bar{t}$ decay chain, accomplished by assuming on-shell top quarks that each decay to three final state fermions. These assumptions limit the validity of a comparison of data to calculations that include higher orders in top quark decay and off-shell top quarks (e.g., in loops). These limitations are not intrinsic to the lepton-based asymmetry. Although we make some use of $t\bar{t}$ reconstruction, the effects of off-shell top quarks and of decays with additional final-state partons on the measurement of $A_{FB}^t$ are negligible.

Experimentally, the direction of a lepton is determined with far greater precision than that of a top quark. Thus, corrections for the detector acceptance and experimental resolutions are simpler. Furthermore, with no need for full reconstruction of the $t\bar{t}$ system, the $l+3\text{jet}$ sample can be used for this measurement in addition to the previously used $l+\geq4\text{jet}$ sample. This addition almost doubles the number of $t\bar{t}$ events analyzed, at the expense of a lower signal-to-background ratio. The inclusion of the $l+3\text{jet}$ sample also reduces the acceptance corrections, which are a leading source of systematic uncertainty in Ref. \cite{3}.

In Ref. \cite{3}, the CDF Collaboration reported a strong increase of $A_{FB}^t$ with the invariant mass of the $t\bar{t}$ system, $m_{t\bar{t}}$. The dependence of the asymmetry on $m_{t\bar{t}}$ observed in the previous D0 measurement \cite{3} is statistically consistent with both the SM prediction and the CDF result. Measuring $m_{t\bar{t}}$ requires full reconstruction of the $t\bar{t}$ system, but we can also study the dependence of the asymmetry on the $t\bar{t}$ kinematics by relying on the transverse momentum of the lepton, $p_T^l$. This observable can readily be studied in $l+3\text{jet}$ events and is measured with far greater precision than $m_{t\bar{t}}$. Furthermore, $p_T^l$ is strongly correlated with $m_{t\bar{t}}$, and is useful in comparing data to the predictions of different models \cite{17}. This differential measurement is therefore well motivated both experimentally and as a test of new physics models.

We report here an updated measurement of $A_{FB}^t$, using the full dataset collected by the D0 experiment during Run II of the Fermilab Tevatron Collider at $\sqrt{s} = 1.96\text{TeV}$. We extend the measurement to include $l+3\text{jet}$ events, which required improvements in the background modeling, and measure the $p_T^l$ dependence of $A_{FB}^t$ for the first time. The measurement reported in this paper supersedes the results of Ref. \cite{3}.

II. THE D0 DETECTOR

The D0 detector \cite{18} is a multipurpose particle detector with the following main components. At the core is a silicon microstrip tracker \cite{19}, surrounded by a scintillating-fiber tracker. The tracking detectors are located within a superconducting solenoid magnet that provides a 2T axial magnetic field and is surrounded by liquid argon / uranium calorimeters. The calorimeters \cite{20} are enclosed within a central barrel cryostat and two end-cap cryostats. A muon detection system \cite{21} surrounds the calorimetry, and consists of an iron toroidal magnet, tracking detectors, and scintillation trigger detectors. Data collection is triggered by a three-level system.

The D0 coordinate system has the $z$ axis along the direction of the proton beam. Particle directions are presented in terms of their azimuthal angle $\phi$ and their pseudorapidity $\eta = -\ln \left[ \tan \left( \frac{\theta}{2} \right) \right]$, where $\theta$ is the polar angle.

III. DEFINING THE LEPTON-BASED ASYMMETRY

In this analysis, we measure the charge ($q_y$) and rapidity ($y_l$) of the electron or muon that originates from the $W$ boson from top quark decay. Events with $q_y y_l > 0$ are defined as forward and events with $q_y y_l < 0$ are defined as backward. We define the lepton-based forward-backward
asymmetry as
\[ A_{\text{FB}}^l = \frac{N^l_F - N^l_B}{N^l_F + N^l_B}, \]
where \( N^l_F \) and \( N^l_B \) are the number of forward and backward events, respectively. All asymmetries are reported after subtracting the estimated background.

The asymmetry can be defined at the “reconstruction level,” which refers to the measured lepton parameters and is affected by acceptance and resolution. To enable direct comparisons with SM and non-SM calculations, the asymmetry can also be defined at the “production level,” before acceptance and resolution effects take place. The production level is sometimes also denoted as the generator level, or the parton level.

Though the rapidity coverage differs for electrons and muons, we assume lepton flavor universality and define \( A_{\text{FB}}^l \) and the acceptance in terms of \(|y|\). To avoid large acceptance corrections, only events with \(|y| < 1.5\) are used (see Ref. [2]). Throughout most of this paper, the production-level \( A_{\text{FB}}^l \) is defined counting only leptons produced within this lepton coverage. However, in Section [X] we also discuss asymmetries extrapolated to the full acceptance.

IV. ANALYSIS STRATEGY

The selection focuses on \( t\bar{t}(X) \rightarrow W^+bW^-\bar{b}(X) \) events in the \( l+3\) jets decay mode, where one W boson decays hadronically (\( q\bar{q}' \)) and the other decays leptonically (\( l\nu_l \)). The experimental signature of this decay mode is one isolated lepton (\( e, \mu \)) with a large \( p_T \), a significant imbalance in transverse momentum (\( E_T, \) with the letter \( E \) indicating a calorimetry-based observable) from the undetected neutrino, and jets arising from the two b quarks and from the two quarks from \( W \rightarrow q\bar{q}' \) decay. We select electrons and muons, which arise either directly from the W boson decay or through an intermediate \( \tau \) lepton.

A prototypical \( l+3\) jets event contains four final state quarks and hence four jets. Previous measurements selected events with at least four jets. Only half of the \( t\bar{t} \) events in the \( l+3\) jets channel have four or more selected jets, as one of the jets may fail the selection criteria due to insufficient transverse momentum (\( p_T \)) or due to large absolute rapidity. In addition, the decay products of two of the final state partons may be clustered into a single jet.

In this measurement we also select events with three jets. The inclusion of three-jet events has the advantages of increasing the statistical power of the measurement and making the measurement less susceptible to biases from selection. However, these additional events have a lower signal-to-background ratio than the events with \( \geq 4 \) jets. To maximize the statistical power of the purer subsets, we separate the measurement into several channels, defined by the number of jets (3 or \( \geq 4 \)) and the number of “\( b\)-tagged” jets (0, 1, or \( \geq 2 \)), that is, jets identified as likely to originate from a \( b \) quark.

We identify variables that discriminate between the \( t\bar{t} \) signal and the production of \( W \) bosons in association with jets (\( W+\)jets), and combine the variables into a single discriminant \( D \) by neglecting the small correlations between them. There are separate discriminants for \( l+3\) jet events and \( l+\geq4\) jet events. We use these discriminants to estimate the number of selected \( t\bar{t} \) events and their reconstruction-level \( A_{\text{FB}}^l \) (see Section [VII]).

