Quark contribution to the evolution of color dipole

Ian Balitsky

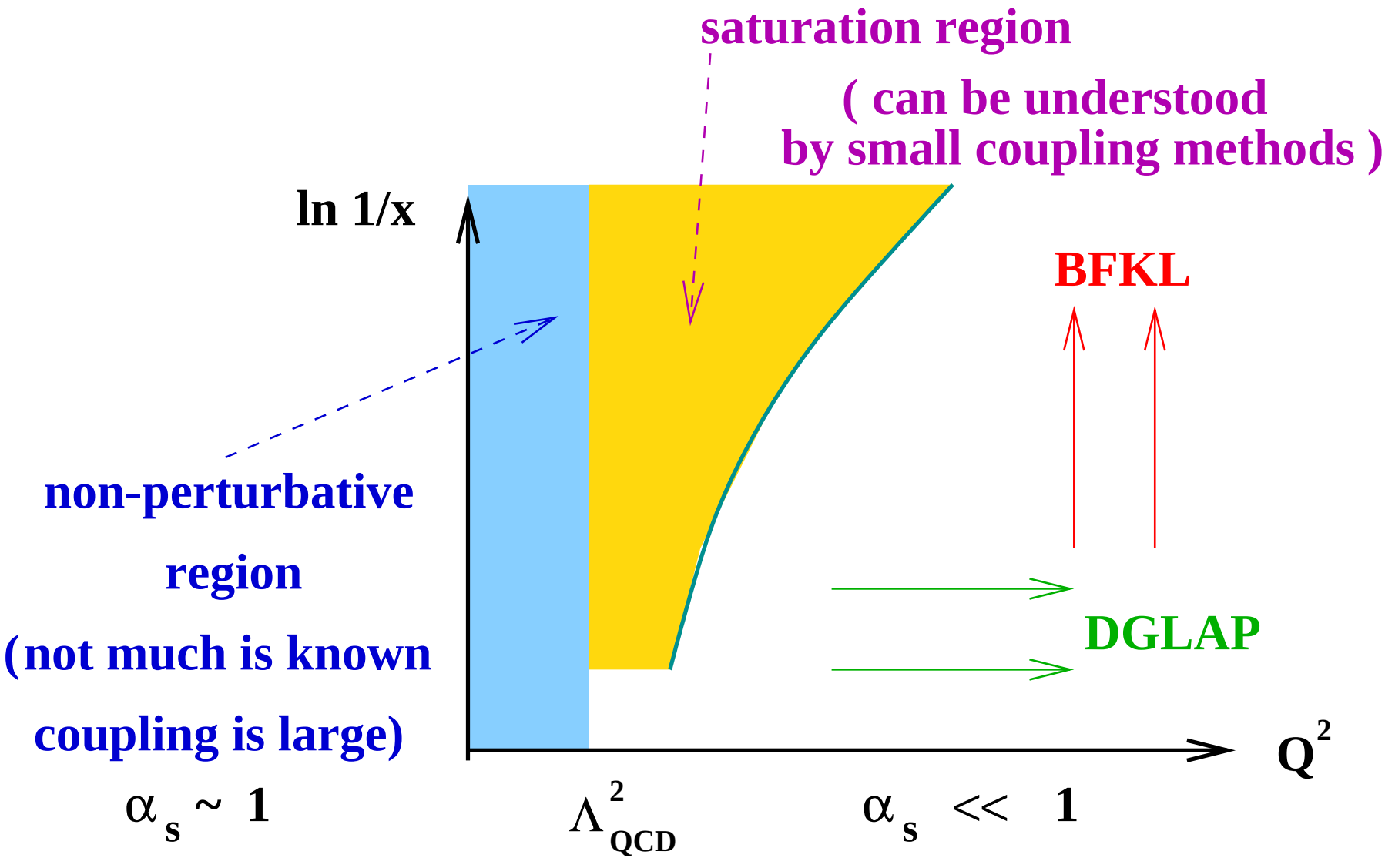
JLab & ODU

Lisbon, 28 June 2006

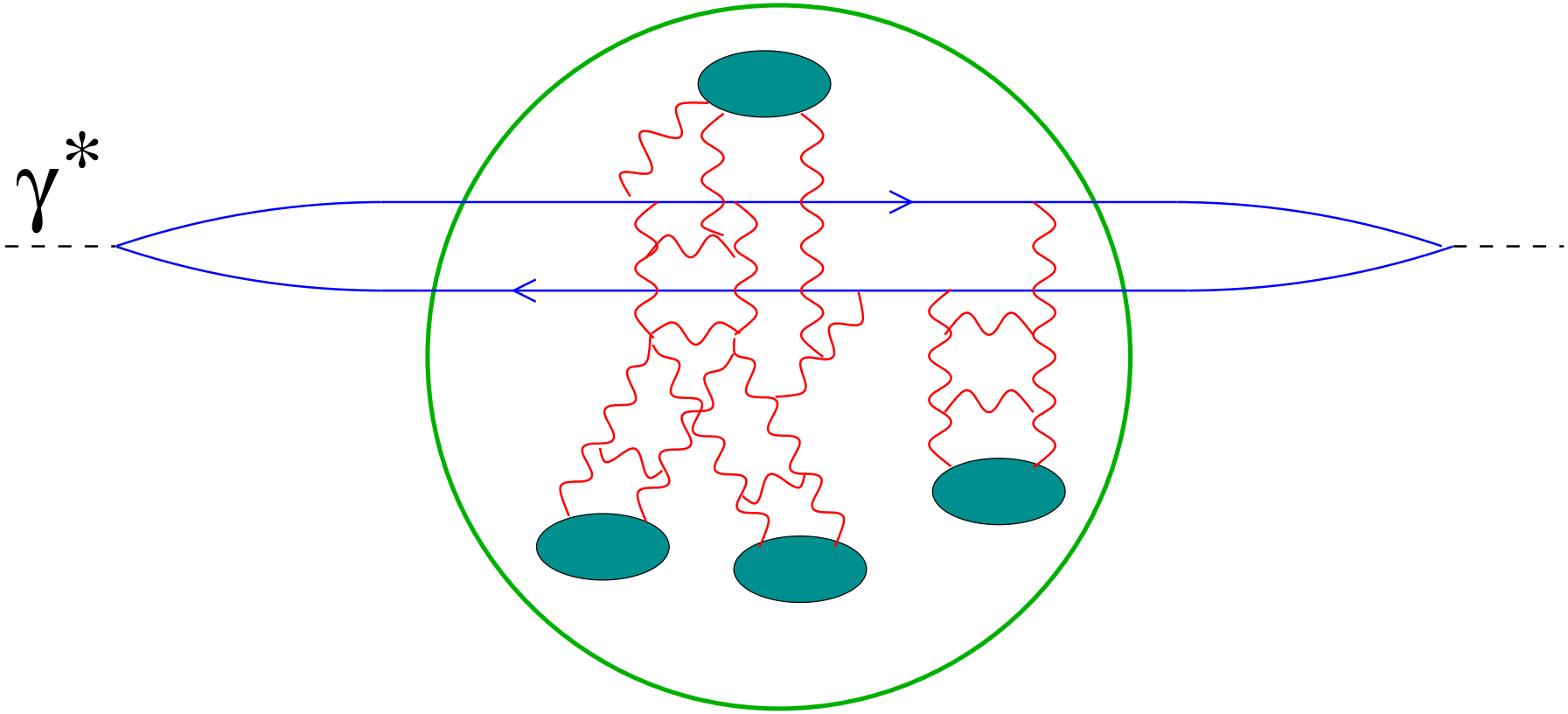
Plan

- Small-x DIS as an evolution of Wilson lines
- Non-linear evolution equation (BK eqn)
- Quark loop contribution to NLO BK.
- Can the high-energy scattering can be described in terms of dipoles?
- Bubble chain and the argument of coupling constant
- Conclusions and outlook

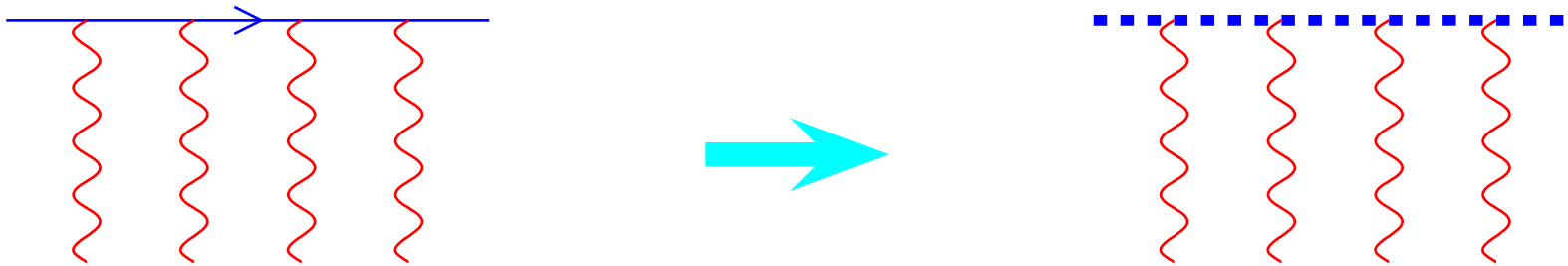
“Phase diagram” for DIS



Small-x DIS from the nucleus

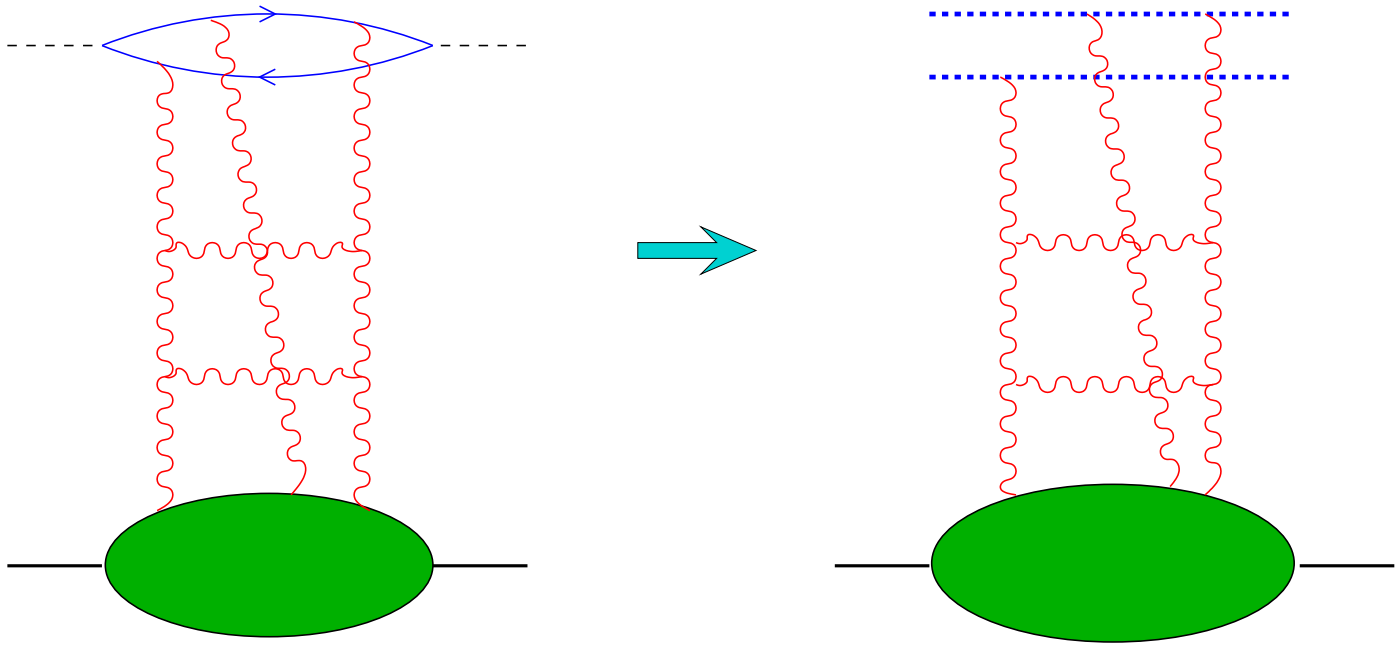


Fast quark moves along the straight line $\Rightarrow$



quark propagator reduces to the Wilson line collinear to quark's velocity

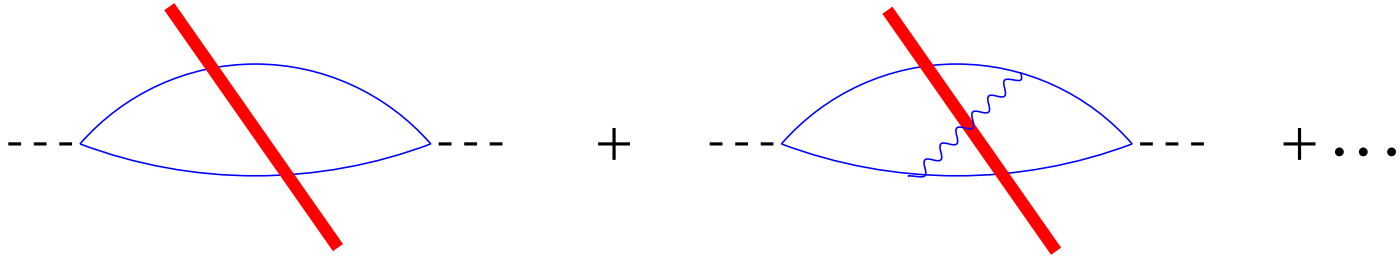
At high energies, the amplitude of $\gamma^* A \to \gamma^* A$ scattering reduces to the matrix element of a two-Wilson-line operator ("color dipole").



