BFKL Pomeron: modeling confinement *

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Abstract

In this talk we introduce the confinement into the kernel of the BFKL equation, assuming that the sizes of produced dipoles cannot be large. The goal of this talk is to share with you our study how this assumption, which leads to a correct exponential decrease of the amplitude at large impact parameters, affects the main properties of the BFKL Pomeron. We solve the equations for total cross section and $\langle |b^2| \rangle$ numerically and developed some methods of analytical solutions. The main result is that the modified BFKL Pomeron has the same intercept and $\alpha'_{I\!P} = 0$ as the BFKL Pomeron. It gives us a hope that the unknown confinement will change only slightly the equations of the CGC/saturation approach

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1 Introduction

In this talk we discuss briefly the results of our paper (see Ref.[1]) in which we address the but still unsolved problem in the CGC/saturation approach[2] : large impact parameter dependence of the scattering amplitude.It was shown in Ref.[3] that CGC/saturation approach that leads to the partial amplitude smaller than unity and satisfies the unitarity constraints, generates the radius of interaction that increases as a power of energy in explicit contradiction to the Froissart theorem[4]. It stems from large b behaviour of the BFKL Pomeron which has the form: $A\left(b \gg 1/Q_s\right) \propto s^{\Delta}/b^2$. Amplitude $A\left(b \gg 1/Q_s\right)$ becomes of the order of unity at typical $b^2 \propto s^{\Delta}$ leading to $\sigma \propto s^{\Delta}$ in the contradiction to the Froissart theorem ($\sigma \propto \ln^2 s$). The power-like

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dependence of the scattering amplitude is a direct consequence of the perturbative QCD technique which is a part of the CGC/saturation approach. Since the lightest hadron (pion) has a finite mass (m_π) we know that the amplitude is proportional to $\exp\left(-2m_\pi\, b\right)$ at large b. This exponential behaviour translates into Froissart theorem. Therefore, we have to find how confinement of quarks and gluons being of non-perturbative nature, will change the large b behaviour of the scattering amplitude in the region where this amplitude is small. Without solving this problem the CGC/saturation approach cannot be considered as self consistent effective theory for high energy QCD.

During the past decade numerous attempts (see our paper [1] for references) have been made to solve this problem without decisive result. However we learned several lessons from these tries: (i) the confinement of quarks and gluon have to be included in the BFKL kernel (to include in the initial conditions is not enough); (ii) suppressing large sizes of the produced dipoles in the decay of one dipole to two dipoles we reproduce correct b-dependence; and since at large b the amplitude is small we do not need to take into account the non-linear corrections.

Therefore, corrections from confinement have to be included in the kernel of the BFKL equation:

$$\begin{aligned}\frac{\partial N\left(x_{10},b,Y\right)}{\partial Y} &= \bar{\alpha}_S \int d^2 x_{12} K\left(x_{12},x_{20}|x_{10}\right)\\ &\times\Big\{2\,N\left(x_{12},\vec{b}-\frac{1}{2}\vec{x}_{02};Y\right)\;-\;N\left(x_{10},b;Y\right)\Big\}\end{aligned}\tag{1}$$

where kernel K describe the decay of one dipole to two dipoles ($x_{10} \to x_{12}+x_{02}$). We solve this equation with the modified BFKL kernel:

$$K\left(x_{12},x_{20}|x_{10}\right) \;=\; \frac{x^2_{10}}{x^2_{12}\,x^2_{02}}\,e^{-B\left(x^2_{12}\,+\,x^2_{02}\right)}\tag{2}$$

The motivation for this behaviour of the wave function of a dipole in the confinement region stems from the Gaussian-like form of the wave functions of mesons in holographic AdS/QCD approach as well as in the phenomenology of the gluon emission at long distances. However, we will argue in conclusions that the main results of this paper do not depend on the particular form of Eq. (2).

2 Summary of the results

Since the lack of the room in this short version of the talk we start with the discussion of the main results of the paper.

