Multi-Jet Predictions in the High Energy Limit of QCD

Jeppe R. Andersen

HEP Group, Cavendish Lab.

UNIVERSITY OF CAMBRIDGE

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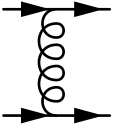
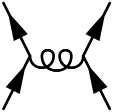
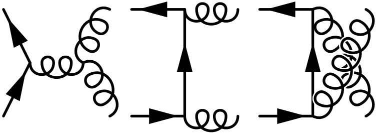
Outline

1. The High Energy Limit of Scattering Processes
   - The High Energy Limit and Full, Fixed Order Results
   - Possibility for $2 \to 2 + n$:
     Reggeisation and Relation to the BFKL Equation
   - Direct Solution of the BFKL Evolution
2. Necessities for a Calculation to NLL Accuracy
   - Building Blocks from Fixed Order Calculations
   - Full Next-to-leading Logarithmic Accuracy:
     Fully Exclusive Final State
3. Summary and Conclusions

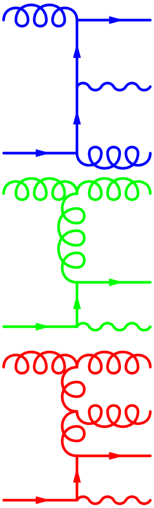
The High Energy Limit of Fixed Order Matrix Elements

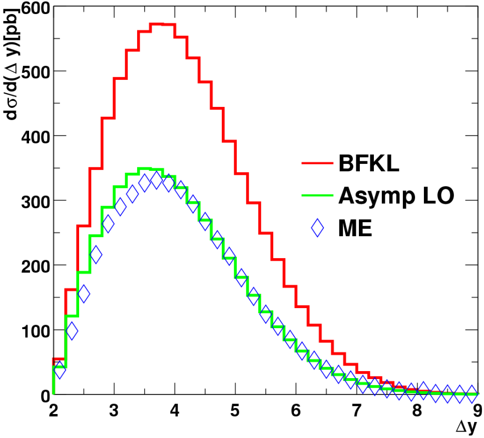
Process	Diagrams	$\overline{\sum}\|\mathcal{M}\|^2/g^4$
$qq' \to qq'$		$\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$
$q\bar{q} \to q'\bar{q}'$		$\frac{4}{9}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$
$q\bar{q} \to gg$		$\frac{32}{27}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$

High Energy Limit: $|\hat{t}|$ fixed, $\hat{s} \to \infty$

t–channel dominance

Example: W+n-jet production at the LHC





$$\Delta y = y_{j_2} - y_{j_1}, y_W, y_{j_2} \geq 1, y_{j_1} \leq -1$$

V. Del Duca, F. Maltoni, W.J. Stirling, JRA

Observations

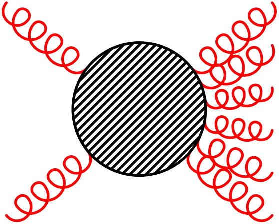
- In the limit of large rapidity spans, the fixed order matrix elements are dominated by contributions from diagrams with a *t*-channel gluon exchange
- This limit will be called The High Energy Limit and is generally characterised by the following phase space configuration of the final state particles

$$y_0 > y_1 > \cdots > y_n > y_{n+1}, \quad |k_0| \sim |k_i| \sim |k_{n+1}|$$

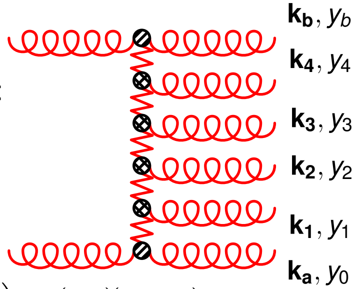
  i.e. multiple, isolated, hard parton production (multiple jets)
- Good agreement ($\sim$ 10%) with the full, fixed order result in the relevant limit

The Possibility for Prediction of n-jet Rates

The Power of Reggeisation



High Energy Limit
$$\xrightarrow{\qquad}$$
$$|\hat{t}| \quad \text{fixed}, \quad \hat{s} \to \infty$$



$$\mathcal{A}^R_{2\to 2+n} = \Gamma_{A'A} \left(\prod_{i=1}^{n} \frac{e^{\omega(q_i)(y_{i-1}-y_i)}}{q_i^2} V^{J_i}(q_i, q_{i+1}) \right) \frac{e^{\omega(q_{n+1})(y_n - y_{n+1})}}{q_{n+1}^2} \Gamma_{B'B}$$

$$\mathbf{q}_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$$

NLL: Fadin, Fiore, Kozlov, Reznichenko

At LL only gluon production; at NLL also quark–anti-quark pairs produced.