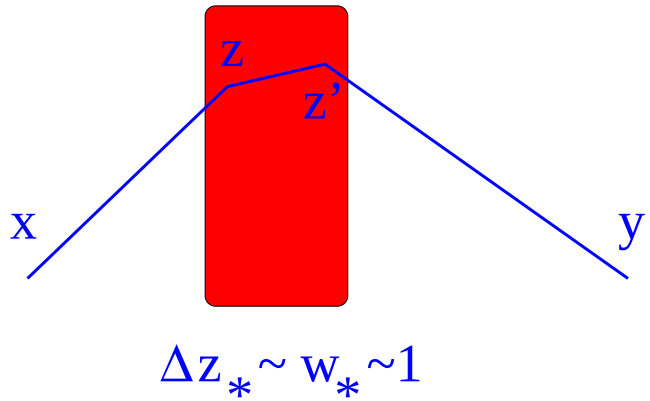
$$A(s) \;=\; \int \frac{d^2 k_\perp}{4\pi^2} I(k_\perp) \langle B | \mathrm{Tr}\{ U^{\eta_A}(k_\perp) U^{\dagger \eta_A}(-k_\perp) \} | B \rangle + \ldots$$

In the spectator frame

High-speed nucleus shrinks to a “pancake” $\Rightarrow$



Quarks (and gluons) do not have time to deviate in the transverse direction



$p = \alpha p_1 + \beta p_2 + p_\perp$ = Sudakov variables

$$(z|\frac{1}{p^2+i\epsilon}|z') = \frac{s}{8\pi x_*}\int_0^\infty \frac{d\alpha}{\alpha} e^{-i\alpha(z-z')_\bullet - i\frac{(z-z')_\perp^2}{4(z-z')_*}\alpha s}$$

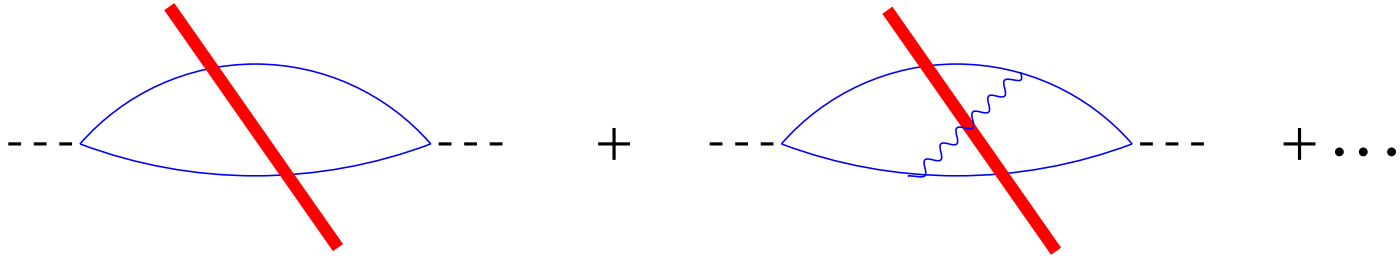
$$\Rightarrow \ |z-z'|_\perp \sim \sqrt{\frac{w_*}{\alpha s}} \sim \sqrt{\frac{1}{s}}$$

$z_* \equiv z\cdot p_1, \quad z_\bullet \equiv z\cdot p_2$

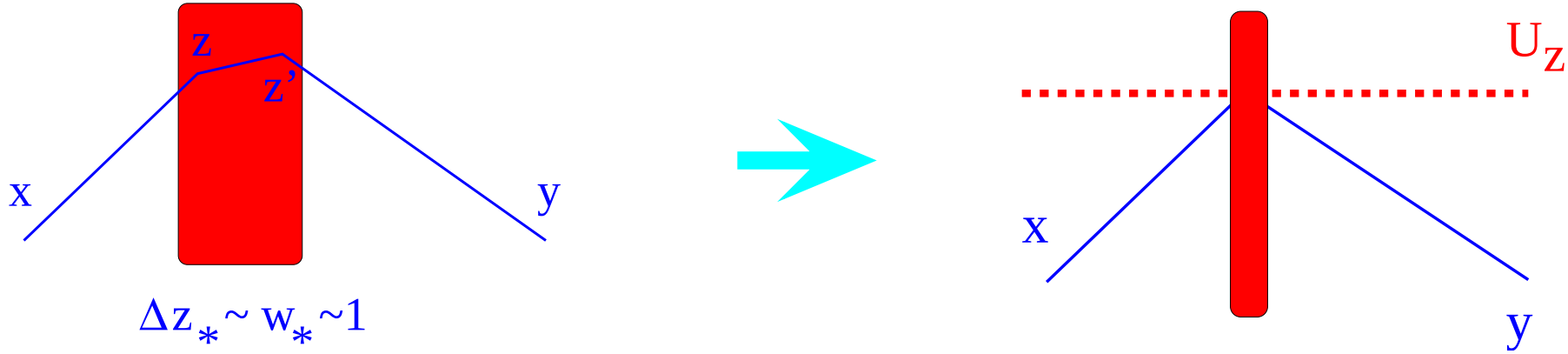
$\Rightarrow$ Interaction $\sim$ U_z = gauge factor ordered along (a segment of) the straight line

In the spectator frame

High-speed nucleus shrinks to a “pancake” $\Rightarrow$



Quarks (and gluons) do not have time to deviate in the transverse direction

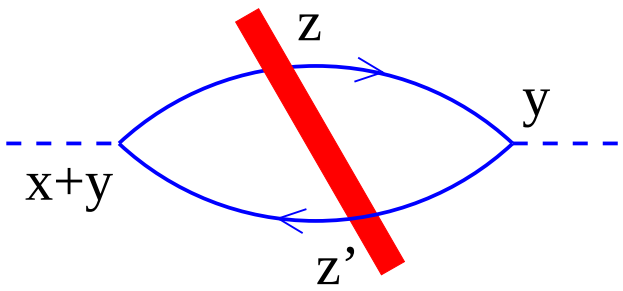


$$G(x,y) = \int dz \delta(z_*)(x|\frac{\not p}{p^2}|z)\ \not p_2 U_z(z|\frac{\not p}{p^2}|y)$$

$U_z = [\infty p_1 + z_\perp, -\infty p_1 + z_\perp]$ – Wilson line

$$[x,y] \equiv P e^{ig\int_0^1 du (x-y)^\mu A_\mu(ux+(1-u)y)}$$

Feynman diagrams in a shock-wave background

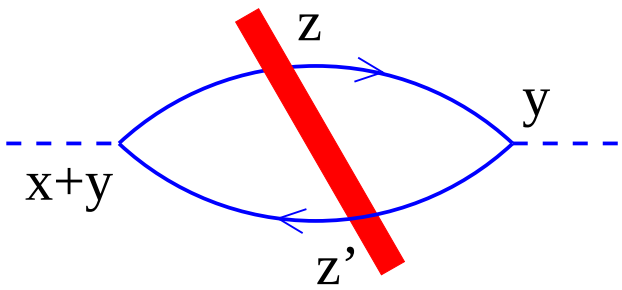


$$\int d^4x d^4y \, e^{-ip_A\cdot x} \langle T\{j_A(x+y) j'_A(y)\}\rangle_A$$

$$= \int \frac{d^2k_\perp}{4\pi^2} I^A(k_\perp) \mathrm{Tr}\{U(k_\perp)U^\dagger(-k_\perp)\} + \ldots \Rightarrow$$

$$A(s) = \int \frac{d^2k_\perp}{4\pi^2} I(k_\perp) \langle B|\mathrm{Tr}\{U(k_\perp)U^\dagger(-k_\perp)\}|B\rangle +$$

Feynman diagrams in a shock-wave background

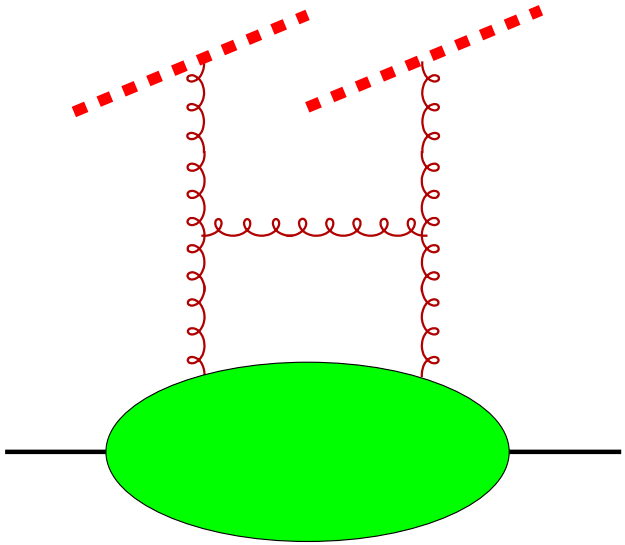


$$\int d^4x d^4y\, e^{-ip_A\cdot x}\langle T\{j_A(x+y)j'_A(y)\}\rangle_A$$

$$= \int \frac{d^2k_\perp}{4\pi^2} I^A(k_\perp)\mathrm{Tr}\{U(k_\perp)U^\dagger(-k_\perp)\} + \ldots \Rightarrow$$

$$A(s) = \int \frac{d^2k_\perp}{4\pi^2} I(k_\perp)\langle B|\mathrm{Tr}\{U(k_\perp)U^\dagger(-k_\perp)\}|B\rangle +$$

Energy dependence of the amplitude $A(s)$ is determined by the dependence of the Wilson lines on the rapidity η_A defined by the slope of the line.



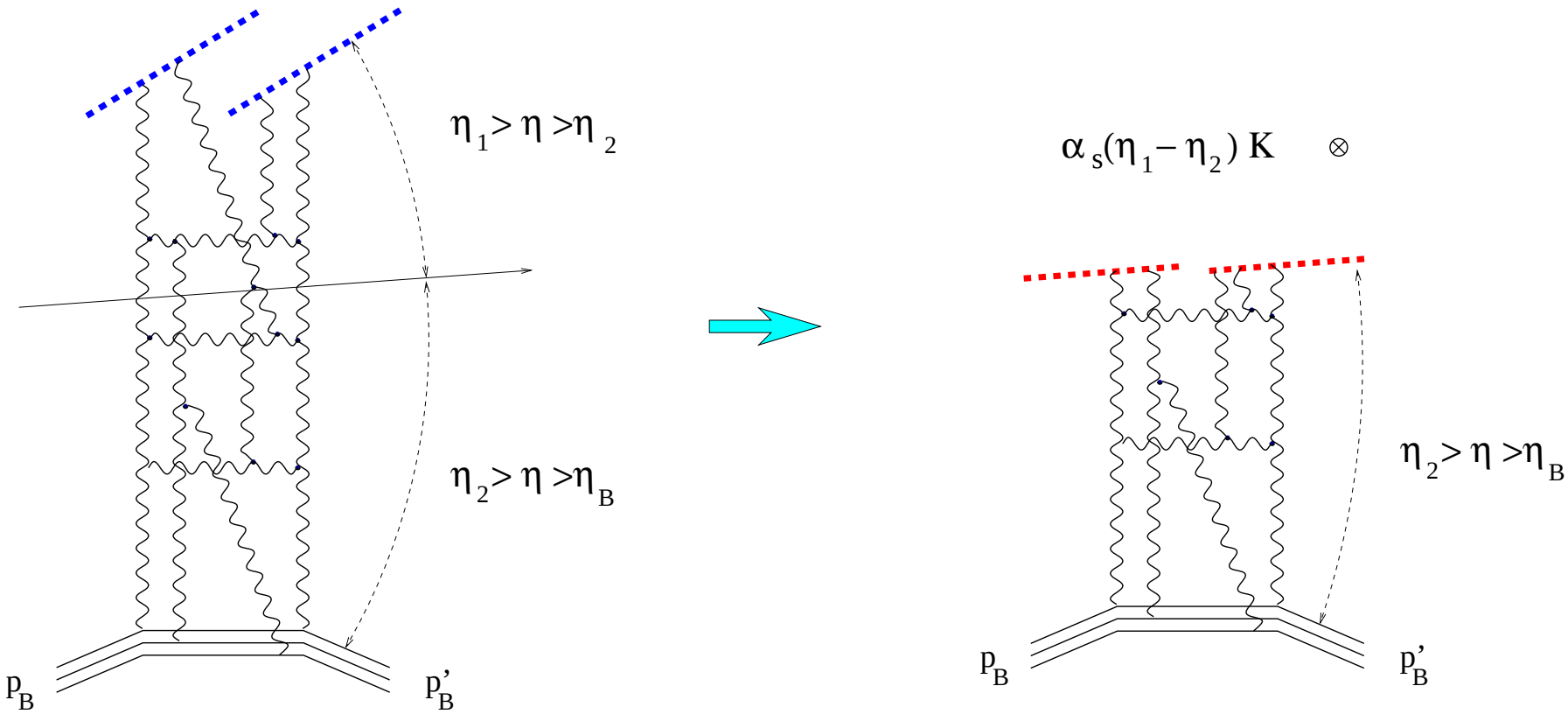
light-like Wilson line $\Rightarrow$ divergence $\sim \int^\infty \frac{d\alpha}{\alpha}$

