ELECTROPRODUCTION OF TWO VECTOR MESONS IN THE NEXT-TO-LEADING ORDER

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- **D.Yu. Ivanov, M.I. Kotsky and A.P., Eur. Phys. J. C 38 (2004) 195, hep-ph/0405297**
- **D.Yu. Ivanov and A.P., work in progress**

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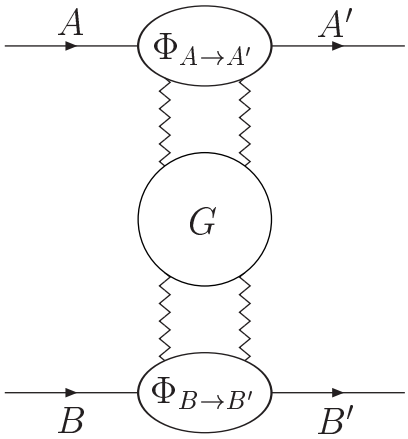
Plan of the talk

- **Introduction and motivations**
- **The $\gamma^* \to V$ impact factor**
  - **Impact factors in the BFKL approach**
  - **Collinear factorization $\longrightarrow$ hard amplitude $\otimes$ distribution amplitude**
  - **Cancellation of divergences**
  - **Final result in the NLA**
- **The $\gamma^*\gamma^* \to VV$ amplitude in the NLA (preliminary)**
  - **amount of the IF corrections**
  - **role of s_0**
  - **renormalization scale dependence**
- **Conclusions**

Introduction

Scattering $A + B \longrightarrow A' + B'$ in the Regge kinematical region $s \to \infty$, t fixed

BFKL approach: convolution of the Green's function of two interacting Reggeized gluons and of the impact factors of the colliding particles.



Valid both in
LLA (resummation of all terms $(\alpha_s \ln(s))^n$)
NLA (resummation of all terms $\alpha_s(\alpha_s \ln(s))^n$).

- The Green's function is determined through the BFKL equation.

  The kernel of the BFKL equation is completely known in the NLA for the forward ($t = 0$) case.

  [V.S. Fadin, L.N. Lipatov (1998)]
  [G. Camici, M. Ciafaloni (1998)]

  In the non-forward case it is completely known for the octet color representation, relevant for the "bootstrap".

  Recently, also the last missing piece for the singlet representation has been determined, that from Reggeon-Reggeon $\to$ gluon-gluon.

  [V.S. Fadin and R. Fiore (2005)]

- Impact factors have been calculated in the NLA for
  colliding partons [V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)]
  forward jet production [J. Bartels, D. Colferai, G.P. Vacca (2003)]

Among the colorless impact factors, the most important is certainly the $\gamma^* \to \gamma^*$ impact factor. Its calculation is rather complicated and so far only partial results are available.

[J. Bartels, D. Colferai, S. Gieseke, A. Kyrieleis (2002)]
[V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2003)]
[J. Bartels, A. Kyrieleis (2004)]

Here, the impact factor for the $\gamma^* \to V$ transition, $V = \rho^0,\ \omega,\ \phi$, is considered. The large photon virtuality Q^2 provides the hard scale for the application of perturbative QCD.

The calculation is simpler than for the $\gamma^* \to \gamma^*$ case and a closed analytical expression can be achieved in the NLA.

Theoretical interest:

for the first time the amplitude of a physical process, $\gamma^*\gamma^* \to VV$, can be written completely within perturbative QCD and with NLA accuracy
(for the LLA case, see [B. Pire, L. Szymanowsky, S. Wallon (2004)])

$\longrightarrow$ possibility to understand the role and the optimal choice of energy scales in the BFKL approach;

$\longrightarrow$ comparison between different approaches, such as BFKL and DGLAP.

Phenomenological interest:

first step toward the application of the BFKL approach to the description of

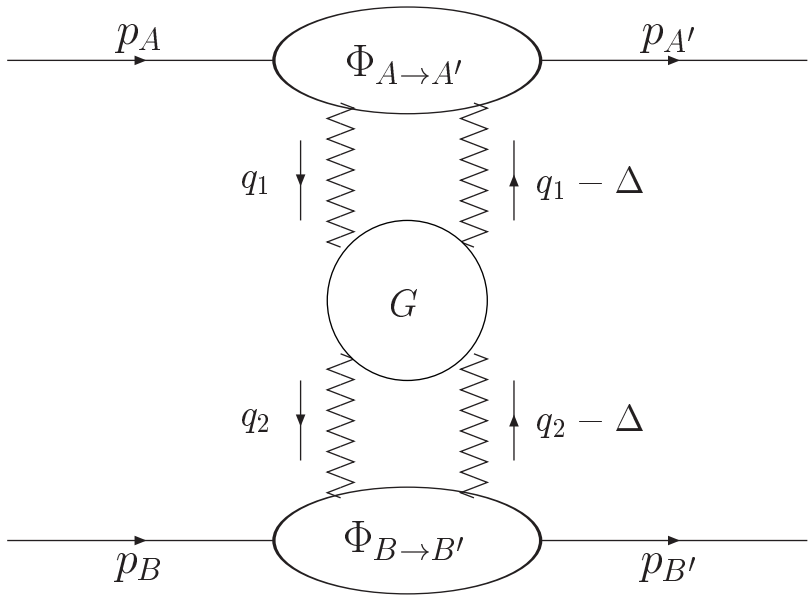
- $\gamma^* p \to Vp$, carried out at HERA;
- $\gamma^*\gamma^* \to VV$ or $\gamma^*\gamma \to VJ/\Psi$, which can be studied at high-energy e^+e^- and $e\gamma$ colliders.

The $\gamma^* \to V$ impact factor

BFKL approach for the amplitude of the process $AB \to A'B'$

A, B, A', B' colorless particles

$\sqrt{s} \to \infty$, fixed momentum transfer $\Delta \approx \Delta_\perp$



$$\mathcal{I}m_s\,(\mathcal{A})_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{\Delta})^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{\Delta})^2}$$

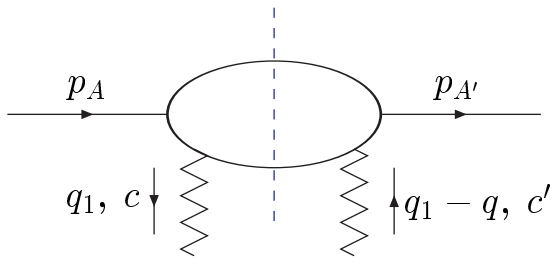
$$\times \Phi_{A\to A'}(\vec{q}_1, \vec{\Delta}, s_0) \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2, \vec{\Delta}) \right] \Phi_{B\to B'}(-\vec{q}_2, -\vec{\Delta}, s_0),$$

$D = 4 + 2\varepsilon$ **(for both UV and IR divergences),** s_0 **energy scale parameter**

The Green's function obeys the (generalized) BFKL equation

$$\omega\, G_\omega(\vec{q}_1, \vec{q}_2, \vec{\Delta}) = \vec{q}_1^{\,2}(\vec{q}_1 - \vec{\Delta})^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}k}{\vec{k}^2(\vec{k} - \vec{\Delta})^2} \mathcal{K}(\vec{q}_1, \vec{k}, \vec{\Delta}) G_\omega(\vec{k}, \vec{q}_2, \vec{\Delta})$$

$\mathcal{K}$ singlet kernel in the NLA



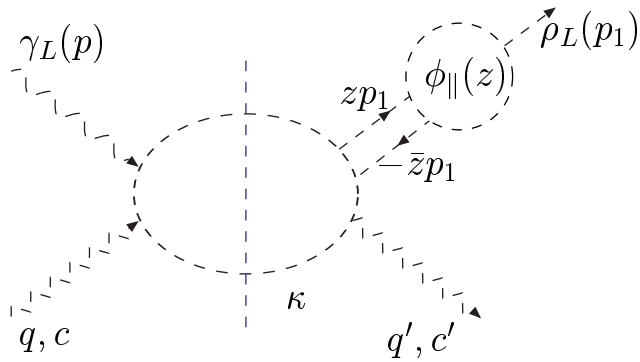
$$\Phi_{A\to A'}(\vec{q}_1,\vec{\Delta},s_0) = \frac{\delta^{cc'}}{\sqrt{N_c^2-1}}\left[\left(\frac{s_0}{\vec{q}_1^{\,2}}\right)^{\frac{1}{2}\omega(-\vec{q}_1^{\,2})}\left(\frac{s_0}{(\vec{q}_1-\vec{\Delta})^2}\right)^{\frac{1}{2}\omega(-(\vec{q}_1-\vec{\Delta})^2)}\right.$$

$$\left.\times\sum_{\{f\}}\int\frac{d\kappa}{2\pi}\theta(s_\Lambda-\kappa)d\rho_f\Gamma^c_{A\{f\}}\left(\Gamma^{c'}_{A'\{f\}}\right)^*\right]$$

$$-\frac{1}{2}\int\frac{d^{D-2}k}{\vec{k}^2(\vec{k}-\vec{\Delta})^2}\Phi^{Born}_{A\to A'}(\vec{k},\vec{\Delta},s_0)\mathcal{K}^{Born}_r(\vec{k},\vec{q}_1,\vec{\Delta})\ln\left(\frac{s_\Lambda^2}{s_0(\vec{k}-\vec{q}_1)^2}\right)$$

$\omega(t)$ **Reggeized gluon trajectory in the LLA;** $\quad s_\Lambda\to\infty$

$\Phi_{\gamma^*\to V}$, $V=\rho^0,\omega,\phi$; **forward case** ($\vec{\Delta}=0$)



Sudakov vectors: $p_1,\ p_2,\quad p_1^2=p_2^2=0,\quad 2(p_1p_2)=s$

$$p = p_1 - \frac{Q^2}{s}\,p_2$$

$$q = \frac{\kappa+Q^2+\vec{q}^{\,2}}{s}p_2 + q_\perp\,,\qquad q^2 = q_\perp^2 = -\vec{q}^{\,2}$$

$$q' = \frac{\kappa+\vec{q}^{\,2}}{s}p_2 + q_\perp\,,$$

$\vec{q}^{\,2},\ Q^2 \gg$ **any hadronic scale; neglected power suppressed contributions**

In this kinematics the impact factor can be calculated in the collinear factorization framework.

[V.L. Chernyak, A.R. Zhitnitsky (1977, 1980)]
[V.L. Chernyak, V.G. Serbo, A.R. Zhitnitsky (1977, 1980)]
[G.P. Lepage and S.J. Brodsky (1979, 1980)]
[S.J. Brodsky, G.P. Lepage, A.A. Zaidi (1981)]
[A.V. Efremov, A.V. Radyushkin (1980)]

The dominant helicity amplitude is $\gamma_L^* \to V_L$.

Factorization, both in LLA and in NLA, in the convolution (here, $\Phi_{\gamma_L^* \to V_L}$ is the unprojected impact factor)

$$\Phi_{\gamma_L^* \to V_L}(\alpha, s_0) = -\frac{4\pi e_q f_V \delta^{cc'}}{N_c Q} \int_0^1 dz\, T_H(z, \alpha, s_0, \mu_F, \mu_R)\, \phi_{\parallel}(z, \mu_F)$$

$$f_V \simeq 200\,\mathrm{MeV}\,, \qquad \alpha = \frac{\vec{q}^{\,2}}{Q^2}$$

T_H **hard-scattering amplitude, perturbative series in $\alpha_S(\mu_R)$, $\mu_R^2 \sim Q^2, \vec{q}^{\,2}$**

$\phi_{\parallel}(z)$ **meson twist-2 distribution amplitude**

$$\langle 0|\bar{\Psi}(0)\gamma^\mu \Psi(y)|V_L(p_1)\rangle_{y^2\to 0} = f_V\, p_1^\mu \int_0^1 dz\, e^{-iz(p_1 y)}\, \phi_{\parallel}(z, \mu_F)$$

(path-ordered gauge factor between quark operators implied)

The distribution amplitude has the meaning of probability amplitude to find a meson in a state with minimal number of constituents – quark and antiquark – when they are separated by small transverse distances, $r_\perp \sim 1/\mu_F$

$\mu_F^2 \sim Q^2, \vec{q}^{\,2}$ **factorization scale**

z, $\bar{z} \equiv 1 - z$ **longitudinal momentum fractions carried by quark and antiquark**

$$|\rho^0\rangle = \frac{1}{\sqrt{2}}\left(|\bar{u}u\rangle - |\bar{d}d\rangle\right) \qquad |\omega\rangle = \frac{1}{\sqrt{2}}\left(|\bar{u}u\rangle + |\bar{d}d\rangle\right) \qquad |\phi\rangle = |\bar{s}s\rangle$$

$$e_q \longrightarrow \qquad \frac{e}{\sqrt{2}} \qquad \frac{e}{3\sqrt{2}} \qquad -\frac{e}{3}$$

The distribution amplitude defined by

$$\langle 0|\bar{\Psi}(0)\gamma^\mu\Psi(y)|V_L(p_1)\rangle_{y^2\to 0} = f_V\, p_1^\mu \int\limits_0^1 dz\, e^{-iz(p_1 y)}\, \phi_\parallel(z,\mu_F)$$

is a non-perturbative function, but its dependence on μ_F is perturbative

$$\mu_F^2 \frac{d\phi_\parallel(z,\mu_F)}{d\mu_F^2} = \int\limits_0^1 V(z,z')\phi_\parallel(z',\mu_F)dz' \,.$$

Local limit $y\to 0$, $\bar{\Psi}\gamma^\mu\Psi$ conserved current $\longrightarrow \int\limits_0^1 dz\,\phi_\parallel(z,\mu_F)$ renorm-invariant; $\int\limits_0^1 dz\,\phi_\parallel(z,\mu_F)=1$ is our convention for f_V.