1. The scattering amplitude
   $N\left(x_{10},b,Y\right) \xrightarrow{Bb^2\,\gg\,1} e^{-4B\,b^2}$ (expected);

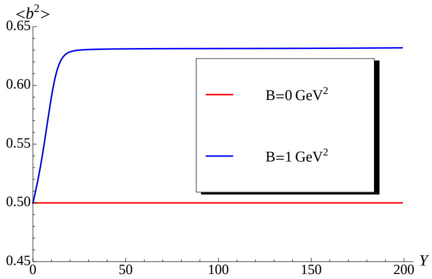

Figure 1: $\langle |b^2| \rangle$ versus Y for the modified (upper curve) and for the bare BFKL pomeron

2. $N\left(x_{10}, b, Y\right) \xrightarrow{Y \gg 1} e^{\omega_0 Y}$ with $\omega_0 = \omega_{\rm BFKL}$ (unexpected !!!);
3. $\langle |b^2| \rangle =$ Constant (Y) (expected);
4. Saturation scale $Q_s^2 \propto e^{\lambda Y}$
   $\lambda = \lambda_{\rm BFKL}$ (expected);
5. The modified BFKL Pomeron looks similar to the Pomeron in N=4 SYM[5] and in high energy phenomenology[6]: $\Delta_{I\!P} \sim 0.3$; $\alpha'_{I\!P} = 0$ (unexpected).

We postpone the discussion of the point 2 to the next section and briefly outline the results which were expected.

Large b behaviour of the amplitude. The large b behaviour of the amplitude, we can derive directly from Eq. (1) . Indeed, one can see that the main contribution at large b stems from the region where $|\vec{b} - \vec{x}_{12}| \leq x_{10}$. At such x_{12} the equation takes the form

$$\frac{\partial N\left(x_{10} \approx 2b, b; Y\right)}{\partial Y} = \qquad (3)$$
$$4\bar{\alpha}_S \int \frac{d^2 x_{12}}{4 b^2} e^{-4\,B\,b^2 - B\,x^2_{12}}\, 2\, N\left(x_{12}, 0; Y\right) \;\propto\; e^{-4\,B\,b^2}$$

$\langle |b^2| \rangle$ **dependence versus Y.** The general origin of the energy dependence of $\langle |b^2| \rangle$ was found by Gribov[7] (Gribov's diffusion) and it stems from the uncertainty principle, that at each emission the shift in impact parameter (Δb) is proportional to $1/p_T$ where p_T is the average transverse momentum of emitted parton (gluon). In the parton model p_T is a dimensional scale of the model that does not depend on energy (rapidity) of the parton. In this case after n emission the average shift in b is equal to $\langle b^2 \rangle_n \propto \Delta b^2\, n$ and since $n \propto \ln s$ (s is the energy) we have the well known result that the interaction radius $R^2 \propto \ln s$. However, in QCD p_T increases with Y (energy) and after first several emission it becomes so large that we can neglect that shift in b. Our numerical calculation shows that, indeed, $\langle |b^2| \rangle$ is constant at large Y (see Fig. 1). **Saturation scale.** The energy behaviour of the new dimensional scale (saturation moment) can be found from the linear equation (see Ref.[2]). This scale is the solution of the equation

$$\frac{4}{Q^2_s\left(Y\right)}\, N\left(\frac{2}{Q_s\left(Y\right)}; Y\right) \;=\; N_0 \;\leq\; 1 \qquad (4)$$

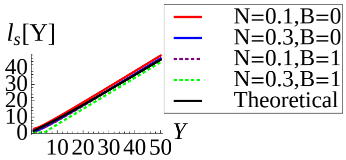

Figure 2: $l_s\left(Y\right) \;=\; \ln\left(Q^2_s(Y)\Big/Q^2_s(Y_0)\right)$, where $Q_s\left(Y\right)$ is the solution to Eq. (4), versus Y for different values of N_0. $\bar{\alpha}_S = 0.2$. The red lines correspond to solution of the BFKL equation, the solid dark brown line describes the theoretical predictions for the saturation scale for the BFKL equation while the blue line is the solution to Eq. (4) for the modified BFKL equation.

where N_0 is a constant. Fig. 2 shows that the modified BFKL equation generates the same saturation momentum which is close to the theoretical predictions.