$\Rightarrow$ replace U by U^{η_A} ordered along the line $\|$ $n_A = p_1 + e^{-\eta_A} p_2$

$$\langle B|U_x^{\eta_A} U_y^{\dagger\eta_A}|B\rangle \sim \int_{\eta_B}^{\eta_A} d\eta$$

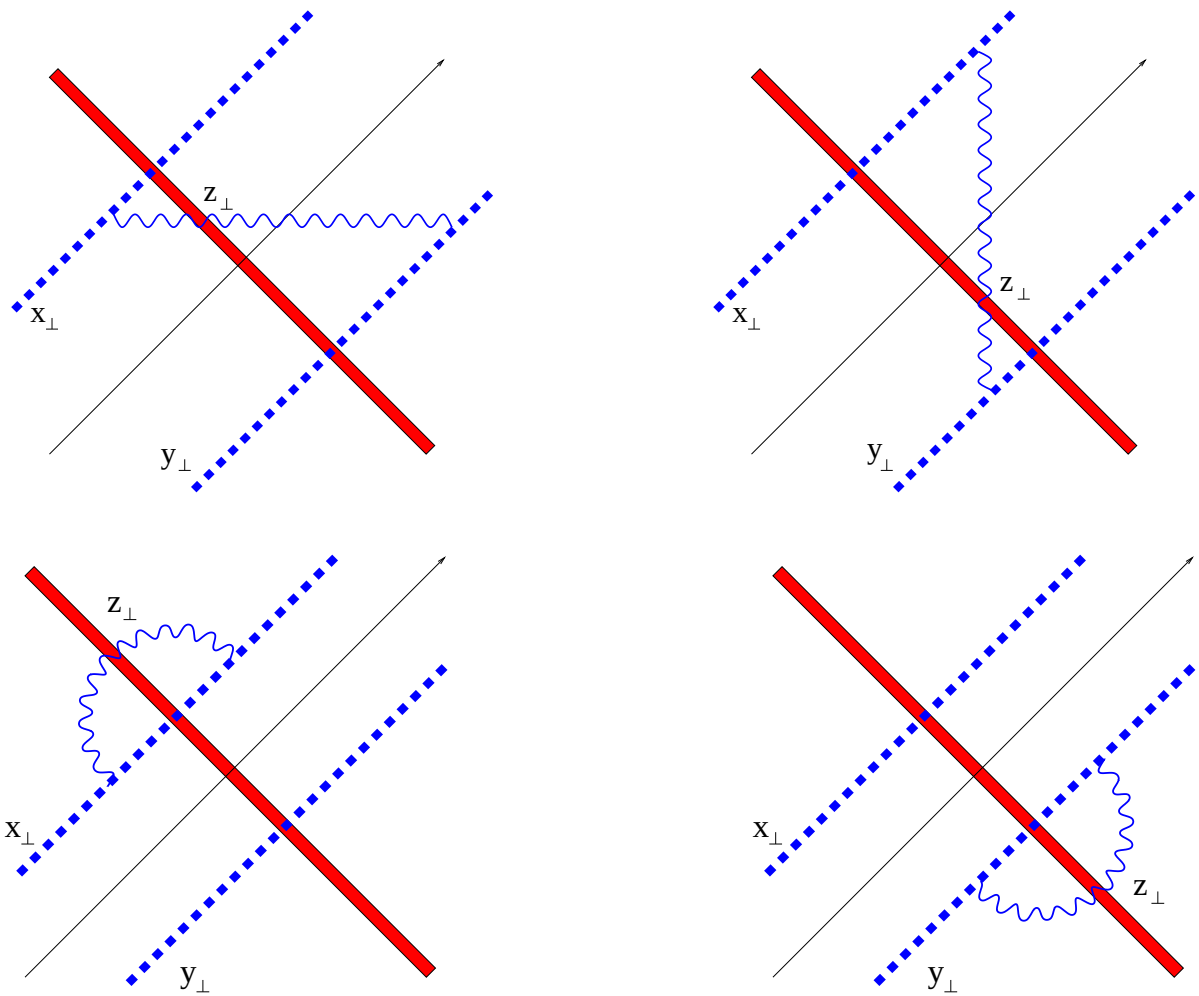
Evolution equation

To get the evolution equation, consider the dipole with the slope $\parallel \ \eta_1$ and integrate over the gluons with rapidities $\eta_1 > \eta > \eta_2$. This integral gives the kernel of the evolution equation (multiplied by the dipole(s) with the slope corresponding to η_2).



In the frame $\parallel \ \eta_1$ the gluons with $\eta < \eta_2$ are seen as a pancake $\Rightarrow$

One-loop evolution



The structure is

[$x \longrightarrow z$: free propagation]
$\times$
[$U^{ab}(z_\perp)$ - instantaneous interaction with the $\eta < \eta_2$ shock wave]
$\times$
[$z \longrightarrow y$: free propagation]

$$U_z^{ab} = \mathrm{Tr}\{t^a U_z t^b U_z^\dagger\} \;\Rightarrow (U_x U_y^\dagger)^{\eta_1} = (U_x U_y^\dagger)^{\eta_1} + \alpha_s(\eta_1 - \eta_2)(U_x U_z^\dagger U_z U_y^\dagger)^{\eta_2}$$

$\Rightarrow$ non-linear evolution

Non-linear evolution equation

$$\frac{\partial}{\partial\eta}\mathcal{U}(x_\perp, y_\perp) \;=$$

$$-\frac{\bar{\alpha}}{4\pi}\int dz_\perp \frac{(\vec{x}-\vec{y})_\perp^2}{(\vec{x}_\perp-\vec{z}_\perp)^2(\vec{z}_\perp-\vec{y}_\perp)^2}$$

$$\times\left\{\mathcal{U}(x_\perp, z_\perp)+\mathcal{U}(z_\perp, y_\perp)-\mathcal{U}(x_\perp, y_\perp)-\mathcal{U}(x_\perp, z_\perp)\mathcal{U}(z_\perp, y_\perp)\right\}$$

$$\mathcal{U}(x_\perp, y_\perp) \equiv \frac{1}{N_c}(N_c - \mathrm{Tr}\{U(x_\perp)U^\dagger(y_\perp)\})$$

LLA for DIS in pQCD $\Rightarrow$ BFKL
LLA for DIS in sQCD $\Rightarrow$ NL eqn
(s for semiclassical)

Non-linear evolution equation

$$\frac{\partial}{\partial\eta}\mathcal{U}(x_\perp, y_\perp) =$$

$$-\frac{\bar{\alpha}}{4\pi}\int dz_\perp \frac{(\vec{x}-\vec{y})_\perp^2}{(\vec{x}_\perp-\vec{z}_\perp)^2(\vec{z}_\perp-\vec{y}_\perp)^2}$$

$$\times\left\{\underbrace{\mathcal{U}(x_\perp, z_\perp)+\mathcal{U}(z_\perp, y_\perp)-\mathcal{U}(x_\perp, y_\perp)}-\mathcal{U}(x_\perp, z_\perp)\mathcal{U}(z_\perp, y_\perp)\right\}$$

$$\mathcal{U}(x_\perp, y_\perp) \equiv \frac{1}{N_c}(N_c - \mathrm{Tr}\{U(x_\perp)U^\dagger(y_\perp)\})$$

LLA for DIS in pQCD $\Rightarrow$ BFKL

LLA for DIS in sQCD $\Rightarrow$ NL eqn

(s for semiclassical)

Non-linear evolution equation

$$\frac{\partial}{\partial\eta}\mathcal{U}(x_\perp, y_\perp) =$$

$$-\frac{\bar{\alpha}}{4\pi}\int dz_\perp \frac{(\vec{x}-\vec{y})_\perp^2}{(\vec{x}_\perp-\vec{z}_\perp)^2(\vec{z}_\perp-\vec{y}_\perp)^2}$$

$$\times\left\{\mathcal{U}(x_\perp, z_\perp)+\mathcal{U}(z_\perp, y_\perp)-\mathcal{U}(x_\perp, y_\perp)-\underbrace{\mathcal{U}(x_\perp, z_\perp)\mathcal{U}(z_\perp, y_\perp)}\right\}$$

$$\mathcal{U}(x_\perp, y_\perp) \equiv \frac{1}{N_c}(N_c - \mathrm{Tr}\{U(x_\perp)U^\dagger(y_\perp)\})$$

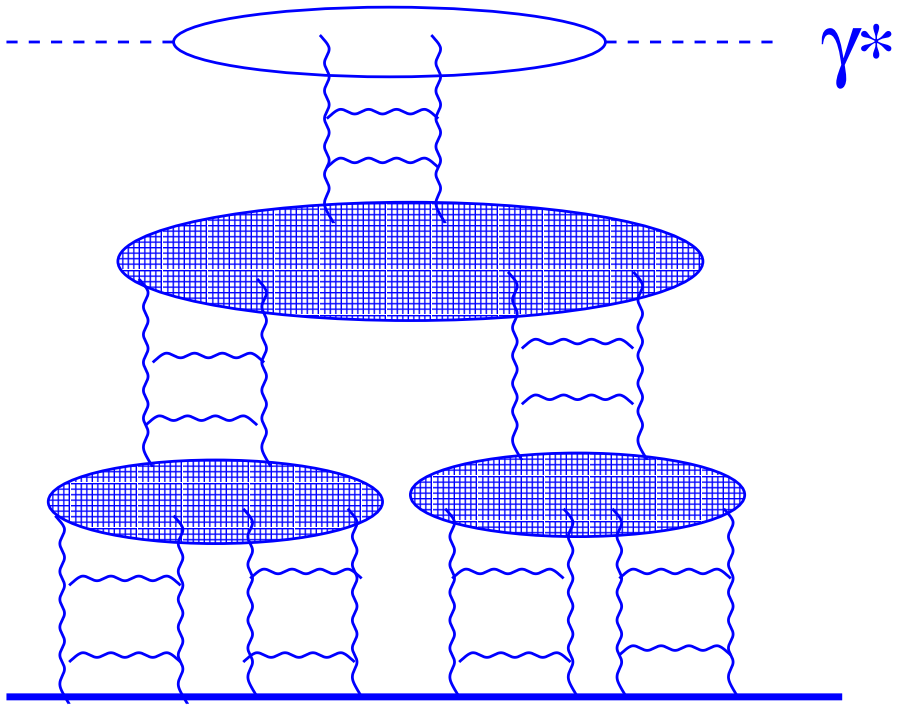
LLA for DIS in pQCD $\Rightarrow$ BFKL
LLA for DIS in sQCD $\Rightarrow$ NL eqn
(s for semiclassical)

Example - LLA for the structure functions of large nuclei: $\alpha_s \ln\frac{1}{x} \sim 1$, $\alpha_s^2 A^{1/3} \sim 1$

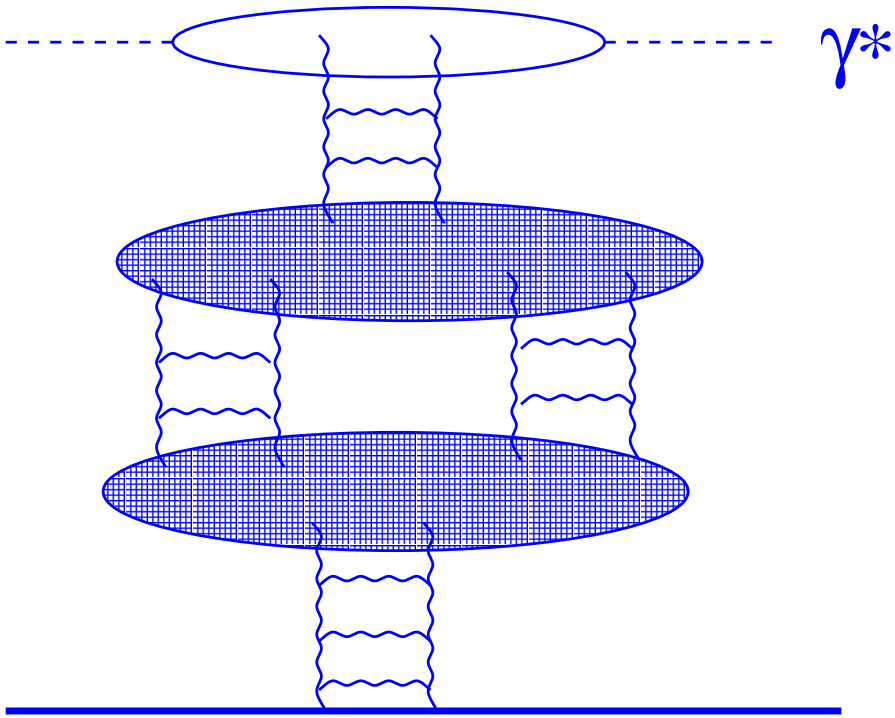
References

- Gribov, Levin, Ryskin (1983) - GLR eqn suggested
- Mueller, Qui (1986) - DLA limit of GLR eqn proved
- I.B. (1996) - the above eqn derived
- Kovchegov (1999) - the above eqn rederived (in the dipole model) and used for the large nuclei
- Braun (M.A.) (2000) - NL = GLR + 3-pomeron vertex from Bartels *et. al.*
- JIMWLK(2000) - obtained from the RG eqn for Color Glass Condensate