The addition of three jet events increases the sensitivity of the analysis to the modeling of \( W+\)jets production, which contributes most of the selected three jet events but only a minority of the selected \( \geq 4 \) jet events. We study and improve the modeling of \( A_{\text{FB}}^l \) in the \( W+\)jets background using a top-depleted control sample (see Section [VII]).

We then correct the \( q\bar{y}_l \) distribution to the production level within the lepton coverage, and measure the production-level \( A_{\text{FB}}^l \). Since the angular resolutions for electrons and muons are excellent, the incorrect classification of events as forward or backward is negligibly small. We therefore correct \( A_{\text{FB}}^l \) only for acceptance effects (see Section [VIII]).

In addition to measuring the inclusive \( A_{\text{FB}}^l \), we also measure \( A_{\text{FB}}^l \) in three \( p_T \) regions: \( 20 \leq p_T < 35 \) GeV, \( 35 \leq p_T < 60 \) GeV, and \( p_T \geq 60 \) GeV. To measure the \( p_T \) dependence, we first correct for migrations between different \( p_T \) regions and then correct for the effects of acceptance (see Section [VIII]).

V. EVENT SELECTION

The event selection criteria used in this analysis are similar to those used to measure the \( t\bar{t} \) production cross section in the \( l+\)jets channel [22]. In particular, we also accept events with three selected jets. The reconstruction and identification of jets, isolated leptons, and \( E_T \) is described in Ref. [22].

Only jets with transverse momentum \( p_T > 20 \) GeV and \(|\eta| < 2.5\) are considered for further analysis, and events are required to contain at least three such jets. The leading jet, that is the jet with the largest \( p_T \), is also required to have \( p_T > 40 \) GeV. As in Ref. [22], we minimize the effect of multiple \( pp \) collisions in the same bunch crossing by requiring that jets are vertex confirmed [24], i.e., have at least two tracks within the jet cone pointing back to the primary \( pp \) collision vertex (PV).

A typical decay of a \( b \) hadron occurs at a distance of the order of 1 mm from the PV and results in charged tracks that are detected by the tracking system and form a displaced secondary vertex. Thus, jets that originate from a \( b \) quark can be identified by the properties of the tracks reconstructed within the jet cone, in particular by their displacement from the PV, and by the reconstruction of displaced secondary vertices. Several observables useful for identifying such jets are combined into a mult-
tivariate discriminant that is used in this analysis to tag b jets by selecting jets likely to originate from a b quark among the three or four jets with the highest transverse momentum.

The $e+$jets and $\mu+$jets channels have similar event selection requirements. Only events collected with single-lepton or lepton+jet triggers are used. The criteria for selecting $e+$jets events are:

- one isolated electron with $p_T > 20$ GeV and $|\eta| < 1.1$,
- $|E_T| > 20$ GeV, and
- $\Delta \phi(e,E_T) > (2.2 - 0.045 \cdot E_T/\text{GeV})$ radians.

For $\mu+$jets events, the criteria are:

- one isolated muon with $p_T > 20$ GeV and $|\eta| < 1.5$,
- $25$ GeV $< |E_T| < 250$ GeV, and
- $\Delta \phi(\mu,E_T) > (2.1 - 0.035 \cdot E_T/\text{GeV})$ radians.

Events with a second isolated electron or muon passing the selection are rejected. The $|E_T| < 250$ GeV cut suppresses events where the $E_T$ is due to a mismeasurement of the $p_T$ of the muon, which is reconstructed from the curvature of the track of the muon. The $\Delta \phi(l,E_T)$ cuts reduces the background from QCD multijet (MJ) production. MJ events can pass the selection when a jet is misidentified as an isolated lepton. This often results in spurious reconstructed $E_T$ along the lepton’s direction.

In addition to the above criteria of Ref. [22], we also require that the curvature of the track associated with the lepton is well measured. This requirement, while $\approx 97\%$ efficient for leptons produced in $t\bar{t}$ decay, suffices to lower the lepton charge misidentification rate to less than one part in a thousand. It also reduces the migration of events among the three $p_T$ regions.

For events with muons with $p_T > 60$ GeV, we also require that the magnitude of the vector sum of the muon momentum and missing transverse energy is greater than 20 GeV. This requirement rejects events consistent with low energy muons from low energy jets that are badly reconstructed as having high $p_T$, leading to their misclassification as isolated leptons. Such events are part of the MJ background, but their modeling as part of that background, using the technique described in Section VIIA, is problematic. To limit any possible mismodeling, we also suppress these events with additional requirements on the track associated with the muon. Leptons from signal events pass these additional requirements with $\approx 85\%$ efficiency.

The main background after this event selection is due to $W+$jets production. There is a smaller contribution from MJ production. Other small backgrounds from single top quark, $Z+$jets and diboson production are also present.

We use the MC@NLO event generator [14] combined with HERWIG showering [26] to model the behavior of $t\bar{t}$ events, and ALPGEN [27] combined with PYTHIA [28] to simulate the $W+$jets background. The rate of inclusive $W+\ell\bar{\ell}$ and $W+b\bar{b}$ production predicted by ALPGEN is scaled up by a factor of 1.47, so that the ratio of the heavy flavor production rate to the inclusive $W+$jets production rate agrees with the ratio calculated at next-to-leading-order (NLO) [24, 29]. The simulated W-boson $p_T$ distribution is reweighted to match the product of the measured $Z$-boson $p_T$ distribution from D0 data [31] and the SM ratio of the distributions of the $W$-boson $p_T$ and the $Z$-boson $p_T$, as calculated at NLO with RESBOS [31].

For the other backgrounds, $Z+$jets events are simulated with ALPGEN, diboson events are simulated with PYTHIA and events from single top quark production are simulated with COMPHEP [32]. The normalizations for the last three background processes are taken from NLO calculations [29]. For all simulated events, event generation is followed by the D0 detector simulation and reconstruction programs. To model energy depositions from noise and additional $p\bar{p}$ collisions within the same bunch crossing, simulated events are overlaid with data from random $p\bar{p}$ collisions. The properties of the MJ background are evaluated using control samples from collider data.

VI. THE PREDICTED ASYMMETRIES

As the asymmetry first appears at order $\alpha_s^3$, with the largest contribution due to a loop diagram, it is not fully simulated by tree-level event generators. In addition, the modeling of selection and reconstruction effects requires full Monte Carlo (MC) simulation. The MC@NLO event generator is well suited for this measurement as it couples a NLO calculation of $t\bar{t}$ production with subsequent parton showers to fully simulate $t\bar{t}$ events. The asymmetries of simulated $t\bar{t}$ and $W+$jets production are listed in Table III for different jet and $b$-tag multiplicities.

The two leading contributions to the $t\bar{t}$ asymmetry, at order $\alpha_s^3$, are as follows. In events with no additional radiated partons, the interference between the Born and box diagrams leads to a positive asymmetry. At order $\alpha_s^3$, the $p_T$ of the $t\bar{t}$ system for these events is $p_T^{t\bar{t}} = 0$. However, realistic simulation of these processes using parton showers allows for some transverse boost of the $t\bar{t}$ system. The interference between diagrams containing initial or final state radiation decreases the asymmetry. These events have non-zero $p_T^{t\bar{t}}$ at order $\alpha_s^3$, and $p_T^{t\bar{t}}$ is usually increased by the parton showers. Thus the predicted asymmetry decreases as a function of the $p_T$ of the $t\bar{t}$ system, $p_T^{t\bar{t}}$, which can also be seen here as a decrease with increasing jet multiplicity.