Evolution kernel known up to the second order:

$$V(z,z') = \frac{\alpha_s(\mu_F)}{2\pi}V^{(1)}(z,z') + \left(\frac{\alpha_s(\mu_F)}{2\pi}\right)^2 V^{(2)}(z,z') + \ldots$$

$$V^{(1)}(z,z') = C_F\left[\frac{1-z}{1-z'}\left(1+\frac{1}{z-z'}\right)\theta(z-z') + \frac{z}{z'}\left(1+\frac{1}{z'-z}\right)\theta(z'-z)\right]_+$$

$$[f(z,z')]_+ \equiv f(z,z') - \delta(z-z')\int_0^1 dt\, f(t,z)$$

Cancellation of divergences:

T_H calculated for bare α_S and meson distribution amplitude $\phi_\parallel^{(0)}(z)$
$\longrightarrow$ UV and IR divergences, common parameter ε

UV divergences disappear after coupling constant renormalization ($\overline{\rm MS}$)

$$\alpha_s = \alpha_s(\mu_R)\left[1+\frac{\alpha_s(\mu_R)}{4\pi}\beta_0\left(\frac{1}{\hat{\varepsilon}}+\ln\left(\frac{\mu_R^2}{\mu^2}\right)\right)\right]$$

$$\beta_0 = \frac{11\,N_c}{3} - \frac{2n_f}{3}\,, \qquad \frac{1}{\hat{\varepsilon}} = \frac{1}{\varepsilon} + \gamma_E - \ln(4\pi)$$

IR divergences

soft: cancel in the sum of "virtual" and "real" parts of radiative corrections

collinear: absorbed into the definition of the distribution amplitude ($\overline{\rm MS}$)

$$\phi_\parallel^{(0)}(z) \to \phi_\parallel(z,\mu_F) - \frac{\alpha_s(\mu_F)}{2\pi}\left(\frac{1}{\hat{\varepsilon}}+\ln\left(\frac{\mu_F^2}{\mu^2}\right)\right)\int\limits_0^1 V^{(1)}(z,z')\phi_\parallel(z',\mu_F)dz'$$

Impact factor in the LLA

$$\Phi_{\gamma^*\to V} = -i\int \frac{d\kappa}{2\pi}\, Disc_\kappa\, \mathcal{A}_{\gamma^* R\to V R'} \ ,$$

$$(2\pi)^4\delta^4(p+q-p_1-q')\mathcal{A}_{\gamma^* R\to V R'} = \int d^4x\, e^{-i(px)} e_\mu \langle V(p_1)R'|J^\mu_{\rm em}(x)|R\rangle$$

$$J^\mu_{\rm em} = -e_q\bar{\Psi}\gamma^\mu\Psi$$

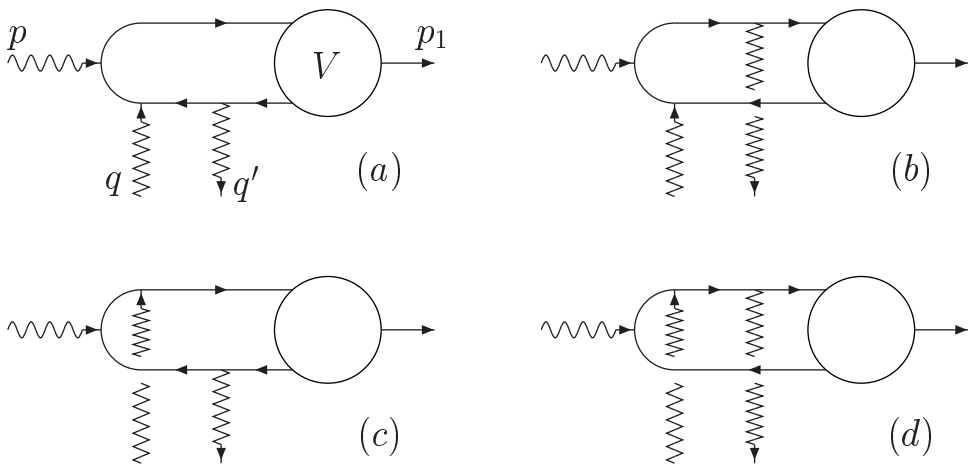


Diagram (a):

$$(2\pi)^4\delta^4(p+q-p_1-q')\mathcal{A}^{(a)}_{\gamma^* R\to V R'} = -e_q(ig)^2\int d^4x d^4y d^4y_1 e^{-i(px)-i(qy_1)+i(q'y)}$$

$$\times\left(t^c t^{c'}\right)_{ij} e_\mu\langle V(p_1)|T\left[\overline{\Psi}^i(x)\gamma^\mu\ \Psi(x)\overline{\Psi}(y_1)\ \frac{\not{p}_2}{s}\ \Psi(y_1)\overline{\Psi}(y)\ \frac{\not{p}_2}{s}\Psi^j(y)\right]|0\rangle$$

Translational invariance, color neutrality of the meson, Fierz identity

$$\begin{aligned}\longrightarrow \langle V(p_1)|\overline{\Psi}^i_\alpha(x)\Psi^j_\beta(y)|0\rangle &= e^{i(p_1 y)}\langle V(p_1)|\overline{\Psi}^i_\alpha(x-y)\Psi^j_\beta(0)|0\rangle \\ &= e^{i(p_1 y)}\frac{\delta^{ij}}{N_c}\frac{1}{4}\left\{(\gamma^\mu)_{\beta\alpha}\langle V(p_1)|\overline{\Psi}(x-y)\gamma_\mu\Psi(0)|0\rangle + \ldots\right\} ,\end{aligned}$$

Other Fierz structures:

- **even number of γ's $\longrightarrow$ chiral-odd quark-pair $\longrightarrow$ no contribution (chirality conserved in massless limit)**
- **$\gamma_\mu\gamma_5$ $\longrightarrow$ twist-3 (important for transverse VM production, suppressed by m_V/Q)**

[P. Ball, V.M. Braun, Y. Koike and K. Tanaka (1998)]

$$\mathcal{A}^{(a)}_{\gamma^* R\to V_L R'} = -\frac{e_q g^2 \delta^{cc'}}{8N_c}\int \frac{d^4 l_1 d^4 l_2 d^4(x-y)}{(2\pi)^4}\delta^4(l_1-l_2-q)e^{-i((p+l_1)(x-y))}$$
$$\times\ \mathrm{Sp}\left[\not{e}\frac{\not{l}_1}{l_1^2+i\epsilon}\frac{\not{p}_2}{s}\frac{\not{l}_2}{l_2^2+i\epsilon}\frac{\not{p}_2}{s}\gamma^\mu\right]\langle V_L(p_1)|\overline{\Psi}(x-y)\gamma_\mu\Psi(0)|0\rangle\,.$$

Leading power asymptotics from the region $(x-y)^2\to 0$, where

$$\langle V_L(p_1)|\overline{\Psi}(y)\gamma^\mu\Psi(0)|0\rangle_{y^2\to 0} = f_V p_1^\mu \int\limits_0^1 dz\, e^{iz(p_1 y)}\phi_\parallel(z)$$

$$\longrightarrow \mathcal{A}^{(a)}_{\gamma^* R\to V_L R'} = -\frac{e_q g^2 f_V \delta^{cc'}}{8N_c}\int\limits_0^1 dz\phi_\parallel(z)\int d^4 l_1 \delta^4(p_1 z - p - l_1)$$
$$\times\ \mathrm{Sp}\left[\not{e}\frac{\not{l}_1}{l_1^2+i\epsilon}\frac{\not{p}_2}{s}\frac{\not{l}_1-\not{q}}{(l_1-q)^2+i\epsilon}\frac{\not{p}_2}{s}\ \not{p}_1\right]\,.$$

The hard-scattering amplitude is calculated for on-shell quark and antiquark momenta, $p_1 z$ and $p_1\bar{z}$.

The κ-channel discontinuity comes only from the antiquark cut

$$\frac{i}{2\pi}Disc_\kappa\frac{1}{(l_1-q)^2+i\epsilon} = \delta((l_1-q)^2) = \delta((p_1 z - p - q)^2) = \delta(\bar{z}\kappa - z\vec{q}^{\,2})$$

$$\Phi^{(a)}_{\gamma_L^*\to V_L} = -\frac{e_q g^2 f_V \delta^{cc'}(ep_1)}{2N_c Q^2}\int\limits_0^1 dz\,\phi_\parallel(z)$$

only longitudinal photon polarization $\qquad e_L = \frac{1}{Q}p + 2\frac{Q}{s}p_2 \to 2\frac{Q}{s}p_2$

Taking into account also diagrams (b), (c), (d), one gets

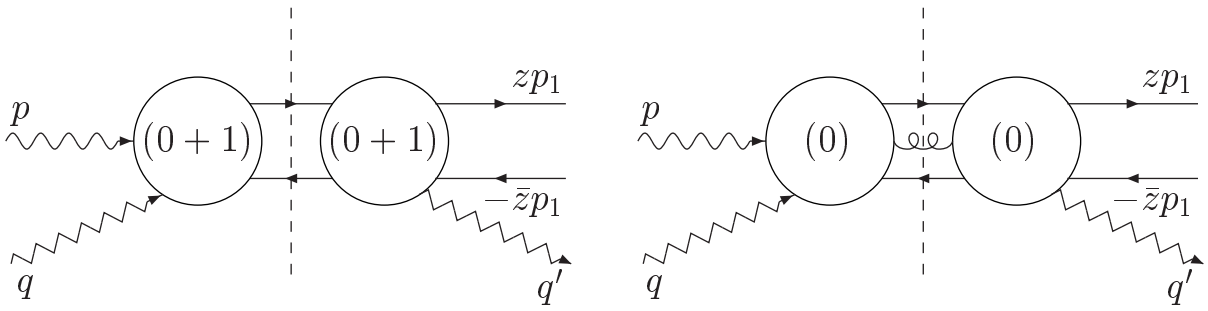
$$T_H^{(0)}(z,\alpha,s_0,\mu_F,\mu_R) = \alpha_S\frac{\alpha}{\alpha+z\bar{z}}\,, \qquad \alpha = \frac{\vec{q}^{\,2}}{Q^2}$$