3 The intercept of the modified BFKL Pomeron

In this section we wish to discuss the main result of our approach: the intercept of the modified BFKL Pomeron turns out to be the same as the BFKL Pomeron. Going to ω-representation ($N\left(x_{01};Y\right) \;=\; \int \frac{d\omega}{2\pi i} e^{\omega Y}\, N_\omega\left(x_{01}\right)$) one can see that the modified BFKL equation takes the form

$$\omega N_\omega\left(x_{01}\right) \;=\; -\,\bar{\alpha}_S \mathcal{H}\, N_\omega\left(x_{01}\right) \tag{5}$$

or

$$E\, N_\omega\left(x_{01}\right) \;=\; \mathcal{H}\; N_\omega\left(x_{01}\right) \tag{6}$$

For finding the eigenfunctions and eigenvalues of Eq. (6) we need to specify the boundary conditions. At short distances Eq. (2) has the same form as the BFKL kernel and, therefore, we have

$$N_\omega\left(x_{01}\right) \xrightarrow{x^2_{01} \ll 1/B} N^{BFKL}_\nu\left(x_{01}\right) \;=\; \left(\frac{1}{x^2_{01}}\right)^{\frac{1}{2}+i\nu} \tag{7}$$

Note that the BFKL spectrum is

$$E\left(\nu\right) \;=\; 2\psi\left(1\right) - \psi\left(-\frac{1}{2} + i\nu\right) - \psi\left(\frac{1}{2} - i\nu\right) \tag{8}$$

For long distances the confinement modification of the kernel does not allow for dipoles to have large sizes leading to the boundary condition

$$N_\omega\left(x_{01}\right) \xrightarrow{x^2_{01} \geq 1/B} \text{Constant} \tag{9}$$

Theory estimates.

Variational method. It is well known that the energy of the ground state is less or equal to

$$E_{\text{ground}} \equiv -\omega_0 \;\;\leq\;\; F[\{N\}] \tag{10}$$

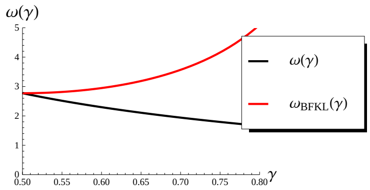

Figure 3: Comparison of $\omega\left(\gamma\right) = \frac{1}{2} + i\nu = F_{min}[\{N\}]$ with ω_{BFKL} given by Eq. (8).

where $F[\{N\}]$ is the following functional

$$F[\{N\}] = \frac{\left\langle N^*(x_{01}) \middle| \mathcal{H} \middle| N(x_{01}) \right\rangle}{\left\langle N^*(x_{01}) \middle| N(x_{01}) \right\rangle}$$

Choosing the set of the BFKL functions of Eq. (7) which is the complete and normalized set of functions, we find that $F_{min}[\{N\}] = \omega_{NFKL} = 4\ln 2\,\bar{\alpha}_S($ see Eq. (8) at $\nu = 0$ (see Fig. 3)

Therefore, the intercept of the modified BFKL equation could be only larger or equal to the intercept of the BFKL one.

In our paper [1] we developed two theoretical approaches: *semi-classical approach* and *diffusion approximation* which show that the modified BFKL equation does not have an intercept larger than the BFKL one. However, due to lack of room we cannot discuss these approaches here but they have been considered in our paper[1].

Numerical calculations.

Solving numericall we face two problems:

1. The kernel is not Fredholm type

$$\int d^2x_{01} d^2x_{12}\, K\left(x_{12}, x_{02} | x_{01}\right) \implies \infty$$

2. The kernel is singular at $x_{12} \quad \rightarrow \quad x_{01}$

We use the following checks of our numerical procedure:

1. Independence on the choice of x_{min} and x_{max};
2. Numerical solution to the BFKL equation coincide with the analytic one;
3. Independence on value of the regulator R;

$$\int d^2x_{13} K^B_R\left(x_{12}, x_{02} | x_{10}\right)\, N\left(x_{12}; Y\right) \equiv$$
$$\int d^2x_{12}\, \frac{e^{-B\left(x^2_{12} + x^2_{02}\right)}}{x^2_{12} + R^2} \Big\{ 2\,N\left(x_{12}; Y\right) - 2\,\frac{x^2_{10}}{x^2_{12} + x^2_{02} + 2\,R^2} N\left(x_{10}; Y\right) \Big\}$$

4. Independence on value of B;

The result is plotted in Fig. 4.