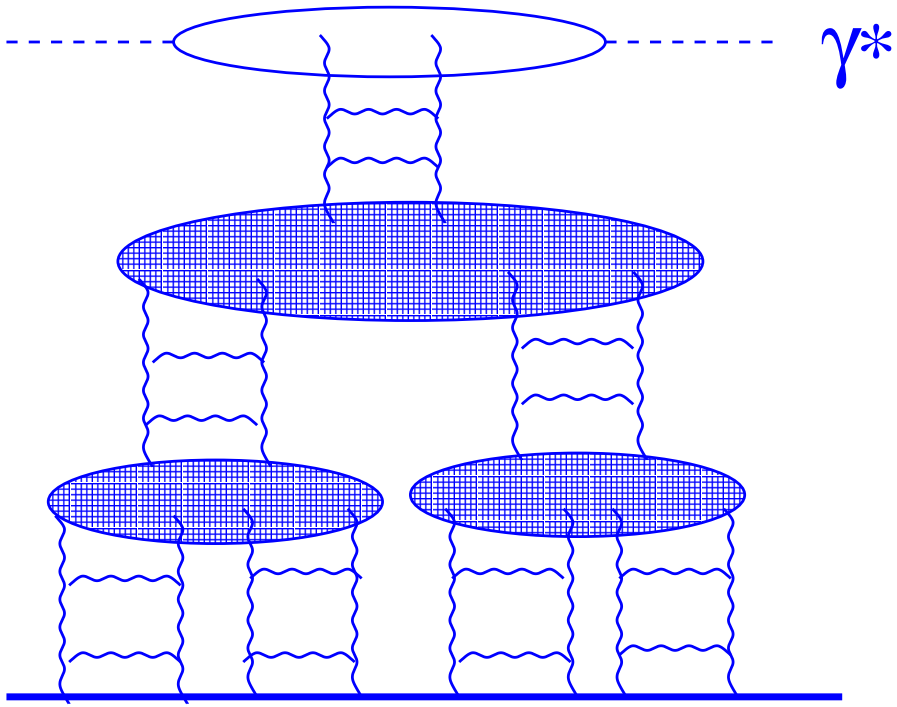
Non-linear equation sums up the "fan" diagrams



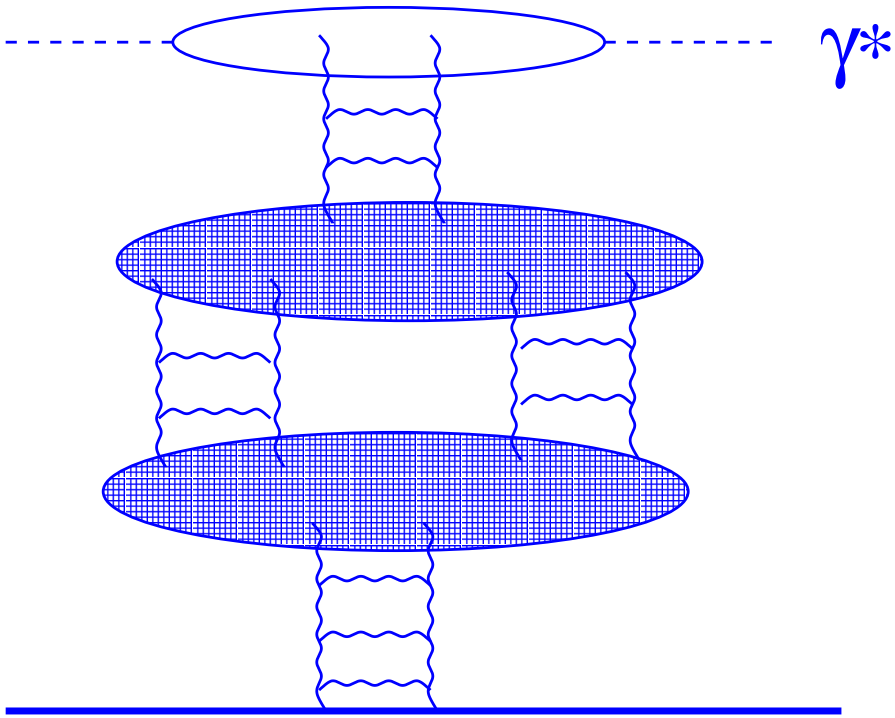
Example of the diagrams left behind by the NL eqn: pomeron loops



Non-linear equation sums up the "fan" diagrams



Example of the diagrams left behind by the NL eqn: pomeron loops

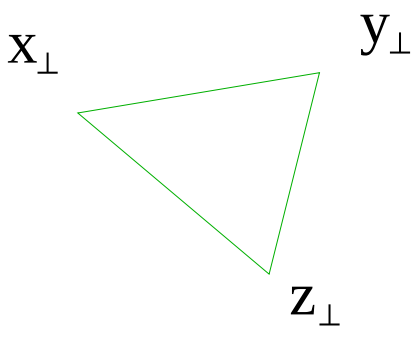


$$x_B \to 0 \overset{\text{BFKL}}{\Rightarrow} \text{gluon density increases} \overset{\text{BK}}{\Rightarrow} \text{saturation} \Rightarrow \text{CGC}$$

Scattering of two ions (CDG's) - pomeron loops, glasma etc. (wait for Friday)

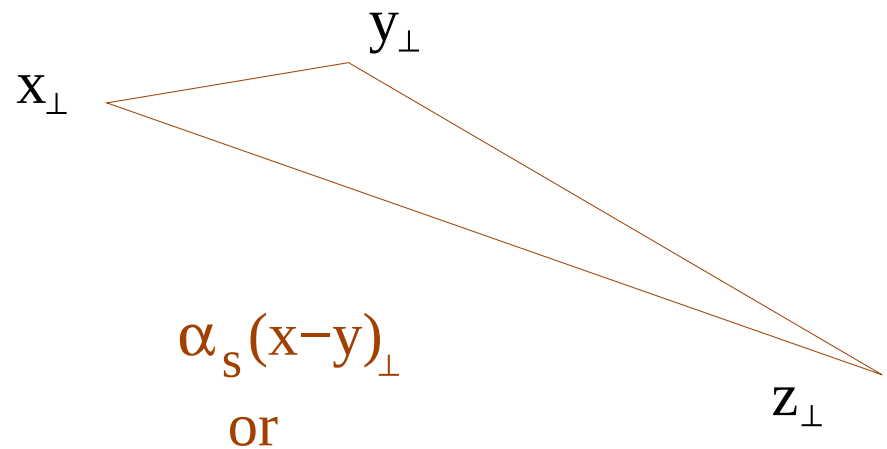
Argument of the α_s in the BK equation

$$\frac{\partial}{\partial\eta}\mathcal{U}(x_\perp,y_\perp) = \frac{\alpha_s(?_\perp)}{2\pi^2}\int dz_\perp \frac{(\vec{x}-\vec{y})_\perp^2}{(\vec{x}_\perp-\vec{z}_\perp)^2(\vec{z}_\perp-\vec{y}_\perp)^2}$$
$$\times\left[\mathcal{U}(x_\perp,z_\perp)+\mathcal{U}(z_\perp,y_\perp)-\mathcal{U}(x_\perp,y_\perp)-\mathcal{U}(x_\perp,z_\perp)\mathcal{U}(z_\perp,y_\perp)\right]$$



$(x-y)_\perp \sim (y-z)_\perp \sim (x-z)_\perp$

$\rightarrow$ α_s(whatever)



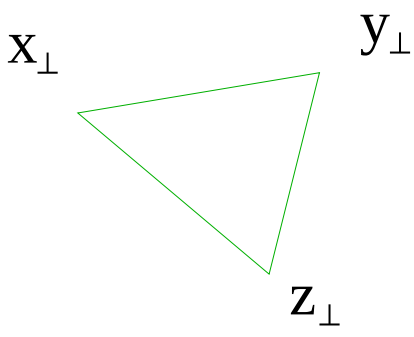
$\alpha_s(x-y)_\perp$

or

$\alpha_s(x-z)_\perp$?

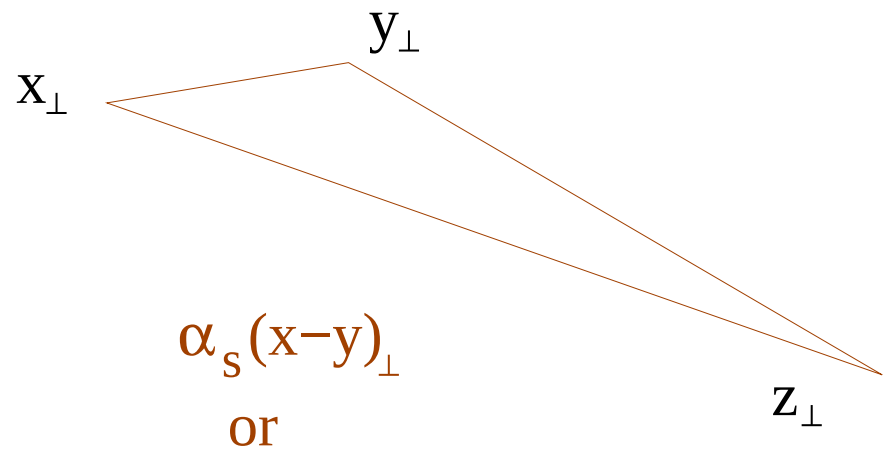
Argument of the α_s in the BK equation

$$\frac{\partial}{\partial\eta}\mathcal{U}(x_\perp,y_\perp) = \frac{\alpha_s(?_\perp)}{2\pi^2}\int dz_\perp \frac{(\vec{x}-\vec{y})_\perp^2}{(\vec{x}_\perp-\vec{z}_\perp)^2(\vec{z}_\perp-\vec{y}_\perp)^2}$$
$$\times\left[\mathcal{U}(x_\perp,z_\perp)+\mathcal{U}(z_\perp,y_\perp)-\mathcal{U}(x_\perp,y_\perp)-\mathcal{U}(x_\perp,z_\perp)\mathcal{U}(z_\perp,y_\perp)\right]$$



$(x-y)_\perp \sim (y-z)_\perp \sim (x-z)_\perp$

$\rightarrow$ α_s(whatever)



$\alpha_s(x-y)_\perp$

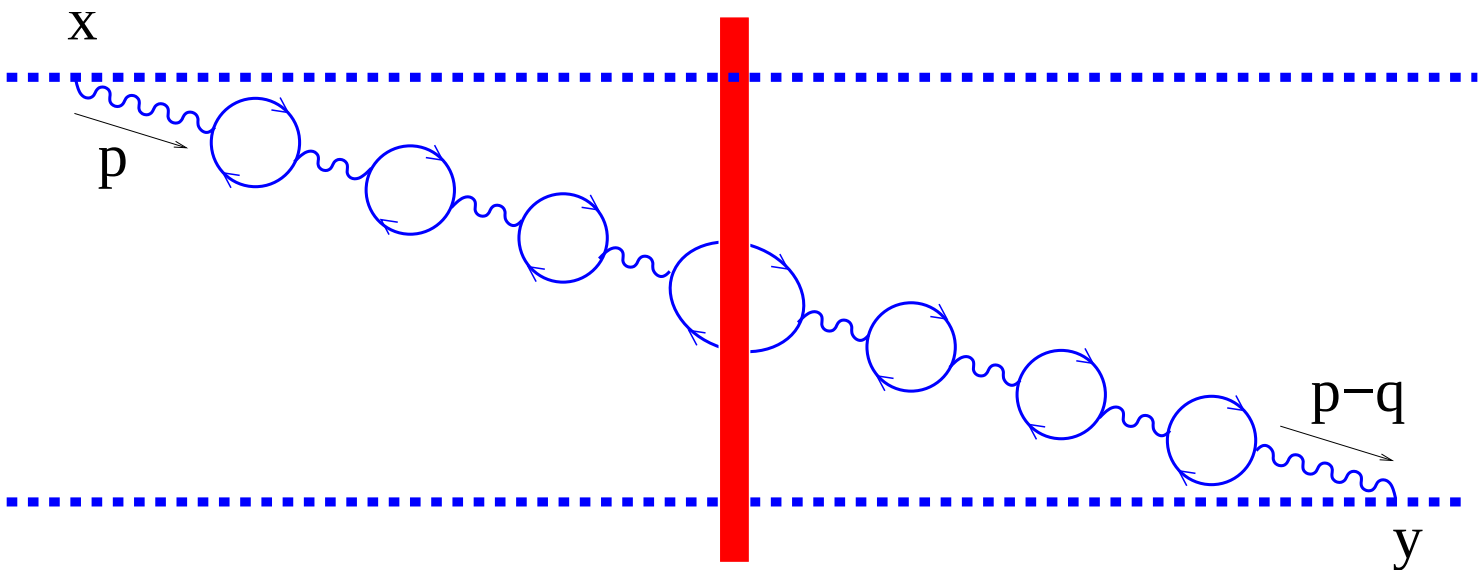
or

$\alpha_s(x-z)_\perp$?

