On the evolution equation for the 3-quark Wilson loop operator *

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Abstract

A nonlinear evolution equation for the 3 quark Wilson loop operator is presented. The calculation of its leading order (LO) kernel and the connected contribution to its next to leading order (NLO) kernel is discussed. It is argued that the connected contribution in the C-odd case after the linearization and transfer to the momentum space does not coincide with the recently obtained 3→3 contribution to the odderon kernel.

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1 Introduction

Theoretical study of scattering in the Regge limit is based mainly on the Balitsky - Fadin - Kuraev - Lipatov (BFKL) approach [1], dipole picture and high energy operator expansion, resulting in the Balitsky - Kovchegov (BK) equation [2, 3], and the JIMWLK functional integral formalism [4]. While the BK equation is written for the color dipole Green function, the BFKL one describes the scattering of arbitrary particles in the linear limit. The easiest object more general than the color dipole is the 3 quark Wilson loop (3QWL) operator $\varepsilon^{i'j'h'}\varepsilon_{ijh}U^i_{1i'}U^j_{2j'}U^h_{3h'}$. This object may model a baryon in the Regge limit.

The first studies of the evolution equation for a baryon Green function were done in [5], where the linear evolution was studied. It was shown that at each gluon emission new color structures appear, which makes it difficult to write an evolution equation in a closed form. Within the JIMWLK approach the C-odd exchange with a baryon initial state was studied in [6]. There the linear evolution equation for the three point C-odd baryon Green function was worked out. Later in [7] the nonlinear evolution equation and impact factors for baryon scattering were obtained.

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This presentation summarizes the studies [8, 9], where the LO evolution equation for the 3QWL and the NLO connected contribution to its kernel were calculated within Balitsky high energy OPE [2, 10].

2 LO equation for 3QWL

We use the light cone variables n_1 and n_2

$$n_1 = (1, 0, 0, 1)\,, \quad n_2 = \frac{1}{2}(1, 0, 0, -1)\,, \quad n_1^+ = n_2^- = n_1 n_2 = 1 \tag{1}$$

and for any vector p have

$$p = p^+ n_1 + p^- n_2 + p_\perp, \qquad p^2 = 2p^+ p^- - \vec{p}^{\,2}, \tag{2}$$

$$p\,k = p^\mu k_\mu = p^+ k^- + p^- k^+ - \vec{p}\vec{k} = p_+ k_- + p_- k_+ - \vec{p}\vec{k}. \tag{3}$$

We define the 3-quark Wilson loop (3QWL) operator

$$B_{123}^\eta = \varepsilon^{i'j'h'} \varepsilon_{ijh} U(\vec{z}_1, \eta)^i_{i'}\, U(\vec{z}_2, \eta)^j_{j'}\, U(\vec{z}_3, \eta)^h_{h'} \tag{4}$$

and use the following notation for such convolutions

$$\varepsilon^{i'j'h'} \varepsilon_{ijh} U^i_{1i'} U^j_{2j'} U^h_{3h'} = U_1 \cdot U_2 \cdot U_3. \tag{5}$$

Here

$$U(\vec{z}, \eta) = P e^{ig \int_{-\infty}^{+\infty} b_\eta^-(z^+, \vec{z}) dz^+}, \tag{6}$$

and b_η^- is the external shock wave field built from only slow gluons

$$b_\eta^- = \int \frac{d^4 p}{(2\pi)^4} e^{-ipz} b^-(p)\, \theta(e^\eta - p^+). \tag{7}$$

The rapidity parameter η separates the slow gluons entering the Wilson lines from the fast ones in the impact factors and the coordinates $\vec{z}_i$ are the points in the transverse plane where the quarks intersect the shockwave. The shape of the path at $z^+ = \pm\infty$ in (6) is not important because the field is concentrated at $z^+ = 0$

$$b^\mu(z) = b^-(z^+, \vec{z}) n_2^\mu = \delta(z^+) b(\vec{z})\, n_2^\mu. \tag{8}$$

Therefore one can connect the three lines in (4) in one point x at $z^+ = +\infty$ and in one point y at $z^+ = -\infty$.

The gluon propagator in the shock wave background reads

$$G^\eta_{\mu\nu}(x, y)^{ab}|_{x^+>0>y^+}$$

$$= -\int \frac{dp^+}{(2\pi)^3} \frac{p^+ \theta(p^+)}{2x^+ y^+} \int d\vec{z} e^{-ip^+\left\{x^- - y^- + \frac{(\vec{z}-\vec{y})^2 + i0}{2y^+} - \frac{(\vec{x}-\vec{z})^2 + i0}{2x^+}\right\}}$$

$$\times \frac{g_{\perp\mu}^{\ \ \alpha} x^+ - (x-z)_\perp^\alpha n_{2\mu}}{x^+} U^{ab}(\vec{z},\eta) \frac{g_{\perp\alpha\nu}(-y^+) - (z-y)_{\perp\alpha} n_{2\nu}}{-y^+}. \tag{9}$$

To derive the evolution equation we have to change $\eta \to \eta + \Delta\eta$

$$b^-_{\eta_1} = b^-_{\eta_2} + b^-_{\Delta\eta}, \quad b^-_{\Delta\eta}(z^+,\vec{z}) = \int \frac{d^4p}{(2\pi)^4} e^{-ipz} b^-\,(p)\,\theta(e^{\eta_1} - p^+)\theta(p^+ - e^{\eta_2}). \tag{10}$$

Therefore we have to include the gluons with $\eta_1 > \ln p^+ > \eta_2$ into the Wilson lines, namely

$$\Delta B^{\eta_1}_{123} = \frac{\langle 0|T(B^{\Delta\eta}_{123} e^{i\int \mathcal{L}(z)dz})|0\rangle}{\langle 0|T(e^{i\int \mathcal{L}(z)dz})|0\rangle}. \tag{11}$$