Prediction of any-jet rate possible.

Reggeisation and the BFKL Equation

The **evolution of the reggeised gluon** is described by the BFKL equation

$$\omega f_\omega\left(\mathbf{k}_a, \mathbf{k}_b\right) = \delta^{(2+2\epsilon)}\left(\mathbf{k}_a - \mathbf{k}_b\right) + \int d^{2+2\epsilon}\mathbf{k}' \mathcal{K}_\epsilon\left(\mathbf{k}_a, \mathbf{k}'\right) f_\omega\left(\mathbf{k}', \mathbf{k}_b\right)$$

ω: Mellin conjugated variable to the rapidity y along the evolution.

- The kernel $\mathcal{K}_\epsilon$ consists of the **virtual** corrections of the trajectory and the **real** corrections from the Lipatov vertices.
- The BFKL equation provides a very convenient framework for organising the divergences in the factorised form of the $|\mathcal{M}|^2$ on the previous slide.

Energy and Momentum Conservation in an Inclusive Framework

One of the benefits of BFKL : **Fully inclusive** any-jet **partonic** cross sections **can be calculated analytically** $(p'_a, p'_b \to p_a, \{p_i\}, p_b)$

$$\mathrm{d}\hat{\sigma}(p_a, p_b) = \Gamma_a(\mathbf{p}_a)\; f(\mathbf{p}_a, -\mathbf{p}_b, \Delta)\; \Gamma_b(\mathbf{p}_b)$$

Inclusive partonic cross section depending on the momentum of two **final state** particles.
In order to **reconstruct the initial state** (impose energy and momentum conservation, correct parton momentum etc.) and calculate the convolution with pdfs, we **need the full final state information**[1]!

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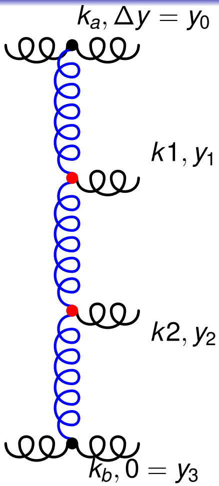
[1] Not resummation of soft, colinear radiation: large contribution to energy

Iteration at (Next to) Leading Logarithmic Accuracy

$$
\begin{aligned}
f(\mathbf{k}_a,\mathbf{k}_b,\Delta) &= \exp\left(\omega_0\left(\mathbf{k}_a^2,\lambda^2,\mu\right)\Delta\right)\delta^{(2)}(\mathbf{k}_a-\mathbf{k}_b)\\
&+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\int d^2\mathbf{k}_i\int_0^{y_{i-1}} dy_i\left[V\left(\mathbf{k}_i,\mathbf{k}_a+\sum_{l=0}^{i-1}\mathbf{k}_l,\mu\right)\right]\\
&\times\exp\left[\omega_0\left(\left(\mathbf{k}_a+\sum_{l=1}^{i-1}\mathbf{k}_l\right)^2,\lambda^2,\mu\right)(y_{i-1}-y_i)\right]\\
&\times\exp\left[\omega_0\left(\left(\mathbf{k}_a+\sum_{l=1}^{n}\mathbf{k}_l\right)^2,\lambda^2,\mu\right)(y_n-0)\right]\\
&\times\delta^{(2)}\left(\sum_{l=1}^{n}\mathbf{k}_l+\mathbf{k}_a-\mathbf{k}_b\right)
\end{aligned}
$$



Two problems:

1) Uses the integrated NLL vertices

(cannot resolve full final state)