In the case of $W+$jets background production, $W$ bosons produced by interactions involving gluons or sea quarks contribute positively to the asymmetry. On the other hand, $W$ bosons produced by valence-valence collisions contribute negatively to the overall asymmetry. The production of $W$ bosons in association with heavy flavor quarks occurs predominantly due to valence-
TABLE I: Simulated reconstructed asymmetries for selected \( t\bar{t} \) and \( W^{+}\text{jets} \) events, by event category. The quoted uncertainties are due to the finite sizes of the simulated samples.

<table>
<thead>
<tr>
<th>Channel</th>
<th>( t\bar{t} ) signal</th>
<th>( W^{+}\text{jets} ) background</th>
<th>( A_{FB}^{t\bar{t}} ), %</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t\bar{t} ) jets, all channels</td>
<td>1.6 ± 0.1</td>
<td>13.0 ± 0.2</td>
<td>13.0 ± 0.2</td>
</tr>
<tr>
<td>( t\bar{t} ) jets, 0 b tags</td>
<td>2.3 ± 0.3</td>
<td>13.5 ± 0.3</td>
<td>13.5 ± 0.3</td>
</tr>
<tr>
<td>( t\bar{t} ) jets, 1 b tag</td>
<td>2.7 ± 0.3</td>
<td>11.6 ± 0.4</td>
<td>11.6 ± 0.4</td>
</tr>
<tr>
<td>( t\bar{t} ) jets, ≥2 b tags</td>
<td>2.8 ± 0.2</td>
<td>7.4 ± 0.9</td>
<td>7.4 ± 0.9</td>
</tr>
<tr>
<td>( t\bar{t} ) ≥4 jets, 0 b tags</td>
<td>−0.9 ± 0.4</td>
<td>14.1 ± 0.9</td>
<td>14.1 ± 0.9</td>
</tr>
<tr>
<td>( t\bar{t} ) ≥4 jets, 1 b tag</td>
<td>0.5 ± 0.2</td>
<td>14.5 ± 1.0</td>
<td>14.5 ± 1.0</td>
</tr>
<tr>
<td>( t\bar{t} ) ≥4 jets, ≥2 b tags</td>
<td>1.1 ± 0.2</td>
<td>8.8 ± 1.9</td>
<td>8.8 ± 1.9</td>
</tr>
</tbody>
</table>

VII. MEASURING THE RECONSTRUCTED \( A_{FB}^{t\bar{t}} \)

We construct a discriminant (see Section VIIA), and extract the sample composition and the asymmetry using a maximum likelihood fit to the distribution of the discriminant and the distribution of the sign of \( q_{yy} \) (see Section VIIA). The asymmetry values measured at this stage rely on the simulated asymmetry of the \( W^{+}\text{jets} \) background. We then use the estimated sample composition to derive weights for the simulated \( W^{+}\text{jets} \) background that are based on the accuracy of control data, as described in Section VIIA. We apply this reweighting, which does not affect the estimation of the sample composition, and repeat the maximum likelihood fit to measure the reconstructed \( A_{FB}^{t\bar{t}} \) for \( t\bar{t} \) events.

A. The discriminant

We choose input variables that (a) provide good separation between \( t\bar{t} \) signal and \( W^{+}\text{jets} \) production, (b) are well modeled, and (c) have little correlation with each other, and with \( y_{t}, q_{y}, \) and \( p_{T}^{l} \). We combine the input variables to form a discriminant \( D \) (as in Refs. [10, 33]) based on the approximate likelihood ratio between the \( t\bar{t} \) and \( W^{+}\text{jets} \) hypotheses. Small correlations between the input variables are neglected, so only their simulated one-dimensional distributions enter \( D \).

For the \( l\geq4 \) jet channels, \( D \) is constructed exactly as in Ref [3]. We first reconstruct the full \( t\bar{t} \) decay chain using a constrained kinematic fit algorithm [34]. For each assignment of the four leading jets to the four quarks from \( t\bar{t} \) decay, the algorithm scales the four-momenta of the observed objects to minimize a \( \chi^{2} \) test statistic. The \( \chi^{2} \) test statistic measures the consistency of the scaled four-momenta with the constraints imposed by the known \( W \) boson and top quark masses, given the experimental resolutions. Only assignments that are consistent with the observed \( b \) tags are considered. The most likely assignment and the scaled four-momenta that minimize \( \chi^{2} \) are used to reconstruct the \( W \)-boson and top-quark resonances. We then build the discriminant from the following variables:

- \( \chi^{2} \) of the likeliest assignment. Low values indicate a \( t\bar{t} \) event.
- \( p_{T}^{l} \), the transverse momentum of the leading \( b \)-tagged jet, or when no jets are \( b \)-tagged, the \( p_{T} \) of the leading jet. Values below \( p_{T}^{l} \approx 50 \) GeV are indicative of \( W^{+}\text{jets} \) production.
- \( k_{R}^{\min} = \min \left( p_{T,a}, p_{T,b} \right) \cdot \Delta R_{ab} \), where \( \Delta R_{ab} = \sqrt{(\eta_{a} - \eta_{b})^{2} + (\phi_{a} - \phi_{b})^{2}} \) is the angular distance between the two closest jets, \( a \) and \( b \), and \( p_{T,a} \) and \( p_{T,b} \) are their transverse momenta.
- \( M_{jj} \), the invariant mass of the jets assigned to the \( W \rightarrow q\bar{q}' \) decay in the kinematic fit, calculated using kinematic quantities before the fit.

Of these variables, only \( \chi^{2} \) depends on the lepton, and that dependence is small as it also depends on the kinematics of the four leading jets. Thus, this discriminant has little correlation with \( p_{T}^{l} \) and \( q_{yy} \).

The variables \( \chi^{2} \) and \( M_{jj} \) are based on the full \( t\bar{t} \) reconstruction, so for the \( l\geq3 \) jet channels we construct a different discriminant. It is constructed in the same manner, but with the following variables:

- \( S \), the sphericity, defined as \( S = \frac{3}{2}(\lambda_{2} + \lambda_{3}) \), where \( \lambda_{2} \) and \( \lambda_{3} \) are the largest two out of the three eigenvalues of the normalized quadratic momentum tensor \( M \). The tensor \( M \) is defined as
  \[
  M_{ij} = \frac{\sum_{a} p_{a}^{i} p_{a}^{j}}{\sum_{a} |p_{a}|^{2}},
  \]
  where \( p_{a}^{\mu} \) is the momentum vector of a reconstructed object \( a \), and \( i \) and \( j \) are the three Cartesian coordinates. The sum over objects includes the three selected jets and the selected charged lepton. Due to the high mass of the top quarks, \( t\bar{t} \) events tend to be more spherical than background events.
- \( p_{T}^{3,\text{rd}} \), the transverse momentum of the third leading jet. This variable tends to have higher values for signal than for background.
- \( M_{jj}^{\min} \), the lowest of the invariant masses of two jets, out of the three possible jet pairings. The simulation of this variable in \( W^{+}\text{jets} \) production is discussed in Section IX.
- \( p_{T}^{l,\text{IB}} \), defined as for the \( l\geq4 \) jet channel, above.
- \( \Delta\phi(\text{jet}_{1}, \text{jet}_{2}) \), the difference in azimuthal angle between the leading jet and the transverse momentum imbalance. This variable provides additional
discrimination between the MJ background and signal. In MJ events the missing energy often originates from jet energy mismeasurement and therefore tends to be directed opposite to the direction of the leading jet, whereas in $t\bar{t}$ events the missing energy is generated by an escaping neutrino.