By collinear factorization, some fermion lines are effectively put on mass-shell; in practice

two-particle states $\longrightarrow$ one-particle state

three-particle states $\longrightarrow$ two-particle states

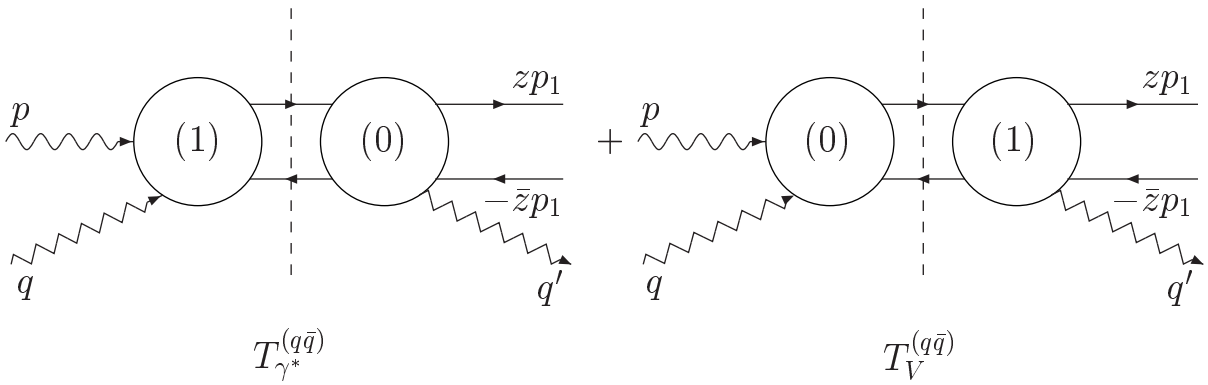
Impact factor in the NLA



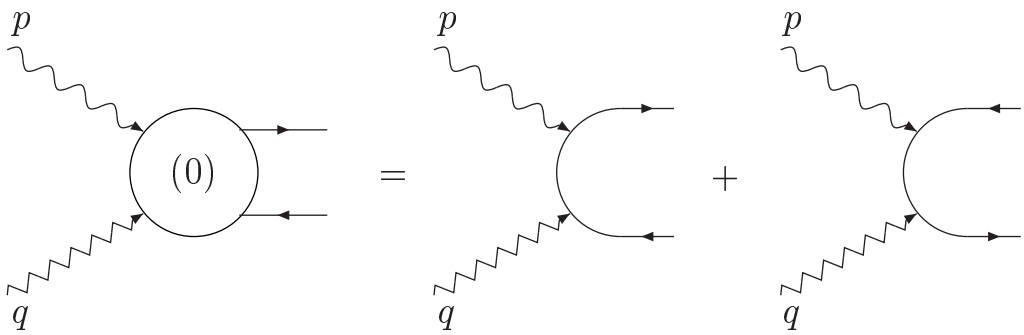
$q\bar{q}$ **intermediate state** $\qquad$ $q\bar{q}g$ **intermediate state**

$$T_H^{(1)} = T^{(q\bar{q})} + T^{(q\bar{q}g)}$$

Quark-antiquark intermediate state

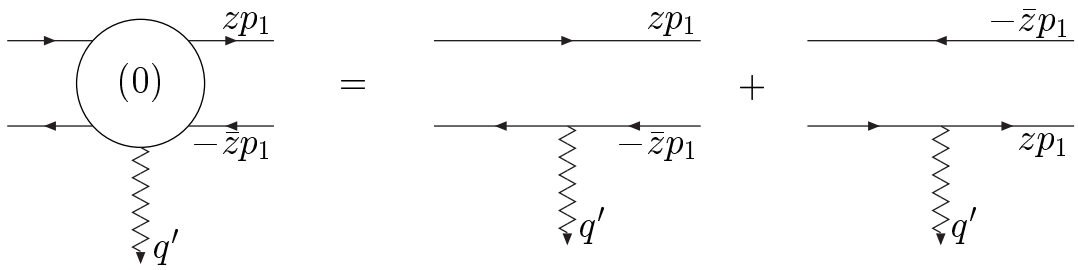


$T_{\gamma^*}^{(q\bar{q})}$ $\qquad$ $T_V^{(q\bar{q})}$

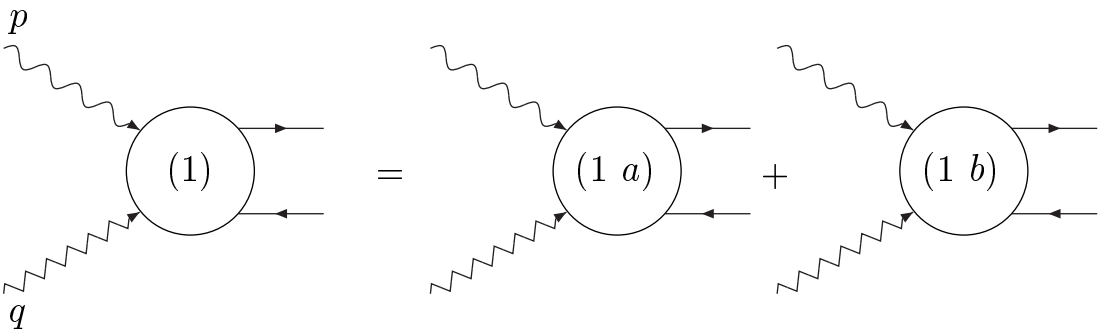


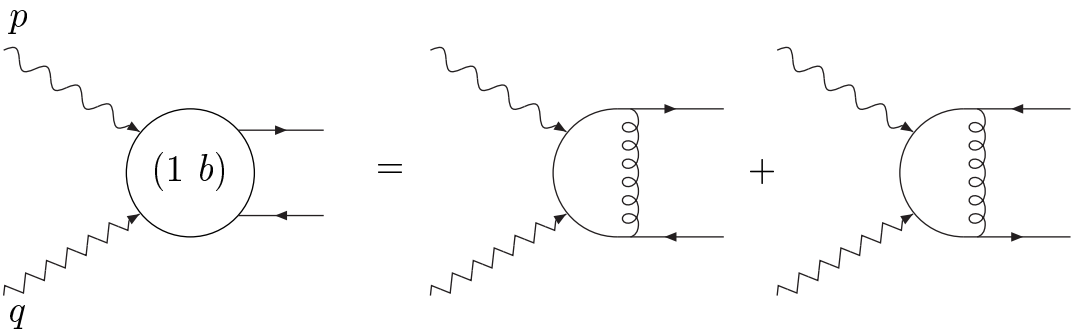
$$\Gamma^{(0)}_{\gamma_L^* q\bar{q}} = 2e_q g Q (t^c)_{ij} z\bar{z} \left(\frac{1}{\vec{q}_1^{\,2} + Q^2 z\bar{z}} - \frac{1}{\vec{q}_2^{\,2} + Q^2 z\bar{z}} \right) \bar{u}^i_\alpha(q_1) \left(\frac{\not{p}_2}{s} \right)_{\alpha\beta} v^j_\beta(q_2)$$

[I.F. Ginzburg, D.Yu. Ivanov (1996)]
[V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2002)]



$$\left(\Gamma^{(0)}_{V_L q\bar{q}}\right)^* = \frac{g f_V}{4N_c}\,(t^{c'})_{ji}\,\phi_{\|}(z)\,dz\left[\bar{v}^j_\beta(q'+p_1\bar{z})\left(\frac{\not{p}_2}{s}\ \not{p}_1\right)_{\beta\alpha} + \left(\not{p}_1\frac{\not{p}_2}{s}\right)_{\beta\alpha} u^i_\alpha(p_1 z + q')\right]$$



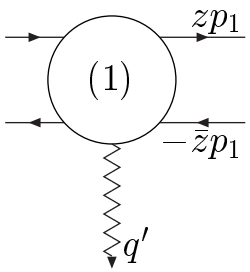


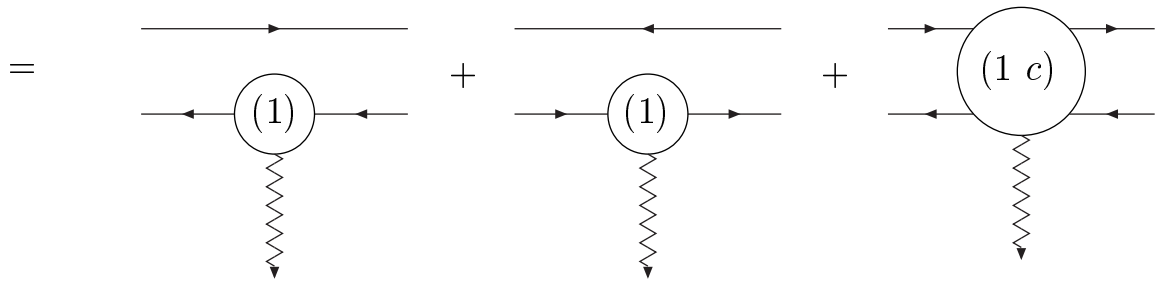
[V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2002)]

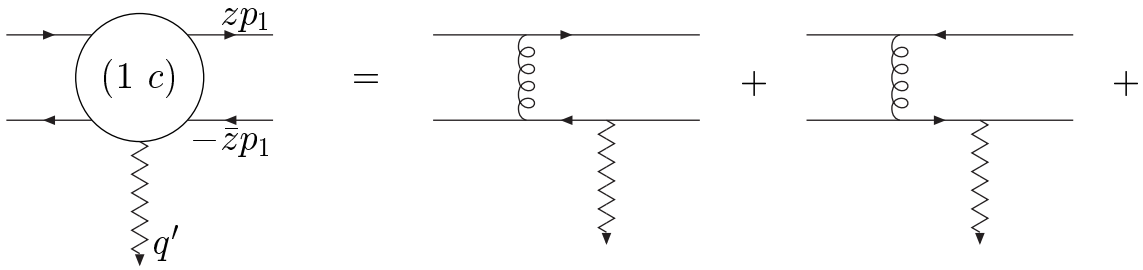
$$T^{(q\bar{q})}_{\gamma^*} = T^{(1a)}_{\gamma^*} + T^{(1b)}_{\gamma^*}$$

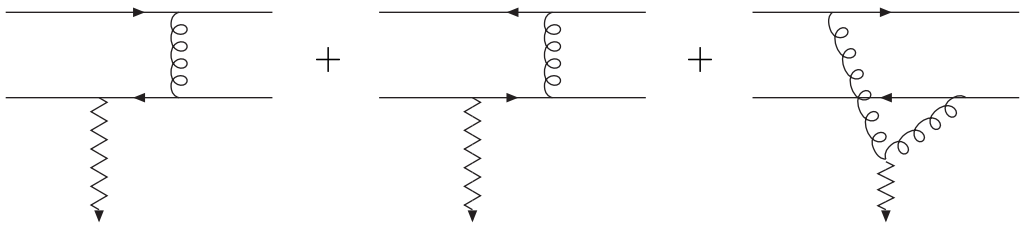
$T^{(1b)}_{\gamma^*}$ = **"box diagrams"**

$T^{(1a)}_{\gamma^*}$ = **"other contributions"**









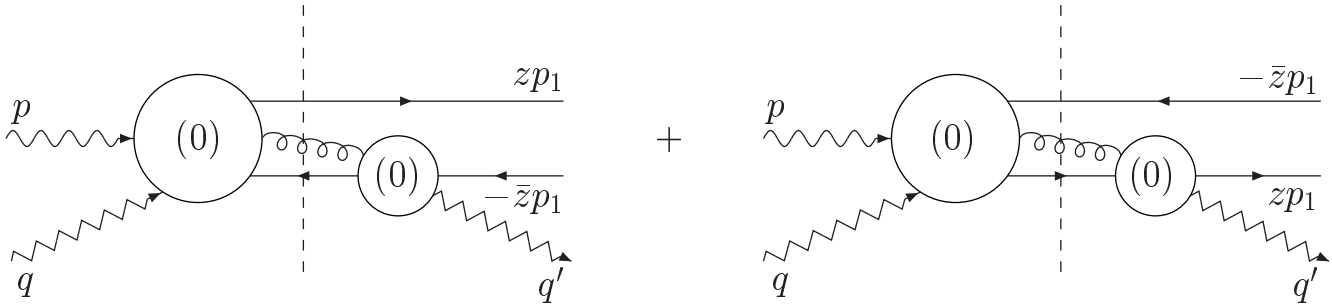
$$T_V^{(q\bar{q})} = T_V^{(1\bar{q})} + T_V^{(1q)} + T_V^{(1c)}$$

$$T^{(q\bar{q})} = T_1 + T_2 + T_3$$

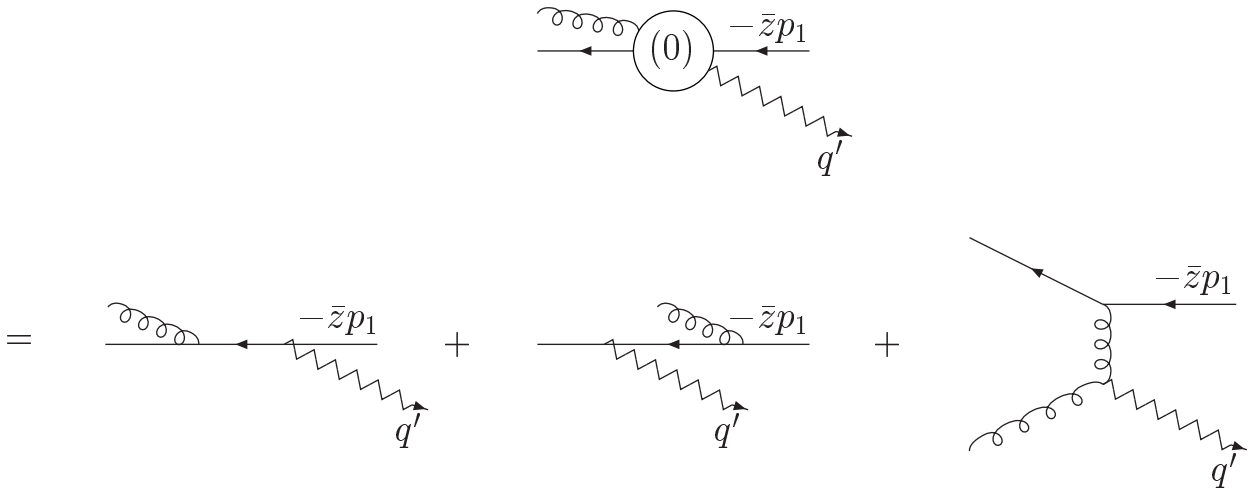
$T_1 = T_{\gamma^*}^{(1a)}$ $\qquad$ $T_2 = T_V^{(1c)} + T_{\gamma^*}^{(1b)}$ $\qquad$ $T_3 = T_V^{(1\bar{q})} + T_V^{(1q)}$

The imaginary parts in $T_V^{(1c)}$ and $T_{\gamma^*}^{(1b)}$ cancel after the sum.