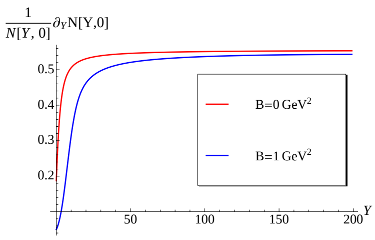

Figure 4: $d\ln N/dY$ versus Y for the BFKL (B=0) and modified BFKL (B=1) equations.
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4 Conclusions

We found out that the modified BFKL Pomeron has the same intercept Δ as the BFKL Pomeron ($\Delta = \omega_{\mathrm{BFKL}} = 4\ln 2\bar{\alpha}_S$) and $\alpha'_{I\!P} = 0$. Therefore, the BFKL Pomeron with the modified kernel reproduces the main features of the soft Pomeron that has been found both from N=4 SYM theory[5] and from the high energy Reggeon phenomenology[6].

Actually, we were surprised that the model for confinement changed so little in the BFKL Pomeron and on qualitative level, the Pomeron that emerges from the modified BFKL equation, looks quite the same at the BFKL Pomeron, both in parameters and in character of the energy behaviour. It seems that the only difference between the BFKL Pomeron and the modified BFKL Pomeron is that the second has a correct large impact parameter behaviour.

We believe that this statement does not depend on the particular form of Eq. (2). As we have mentioned the spectrum of the modified BFKL equation is determined by Eq. (6) with two boundary conditions of Eq. (7) at short distances and Eq. (9) at long distances. At first sight the condition at long distances will restrict the values of ν in the comparison with the BFKL equation. However, it is not the case. Indeed, the eigenvalues of the BFKL equation is degenerate having two eigenfunctions with positive and negative ν. One can see that we can find the sum of these two eigenfunction ($N(x_{12},\omega) = \left(1/x^2_{12}\right)^{\frac{1}{2}} \sin\left(\nu \ln\left(1/(Bx^2_{12})\right) + \phi_0\right)$ which is equal to constant $(\sin\phi_0)$ at $x_{12} = 1/B$. Therefore, at any ν we can satisfy the condition of Eq. (9). On the other hand for $\nu = i\kappa$ ($\kappa > 0$) we have two eigenfunctions: $\left(x^2_{12}\right)^{-\frac{1}{2}-\kappa}$ and $\left(x^2_{12}\right)^{-\frac{1}{2}+\kappa}$. The normalization condition selects out the only eigenfunction $\left(x^2_{12}\right)^{-\frac{1}{2}+\kappa}$ which has no divergency at $x_{12} \to 0$. Using this function we cannot satisfy the condition at $x_{12} \to \infty$ and, therefore, we have no solution of Eq. (6) for $\nu = i\kappa$. Hence, we expect that the spectrum for the modified Hamiltonian will be the same as the BFKL spectrum.

The independence of the spectrum of the BFKL Pomeron on the models for the confinement gives us a hope that the unknown confinement will change only slightly the equations of the CGC/saturation approach and these changes will not depend on the particular way of taking into account the long distances physics. In simple words, this paper gives a hope that the CGC/saturation approach will be still a theory in spite of needed model modifications due to confinement.

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References

[1] E. Levin and S. Tapia, *"BFKL Pomeron: modeling confinement,"*, JHEP *(in press)*, arXiv:1304.8022 [hep-ph],

[2] Yuri V Kovchegov and Eugene Levin, *" Quantum Choromodynamics at High Energies"*, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, Cambridge University Press, 2012 and references therein.

[3] A. Kovner and U. A. Wiedemann, Phys. Rev. D **66**, 051502 (2002) [hep-ph/0112140].

[4] M. Froissart, Phys. Rev. **123** (1961) 1053.

[5] R. C. Brower, J. Polchinski, M. J. Strassler and C. I. Tan, JHEP **0712** (2007) 005 [arXiv:hep-th/0603115].

[6] E. Gotsman, E. Levin and U. Maor, Phys. Lett. B **716** (2012) 425, [arXiv:1208.0898 [hep-ph]] and references therein.

[7] V. N. Gribov, *"Space-time description of hadron interactions at high-energies,"* hep-ph/0006158; Sov. J. Nucl. Phys. **9** (1969) 369 [Yad. Fiz. **9** (1969) 640].