2) Huge variance from nested rapidity integral

LL: J. Kwiecinski, C. Lewis, A. Martin; C.R. Schmidt; L. Orr, W.J. Stirling

NLL: A. Sabio-Vera, JRA

Direct BFKL Evolution @ LL&NLL

Solution to the BFKL equation at fixed Δ at both LL and NLL:

$$f(\mathbf{k}_a, \mathbf{k}_b, \Delta) = \sum_{n=0}^{\infty} \int \mathrm{d}\mathcal{P}_n \, \mathcal{F}_n,$$

$$\int \mathrm{d}\mathcal{P}_n = \left(\int \prod_{i=1}^{n} \mathrm{d}\mathbf{k}_i \int_0^{y_0} \mathrm{d}y_1 \int_0^{y_1} \mathrm{d}y_2 \cdots \int_0^{y_{n-1}} \mathrm{d}y_n \right) \delta^{(2)} \left(\mathbf{k}_a + \sum_{l=1}^{n} \mathbf{k}_l - \mathbf{k}_b \right)$$

$$\mathcal{F}_n = \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)(y_{i-1}-y_i)} \, V(\mathbf{q}_i, \mathbf{q}_{i+1}) \right) e^{\omega(\mathbf{q}_{n+1})(y_n - y_{n+1})}$$

$$\int_0^{y_0} \mathrm{d}y_1 \int_0^{\mathrm{d}y_1} \mathrm{d}y_2 \cdots \int_0^{\mathrm{d}y_{n-1}} \mathrm{d}y_n \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)(y_{i-1}-y_i)} \right) e^{\omega(\mathbf{q}_{n+1})(y_n - y_{n+1})}$$

$$= \int_0^{\Delta} \mathrm{d}\delta y_n \int_0^{\Delta - y_n} \mathrm{d}\delta y_{n-1} \cdots \int_0^{\Delta - y_n - \cdots - y_2} \mathrm{d}\delta y_1 \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)\delta y_i} \right) e^{\omega(\mathbf{q}_{n+1})\delta y_{n+1}}$$

$$= \int_0^{\infty} \mathrm{d}\delta y_{n+1} \int_0^{\infty} \mathrm{d}\delta y_n \cdots \int_0^{\infty} \mathrm{d}\delta y_1 \; \delta(\Delta - \sum_{i=1}^{n+1} \delta y_i) \prod_{i=1}^{n+1} e^{\omega(\mathbf{q}_i)\delta y_i}$$

Direct BFKL Evolution, 2

$$f(\mathbf{k}_a, \mathbf{k}_b, \Delta) = \sum_{n=0}^{\infty} \int \mathrm{d}\mathcal{P}_n \, \mathcal{F}_n,$$

$$\int \mathrm{d}\mathcal{P}_n = \left(\prod_{i=1}^{n} \int \mathrm{d}\mathbf{k}_i \int_0^{\infty} \mathrm{d}\delta y_i \right) \int_0^{\infty} \mathrm{d}\delta y_{n+1} \; \delta^{(2)} \left(\mathbf{k}_a + \sum_{l=1}^{n} \mathbf{k}_l - \mathbf{k}_b \right) \delta \left(\Delta - \sum_{i=1}^{n+1} \delta y_i \right)$$

$$\mathcal{F}_n = \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)\delta y_i} \; V(\mathbf{q}_i, \mathbf{q}_{i+1}) \right) \; e^{\omega(\mathbf{q}_{n+1})\delta y_{n+1}}$$

$f(\mathbf{k}_a, \mathbf{k}_b, \Delta)$: the value at $\Delta \equiv \sum_{i=1}^{n+1} \delta y_i$ of the product of vertices $V(\mathbf{q}_i, \mathbf{q}_{i+1})$ at rapidity $y_i = \sum_{j=1}^{i} \delta y_j$ connected with Regge factors $e^{\omega(\mathbf{q}_i)\delta y_i}$ describing the probability of no (resolved) emission between two adjacent (in rapidity) vertices

Direct BFKL Evolution, 3

1. Choose a random number of vertices for the evolution, $n \geq 0$
2. Generate a set $\{\mathbf{k}_i\}_{i=1,\ldots,n}$ of transverse momenta (the outgoing momenta are $\{-\mathbf{k}_i\}_{i=1,\ldots,n}$)
3. Calculate the corresponding set of trajectories $\{\omega(\mathbf{q}_i)\}_{i=1,\ldots,n+1}$, and vertex factors $\{V(\mathbf{q}_i, \mathbf{q}_{i+1})\}_{i=1,\ldots,n}$, $\mathbf{q}_i = k_a + \sum_{l=1}^{i-1} \mathbf{k}_l$
4. Generate the inter-vertex rapidity separations $\{\delta y_i\}$ according to the distributions $e^{\omega(\mathbf{q}_i)\delta y_i}$
5. Calculate the corresponding $\Delta = \sum_{i=1}^{n+1} \delta y_i$ and return $\prod_{i=1}^{n} V(\mathbf{q}_i, \mathbf{q}_{i+1})$