Quark-antiquark-gluon intermediate state



$$\Gamma_{V_L q\bar{q}g} = \Gamma^{q}_{V_L q\bar{q}g} + \Gamma^{\bar{q}}_{V_L q\bar{q}g}$$



$$\left(\Gamma^{q}_{V_L q\bar{q}g}\right)^{*} = -\frac{g^2 f_V}{4N_c}\bar{v}^{j}_{\beta}(q_2)\left(\left[2\bar{z}\left(\vec{T}\vec{e}^{\,a}\right) + z_3\ \not{e}^{a}_{\perp}\not{T}\right]\frac{\not{p}_2}{s}\ \not{p}_1\right)_{\beta\alpha}\ \phi_{\parallel}(z)\,dz$$

$$q_2 = z_2 p_1 + \frac{\vec{q}_2^{\,2}}{z_2}\frac{p_2}{s} + q_{2\perp} \qquad \textbf{momentum of cut antiquark}$$

$$k = z_3 p_1 + \frac{\vec{k}^2}{z_3}\frac{p_2}{s} + k_{\perp} \qquad \textbf{momentum of cut gluon}$$

$$e^a = \frac{2(\vec{e}^{\,a}\vec{k})}{z_3\, s}p_2 + e^a_{\perp}\,, \qquad e^a k = 0 \qquad \textbf{polarization of cut gluon}$$

$$\vec{T} = \left(\frac{\bar{z}\vec{q}_2 - z_2\vec{q}}{(\bar{z}\vec{q}_2 - z_2\vec{q})^2} - \frac{\vec{q}_2}{\bar{z}\vec{q}_2^{\,2}}\right)(t^a t^{c'})_{ji} + \frac{1}{\bar{z}}\left(\frac{\vec{k}}{\vec{k}^2} + \frac{\vec{q}_2}{\vec{q}_2^{\,2}}\right)(t^{c'} t^a)_{ji}$$

$$\Gamma_{\gamma_L^* q\bar{q}g} = -2\, e_q\, g^2\, Q\, \bar{u}^i_\alpha(zp_1) \left(\left[2\left(\vec{e}^{\,a} (\vec{T}_q \bar{z} - \vec{T}_{\bar{q}} z) \right) + z_3 \left(\not{T}_q + \not{T}_{\bar{q}} \right) \not{e}^a_\perp \right] \frac{\not{p}_2}{s} \right)_{\alpha\beta} v^j_\beta(q_2)\,,$$

$$\vec{T}_q = \left(\frac{\bar{z}\vec{q}_2 - z_2\vec{q}}{(\bar{z}\vec{q}_2 - z_2\vec{q})^2} \left(\frac{1}{Q^2} - \frac{1}{Q^2 + \frac{\vec{q}^{\,2}}{z} + \frac{\vec{q}_2^{\,2}}{z_2} + \frac{\vec{k}^2}{z_3}} \right) - \frac{\vec{q}_2}{Q^2 + \frac{\vec{q}_2^{\,2}\bar{z}}{z_2 z_3}} \frac{1}{Q^2 z_2 z_3} \right) (t^c t^a)_{ij}$$

$$+ \frac{1}{Q^2 z_2 z_3} \left(\frac{\vec{k}}{Q^2 + \frac{\vec{k}^2 \bar{z}}{z_2 z_3}} + \frac{\vec{q}_2}{Q^2 + \frac{\vec{q}_2^{\,2}\bar{z}}{z_2 z_3}} \right) (t^a t^c)_{ij}\,,$$

$$\vec{T}_{\bar{q}} = (t^c t^a)_{ij} \left(\frac{z\vec{q}_2 - \bar{z}_2\vec{q}}{(z\vec{q}_2 - \bar{z}_2\vec{q})^2} \left(\frac{1}{Q^2 + \frac{\vec{q}_2^{\,2}}{z_2\bar{z}_2}} - \frac{1}{Q^2 + \frac{\vec{q}^{\,2}}{z} + \frac{\vec{q}_2^{\,2}}{z_2} + \frac{\vec{k}^2}{z_3}} \right) + \frac{\vec{q}_2}{Q^2 + \frac{\vec{q}_2^{\,2}}{z_2\bar{z}_2}} \frac{1}{Q^2 z z_2 \bar{z}_2} \right.$$

$$\left. - \frac{\vec{q}_2}{Q^2 + \frac{\vec{q}_2^{\,2}\bar{z}}{z_2 z_3}} \frac{\bar{z}}{Q^2 z z_2 z_3} \right) + \frac{(t^a t^c)_{ij}}{z} \left(\frac{\vec{k}}{\vec{k}^2} \left(\frac{1}{Q^2 + \frac{\vec{q}_2^{\,2}}{z_2\bar{z}_2}} - \frac{1}{Q^2} \right) \right.$$

$$\left. + \frac{\bar{z}}{Q^2 z_2 z_3} \left(\frac{\vec{k}}{Q^2 + \frac{\vec{k}^2\bar{z}}{z_2 z_3}} + \frac{\vec{q}_2}{Q^2 + \frac{\vec{q}_2^{\,2}\bar{z}}{z_2 z_3}} \right) - \frac{\vec{q}_2}{Q^2 + \frac{\vec{q}_2^{\,2}}{z_2\bar{z}_2}} \frac{1}{Q^2 z_2 \bar{z}_2} \right)\,,$$

[V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2002)]

After convolution and summation over Dirac and color indices of the cut antiquark and gluon, we get

$$-\frac{e_q g^4 f_V Q}{N_c (2\pi)^{3+2\varepsilon}} \int_0^1 \phi_\parallel(z)\, dz \int_0^{\bar{z}} \frac{dz_3}{z_3} d^{2+2\varepsilon}\vec{q}_2\, \theta(s_\Lambda - \kappa)$$

$$\left[(\vec{T}_q \vec{T})(2\bar{z} z_2 + (1+\varepsilon) z_3^2) + (\vec{T}_{\bar{q}} \vec{T})(z_3(2z-1) - 2z\bar{z} + (1+\varepsilon) z_3^2) \right]$$

The divergence for $z_3 \to 0$ is cured by the $\theta(s_\Lambda - \kappa)$:

$$\kappa = \frac{\vec{k}^2}{z_3} + \frac{\vec{q}_2^{\,2}}{z_2} - \vec{q}^{\,2}$$

$$\longrightarrow z_3^{min} = \frac{\vec{k}^2}{s_\Lambda}$$

The contribution to the impact factor from $q\bar{q}g$ intermediate state can be separated in two terms:

$$T^{(q\bar{q}g)} = T_4 + T_5$$

$$T_4 = \text{“divergent part for } z_3 \to 0 + s_\Lambda\text{-counterterm”}$$
$$T_5 = \text{“remaining part (here } z_3^{min} \text{ can be put equal to 0)”}$$

Final result for the NLA impact factor

$$T_H^{(1)} = \frac{g^2 \Gamma[1-\varepsilon]\left(Q^2/\mu^2\right)^\varepsilon}{(4\pi)^{2+\varepsilon}} T_H^{(0)} \left[\, \tilde{\tau}(z) + \tilde{\tau}(\bar{z}) \,\right]$$

$$\tilde{\tau}(z) = \frac{C_F}{\varepsilon}\left[\frac{3}{2} + \frac{(1-2\,z)\,(\alpha - \bar{z}z)}{4\,c\,\bar{z}\,z}\ln\left(\frac{1+2\,c-2\,z}{2\,c-1+2\,z}\right) - \frac{(\alpha+\bar{z}z)}{4\,c\,\bar{z}\,z}\ln\left(\frac{2\,c+1}{2\,c-1}\right)\right.$$
$$\left. - \frac{(\alpha-\bar{z}z)}{2\,\bar{z}\,z}\ln\left(\frac{\alpha+\bar{z}z}{\alpha}\right)\right] - \frac{\beta_0}{2\,\varepsilon} + \ldots\,, \qquad c = \sqrt{\alpha + 1/4}$$

Double poles in ε which appeared in intermediate steps canceled in the sum.

Renormalization:

$$\Delta^{\alpha_s} T_H(z) = \frac{\alpha_s(\mu_R)}{4\pi}\beta_0\left[\frac{1}{\hat{\varepsilon}} + \ln\left(\frac{\mu_R^2}{\mu^2}\right)\right] T_H^{(0)}(z)\,,$$

$$\Delta^{coll} T_H(z) = -\frac{\alpha_s(\mu_F)}{2\pi}\left[\frac{1}{\hat{\varepsilon}} + \ln\left(\frac{\mu_F^2}{\mu^2}\right)\right]\int_0^1 du\, T_H^{(0)}(u)\, V^{(1)}(u,z)\,,$$

The effect of renormalization is given by the replacements

$$\frac{\beta_0}{\varepsilon} \to \beta_0 \ln\left(\frac{Q^2}{\mu_R^2}\right)\,, \qquad \frac{C_F}{\varepsilon} \to C_F \ln\left(\frac{Q^2}{\mu_F^2}\right)\,.$$

$$T_H(z,\alpha,s_0,\mu_F,\mu_R)|_{\alpha\to 0} = \alpha_S(\mu_R)\frac{\alpha}{\alpha + z\bar{z}}\left\{1 + \frac{\alpha_s(\mu_R)}{4\pi}\left[\tau(z) + \tau(\bar{z})\right]\right\}$$