Result: $\alpha_s = \alpha_s(|x-y|_\perp)$

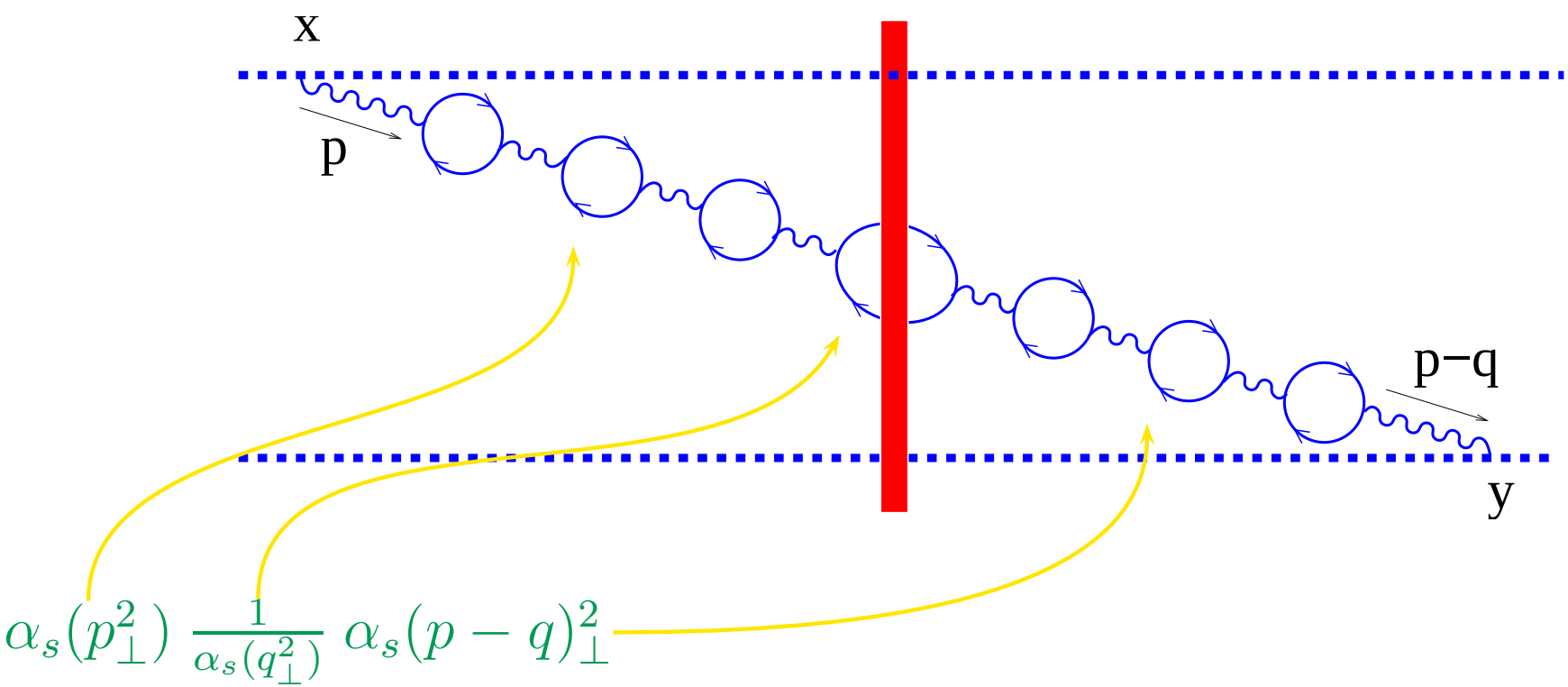
Quark bubble chain and the argument of α_s

$$\alpha_s(p_\perp^2) = \frac{\alpha_s(\mu^2)}{1 + \left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)\frac{\alpha_s}{4\pi}\ln\frac{p_\perp^2}{\mu^2}}$$



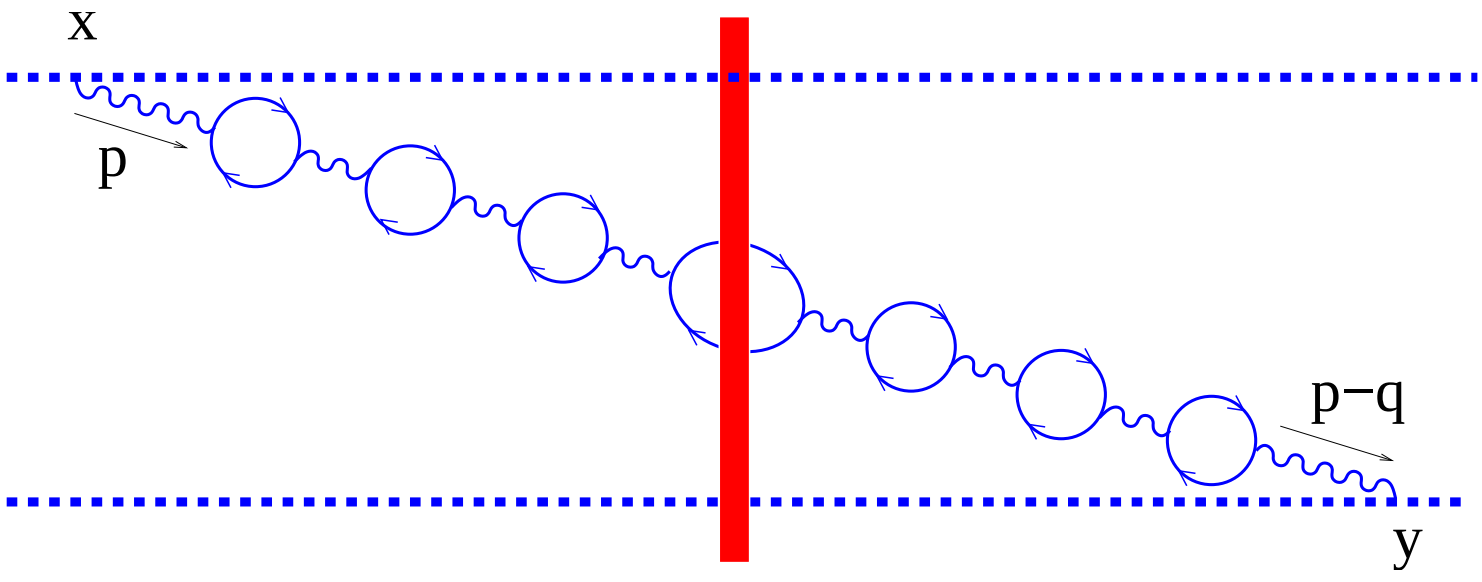
Quark bubble chain and the argument of α_s

$$\alpha_s(p_\perp^2) = \frac{\alpha_s(\mu^2)}{1 + \left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)\frac{\alpha_s}{4\pi}\ln\frac{p_\perp^2}{\mu^2}}$$



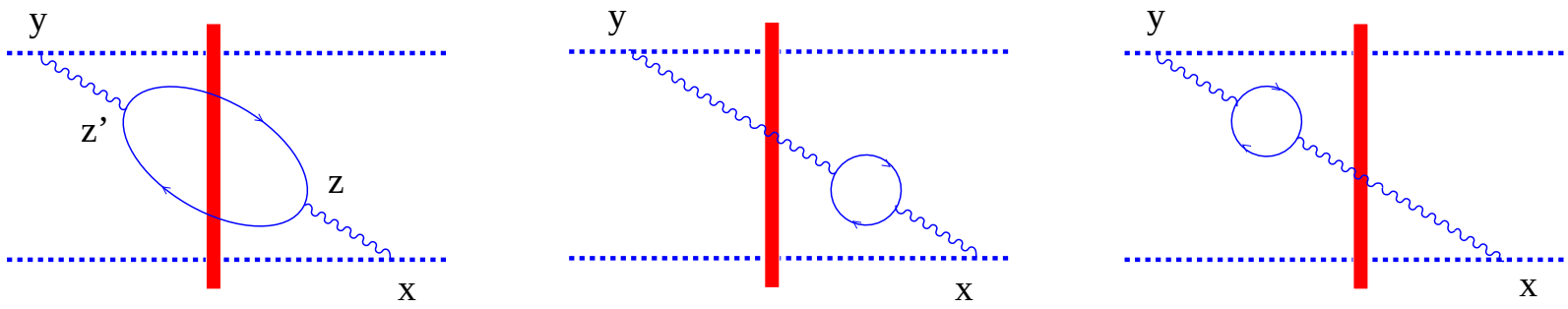
Quark bubble chain and the argument of α_s

$$\alpha_s(p_\perp^2) = \frac{\alpha_s(\mu^2)}{1 + \left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)\frac{\alpha_s}{4\pi}\ln\frac{p_\perp^2}{\mu^2}}$$



$\alpha_s(p_\perp^2)\ \frac{1}{\alpha_s(q_\perp^2)}\ \alpha_s(p-q)_\perp^2 \quad \Rightarrow \quad \alpha_s(x-y)_\perp^2$ after the Fourier transformation

Quark contribution to the NLO kernel



(a) (b) (c)

$$(a) \;=\; -\frac{4\alpha_s^2}{\pi} n_f \Delta\eta t^a U_x \otimes t^b U_y^\dagger \int đ^d p\, đ^d q\, đ^d q' \, \frac{e^{i(p,x)_\perp - i(p-q-q',y)_\perp}}{p^2(p-q-q')^2}$$

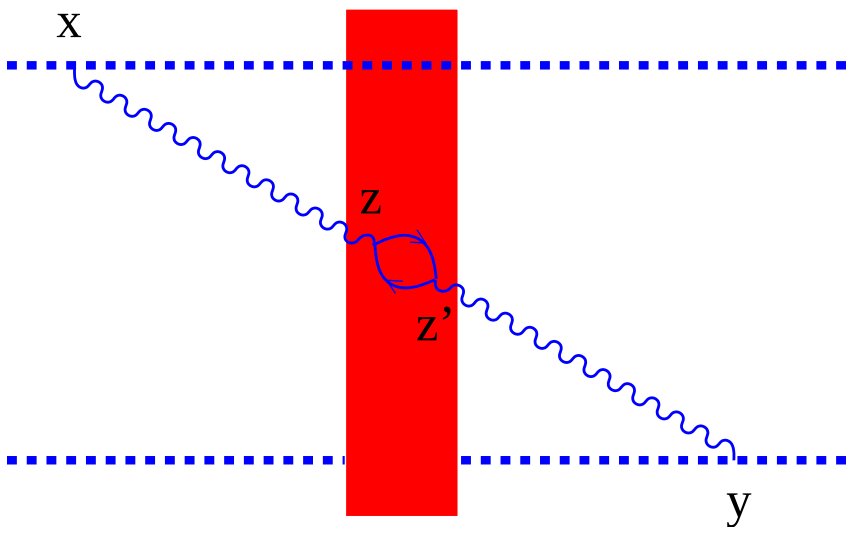
$$\mathrm{Tr}\{t^a U(q) t^b U^\dagger(q')\} \int_0^1 dv du \left[-(q+q')^2 \frac{\bar{u}u\Gamma(1-\frac{d}{2})\mu^{2-d}}{(P^2\bar{v}v + Q^2\bar{u}u)^{1-\frac{d}{2}}} \right.$$

$$+ \frac{\Gamma(2-\frac{d}{2})\mu^{2-d}}{(P^2\bar{v}v + Q^2\bar{u}u)^{2-\frac{d}{2}}} \Big\{ P^2[\bar{v}v(q,q') - \bar{u}uQ^2 + 2\bar{u}u\bar{v}v(q^2 + q'^2)] - 2\bar{u}u\bar{v}v(P,q)(P,q'$$

$$\left. + \bar{u}u(1-2u)[\bar{v}v(q,q')(P,q+q') + \bar{v}q^2(P,q') + vq'^2(P,q)] + \bar{u}^2u^2Q^2(q+q')^2 \Big\} \right]$$