Jets that arise from gluon splitting are typical of $W+$jets and MJ production, and tend to have a low invariant mass and somewhat lower $p_T$ than jets in $t\bar{t}$ events. Thus, low $M_{jj}^\text{min}$, $M_{jj}$, and $k_T^\text{min}$ values are indicative of background.

The distributions of these variables in data and their modeling are shown in Fig. 1 for $l+\geq4\text{jet}$ events and in Fig. 2 for $l+3\text{jet}$ events. The fractions of $t\bar{t}$ signal, $W+$jets background and MJ background are taken from the results of the fit described in the next subsection. The number of events ($N\text{OB}$) due to the other background processes, $Z+$jets, single top quark and diboson production, is fixed to the predicted value.

### B. Maximum likelihood fit

Selected events are categorized into six channels by the number of jets and $b$ tags. The $l+3\text{jet}$, zero-$b$-tag channel serves as a control region for the asymmetries of the $W+$jets background while the other five “signal” channels are used in the maximum likelihood fit. The number of selected $t\bar{t}$, $W+$jets, and MJ events in the five signal channels, i.e., the sample composition of the data sample, and the reconstructed $A_{FB}^t$ are extracted simultaneously using a maximum likelihood fit to the distributions of $D$ and $\text{sgn}(q_y)$ (the $\text{sgn}$ function is 1 if its operand is positive and $-1$ otherwise) across the five signal channels. The distribution of the discriminant across all channels is shown in Fig. 3. The following four samples are used to construct the templates for the fit:

- simulated $t\bar{t}$ signal events with $q_y > 0$,
- simulated $t\bar{t}$ signal events with $q_y < 0$,
- simulated $W+$jets events,
- a control data sample that has been enriched in MJ production by inverting the lepton isolation requirements [22].

The shape of the discriminant is the same for both signal templates. Thus, their relative contribution is controlled by the $\text{sgn}(q_y)$ distribution, which yields the fitted reconstruction-level asymmetry, after background subtraction.

The normalization of the MJ background is determined using the observed number of events in the MJ-enriched control sample and the probability of a jet to satisfy the lepton quality requirements [22]. The probability for jets to pass lepton quality requirements, particularly in the $\mu+$jets channel, is dependent on $p_T$. We therefore split the MJ background template into six components, one for each lepton flavor and $p_T$ region. The presence of signal in the MJ control sample (“signal contamination”) is accounted for both in the likelihood and when calculating the relative weights of the templates in the data model (e.g., in Figs. 1 and 2). To reduce statistical fluctuations in the $p_T$-dependent measurement, and in other fits of subsamples (see Section X), the number of bins of the MJ discriminant distributions is reduced by a factor of two, and for the $p_T \geq 60\text{GeV}$ measurement, by a factor of three.

The results of this fit are given in Table II, where the measured $A_{FB}^t$ values are from the fit done after the reweighting of the $W+$jets background described below. In Fig. 4 the distributions of $q_y$ are taken from the simulated samples, with the exception of the distribution for MJ production, which is modeled from the MJ-enriched control sample. The distribution for $W+$jets is shown after the reweighting described in the next section.

### C. Reweighting the simulated $W+$jets background

Since both the $W+$jets background and the $t\bar{t}$ signal contribute to the $A_{FB}^t$ of the selected data, accurate modeling of $A_{FB}^t$ in $W+$jets production is required for the measurement of $A_{FB}^t$ in $t\bar{t}$ events. The asymmetry has been measured precisely for inclusive $W$-boson production [35]. However, there are notable differences between inclusive $W$-boson production and the production of a $W$ boson in association with jets, which constitutes the main background in this analysis. In particular, inclusive $W$-boson production is dominated by collisions between valence $u$ and $d$ (or $\bar{u}$ and $\bar{d}$) quarks. As the average momentum carried by $u$ quarks is higher than that carried by $d$ quarks, the $W^+$ bosons are preferentially boosted in the direction of the incoming proton. The boost of the $W$ bosons leads to positive $A_{FB}^t$ in inclusive $W$-boson production, which dominates over the negative contribution to $A_{FB}^t$ due to the $V-A$ nature of $W$-boson decay. But in

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$≥20$</th>
<th>$20–35$</th>
<th>$35–60$</th>
<th>$≥60$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{FB}^t$, %</td>
<td>1.6 ± 0.2</td>
<td>1.2 ± 0.5</td>
<td>1.2 ± 0.4</td>
<td>2.3 ± 0.3</td>
</tr>
<tr>
<td>$N_{W+jets}$</td>
<td>4445 ± 68</td>
<td>1609 ± 40</td>
<td>1842 ± 45</td>
<td>1008 ± 41</td>
</tr>
<tr>
<td>$N_{MJ}$</td>
<td>969 ± 23</td>
<td>325 ± 13</td>
<td>309 ± 14</td>
<td>333 ± 14</td>
</tr>
<tr>
<td>$N_{OB}$</td>
<td>787</td>
<td>271</td>
<td>319</td>
<td>197</td>
</tr>
<tr>
<td>$N_{el}$</td>
<td>4746 ± 64</td>
<td>1341 ± 38</td>
<td>1951 ± 43</td>
<td>1438 ± 38</td>
</tr>
<tr>
<td>$N_{sel}$</td>
<td>10947</td>
<td>3548</td>
<td>4422</td>
<td>2977</td>
</tr>
</tbody>
</table>
the $W$+jets events that pass the selection criteria of this analysis and contribute to the background, the rate and properties of the events that originate from interactions between valence quarks are different. In these events, the production of multiple jets reduces the boost of the $W$ bosons relative to their boost in inclusive $W$ production, leading to negative $A_{FB}$. Only $20\%$–$40\%$ of the $W$+jets background originates from interactions between valence quarks and the rest originates from quark–gluon interactions, which produce a positive $A_{FB}$.

We compare the simulated $A_{FB}$ to the control data sample with three jets and zero $b$ tags, which is dominated by $W$+jets background and is not used for measuring $A_{FB}$ in $t\bar{t}$ events. The composition of the three-jet, zero-$b$-tag control sample cannot be determined reliably by applying the technique of the previous section solely within the control sample itself (see Fig. 3). Using the normalizations of the $t\bar{t}$ signal and MJ background from the fit of Section VII B, we predict their contributions in the control sample. We find that the control sample is dominated by $W$+jets background, with about $75\%$ of events from $W$+jets production, $12\%$ from MJ production, $9\%$ from other backgrounds and $4\%$ from $t\bar{t}$ production.