$$
\begin{aligned}
\tau(z) = C_F \ln\left(\frac{Q^2}{\mu_F^2}\right) \Bigg[& \frac{3}{2} + \frac{(1-2\,z)\,(\alpha-\bar{z}z)}{4\,c\,\bar{z}\,z} \ln\left(\frac{1+2\,c-2\,z}{2\,c-1+2\,z}\right) - \frac{(\alpha+\bar{z}z)}{4\,c\,\bar{z}\,z} \ln\left(\frac{2\,c+1}{2\,c-1}\right) \\
& - \frac{(\alpha-\bar{z}z)}{2\,\bar{z}\,z} \ln\left(\frac{\alpha+\bar{z}z}{\alpha}\right)\Bigg] - \frac{\beta_0}{2} \ln\left(\frac{Q^2}{\mu_R^2}\right) + n_f \left[-\frac{5}{9} + \frac{\ln\alpha}{3}\right] + C_F \Bigg[-\frac{\ln^2\alpha}{2} - 3\ln\left(\frac{\alpha+\bar{z}z}{z}\right) \\
& + 4\ln^2\left(c - z + \frac{1}{2}\right) - 2\ln\left(c-\frac{1}{2}\right)\ln\left(c+\frac{1}{2}\right) + \frac{(\alpha+\bar{z}z)}{2\,\alpha\,\bar{z}\,z}\left(\ln\alpha + 2\,c\ln\left(\frac{2\,c+1}{2\,c-1}\right)\right) \\
& - \frac{(1-2\,z)\,(\alpha+\bar{z}z)}{2\,c\,\bar{z}\,z}\ln(1+2\,c-2\,z) - \frac{(\alpha+\bar{z}z)}{2\,\bar{z}\,z}\ln\left(\frac{\alpha+\bar{z}z}{\alpha}\right) + \frac{(\alpha+\bar{z}z)}{\bar{z}\,(\alpha+z)}\ln\left(\frac{\alpha+\bar{z}z}{\alpha\,z}\right) \\
& + \frac{(\alpha+\bar{z}z)}{2\,c\,\bar{z}\,z}\left(\frac{\pi^2}{6} + \ln 4\ln\alpha + \frac{1}{2}\ln\left(\frac{1+2\,c}{2\,c-1}\right) + 2\ln c\ln\left(\frac{2\,c-1}{4}\right) - \ln^2(2\,c+1)\right. \\
& \left. - \ln^2 c + \frac{1}{4}\ln^2\left(\frac{2\,c+1}{2\,c-1}\right) - 2\mathrm{Li}_2\left(\frac{2\,c-1}{4\,c}\right) - \mathrm{Li}_2\left(\frac{2}{1+2\,c}\right)\right) + \frac{(\alpha+\bar{z}z)}{2\,\bar{z}\,z}\left(\frac{\ln^2\alpha}{2}\right. \\
& - 2\ln\left(c-\frac{1}{2}\right)\ln\left(c+\frac{1}{2}\right) + \ln^2\left(\frac{z}{\alpha}\right) - \ln^2\left(\frac{\alpha+\bar{z}z}{z}\right) + 2\mathrm{Li}_2\left(\frac{2\,z}{1-2\,c}\right) \\
& \left. + 2\mathrm{Li}_2\left(\frac{2\,z}{1+2\,c}\right)\right) - \frac{(1-2\,z)\,(\alpha-\bar{z}z)}{2\,c\,\bar{z}\,z}\left(\ln\left(\frac{z}{4\alpha+1}\right)\ln\left(\frac{2\alpha+(1-2\,c)\,z}{2\alpha+(1+2\,c)\,z}\right)\right. \\
& - \ln(4\alpha+1)\,\ln\left(\frac{2\,c+1}{2\,c-1}\right) + 2\mathrm{Li}_2\left(\frac{1-2\,c-2\,z}{1+2\,c-2\,z}\right) - \mathrm{Li}_2\left(\frac{2\,z}{1-2\,c}\right) \\
& \left. + \mathrm{Li}_2\left(\frac{2\,z}{1+2\,c}\right)\right)\Bigg] + N_c\Bigg[\ln\left(s_0/Q^2\right)\,\ln\left(\frac{(\alpha+\bar{z}z)^2}{z^2\alpha}\right) + \frac{20}{9} - \frac{\ln\alpha}{3} + \frac{1}{2}\ln^2\left(\frac{\bar{z}}{z}\right) \\
& + \frac{1}{2}\ln^2\left(\frac{\alpha+\bar{z}z}{\alpha}\right) - 2\ln\left(\frac{\alpha+z}{z}\right)\ln\left(\frac{\alpha+\bar{z}z}{\alpha\,z}\right) - \ln^2\left(\frac{\alpha+\bar{z}z}{z}\right) + 3\,\ln z\,\ln\left(\frac{\alpha+\bar{z}z}{\alpha\,z}\right) \\
& + 2\,\ln^2 z - \frac{(\alpha+\bar{z}z)}{2\,\alpha\,\bar{z}\,z}\left(z\ln\left(\frac{\alpha}{z}\right) + \sqrt{z\,(4\,\alpha+z)}\ln\left(\frac{z+\sqrt{z\,(4\,\alpha+z)}}{-z+\sqrt{z\,(4\,\alpha+z)}}\right)\right) \\
& + \mathrm{Li}_2\left(\frac{2\,z}{1-2\,c}\right) + \mathrm{Li}_2\left(\frac{2\,z}{1+2\,c}\right) + 2\mathrm{Li}_2\left(-\frac{z^2}{\alpha+\bar{z}z}\right) - 2\mathrm{Li}_2\left(-\frac{z}{\alpha}\right) + 3\,\mathrm{Li}_2\left(-\frac{\bar{z}z}{\alpha}\right)\Bigg] \\
& + \frac{1}{N_c}\Bigg[\frac{5}{2} + \left(\frac{\alpha+\bar{z}z}{\bar{z}\,z} - \frac{3}{2}\right)\ln\alpha + \frac{1}{2}\ln\left(\frac{4\,\alpha}{(2\,c-1)^2}\right)\ln\left(\frac{2\,c+1}{2\,c-1}\right) + \frac{1}{2}\ln^2\left(\frac{1+2\,c-2\,z}{2\,c-1+2\,z}\right) \\
& + \frac{c\,(1-2\,z)\,(\alpha+\bar{z}z)}{\bar{z}^2\,z^2}\ln(1+2\,c-2\,z) + \frac{(1-2\,z)\,(\alpha+\bar{z}z)}{\bar{z}^2 z}\ln\left(\frac{z}{\alpha+\bar{z}z}\right) + \mathrm{Li}_2\left(\frac{2\,z}{1+2\,c}\right) \\
& + \ln z\ln\left(\frac{\alpha+\bar{z}z}{\alpha}\right) + \frac{(\alpha+\bar{z}z)}{4\,\bar{z}^2 z^2}\left(2\,c\,\ln\left(\frac{2\,c+1}{2\,c-1}\right) - \ln\left(\frac{\alpha+\bar{z}z}{\alpha}\right)\right) + \mathrm{Li}_2\left(\frac{2\,z}{1-2\,c}\right) \\
& - \frac{\left(\alpha^2+\alpha\,z-\bar{z}\,z^3\right)}{2\,\bar{z}\,z^3}\left(2\ln\left(c-z+\frac{1}{2}\right)\ln\left(c+z-\frac{1}{2}\right) - 2\ln\left(c-\frac{1}{2}\right)\ln\left(c+\frac{1}{2}\right)\right. \\
& + \ln\left(\frac{2\,c+1}{2\,c-1}\right)\ln\left(\frac{2\,\alpha+(1+2\,c)\,z}{2\,\alpha+(1-2\,c)\,z}\right) - \ln\left(\frac{\alpha\,\bar{z}^2\,z^2}{(\alpha+\bar{z}z)^2}\right)\ln\left(\frac{\alpha+\bar{z}z}{\alpha}\right) \\
& + 2\ln\left(\frac{\alpha+z}{\alpha}\right)\ln\left(\frac{\alpha\,z}{\alpha+\bar{z}z}\right) - 2\mathrm{Li}_2\left(-\frac{z}{\alpha}\right) + 4\mathrm{Li}_2\left(\frac{2\,z}{1-2\,c}\right) + 4\mathrm{Li}_2\left(\frac{2\,z}{1+2\,c}\right) \\
& \left. - 2\mathrm{Li}_2\left(-\frac{\bar{z}z}{\alpha}\right) + 2\mathrm{Li}_2\left(-\frac{z^2}{\alpha+\bar{z}z}\right)\right)\Bigg]
\end{aligned}
$$

$$
c = \sqrt{\alpha + 1/4}
$$

The $\gamma^*\gamma^* \to VV$ amplitude

$$\gamma^*(q_1)\gamma^*(q_2) \to \rho(p_1)\rho(p_2)\ , \qquad\qquad s \gg Q_{1,2}^2 \gg \Lambda_{QCD}^2$$

$$s = (p_1+p_2)^2, \qquad \textbf{photon virtualities: } q_1^2 = -Q_1^2,\ q_2^2 = -Q_2^2$$

- **Longitudinally polarized vector mesons are produced by longitudinally polarized photons; other helicity amplitudes power suppressed by $\sim m_\rho/Q_{1,2}$;**
- **forward scattering, i.e. zero transverse momenta of the produced ρ mesons.**

In the BFKL approach with NLA accuracy

$$\mathcal{I}m_s\,(\mathcal{A}) = \frac{s}{(2\pi)^2}\int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\Phi_1(\vec{q}_1)\int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\Phi_2(\vec{q}_2)\int\limits_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1,\vec{q}_2)$$

$\Phi_1(\vec{q}_1)$, $\Phi_2(\vec{q}_2)$ **impact factors for the $\gamma^*(q_1)\to\rho(p_1)$, $\gamma^*(q_2)\to\rho(p_2)$ transitions.**

The Green's function obeys the BFKL equation

$$\delta^2(\vec{q}_1-\vec{q}_2) = \omega\, G_\omega(\vec{q}_1,\vec{q}_2) - \int d^2\vec{q}\, K(\vec{q}_1,\vec{q})\, G_\omega(\vec{q},\vec{q}_2)$$

Transverse momentum notation:

$$\hat{\vec{q}}\,|\vec{q}_i\rangle = \vec{q}_i|\vec{q}_i\rangle$$

$$\langle\vec{q}_1|\vec{q}_2\rangle = \delta^{(2)}(\vec{q}_1-\vec{q}_2) \qquad \langle A|B\rangle = \langle A|\vec{k}\rangle\langle\vec{k}|B\rangle = \int d^2k A(\vec{k})B(\vec{k})$$

$$\hat{1} = (\omega-\hat{K})\hat{G}_\omega \qquad \longrightarrow \qquad \hat{G}_\omega = (\omega-\hat{K})^{-1}$$

$$\hat{K} = \bar{\alpha}_s\hat{K}^0 + \bar{\alpha}_s^2\hat{K}^1\ , \qquad\qquad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

With NLA accuracy

$$\hat{G}_\omega = (\omega-\bar{\alpha}_s\hat{K}^0)^{-1} + (\omega-\bar{\alpha}_s\hat{K}^0)^{-1}\left(\bar{\alpha}_s^2\hat{K}^1\right)(\omega-\bar{\alpha}_s\hat{K}^0)^{-1} + \mathcal{O}\left[\left(\bar{\alpha}_s^2\hat{K}^1\right)^2\right]$$

$$\Phi_{1,2}(\vec{q}) = \alpha_s\, D_{1,2}\left[C^{(0)}_{1,2}(\vec{q}^{\,2}) + \bar{\alpha}_s C^{(1)}_{1,2}(\vec{q}^{\,2})\right] \qquad D_{1,2} = -\frac{4\pi e_q f_V}{N_c Q_{1,2}}\sqrt{N_c^2-1}$$

$$C^{(0)}_{1,2}(\vec{q}^{\,2}) = \int_0^1 dz\, \frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + z\bar{z}Q^2_{1,2}}\phi_{\|}(z)$$

For the asymptotic distribution amplitude $\phi_{\|}(z,\mu_F) = 6z(1-z)$,

$$C^{(0)}\left(\alpha = \frac{\vec{q}^{\,2}}{Q^2}\right) = 6\,\alpha\left[1 - \frac{\alpha}{c}\ln\frac{2c+1}{2c-1}\right]$$

Basis of eigenfunctions of the LLA kernel,

$$\hat{K}^0|\nu\rangle = \chi(\nu)|\nu\rangle\,, \qquad \chi(\nu) = 2\psi(1) - \psi\left(\frac{1}{2}+i\nu\right) - \psi\left(\frac{1}{2}-i\nu\right)$$

$$\langle\vec{q}|\nu\rangle = \frac{1}{\pi\sqrt{2}}\left(\vec{q}^{\,2}\right)^{i\nu-\frac{1}{2}} \qquad \langle\nu'|\nu\rangle = \int\frac{d^2\vec{q}}{2\pi^2}\left(\vec{q}^{\,2}\right)^{i\nu-i\nu'-1} = \delta(\nu-\nu')$$

Action of the full NLA kernel on the LLA eigenfunctions:

$$\begin{aligned}\hat{K}|\nu\rangle &= \bar{\alpha}_s(\mu_R)\chi(\nu)|\nu\rangle + \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(\nu) + \frac{\beta_0}{4N_c}\chi(\nu)\ln(\mu_R^2)\right)|\nu\rangle \\ &+ \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(\nu)\left(i\frac{\partial}{\partial\nu}\right)|\nu\rangle\end{aligned}$$

$$\chi^{(1)}(\nu) = -\frac{\beta_0}{8\,N_c}\left(\chi^2(\nu) - \frac{10}{3}\chi(\nu) - i\chi'(\nu)\right) + \bar{\chi}(\nu)$$

$$\begin{aligned}\bar{\chi}(\nu) &= -\frac{1}{4}\left[\frac{\pi^2-4}{3}\chi(\nu) - 6\zeta(3) - \chi''(\nu) - \frac{\pi^3}{\cosh(\pi\nu)}\right. \\ &\left. + \frac{\pi^2\sinh(\pi\nu)}{2\,\nu\,\cosh^2(\pi\nu)}\left(3 + \left(1+\frac{n_f}{N_c^3}\right)\frac{11+12\nu^2}{16(1+\nu^2)}\right) + 4\,\phi(\nu)\right]\end{aligned}$$

$$\phi(\nu) = 2\int_0^1 dx\,\frac{\cos(\nu\ln(x))}{(1+x)\sqrt{x}}\left[\frac{\pi^2}{6} - \mathbf{Li}_2(x)\right] \qquad \mathbf{Li}_2(x) = -\int_0^x dt\,\frac{\ln(1-t)}{t}$$

$$\chi'(\nu) = d\chi(\nu)/d\nu \qquad \chi''(\nu) = d^2\chi(\nu)/d\nu^2$$

$$\frac{\mathcal{I}m_s\left(\mathcal{A}\right)}{D_1 D_2} = \frac{s}{(2\pi)^2}\int\limits_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(\nu)} \alpha_s^2(\mu_R) c_1(\nu) c_2(\nu)\left[1+\bar{\alpha}_s(\mu_R)\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)}+\frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right)\right.$$
$$\left.+\bar{\alpha}_s^2(\mu_R)\ln\left(\frac{s}{s_0}\right)\left(\bar{\chi}(\nu)+\frac{\beta_0}{8N_c}\chi(\nu)\left[-\chi(\nu)+\frac{10}{3}+i\frac{d\ln(\frac{c_1(\nu)}{c_2(\nu)})}{d\nu}+2\ln(\mu_R^2)\right]\right)\right]$$