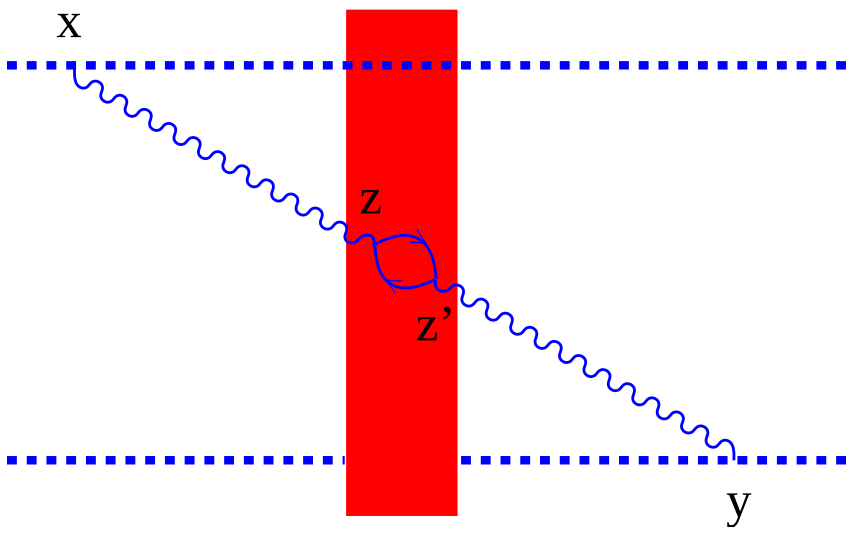
$$P = p - (q+q')u, \;\; Q^2 = q_\perp^2\bar{v} + q'^2_\perp v, \;\; \bar{v} \equiv 1 - v, \;\; d = d_\perp = 2 - \epsilon$$

A problem: quark loop inside the shock wave

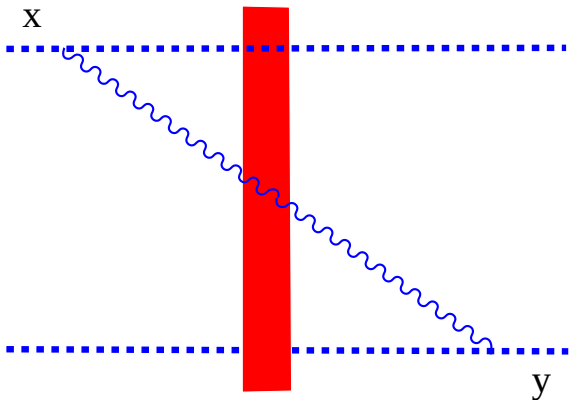


$|z - z'|_\perp^2 \sim \frac{1}{\alpha s} \Rightarrow$ one can expand the quark loop near the light cone $\Rightarrow$ the contribution is local in $z_\perp$.

A problem: quark loop inside the shock wave



Tree-level gluon propagator $G_{\bullet\bullet}(x,y) = \int dz\delta(z_*)(x|\frac{1}{p^2}|z)\frac{1}{\alpha}\partial_\perp^2 U_z(z|\frac{1}{p^2}|y)$



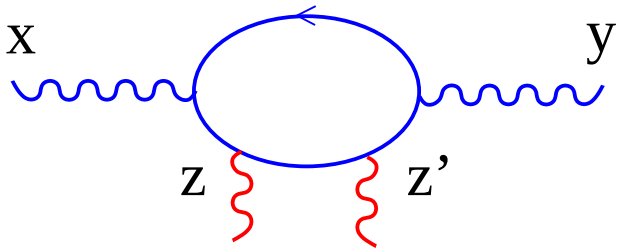
We need the operators up to twist 4 $\Rightarrow$ the candidates for an additional interaction term are

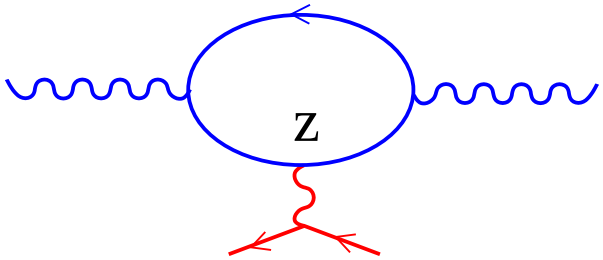
$$[GG] = \int dudv\,[\infty,u]G_{\bullet i}(up_1+z_\perp)[u,v]G_\bullet^{\ i}(vp_1+z_\perp)[v,-\infty]$$

$$[DG] = \int du\,[\infty,u]D^iG_{\bullet i}(up_1+z_\perp)[u,-\infty], \qquad -i[DG]+2[GG]=\partial_\perp^2 U$$

A way to fish out such extra term is to find the light-cone expansion of $U_xU_y^\dagger$ at $x_\perp \to y_\perp$ up to twist-4 terms and compare it with the expansion of the expression from the previous page.

A typical term of the light-cone expansion





$$\langle A^a_\bullet(x_*,x_\perp)A^b_\bullet(y_*,y_\perp)\rangle_A = \frac{1}{8\pi^2}\int_0^\infty \frac{d\alpha}{\alpha^3}(x_\perp|e^{-i\frac{p_\perp^2}{\alpha s}(x-y)_*}|y_\perp)\,\frac{\Gamma(r)\Gamma(r-1)}{\Gamma(2r-1)}$$

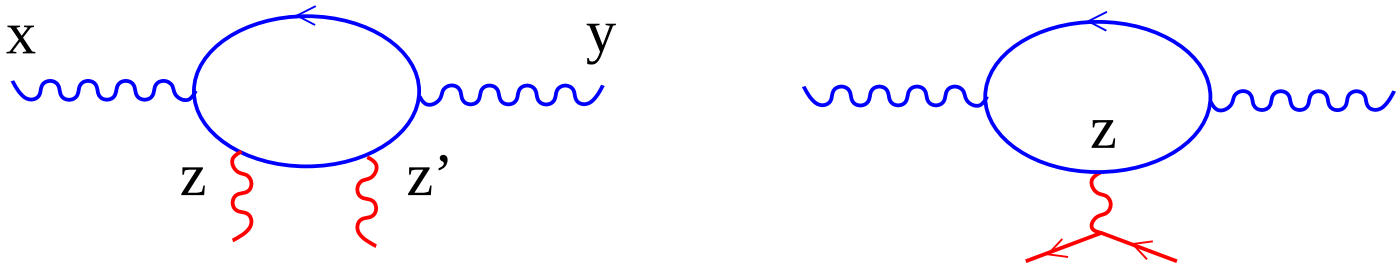
$$\times\left(\int_{-\infty}^{\infty}dz_*\int_{-\infty}^{z_*}d'_*([\infty,z_*]G_{\bullet i}(z_*,x_\perp)[z_*,z'_*]G_\bullet^{\ i}(z'_*,x_\perp)[z',-\infty])^{ab}\right.$$

$$\times\left\{\frac{1}{(2-r)}[(x-z)_*^{2-r}+(z-y)_*^{2-r}-(x-y)_*^{2-r}-(z-z')_*^{2-r}]+\frac{(z-z')_*^{2-r}}{r+1}\right\}$$

$$-\,i\int_{-\infty}^{\infty}dz_*\,([\infty_*,z_*]D^iG_{\bullet i}(z_*,x_\perp)[z_*,-\infty])^{ab}\,\frac{(x-z)_*^{2-r}+(z-y)_*^{2-r}-(x-y)_*^{2-r}}{2-r}$$

$$z_*,z'_*\ll x_*,y_*\Rightarrow$$

A typical term of the light-cone expansion



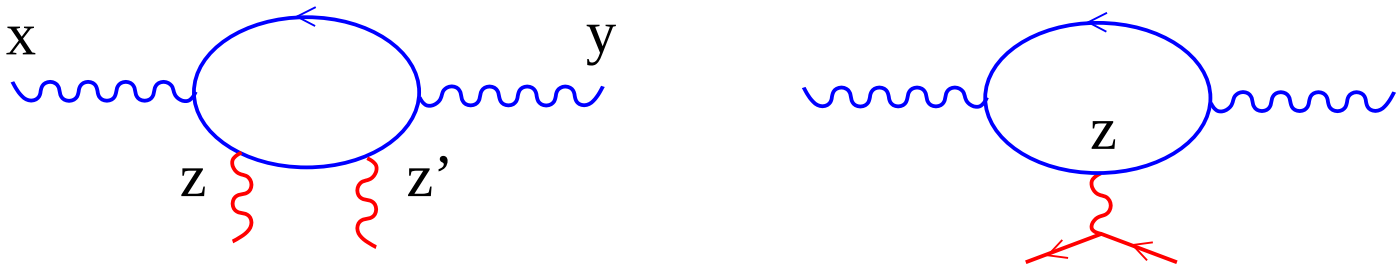
$$\langle A^a_\bullet(x_*,x_\perp)A^b_\bullet(y_*,y_\perp)\rangle_A \;=\; \frac{1}{8\pi^2}\int_0^\infty \frac{d\alpha}{\alpha^3}(x_\perp|e^{-i\frac{p_\perp^2}{\alpha s}(x-y)_*}|y_\perp)\,\frac{\Gamma(r)\Gamma(r-1)}{\Gamma(2r-1)}$$

$$\times\left(\int_{-\infty}^{\infty}dz_*\int_{-\infty}^{z_*}d'_*([\infty,z_*]G_{\bullet i}(z_*,x_\perp)[z_*,z'_*]G_\bullet^{\,i}(z'_*,x_\perp)[z',-\infty])^{ab}\right.$$

$$\times\left\{\frac{1}{(2-r)}[x_*^{2-r}+(-y)_*^{2-r}-(x-y)_*^{2-r}-(z-z')_*^{2-r}]+\frac{(z-z')_*^{2-r}}{r+1}\right\}$$

$$\left.-\,i\int_{-\infty}^{\infty}dz_*\,([\infty_*,z_*]D^iG_{\bullet i}(z_*,x_\perp)[z_*,-\infty])^{ab}\,\frac{x_*^{2-r}+(-y)_*^{2-r}-(x-y)_*^{2-r}}{2-r}\right)$$

A typical term of the light-cone expansion



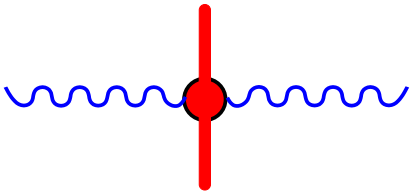
$$\langle A^a_\bullet(x_*,x_\perp)A^b_\bullet(y_*,y_\perp)\rangle_A = \frac{1}{8\pi^2}\int_0^\infty \frac{d\alpha}{\alpha^3}(x_\perp|e^{-i\frac{p_\perp^2}{\alpha s}(x-y)_*}|y_\perp)\,\frac{\Gamma(r)\Gamma(r-1)}{\Gamma(2r-1)}$$

$$\times\left(\int_{-\infty}^{\infty}dz_*\int_{-\infty}^{z_*}d'_*([\infty,z_*]G_{\bullet i}(z_*,x_\perp)[z_*,z'_*]G_\bullet^{\,i}(z'_*,x_\perp)[z',-\infty])^{ab}\right.$$

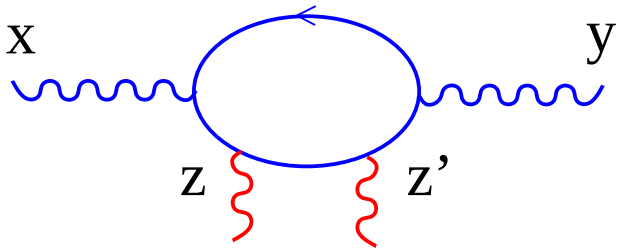
$$\times\left\{\frac{1}{(2-r)}[x_*^{2-r}+(-y)_*^{2-r}-(x-y)_*^{2-r}-(z-z')_*^{2-r}]+\frac{(z-z')_*^{2-r}}{r+1}\right\}$$

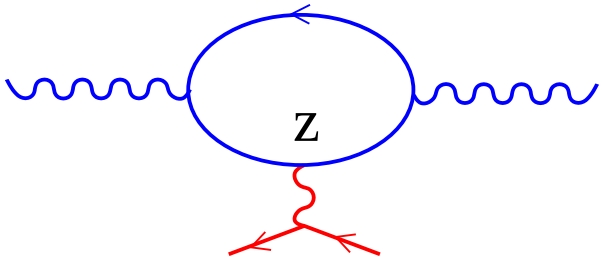
$$\left.-\,i\int_{-\infty}^{\infty}dz_*\,([\infty_*,z_*]D^iG_{\bullet i}(z_*,x_\perp)[z_*,-\infty])^{ab}\,\frac{x_*^{2-r}+(-y)_*^{2-r}-(x-y)_*^{2-r}}{2-r}\right)$$

Term $\sim (z-z')^{2-r}$ = contribution from large $p_\perp^2 \sim M^2_{\rm reg}$ $\Rightarrow$

an additional vertex  $\sim [GG]$

A typical term of the light-cone expansion





$$\langle A^a_\bullet(x_*,x_\perp)A^b_\bullet(y_*,y_\perp)\rangle_A = \frac{1}{8\pi^2}\int_0^\infty \frac{d\alpha}{\alpha^3}(x_\perp|e^{-i\frac{p_\perp^2}{\alpha s}(x-y)_*}|y_\perp)\,\frac{\Gamma(r)\Gamma(r-1)}{\Gamma(2r-1)}$$

$$\times\Bigg(\int_{-\infty}^{\infty}dz_*\int_{-\infty}^{z_*}d'_*([\infty,z_*]G_{\bullet i}(z_*,x_\perp)[z_*,z'_*]G^i_\bullet(z'_*,x_\perp)[z',-\infty])^{ab}$$