The differential asymmetry $A_{FB}^{t\bar{t}}(|y|)$ is constrained by continuity to be zero at $|y| = 0$, hence at first order it is proportional to $|y|$. Figure 3 shows $A_{CR}(|y|)$, where $A_{CR}$ is the $A_{FB}$ of the $W$+jets background in the control region, and fits of $A_{CR}(|y|)$ to a line that passes through the origin for the $W$+jets simulation and for data. The control data is shown after subtraction of the estimated contributions from $t\bar{t}$, MJ, and other-background production.

We weight each event of the simulated $W$+jets background, regardless of its jet and $b$-tag multiplicities, using the function

$$w = 1 + \alpha q\chi y,$$

(3)

choosing $\alpha$ so that the simulated slope of $A_{CR}(|y|)$ (shown in Fig. 3 for $\alpha = 0$) agrees with the observed slope. Thus, we rely on the MC generators to describe the dependence of the $W$+jets $A_{FB}$ on the number of jets and the number of $b$ tags, and the resulting channel-to-channel differences in this asymmetry. The statistical uncertainty on $\alpha$ is taken from the statistical uncertainties on the slopes of the fits to both data and MC. The resulting differences $\Delta A_{CR}$ between the $A_{CR}$ before and after the reweighting are larger than expected from the PDF uncertainties on $A_{CR}$ (see Table III). This raises the possibility that this tension is not entirely due to the choice of PDFs and leads us to assign the entire effect of the reweighting as a systematic uncertainty.

The $q\chi y$ reweighting, using the control data, reduces the PDF uncertainties by at least a factor of three in each $p_T$ range. However, for each $p_T$ bin the uncertainties on the $A_{FB}$ of the $W$+jets background from the reweighting procedure are more than twice the size of the simulated PDF uncertainties evaluated without the reweighting procedure.

The production-level measurement is affected by this $q\chi y$ reweighting through the $q\chi y$ distribution of the subtracted $W$+jets background. The reconstruction-level
FIG. 2: Input variables to the discriminant in the $l+3$-jet sample (see Section VII A for definitions of variables) for events with at least one $b$ tag. Overflows are shown in the extreme bins. The ratios between the data counts and the model expectations are shown in the lower panel of each figure. The hashed area indicates the systematic uncertainties on the model expectations.

FIG. 3: The discriminants for all channels, concatenated into a single variable $D_c$. Each unit of $D_c$ corresponds to a channel, as labeled in the plot. The region $D_c < 1$ is not used in the fit for sample composition and $A_F^{1B}$. The ratio of the data counts and the model expectation is shown below. The hashed area indicates the systematic uncertainties on the model expectations.
FIG. 4: The $q_{yy}$ distribution for: (a) $l+3$ jets and 0 $b$ tags, (b) $l+\geq4$ jets and 0 $b$ tags, (c) $l+3$ jets and 1 $b$ tag, (d) $l+\geq4$ jets and 1 $b$ tag, (e) $l+3$ jets and $\geq2$ $b$ tags, and (f) $l+\geq4$ jets and $\geq2$ $b$ tags. The ratio between the data counts and the model expectation is shown below each plot. The hashed area indicates the systematic uncertainties on the model expectations.
measurement is affected by this reweighting through $\Delta A_{CR}$. The effect of the reweighting on the $W+\text{jets}$ in each signal channel is within 0.3% of $\Delta A_{CR}$.

### VIII. UNFOLDING THE ASYMMETRIES

The inclusive $A_{FB}^l$ is unfolded separately in each channel, and the measured $A_{FB}^l$ values are then combined to form the inclusive measurement. Due to the excellent angular resolution for leptons, migrations in rapidity are negligible, and unfolding the $A_{FB}^l$ reduces to correcting for acceptance effects. The inverse of the simulated selection efficiency is taken as a weight for each bin in $q/y_l$ to correct for acceptance effects. These corrections therefore assume the SM as modeled in MC@NLO. We restrict the selection to $|y_l| < 1.5$ to avoid the region of low acceptance, and compute the weights in 48 bins as in the previous $A_{FB}^l$ measurement [3].

For the differential $A_{FB}^l$ measurement, we define for each of the 48 $q/y_l$ bins a vector $\bar{r}$ of observed counts in the three $p_T^l$ bins. The observed counts are affected by the migration of $\approx 10\%$ of the events over the bin boundaries in $p_T^l$. The expectation value of $\bar{r}$ is $\langle \bar{r} \rangle = A M \bar{p}$, where $A$ is the acceptance matrix, $M$ is the $3 \times 3$ migration matrix and $\bar{p}$ is the vector of production-level event counts. The acceptance matrix is a $3 \times 3$ diagonal matrix with the three acceptance probabilities embedded in its diagonal. The vector of the unfolded production-level counts that best estimates the vector $\bar{p}$ is $\bar{u} = A^{-1} M^{-1} \bar{r}$. With a nearly diagonal migration matrix and only three bins, the above matrix inversion yields stable solutions.

We evaluate the statistical uncertainty of the unfolded $A_{FB}^l$ from each channel using an ensemble of pseudo-datasets that match the sample composition in data, with the signal simulated according to MC@NLO. The pseudo-datasets are simulated using Poisson fluctuations both on the selected sample and on the MJ control sample. The statistical uncertainties on $A_{FB}^l$ for each channel and the weight of each channel in the combined measurement are listed in Table IV.
TABLE IV: Statistical uncertainty ($\sigma$) on the measured $A_{FB}^l$ and weight for each channel (where applicable). The weight for each channel is proportional to $\sigma^{-2}$.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma$</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t + 3$ jets, 0 b tags</td>
<td>24%</td>
<td>n/a</td>
</tr>
<tr>
<td>$t + 3$ jets, 1 b tag</td>
<td>6.8%</td>
<td>0.11</td>
</tr>
<tr>
<td>$t + 3$ jets, $\geq 2$ b tags</td>
<td>4.7%</td>
<td>0.24</td>
</tr>
<tr>
<td>$t + \geq 4$ jets, 0 b tags</td>
<td>13.9%</td>
<td>n/a</td>
</tr>
<tr>
<td>$t + \geq 4$ jets, 1 b tag</td>
<td>4.7%</td>
<td>0.24</td>
</tr>
<tr>
<td>$t + \geq 4$ jets, $\geq 2$ b tags</td>
<td>3.6%</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Since the $l + 3$ jet, zero-b-tag channel is used to tune the modeling of the $W^+\text{jets}$ background, it cannot be used to extract the signal $A_{FB}^l$. We also do not use the $l + \geq 4$ jet, zero-b-tag channel for the unfolded result, due to its low purity and the large uncertainty on $A_{FB}^l$. The weighted average of the four remaining b-tagged channels gives our combined value for $A_{FB}^l$.