$|\nu\rangle$ representation for impact factors:

$$\frac{C_1^{(0)}(\vec{q}^{\,2})}{\vec{q}^{\,2}} = \int\limits_{-\infty}^{+\infty} d\,\nu'\, c_1(\nu')\langle\nu'|\vec{q}\rangle \qquad \frac{C_2^{(0)}(\vec{q}^{\,2})}{\vec{q}^{\,2}} = \int\limits_{-\infty}^{+\infty} d\,\nu\, c_2(\nu)\,\langle\vec{q}|\nu\rangle$$

$$c_1(\nu) = \int d^2\vec{q}\; C_1^{(0)}(\vec{q}^{\,2})\frac{\left(\vec{q}^{\,2}\right)^{i\nu-\frac{3}{2}}}{\pi\sqrt{2}} \qquad c_2(\nu) = \int d^2\vec{q}\; C_2^{(0)}(\vec{q}^{\,2})\frac{\left(\vec{q}^{\,2}\right)^{-i\nu-\frac{3}{2}}}{\pi\sqrt{2}}$$

$$c_{1,2}(\nu) = \frac{\left(Q_{1,2}^2\right)^{\pm i\nu-\frac{1}{2}}}{\sqrt{2}}\frac{\Gamma^2[\frac{3}{2}\pm i\nu]}{\Gamma[3\pm 2i\nu]}\frac{6\pi}{\cosh(\pi\nu)}$$

$$c_1(\nu)c_2(\nu) = \frac{1}{Q_1 Q_2}\left(\frac{Q_1^2}{Q_2^2}\right)^{i\nu}\frac{9\,\pi^3(1+4\nu^2)\sinh(\pi\nu)}{32\,\nu\,(1+\nu^2)\cosh^3(\pi\nu)}$$

$$i\frac{d\ln(\frac{c_1(\nu)}{c_2(\nu)})}{d\nu} = 2\left[\psi(3+2i\nu)+\psi(3-2i\nu)-\psi\left(\frac{3}{2}+i\nu\right)-\psi\left(\frac{3}{2}-i\nu\right)-\ln\left(Q_1Q_2\right)\right]$$

$$C^{(1)}(\vec{q}^{\,2}) = \int\limits_0^1 dz\,\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+z\bar{z}Q^2}\phi_{||}(z)$$
$$\times\left[\frac{1}{4}\ln\left(\frac{s_0}{Q^2}\right)\ln\left(\frac{(\alpha+z\bar{z})^4}{\alpha^2 z^2\bar{z}^2}\right)+\frac{\beta_0}{4N_c}\left(\ln\left(\frac{\mu_R^2}{Q^2}\right)+\frac{5}{3}-\ln(\alpha)\right)+\ldots\right]$$

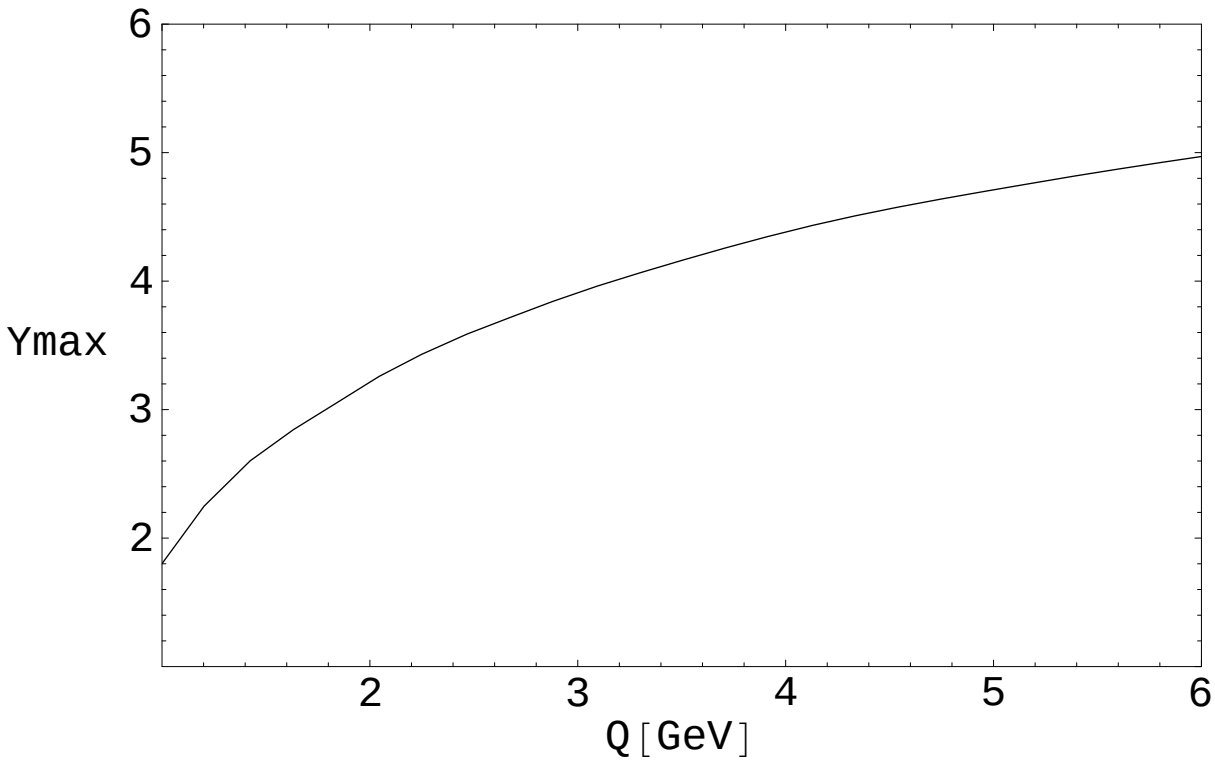
$$c_{1,2}^{(1)}(\nu) = \tilde{c}_{1,2}^{(1)}(\nu)+\bar{c}_{1,2}^{(1)}(\nu)$$

$$\begin{aligned}\frac{\tilde{c}_1^{(1)}(\nu)}{c_1(\nu)}+\frac{\tilde{c}_2^{(1)}(\nu)}{c_2(\nu)} &= \ln\left(\frac{s_0}{Q_1Q_2}\right)\chi(\nu)+\frac{\beta_0}{2N_c}\left[\ln\left(\frac{\mu_R^2}{Q_1Q_2}\right)+\frac{5}{3}\right.\\ &+ \left.\psi(3+2i\nu)+\psi(3-2i\nu)-\psi\left(\frac{3}{2}+i\nu\right)-\psi\left(\frac{3}{2}-i\nu\right)\right]\end{aligned}$$

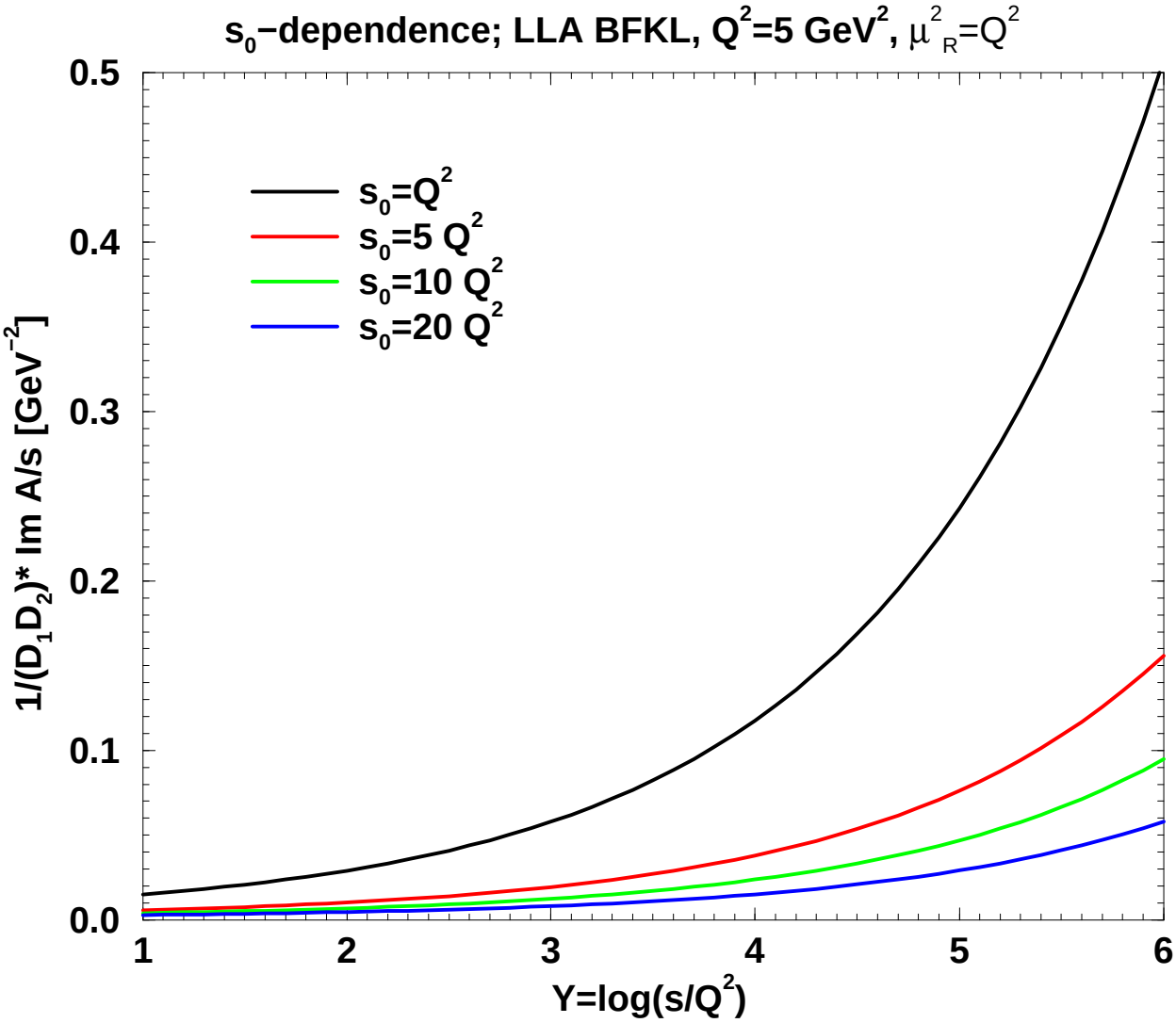
Numerical results: $Q_1^2 = Q_2^2 \equiv Q^2$