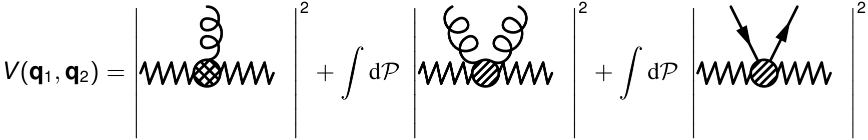
Possibility to construct full final state[2]! Trivial to impose energy and momentum conservation and do **proper jet studies.**

[2] See later

Observation

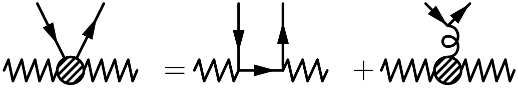
1. Imposing Energy and Momentum conservation (i.e. restricting phase space integral to that accessible at a given energy) is completely unrelated to the NLL corrections to the evolution.
2. To calculate an observable to full NLL accuracy, three ingredients are necessary:
   - NLL Impact Factors
   - NLL Evolution
   - Energy and Momentum Conservation

The Ingredients of the NLL Vertex

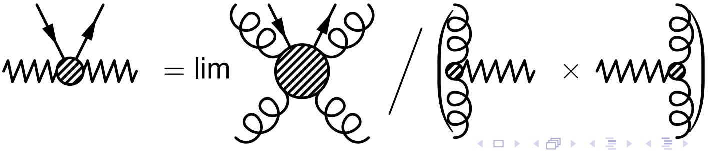


Two methods for obtaining the vertices at NLL:

- Fadin & Lipatov:

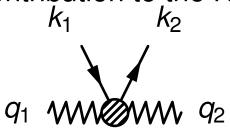


- V. Del Duca:



Divergences and Strategy

Quark contribution to the NLL vertex:



Divergences separate into two categories:
$\Delta = \mathbf{q}_1 - \mathbf{q}_2 = 0$: Regulated by the **NLL Trajectory**
$\mathbf{k}_1 \to x\Delta$: Regulated by the quark contribution to the NLL corrections to the one-gluon production vertex (x is the light-cone momentum fraction of the anti-quark)

Strategy:

1. Implement Lipatov Vertices and perform integration, while having access to **full final state information**. Only possibility of **combining energy and momentum conservation** with **NLL evolution**.
2. **Check** that the **numerical integration over full phase space** agrees with the result of Fadin & Lipatov (or Camici & Ciafaloni)

Structure of the Amplitude

$$K_r^{(2),q\bar{q}}(\mathbf{q}_1,\mathbf{q}_2) \propto \int \mathrm{d}\kappa\, \mathrm{d}\rho_f\, \delta^{(D)}(q_1 - q_2 - k_1 - k_2) \sum_{i_1,i_2,f} \left|\gamma^{q\bar{q}}_{i_1 i_2}(q_1,q_2,k_1,k_2)\right|^2$$

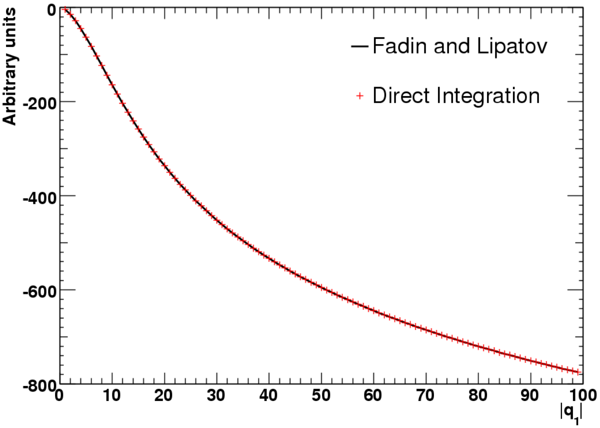
$$\sum_{i_1,i_2,f} \left|\gamma^{q\bar{q}}_{i_1 i_2}(q_1,q_2,k_1,k_2)\right|^2$$
$$\propto \left[(N_c^2-1)\left(\left|A^{\bar{q}q}_{+-}(k_1,k_2)\right|^2 + \left|A^{\bar{q}q}_{+-}(k_2,k_1)\right|^2\right) + 2A^{\bar{q}q}_{+-}(k_1,k_2)A^{*\bar{q}q}_{+-}(k_2,k_1)\right]$$

Symmetry properties of the divergent part of the amplitude ensures that the $1/N_c^2$ suppressed contribution is **finite**.