The lepton-based asymmetries unfolded to the production level are summarized in Table V and shown in Fig. 6. The results are compared to MC@NLO-based predictions.

TABLE V: Predicted and observed production-level asymmetries. The first uncertainty on the measured $A_{FB}^l$ is statistical and the second systematic. The statistical uncertainties on the MC predictions are less than 0.1%, while the scale and PDF uncertainties are estimated to be < 1%.

<table>
<thead>
<tr>
<th>$p_T$ range, GeV</th>
<th>Data $A_{FB}^l$, %</th>
<th>MC@NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclusive</td>
<td>4.2 $^{+1.7}_{-2.0}$</td>
<td>2.0</td>
</tr>
<tr>
<td>20–35</td>
<td>$-0.3 \pm 4.1 \pm 3.6$</td>
<td>1.6</td>
</tr>
<tr>
<td>35–60</td>
<td>4.8 $^{+2.2}_{-2.1}$</td>
<td>2.3</td>
</tr>
<tr>
<td>$\geq 60$</td>
<td>9.3 $^{+2.7}_{-2.3}$</td>
<td>3.1</td>
</tr>
</tbody>
</table>

FIG. 6: Predicted and observed production-level asymmetries as a function of absolute lepton rapidity. Statistical uncertainties are indicated by the inner, and the total uncertainties by the outer error bars.

TABLE VI: Predicted and observed production-level asymmetries as a function of $|y|$. The first uncertainty on the measured values is statistical and the second is systematic. The statistical uncertainties on the MC predictions are less than 0.1%, while the scale and PDF uncertainties are estimated to be < 1%.

| $|y|$ range     | Data $A_{FB}^l$, % | MC@NLO |
|----------------|-------------------|--------|
| 0.0 – 0.125    | 0.5 $^{+0.8}_{-0.7}$ | 0.2    |
| 0.125 – 0.375  | 0.5 $^{+1.3}_{-1.8}$ | 0.9    |
| 0.375 – 0.625  | 2.6 $^{+1.7}_{-1.5}$ | 1.8    |
| 0.625 – 1      | 1.9 $^{+2.0}_{-2.3}$ | 2.7    |
| 1.0 – 1.5      | 13.2 $^{+2.6}_{-2.9}$ | 3.7    |

FIG. 7: Predicted and observed production-level asymmetries as a function of lepton transverse momentum. The last bin extends beyond the edge of the plot and has no upper boundary. Statistical uncertainties are indicated by the inner, and the total uncertainties by the outer error bars.

IX. SYSTEMATIC UNCERTAINTIES

We consider several sources of systematic uncertainty. For most sources, we vary the modeling according to the evaluated uncertainty in the relevant parameters of the model, repeat the entire analysis and propagate the effect to the final result. This accounts for the correlations between the channels and between the various steps of the analysis, such as the maximal likelihood fit, the fit for $\alpha$, and the unfolding. Some sources are quantified using more specialized procedures, as described below. Systematic uncertainties from different sources are added in quadrature to yield the total systematic uncertainty. Table VII lists the systematic uncertainties on the predicted reconstruction-level $A_{FB}^l$ (as listed in Tables I and II), on the measured reconstruction-level $A_{FB}^l$, and on the measured production-level $A_{FB}^l$. The systematic sources are classified into the following categories:
Jet reconstruction (reco)
This source includes the jet reconstruction and identification efficiencies, as well as the efficiency of the vertex confirmation described in Section VI. The simulated efficiencies are calibrated on dijet data. Additional $p\bar{p}$ collisions within the same bunch crossing can yield additional jets, and their modeling is also included in this category. The rate of additional $p\bar{p}$ collisions is derived from the number of reconstructed vertices per event.

Jet energy measurement
The jet energy scale (JES) is measured using dijet and photon+jet samples \[^{[36]}\]. The simulated jet energy resolution (JER) is calibrated using $Z+\text{jet}$ data. Their uncertainties are propagated to the measured asymmetry.

Signal modeling
As discussed in Section VI, the SM predicts a negative asymmetry for events with additional final state gluons (and hence with larger $p_T^t$). Thus, event selection introduces a bias on the measured asymmetry, in particular making it sensitive to the jet multiplicity. The inclusion of $l+3$ jet events in the analysis reduces this correlation. To evaluate the size of this systematic effect we vary the amount of initial-state radiation (ISR) within an uncertainty range established from a measurement of ISR rates \[^{[37]}\].

Forward-backward differences in the amount of additional radiation can also affect the measurement through $p_T^{t\bar{t}}$, which is correlated with the acceptance \[^{[3]}\]. These differences are controlled by the simulated color coherence of the partonic showers \[^{[3]}\]. QCD predicts that parton showers in angular order are more likely, while the simulation enforces strict angular ordering \[^{[38]}\]. NLO event generators calculate the first QCD emission analytically, reducing the reliance on the modeling of the parton showers. To quantify this uncertainty, we consider the possibility that this mismodeling also result in poor modeling of this distribution \[^{[3]}\]. We estimate the uncertainty range established from a measurement \[^{[37]}\].

The $p_T^{t\bar{t}}$ distribution was studied in Refs. \[^{[3, 32]}\] and found to be well modeled. This is in contrast to the limitations of the D0-detector simulation, which result in poor modeling of this distribution \[^{[3]}\]. We consider the possibility that this mismodeling also affects $A_{FB}^{l\bar{t}}$ by reweighting the simulated events as a function of the reconstructed $p_T^{t\bar{t}}$ so that the $p_T^{t\bar{t}}$ distribution agrees with data.

The mass of the top quark was varied from its value in the nominal simulation of 172.5 GeV according to the latest measurement \[^{[8]}\]. To quantify additional systematic uncertainties due to the modeling of signal, we repeat the analysis using signal events simulated with ALPGEN combined with PYTHIA. As the box diagram is not included in ALPGEN, ALPGEN predictions are missing the largest contribution to the $t\bar{t}$ asymmetry \[^{[1]}\]. Furthermore, this missing contribution peaks at low $p_T^t$, where acceptance is low, making the acceptance predicted by ALPGEN unrealistic. Therefore we use the acceptance predicted by MC@NLO instead of the one predicted by ALPGEN in evaluating this uncertainty.

The uncertainties on the production-level inclusive $A_{FB}^{l\bar{t}}$ due to the top quark mass, the choice of MC generator, and the overall amount of ISR are similar. The systematic uncertainties due to the forward-backward differences in the additional radiation and due to $p_T^{t\bar{t}}$ reconstruction have negligible effect on the inclusive $A_{FB}^{l\bar{t}}$.

$\beta$ tagging
The $\beta$-tagging efficiency and mistagging probability, which are determined from dijet data with at least one muon identified within a jet, affect the division of events between 0, 1, and $\geq 2$ $\beta$-tag sub-samples. Due to this division of channels, the analysis is now more sensitive to systematic variations on $\beta$-tagging than the previous measurement \[^{[3]}\].