1. Naive validity range of the BFKL approach: $1 = \bar{\alpha}_s(Q^2) \ln\left(\frac{s}{Q^2}\right)$

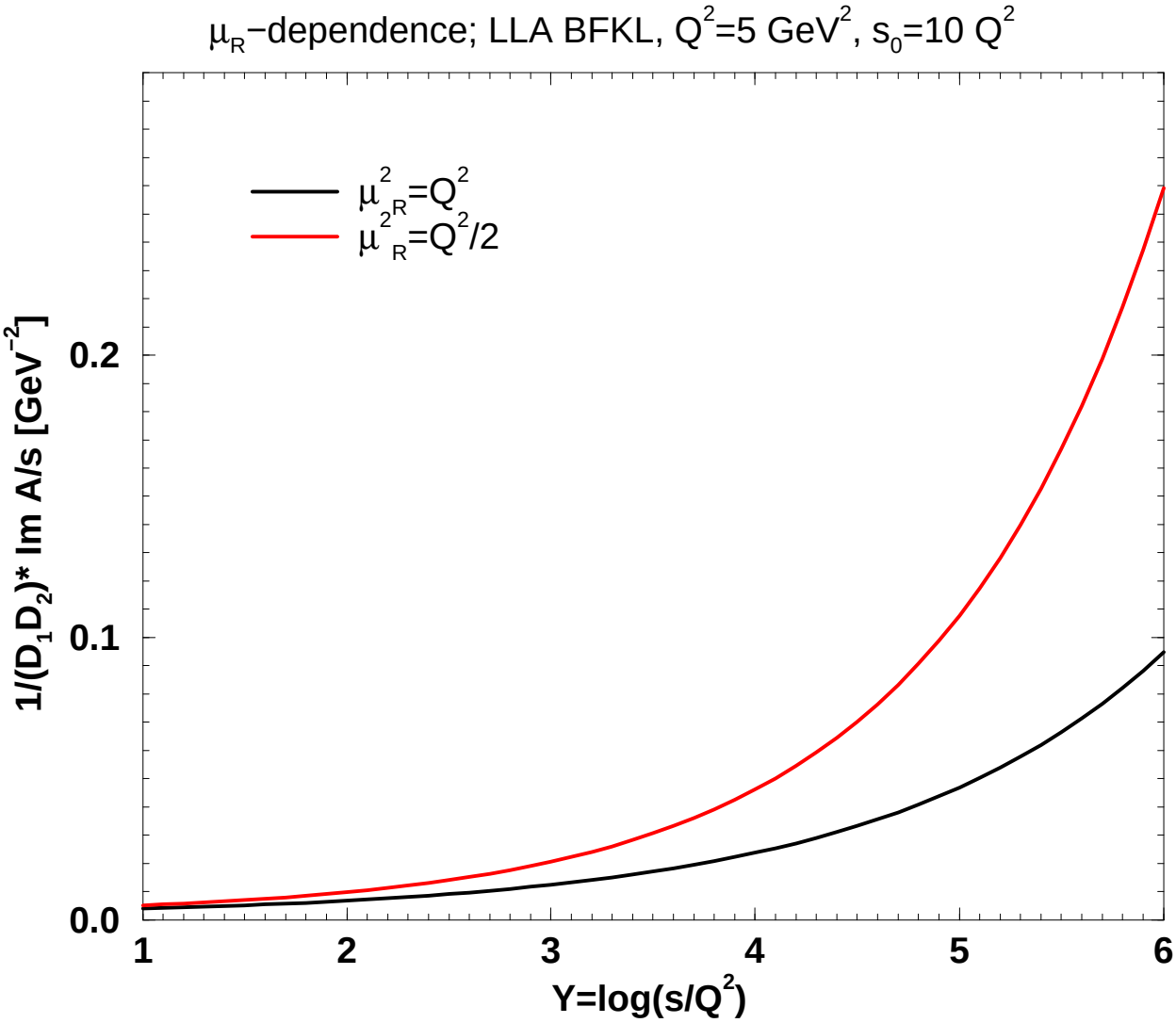
$$Y \equiv \ln\left(\frac{s}{Q^2}\right)$$



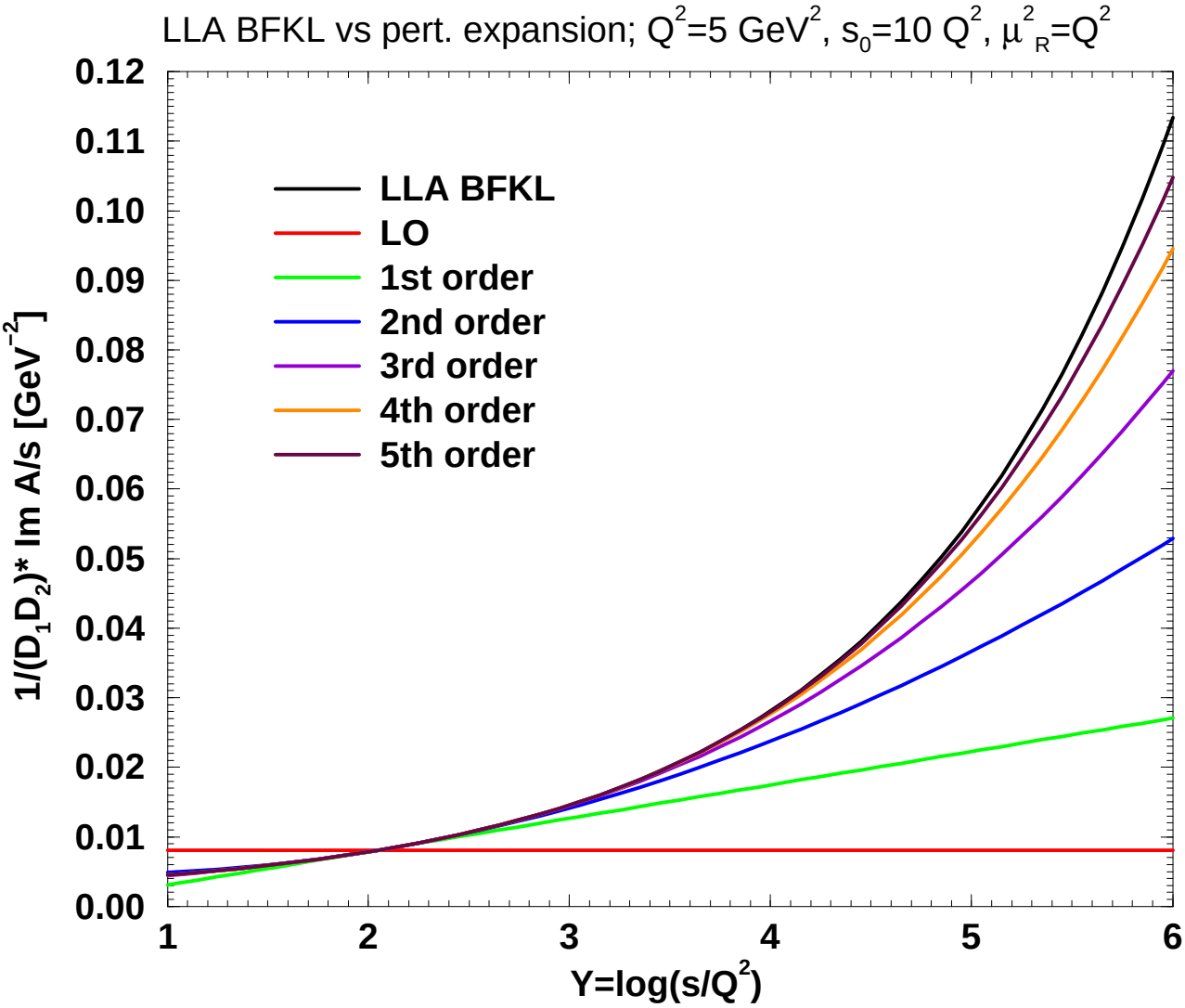
2. s_0-dependence of the LLA amplitude



3. μ_R-dependence of the LLA amplitude

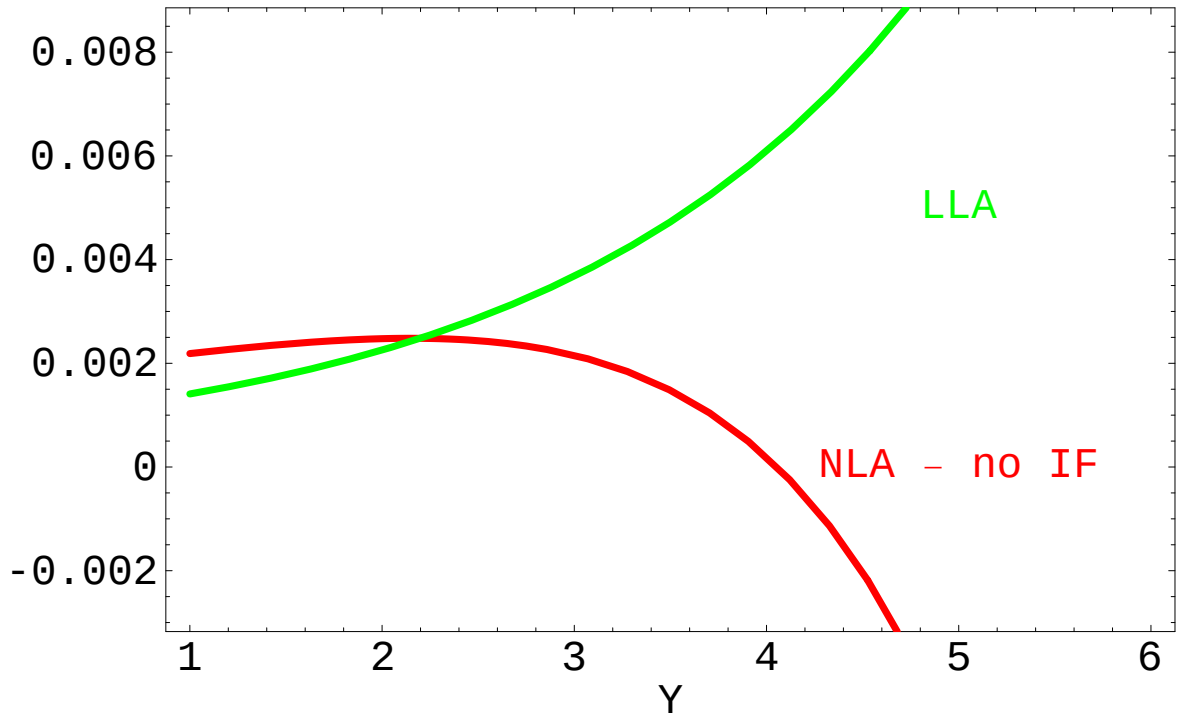


4. perturbative *vs* full result of the LLA amplitude



5. NLA *vs* LLA amplitude "without" IF corrections

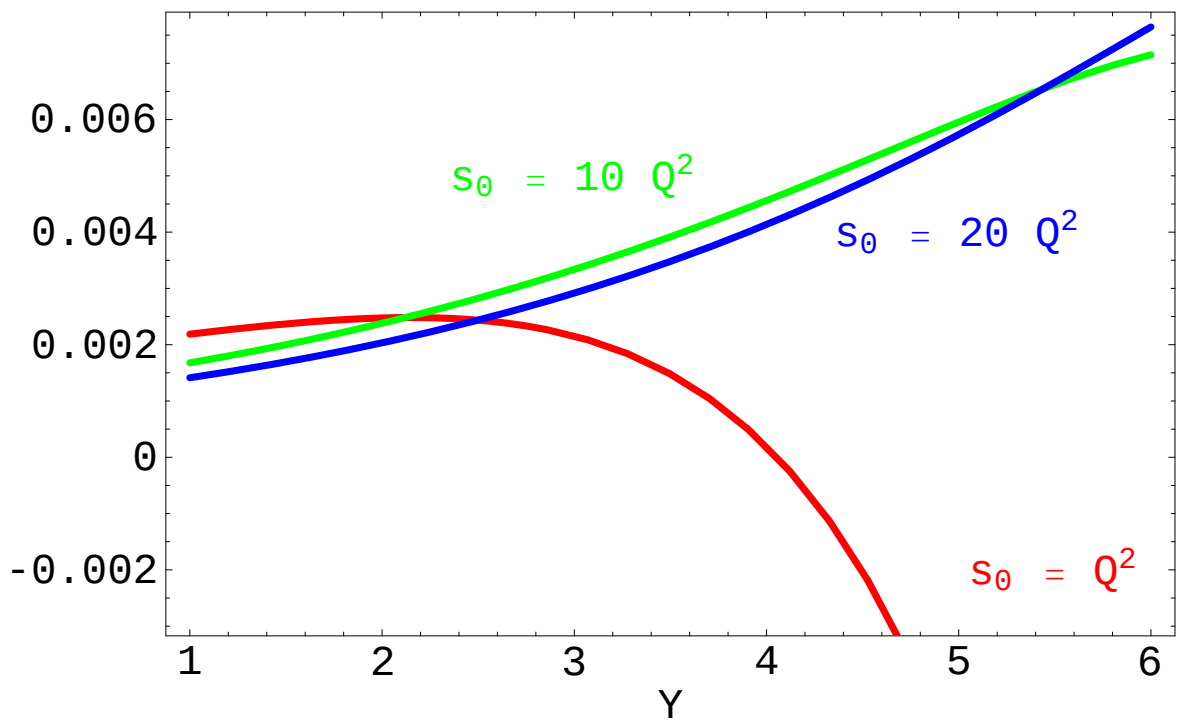
$$\frac{1}{D_1 D_2}\mathcal{I}m_s\mathcal{A}\ [\mathbf{GeV}^{-2}]\ vs\ Y\ \textbf{at}\ Q^2 = 24\ \mathbf{GeV}^2,\ s_0 = Q^2$$



Here and in the following, "without" or "no" IF corrections means with $\tilde{c}_1$ and $\tilde{c}_2$ put equal to zero.