Phase space slice regularisation of the divergent pieces ensures cancellation between the divergences from the quark production and the quark contribution to the NLL corrections to the one-gluon production.

First Check. . .

Check of finite part

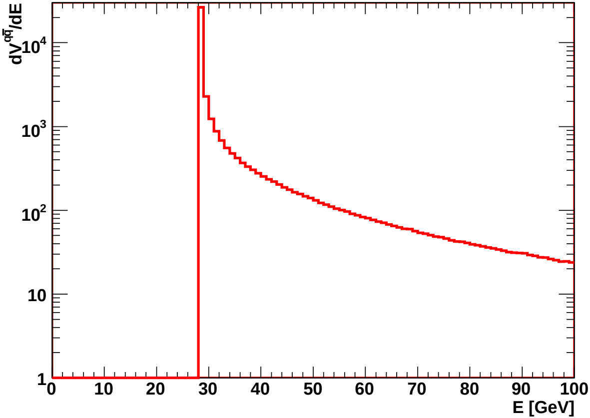


$1/N_c^2$ suppressed terms: 100% agreement.

Calculation under control.

Properties of the $q\bar{q}$-Vertex

$$q_1 = (20, 0)\mathrm{GeV}, q_2 = (0, 20)\mathrm{GeV}:$$



Antiquark: $(\mathbf{k_1}, \eta)$, quark: $(\mathbf{k_2}, -\eta)$, $E = E_1 + E_2$,
$\langle E \rangle \approx 40\mathrm{GeV}$, $\langle \Delta_{12} = 2\eta \rangle \approx .56$

Summary and Conclusions

- Have constructed a **very efficient** method for obtaining the BFKL evolution as an approximation to multi-leg processes
  Also applicable to small-x studies etc.
- Have started the program to obtain **fully exclusive final state information** of the NLL BFKL Evolution **necessary for** energy and momentum conservation and thus **full NLL accuracy**
- Conclusion from the study of the exclusive NLL quark–anti-quark vertex:
  Exclusive information absolutely **crucial for realistic phenomenology**, since the $q\bar{q}$-vertex gets contributions from relatively large invariant masses of the $q\bar{q}$-pair.
  Cannot assign a single rapidity to the quark and the anti-quark.
- `http://www.hep.phy.cam.ac.uk/~andersen/BFKL`

Why do I say we need Energy and Momentum Conservation to obtain full NLL accuracy?

People who do not care about Energy and Momentum conservation in the application of BFKL to the description of colour octet exchange (leading to multiple emissions) often equal the evolution variable Δ to

$$\Delta = \ln \frac{s}{s_0}$$

where s is the total energy and s_0 the Regge scale.
However, we have clearly demonstrated that there is no one-to-one correspondence between the centre of mass energy and the rapidity Δ of the evolution.
Energy and Momentum Conservation may simply suppress the kinematic region of the leading logarithms.
Distinguish NLL accuracy of the *evolution* from NLL accuracy of an *observable*

But I thought E&M-conservation *was* a NLL effect – Why do you say it is not taken into account by the NLL corrections to the kernel?!

$$\Delta y \approx_{\mathrm{LL}} \ln s/s_0 = \ln s/s_1 + \ln s_1/s_0$$

Identifies **correctly** E&M-conservation as a **NLL effect**.
However, the very form of the BFKL equation means that effects from **constraints in phase space cannot be taken into account by the BFKL kernel**:

1. The kernel evolves between two transverse momenta; the BFKL equation is uniform in rapidity. Emission of (energetic) particles does not influence the phase space of emission later (or earlier) in the evolution
2. The NLL corrections to the kernel itself involves a fully inclusive phase space integral over two-particle production vertices.

Poses no problem in diffractive studies (where no particles are emitted from the evolution) - however, very significant effects in jet studies.