Background (Bkg) subtraction
The subtracted amounts of $W+\text{jets}$ and MJ background are varied within their fitted uncertainties. Uncertainties on the normalization and shape of the MJ background arise from the uncertainties on the lepton selection rates, which are used to evaluate the MJ background. An uncertainty of 20% is assigned to the rate of $W+c\bar{c}$ and $W+b\bar{b}$ production.

Background (Bkg) modeling
The $q_{ij}$ reweighting of the $W+\text{jets}$ background is varied using $\alpha$ values of zero and twice the nominal $\alpha$ (see Table III). The effect of increased MJ production at large $|y_j|$ is considered by reweighting the MJ $q_{ij}$ distribution to better match the data in the $l+3$ jet, zero-$\beta$-tag control region. The possibly underestimated muon background with mismeasured high transverse momentum described in Section VI peaks in that region, and an excess of data events in that region is seen in some channels. We also consider a similar increase in $W+\text{jets}$ production at large $|y_j|$.

We account for the marginal agreement of the di-jet invariant mass (see Fig. 26) and related observables between data enriched in $W+\text{jets}$ production and the ALPGEN simulation of such data \[^{[40]}\] by reweighting the simulated $M_{jj}^{\text{min}}$ distribution of the $W+\text{jets}$ background to match data in the $l+3$ jet, 0-\$\beta$-tag control region. This improves the modeling of $M_{jj}^{\text{min}}$ in all channels, supporting the attribution of this small mismodeling to the modeling of $W+\text{jets}$ production. A small mismodeling of $\Delta\phi(jet_1, E_T)$ is indicated in Fig. 23 but its effect on the discrim-
inant is far smaller than that of the region around $M_{jj}^{\text{min}} = 50$ GeV.

**Parton distribution functions**

Each of the error eigenvectors of the set of PDFs is varied up and down, and the effects are added in quadrature. We also consider an uncertainty due to the choice of PDFs, which we evaluate using the nominal PDFs of the CTEQ6L1 [41] and MRST2003 [42] sets. The MRST2003 set is chosen since its $u$, $d$, $s$, and $g$ PDFs differ significantly from those of the CTEQ6L1 set for values of Bjorken $x$ above 0.01, which are most relevant to this analysis.

<table>
<thead>
<tr>
<th>Source</th>
<th>Absolute uncertainty, %</th>
<th>Recovery level</th>
<th>Prod. level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet reco</td>
<td>−0.1</td>
<td>Prediction</td>
<td>Measurement</td>
</tr>
<tr>
<td>JES/JER</td>
<td>+0.1</td>
<td>+0.1/−0.3</td>
<td>+0.2/−0.3</td>
</tr>
<tr>
<td>Signal modeling</td>
<td>−</td>
<td>−</td>
<td>+0.6/−0.4</td>
</tr>
<tr>
<td>$b$ tagging</td>
<td>±0.1</td>
<td>+0.5/−0.8</td>
<td>+0.8/−1.1</td>
</tr>
<tr>
<td>$B_g$ subtraction</td>
<td>n/a</td>
<td>+0.1/−0.3</td>
<td>+0.1/−0.3</td>
</tr>
<tr>
<td>$B_g$ modeling</td>
<td>n/a</td>
<td>+1.4/−1.5</td>
<td>+1.3/−1.5</td>
</tr>
<tr>
<td>PDFs</td>
<td>−</td>
<td>+0.3/−0.2</td>
<td>+0.1/−0.2</td>
</tr>
<tr>
<td>Total</td>
<td>±0.1</td>
<td>+1.5/−1.7</td>
<td>+1.7/−2.0</td>
</tr>
</tbody>
</table>

**TABLE VII:** Systematic uncertainties on $A_{FB}^{l}$. Uncertainties smaller than 0.1% are omitted.

**X. DISCUSSION**

Using a dataset corresponding to an integrated luminosity of 9.7 fb$^{-1}$, we measure the production-level inclusive $A_{FB}^{l}$ to be $(4.2 \pm 2.3 \text{ (stat.)}^{+1.7}_{−2.0} \text{ (syst.)}) \%$. The previously published value, which was measured using a subset of this dataset corresponding to 5.4 fb$^{-1}$ [3], is $A_{FB}^{l} = (15.2 \pm 3.8) \%$. In this section we further compare these measurements and discuss the reasons for the difference between their results. Unlike the previous measurement, the current result is in good agreement with the MC@NLO prediction for the production-level of $A_{FB}^{l} = 2.0 \%$, which has a statistical uncertainty of less than 0.1%. The statistical uncertainty on the measured $A_{FB}^{l}$ is reduced by a factor of $\approx 1.67$ with respect to Ref. [3]. This reduction is mostly due to the addition of new data (by 1.30) and the inclusion of events with three jets (by 1.25).

The inclusion of $l+3$ jet events, the addition of newer data, the use of better object identification algorithms, and improvements to the analysis technique all decrease the measured asymmetry. Both here and in Ref. [3], the analysis strategy was finalized before analyzing Tevatron data. Together, these changes reduce the measured $A_{FB}^{l}$ by 10.5%, yet no single change accounts for a difference of more than 2.5%.

The $p$-value for the previously published value, assuming the asymmetry predicted by MC@NLO, is $1.7 \times 10^{-3}$, while the $p$-value of the new result is 0.24. These numbers do not account for the systematic uncertainty on the theoretical predictions.

Most of the asymmetry in the previous analysis is contained in the $l+\geq 4$ jet channel for events with exactly one $b$ tag. In the current analysis, the asymmetry in this channel is still high compared to the SM expectation, with $A_{FB}= (16.3 \pm 4.8 \text{ (stat.)}^{+2_{.2}}_{−2.4} \text{ (syst.)}) \%$. The relative weight of this channel decreased from $\approx 50 \%$ in the previous analysis to 24% in the current analysis. The $q\bar{q}g$ distributions in each channel are shown in Fig. 4. The $A_{FB}^{l}$ values of the various channels are compared in Fig. 5, Table VIII and Table IX. The consistency between different channels in Fig. 5 corresponds to a $\chi^2$ value of 8.1 for three degrees of freedom, which corresponds to a tail probability of 4.5%.

**FIG. 8:** Measured production-level $A_{FB}^{l}$ by analysis channel. The vertical line shows the MC@NLO prediction. The $\chi^2$ is of a fit to a single value, shown by the crosshatched band and the dashed line. Statistical uncertainties are indicated by the inner vertical lines, and the total uncertainties by the vertical end lines.