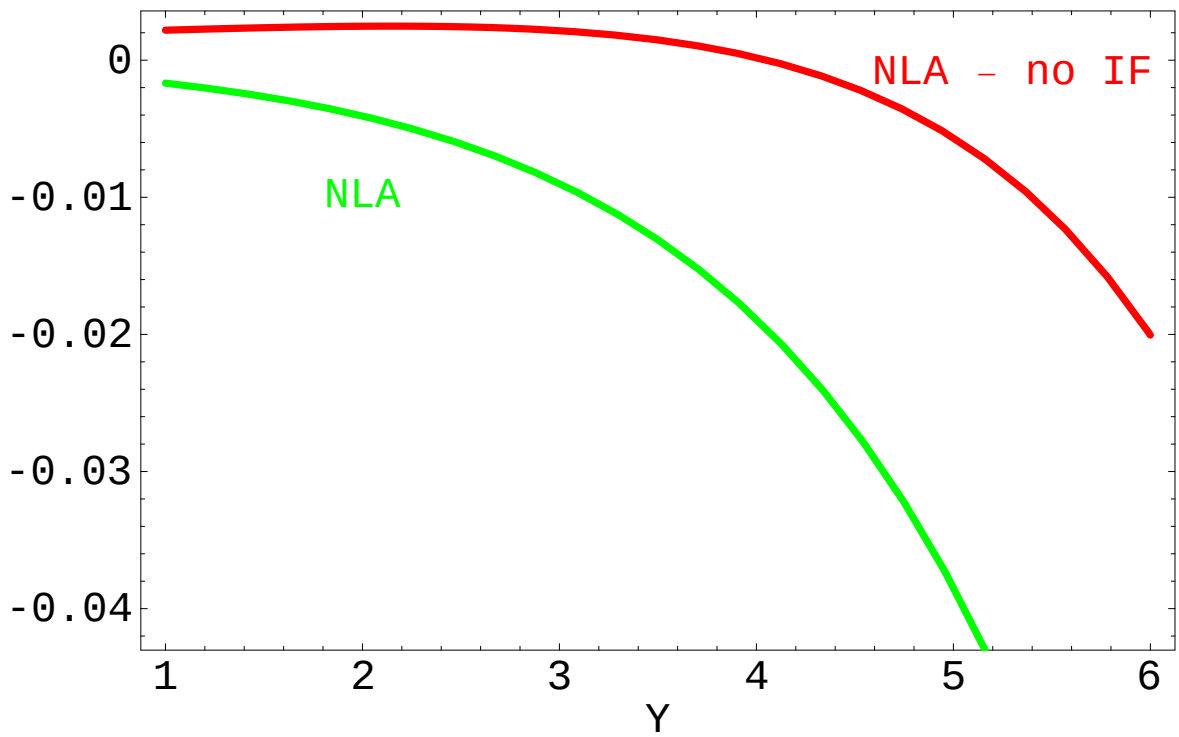
6. NLA "without" IF corrections: s_0-dependence

$$\frac{1}{D_1 D_2}\mathcal{I}m_s\mathcal{A}\ [\mathbf{GeV}^{-2}]\ vs\ Y\ \textbf{at}\ Q^2 = 24\ \mathbf{GeV}^2$$



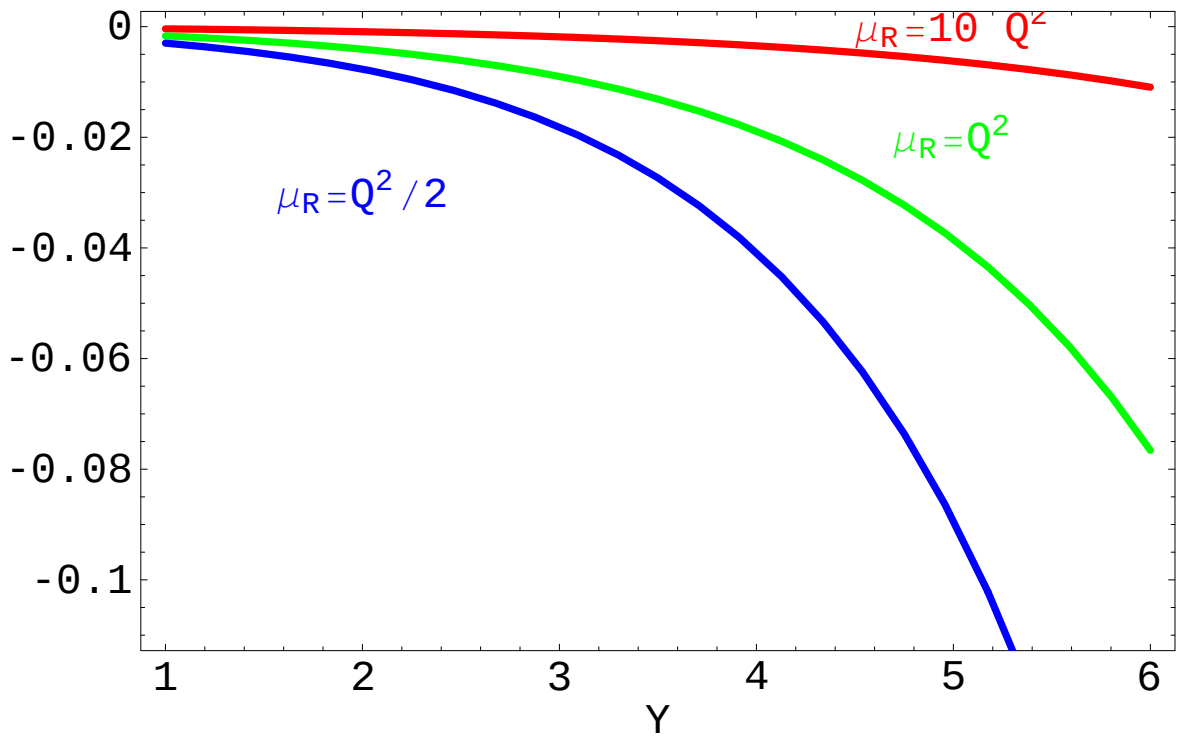
7. complete NLA amplitude: large negative corrections!

$\frac{1}{D_1 D_2}\mathcal{I}m_s\mathcal{A}$ $[\mathbf{GeV}^{-2}]$ vs Y **at** $Q^2 = 24$ $\mathbf{GeV}^2$, $s_0 = Q^2$



8. μ_R-dependence of the NLA amplitude

$\frac{1}{D_1 D_2}\mathcal{I}m_s\mathcal{A}$ $[\mathbf{GeV}^{-2}]$ vs Y **at** $Q^2 = 24$ $\mathbf{GeV}^2$, $s_0 = Q^2$



"Honest" NLA approximation:

$$\frac{1}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s} = \frac{1}{(2\pi)^2}\bar{\alpha}_s(\mu_R)^2 \times \left[b_0 + \bar{\alpha}_s(\mu_R)\, b_1 \left(Y + d_1\right) + \bar{\alpha}_s(\mu_R)^2\, b_2 \left(Y^2 + d_2 Y\right) + \ldots\right]$$

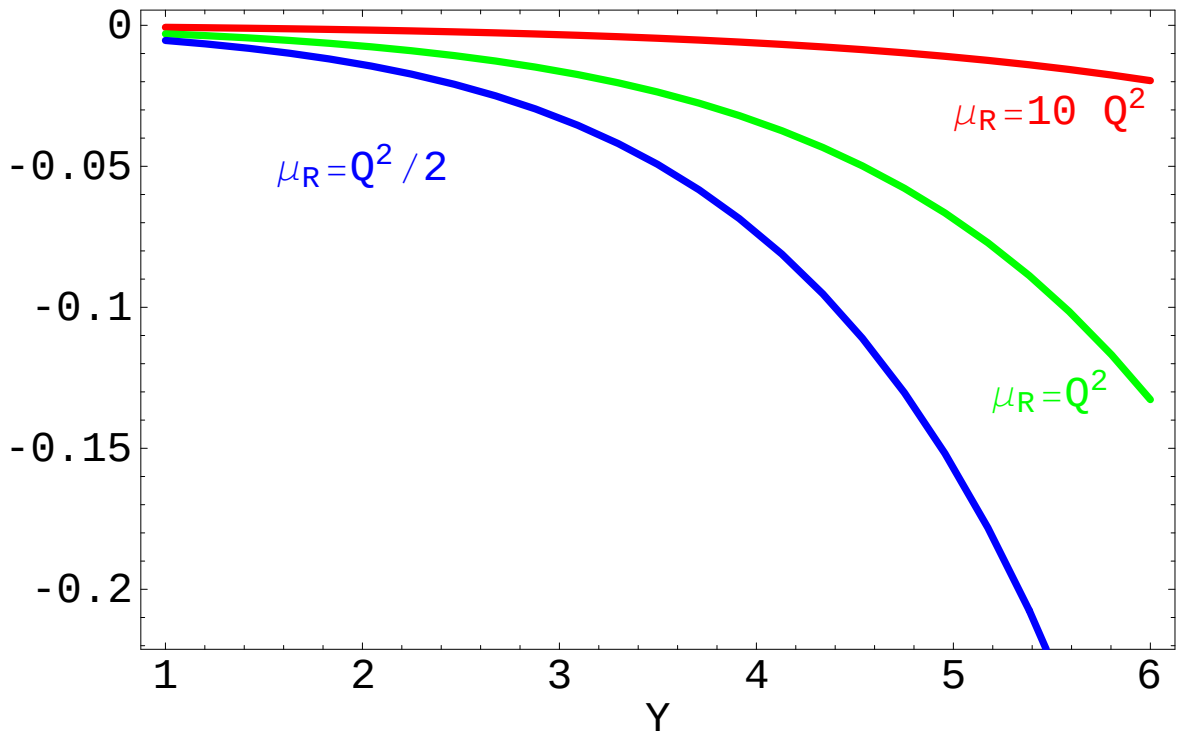
$$Y \equiv \ln\frac{s}{Q^2}$$

No s_0-dependence!

E.g. $Q^2 = 24$ **GeV**^{2}

$$b_0 = 0.711099 \quad b_1 = 1.44133 \quad b_2 = 1.69837 \quad b_3 = 1.37758 \quad b_4 = 0.864446$$
$$b_5 = 0.44041 \quad b_6 = 0.189497 \quad b_7 = 0.0704701 \quad b_8 = 0.0231031$$

$$d_1 = -3.71085 \quad d_2 = -11.3057 \quad d_3 = -23.3879 \quad d_4 = -39.1123$$
$$d_5 = -59.2071 \quad d_6 = -83.0365 \quad d_7 = -111.151 \quad d_8 = -143.06$$



Possible cures? BLM, PMS, ...?

There are indications that a large, "unnatural" value for the renormalization scale μ_R should be used ($\mu_R = \mathcal{O}(10)Q$).

Conclusions (I)

- Up to power suppressed corrections, both in LLA and in NLA, the $\gamma^* \to V$ impact factor factorizes into the convolution of a perturbatively calculable hard-scattering amplitude T_H and a meson twist-2 distribution amplitude.

  T_H has been determined in the NLA by calculating cut diagrams with effectively no more than two-particle intermediate states.

- As it should be, cancellation occurred of the soft infrared divergences between the "real" and the "virtual" parts of the radiative corrections.

  Collinear infrared divergences have been absorbed into the definition of the distribution amplitude.

- A close analytical expression for the impact factor has been obtained.

Conclusions (II)

- After convolution with the Green's function of two interacting Reggeized gluons, the complete amplitude for the $\gamma^*\gamma^* \to VV$ process can be written entirely within perturbative QCD in the NLA.

- The corrections coming from the NLA impact factors are large.

- The s_0-dependence disappears in the "honest" NLA approximation; it survives and is large in the exponentiated form of the amplitude.

- In the "honest" NLA approximation the perturbative series is affected by large and negative NLO coefficients, which may be possibly dumped by a suitable choice of the renormalization scheme.