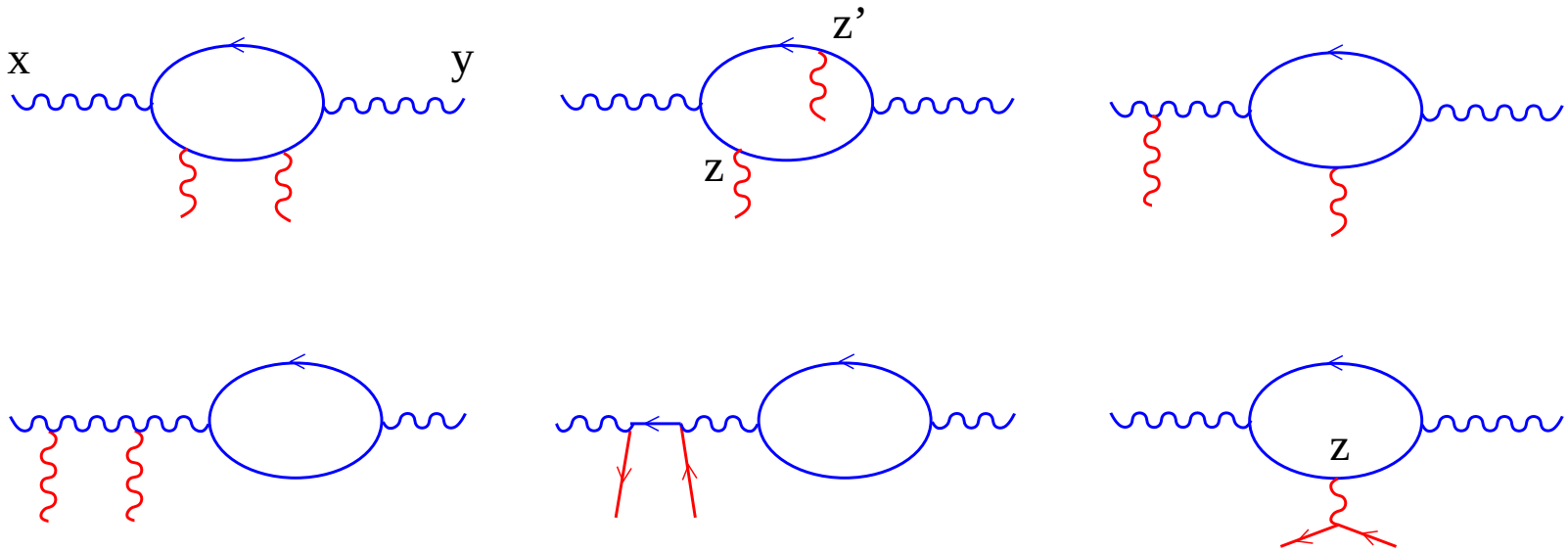
$$\times\left\{\frac{1}{(2-r)}[x_*^{2-r}+(-y)_*^{2-r}-(x-y)_*^{2-r}-(z-z')_*^{2-r}]+\frac{(z-z')_*^{2-r}}{r+1}\right\}$$

$$-i\int_{-\infty}^{\infty}dz_*\,([\infty_*,z_*]D^iG_{\bullet i}(z_*,x_\perp)[z_*,-\infty])^{ab}\,\frac{x_*^{2-r}+(-y)_*^{2-r}-(x-y)_*^{2-r}}{2-r}\Bigg)$$

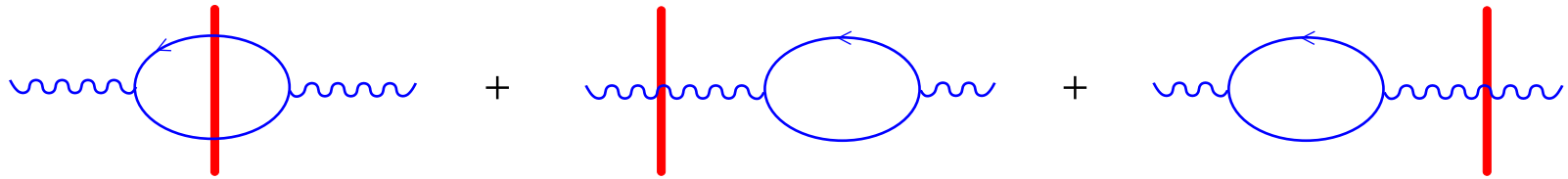
In the sum of all diagrams

- All terms $\sim (z-z')^{2-r}$ cancel
- $-i[DG]+2[GG]$ form $\partial_\perp^2 U$

The light-cone expansion of the sum of the diagrams

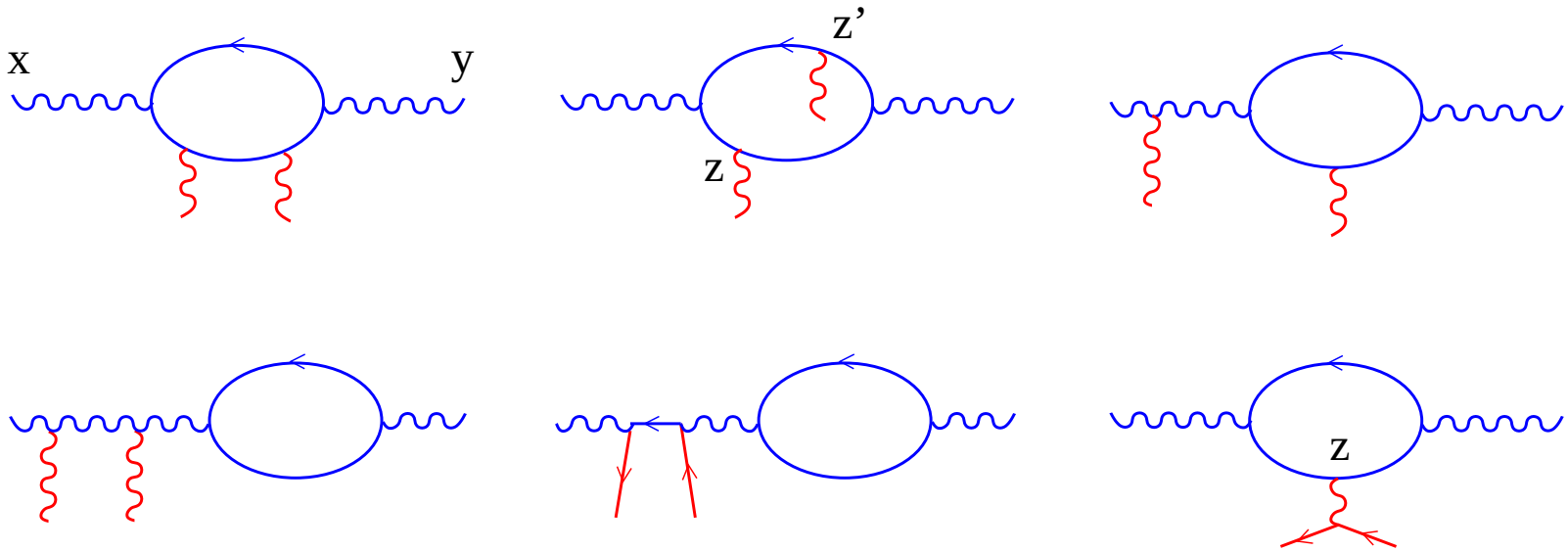


coincides with the expansion of

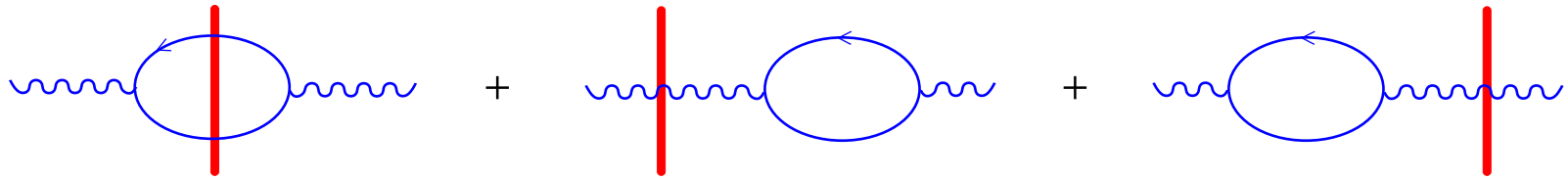


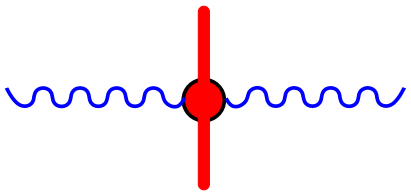
$\Rightarrow$

The light-cone expansion of the sum of the diagrams

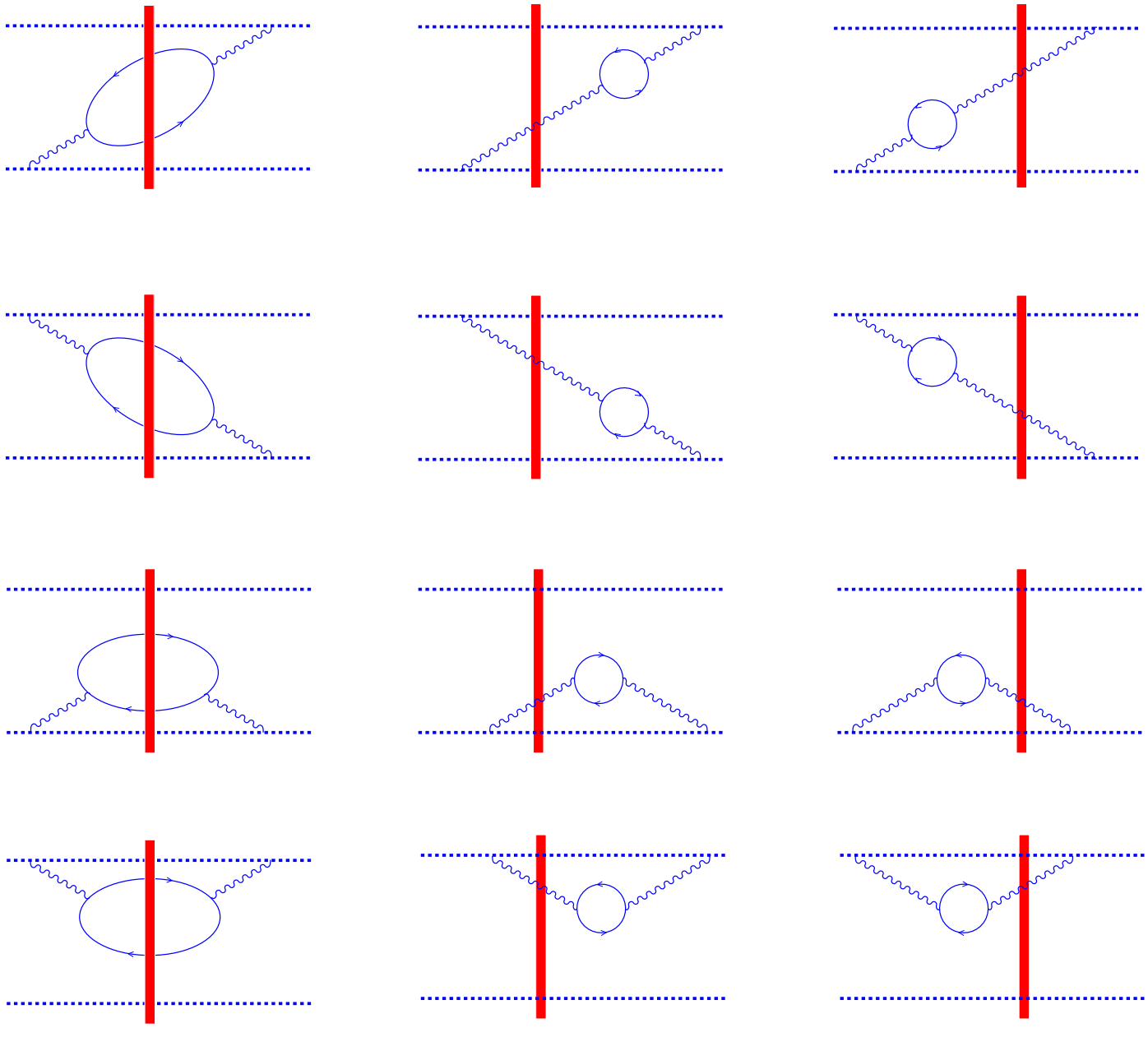


coincides with the expansion of



$\Rightarrow$ no additional vertex  at the one−loop level

Diagrams for the dipole evolution



Quark-loop contribution to NLO BK

$$\frac{d}{d\eta}\mathrm{Tr}\{U_x U_y^\dagger\} = \frac{\alpha_s}{2\pi^2}\int d^2z\,[\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_y^\dagger\} - N_c\mathrm{Tr}\{U_x U_y^\dagger\}]$$

$$\times\left[\frac{(x-y)^2}{X^2Y^2}\left(1-\tfrac{\alpha_s n_f}{6\pi}[\ln(x-y)^2\mu^2+\frac{5}{3}]\right)+\frac{\alpha_s n_f}{6\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2}\right]$$