**TABLE VIII:** Measured and predicted $A_{FB}^{l}$ by channel, at reconstruction level.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$A_{FB}^{l}$, %</th>
<th>MC@NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l+3$ jets, 1 $b$ tag</td>
<td>$-6.8 \pm 6.0 \text{ (stat.)}^{+1.5}_{−1.7} \text{ (syst.)}$</td>
<td>2.7 ± 0.4</td>
</tr>
<tr>
<td>$l+3$ jets, $\geq 2$ $b$ tags</td>
<td>3.7 $\pm 4.3 \text{ (stat.)}^{+1.1}_{−1.2} \text{ (syst.)}$</td>
<td>2.8 $\pm 0.3$</td>
</tr>
<tr>
<td>$l+\geq 4$ jets, 1 $b$ tag</td>
<td>14.8 $\pm 4.2 \text{ (stat.)}^{+1.1}_{−1.2} \text{ (syst.)}$</td>
<td>0.5 $\pm 0.3$</td>
</tr>
<tr>
<td>$l+\geq 4$ jets, $\geq 2$ $b$ tags</td>
<td>$-0.9 \pm 3.2 \text{ (stat.)}^{+0.3}_{−0.2} \text{ (syst.)}$</td>
<td>1.1 $\pm 0.2$</td>
</tr>
<tr>
<td>Total</td>
<td>$2.9 \pm 2.1 \text{ (stat.)}^{+1.5}_{−1.7} \text{ (syst.)}$</td>
<td>1.6 $\pm 0.2$</td>
</tr>
</tbody>
</table>

We also studied the asymmetry in subsamples defined by the charge of the lepton, the flavor of the lepton, and by the polarities of the D0 magnets, which are reversed every two weeks. Reversing the magnet polarities greatly reduces possible experimental biases which involve the lepton. All measurements agree within at most two standard deviations.
TABLE IX: Measured $A_{FB}^i$ by channel at production level. The mc@nlo prediction for $|y_t| < 1.5$ is $A_{FB}^i = 2.0\%$.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Measured $A_{FB}^i$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t+3$-jets, 1 $b$ tag</td>
<td>$-8.2 \pm 6.9$ (stat.)$^{+3.9}_{-9.4}$ (syst.)</td>
</tr>
<tr>
<td>$t+3$-jets, $\geq 2$ $b$ tags</td>
<td>$3.1 \pm 4.7$ (stat.)$^{+1.3}_{-2.4}$ (syst.)</td>
</tr>
<tr>
<td>$t+\geq 4$-jets, 1 $b$ tag</td>
<td>$16.3 \pm 4.8$ (stat.)$^{+2.2}_{-2.1}$ (syst.)</td>
</tr>
<tr>
<td>$t+\geq 4$-jets, $\geq 2$ $b$ tags</td>
<td>$1.3 \pm 3.6$ (stat.)$^{+0.8}_{-0.5}$ (syst.)</td>
</tr>
<tr>
<td>Total</td>
<td>$4.2 \pm 2.3$ (stat.)$^{+1.7}_{-2.0}$ (syst.)</td>
</tr>
</tbody>
</table>

Since the SM-derived corrections to the measured $A_{FB}^i$ are only 1–2%, the dependence of the results on the SM is small and the results may be validly compared to the predictions of beyond the SM predictions. We tested our analysis method using axigluon samples produced using MADGRAPH combined with PYTHIA with axigluon masses of 0.2, 0.4, 0.8, and 2 TeV with completely left-handed, completely right-handed and mixed couplings [43]. In all of these scenarios, the measured production-level $A_{FB}^i$ exhibits additional, model-dependent scatter about the simulated $A_{FB}^i$ values of $<1.5\%$ (absolute).

XI. COMBINATION AND EXTRAPOLATION

The D0 Collaboration measured $A_{FB}^i$ in the dilepton channel, with lepton coverage that extends to $|y_t| = 2$, finding $A_{FB}^i = (4.1 \pm 3.5$ (stat.)$ \pm 1.0$ (syst.))%. To enable a direct combination with the measurements in the $l+$jets channel, the analysis of Ref. [4] is repeated using only leptons with $|y_t| < 1.5$, finding $A_{FB}^i = (4.3 \pm 3.4$ (stat.)$ \pm 1.0$ (syst.))%. The decrease in the statistical uncertainty is due to the removal of the events with $|y_t| > 1.5$, which have a large weight due to the acceptance corrections and thus increase the statistical uncertainty.

This result is combined with the results of the current measurement using the Best Linear Unbiased Estimator (BLUE) method, as described in Ref. [14]. Systematic uncertainties are classified by their source as either completely correlated, e.g., the $b$ tagging uncertainties, or completely uncorrelated, e.g., the background modeling uncertainties. The combination is a weighted average of the input measurements, with the dilepton measurement given a weight of 0.43 and the $l+$jets measurement a weight of 0.57. The combined value of $A_{FB}^i$ for $|y_t| < 1.5$ is $A_{FB}^i = (4.2 \pm 2.0$ (stat.)$ \pm 1.4$ (syst.))% = (4.2 $\pm 2.4$)%.

The measurements are extrapolated to cover the full phase space using the mc@nlo simulation. Extrapolation adds nothing to our experimental measurements of $A_{FB}^i$, but simply extends them by incorporating SM-inspired predictions for $t\bar{t}$ production outside the lepton rapidity coverage, thus facilitating comparison with theoretical calculations and the combination of the measurements. Such extrapolated values should not be compared with non-SM predictions [45]. To include dilepton events with $1.5 < |y_t| < 2$ in the extrapolated values, we extrapolate each result independently before combining them. If we assume a linear dependence of the asymmetry on $|y_t|$, we find an extrapolated asymmetry which is proportional to the measured asymmetry,

$$A_{FB}^{i, ex} = A_{FB}^{i, tot} A_{FB}^{i, pred} / A_{FB}^{i, pred},$$

where $A_{FB}^{i, tot}$ is the simulated $A_{FB}^i$ in the entire phase space and $A_{FB}^{i, pred}$ is the simulated $A_{FB}^i$ within the acceptance. For the simulated $t\bar{t}$ samples, such an extrapolation overestimates the fully inclusive $A_{FB}^i$. Furthermore, only the leading order of the $t\bar{t}$ asymmetry has been calculated, and so the dependence of $A_{FB}^i$ on $|y_t|$ is not known precisely. Therefore we assign a systematic uncertainty to the extrapolation that equals the entire effect of the extrapolation. Using the mc@nlo predictions of $A_{FB}^{i, tot} / A_{FB}^{i, pred} = 1.19$ for the $l+$jets measurement and 1.07 for the dilepton measurement, and find a combined extrapolated asymmetry of $A_{FB}^{i, ex} = (4.7 \pm 2.3$ (stat.)$ \pm 1.5$ (syst.))%.

XII. SUMMARY

Using the full dataset collected by the D0 experiment during Run II of the Tevatron, corresponding to 9.7 fb$^{-1}$, we have measured the forward-backward asymmetry in the direction of leptons from $t\bar{t}$ events in the $l+$jets channel and compared it to a prediction based on mc@nlo. Since the lepton-based asymmetry does not require a full reconstruction of the $t\bar{t}$ event, this measurement also uses events with only three jets. The measured asymmetry at production level for $|y_t| < 1.5$ is $A_{FB}^i = (4.2 \pm 2.4)$%.

We combined this measurement with the measurement in the dilepton channel and found a production-level asymmetry for $|y_t| < 1.5$ of $A_{FB}^i = (4.2 \pm 2.4)$%, to be compared to the mc@nlo prediction of 2.0%. We have presented the first measurement of the differential asymmetry as a function of $p_T$. All results are in agreement with mc@nlo predictions.

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