$$+\frac{\alpha_s^2}{\pi^4}n_f\mathrm{Tr}\{t^a U_x t^b U_y^\dagger\}\int d^2z d^2z'\,\mathrm{Tr}\{t^a U_z t^b U_{z'}^\dagger - t^a U_z t^b U_z^\dagger\}\,\frac{1}{(z-z')^4}$$

$$\times\left\{1-\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\right\}$$

$$X = x-z,\;\; X' = x-z',\;\; Y = y-z,\;\; Y' = y-z'$$

Quark-loop contribution to NLO BK

$$\frac{d}{d\eta}\mathrm{Tr}\{U_x U_y^\dagger\} = \frac{\alpha_s}{2\pi^2}\int d^2z\,[\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_y^\dagger\} - N_c\mathrm{Tr}\{U_x U_y^\dagger\}]$$

$$\times\left[\frac{(x-y)^2}{X^2Y^2}\left(1-\frac{\alpha_s n_f}{6\pi}[\ln(x-y)^2\mu^2+\frac{5}{3}]\right)+\frac{\alpha_s n_f}{6\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2}\right]$$

$$+\frac{\alpha_s^2}{\pi^4}n_f\mathrm{Tr}\{t^a U_x t^b U_y^\dagger\}\int d^2z d^2z'\,\mathrm{Tr}\{t^a U_z t^b U_{z'}^\dagger - t^a U_z t^b U_z^\dagger\}\,\frac{1}{(z-z')^4}$$

$$\times\left\{1-\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\right\}$$

$$X = x-z,\;\; X' = x-z',\;\; Y = y-z,\;\; Y' = y-z'$$

$$\alpha_s(\mu)[1-\frac{\alpha_s n_f}{6\pi}\ln(x-y)^2\mu^2]\;+$$

Quark-loop contribution to NLO BK

$$\frac{d}{d\eta}\mathrm{Tr}\{U_x U_y^\dagger\} = \frac{\alpha_s}{2\pi^2}\int d^2z\,[\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_y^\dagger\} - N_c\mathrm{Tr}\{U_x U_y^\dagger\}]$$

$$\times\left[\frac{(x-y)^2}{X^2Y^2}\left(1-\tfrac{\alpha_s n_f}{6\pi}[\ln(x-y)^2\mu^2+\frac{5}{3}]\right)+\frac{\alpha_s n_f}{6\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2}\right]$$

$$+\frac{\alpha_s^2}{\pi^4}n_f\mathrm{Tr}\{t^a U_x t^b U_y^\dagger\}\int d^2z d^2z'\,\mathrm{Tr}\{t^a U_z t^b U_{z'}^\dagger - t^a U_z t^b U_z^\dagger\}\,\frac{1}{(z-z')^4}$$

$$\times\left\{1-\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\right\}$$

$$X = x-z,\;\; X' = x-z',\;\; Y = y-z,\;\; Y' = y-z'$$

$$\alpha_s(\mu)[1-\tfrac{\alpha_s n_f}{6\pi}\ln(x-y)^2\mu^2] \;+\; \text{gluon loop (work in progress)}$$

Quark-loop contribution to NLO BK

$$\frac{d}{d\eta}\mathrm{Tr}\{U_x U_y^\dagger\} = \frac{\alpha_s}{2\pi^2}\int d^2z\,[\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_y^\dagger\} - N_c\mathrm{Tr}\{U_x U_y^\dagger\}]$$

$$\times\left[\frac{(x-y)^2}{X^2Y^2}\left(1-\tfrac{\alpha_s n_f}{6\pi}[\ln(x-y)^2\mu^2+\frac{5}{3}]\right)+\frac{\alpha_s n_f}{6\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2}\right]$$

$$+\frac{\alpha_s^2}{\pi^4}n_f\mathrm{Tr}\{t^a U_x t^b U_y^\dagger\}\int d^2z d^2z'\,\mathrm{Tr}\{t^a U_z t^b U_{z'}^\dagger - t^a U_z t^b U_z^\dagger\}\,\frac{1}{(z-z')^4}$$

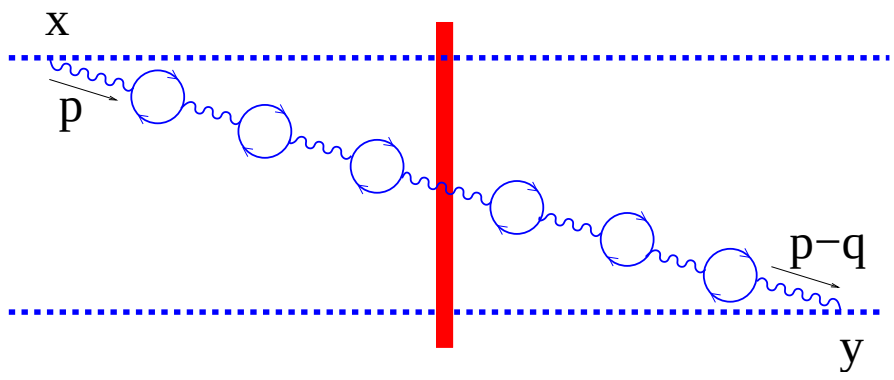
$$\times\left\{1-\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\right\}$$

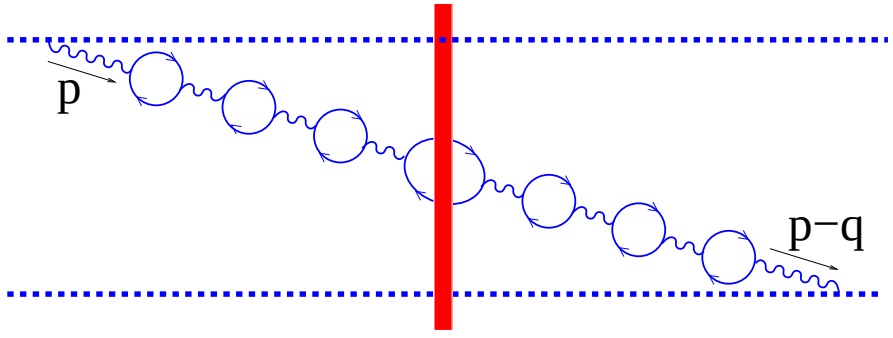
$$X = x-z,\;\; X' = x-z',\;\; Y = y-z,\;\; Y' = y-z'$$

$$\alpha_s(\mu)[1-\tfrac{\alpha_s n_f}{6\pi}\ln(x-y)^2\mu^2] \;+\; \text{gluon loop (work in progress)}$$

$$= \alpha_s(\mu)[1+\frac{b_0\alpha_s}{4\pi}\ln(x-y)^2\mu^2] \;=\; \alpha_s(|x-y|)$$

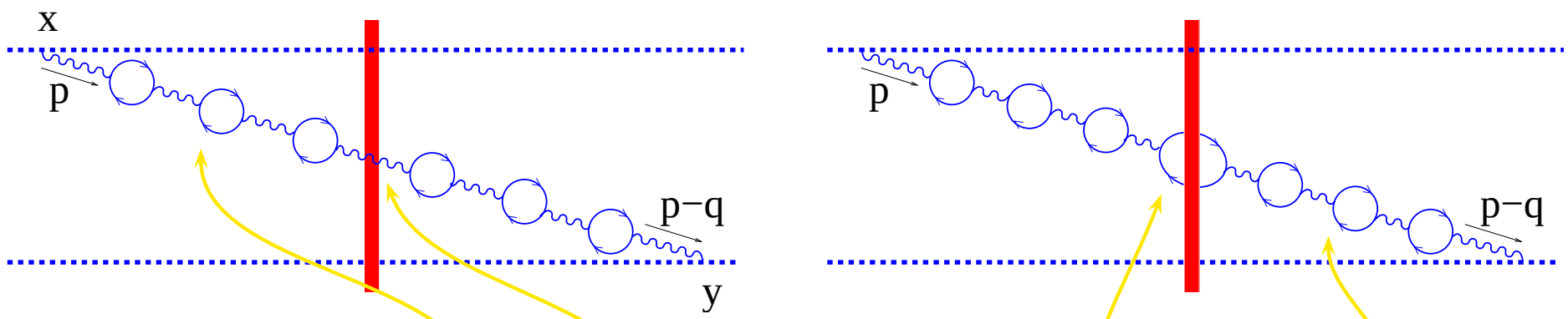
Bubble chain and the argument of coupling constant





Leading log accuracy: $\alpha(\mu) \ll 1$, $\alpha(\mu) \ln \frac{p_\perp^2}{\mu^2} \sim 1$

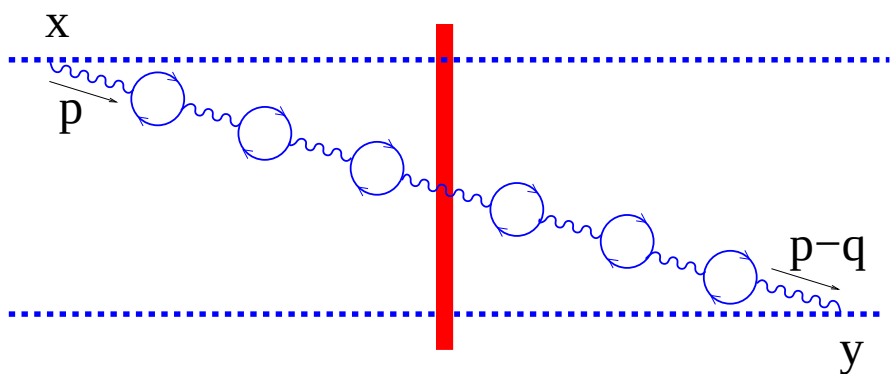
Bubble chain and the argument of coupling constant

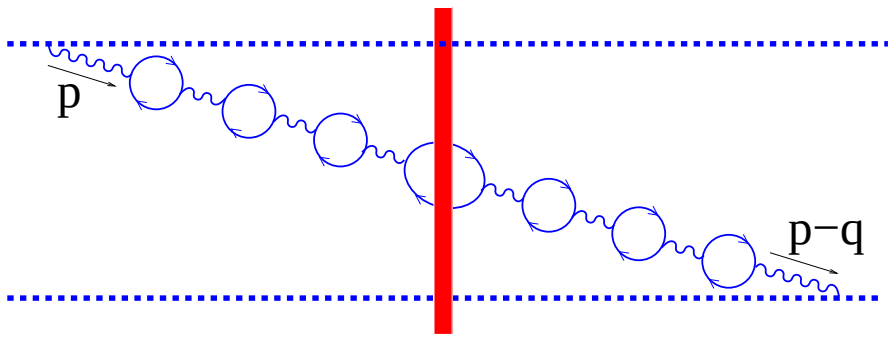


$$LLA \;\Rightarrow\; \frac{\alpha_s(p_\perp^2)}{p_\perp^2}\left[\frac{1}{\alpha_s(\mu^2)} + \left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)\ln\frac{q_\perp^2}{\mu^2}\right]\frac{\alpha_s(p-q)_\perp^2}{(p-q)_\perp^2}$$

$$= \frac{\alpha_s(p_\perp^2)}{p_\perp^2}\,\frac{1}{\alpha_s(q_\perp^2)}\,\frac{\alpha_s(p-q)_\perp^2}{(p-q)_\perp^2}$$

Bubble chain and the argument of coupling constant





$$LLA \Rightarrow \frac{\alpha_s(p_\perp^2)}{p_\perp^2}\left[\frac{1}{\alpha_s(\mu^2)} + \left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)\ln\frac{q_\perp^2}{\mu^2}\right]\frac{\alpha_s(p-q)_\perp^2}{(p-q)_\perp^2}$$

$$= \frac{\alpha_s(p_\perp^2)}{p_\perp^2}\,\frac{1}{\alpha_s(q_\perp^2)}\,\frac{\alpha_s(p-q)_\perp^2}{(p-q)_\perp^2}$$

Fourier transformation (up to $\ln^2$ accuracy) $\Rightarrow$

$$\frac{d}{d\eta}\mathrm{Tr}\{U_xU_y^\dagger\} = \frac{\alpha_s(|x-y|)}{2\pi^2}\int d^2z\,[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_y^\dagger\} - N_c\mathrm{Tr}\{U_xU_y^\dagger\}]$$

$$\times\left[\frac{(x-y)^2}{X^2Y^2} + \frac{1}{X^2}\left(\frac{\alpha_s(X^2)}{\alpha_s(Y^2)} - 1\right) + \frac{1}{Y^2}\left(\frac{\alpha_s(Y^2)}{\alpha_s(X^2)} - 1\right)\right]$$

Conclusions

- High-energy scattering can be described in terms of dipoles (Wilson lines) $U_x U_y^\dagger$ - no new operators at the 1-loop level.
- For the creation of dipoles by the small-x evolution, the coupling constant is determined by the size of the *parent* dipole

Conclusions

- High-energy scattering can be described in terms of dipoles (Wilson lines) $U_x U_y^\dagger$ - no new operators at the 1-loop level.
- For the creation of dipoles by the small-x evolution, the coupling constant is determined by the size of the *parent* dipole

Outlook

- Take into account the gluon (and ghost) loops - hopefully the conclusions will survive