Calculating this matrix element using (9) and the properties of Wilson lines as $SU(3)$ matrices

$$\varepsilon^{i'j'h'} U^i_{i'} U^j_{j'} U^h_{h'} = \varepsilon^{ijh}, \tag{12}$$

$$\varepsilon_{ijh}\varepsilon^{i'j'h'} U^i_{i'} U^j_{j'} = 2(U^\dagger)^{h'}_h, \quad \varepsilon_{ijh}\varepsilon^{i'j'h'} (U^\dagger)^i_{i'} (U^\dagger)^j_{j'} = 2U^{h'}_h, \tag{13}$$

$$U_i \cdot U_j \cdot U_k = (U_i U^\dagger_l)\cdot(U_j U^\dagger_l)\cdot(U_k U^\dagger_l). \tag{14}$$

$$B^\eta_{iij} = U_i \cdot U_i \cdot U_j = 2tr(U_j U^\dagger_i), \tag{15}$$

one gets the following evolution equation

$$\frac{\partial B^\eta_{123}}{\partial\eta} = \frac{\alpha_s 3}{4\pi^2} \int d\vec{z}_4 \left[\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{41}^{\,2}\vec{z}_{42}^{\,2}} (-B^\eta_{123} + \frac{1}{6}(B^\eta_{144}B^\eta_{324} + B^\eta_{244}B^\eta_{314} - B^\eta_{344}B^\eta_{214})) \right.$$
$$\left. + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right]. \tag{16}$$

Equation (16) has several important properties. First, it has no singularities at $\vec{z}_4 = \vec{z}_{1,2,3}$. Then, this equation changes into the BK equation if two of the three quark coordinates coincide. One can check it straightforwardly using (15). It means that $q\bar{q}$ and q-diquark systems obey the same equation.

3 C-even exchange

To separate the C-odd and the C-even contributions we have to write down the evolution equation for $B^\eta_{\bar{1}\bar{2}\bar{3}}$, i.e. the 3-antiquark Wilson loop operator

$$B^\eta_{\bar{1}\bar{2}\bar{3}} = U^\dagger_1 \cdot U^\dagger_2 \cdot U^\dagger_3, \tag{17}$$

describing antibaryon scattering off the shock wave. One can get such an equation from the equation for B^η_{123} changing all the Wilson lines to their conjugates $U_i \leftrightarrow U^\dagger_i$. The C-even Green function has the following form

$$B^+_{123} = B^\eta_{123} + B^\eta_{\bar{1}\bar{2}\bar{3}} - 12. \tag{18}$$

The operator

$$B^{-}_{123} = B^{\eta}_{123} - B^{\eta}_{\bar{1}\bar{2}\bar{3}} \tag{19}$$

changes its sign under C transformation, hence it describes the C-odd Green function. Rewriting evolution equation (16) in terms of (18) and (19), we have

$$\frac{\partial B^{+}_{123}}{\partial \eta} = \frac{\alpha_s 3}{4\pi^2} \int d\vec{z}_4 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2}\vec{z}_{42}^{\,2}} \Big[B^{+}_{144} + B^{+}_{244} - B^{+}_{344}$$

$$+B^{+}_{134} + B^{+}_{234} - B^{+}_{123} - B^{+}_{124} + \frac{1}{12}\left(B^{+}_{144}B^{+}_{324} + B^{+}_{244}B^{+}_{314} - B^{+}_{344}B^{+}_{214}\right)$$

$$+\frac{1}{12}\left(B^{-}_{144}B^{-}_{324} + B^{-}_{244}B^{-}_{314} - B^{-}_{344}B^{-}_{214}\right)\Big] + (2 \leftrightarrow 3) + (1 \leftrightarrow 3). \tag{20}$$

Pomeron exchange starts from the 2-gluon exchange. One can show that

$$B^{+}_{123} = \frac{1}{2}(B^{+}_{133} + B^{+}_{211} + B^{+}_{322}) + \tilde{B}^{+}_{123}. \tag{21}$$

where $\tilde{B}^{+}_{123}$ works from the 4-gluon exchange. As a result, one can write the linear equation in the 2- and 3-gluon approximations for B^{+}_{123}

$$\frac{\partial B^{+}_{123}}{\partial \eta} = \frac{1}{2}\frac{\partial}{\partial \eta}(B^{+}_{133} + B^{+}_{211} + B^{+}_{322})$$

$$= \frac{\alpha_s 3}{4\pi^2} \int d\vec{z}_4 \left[\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2}\vec{z}_{42}^{\,2}} \left(B^{+}_{144} + B^{+}_{244} - B^{+}_{122}\right) + (2 \leftrightarrow 3) + (1 \leftrightarrow 3)\right], \tag{22}$$

which is a sum of 3 independent BFKL equations.

In further approximations one should take into account $\tilde{B}^{+}_{ijk}$, which has the following evolution equation

$$\frac{\partial \tilde{B}^{+}_{123}}{\partial \eta} = \frac{\alpha_s 3}{4\pi^2} \int d\vec{z}_4 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2}\vec{z}_{42}^{\,2}} \Big[(\tilde{B}^{+}_{134} + \tilde{B}^{+}_{234} - \tilde{B}^{+}_{123} - \tilde{B}^{+}_{124})$$

$$+\frac{1}{24}\left(B^{+}_{144}B^{+}_{322} + B^{+}_{244}B^{+}_{311} - B^{+}_{344}B^{+}_{211}\right)$$

$$+\frac{1}{12}\left(B^{+}_{144}\tilde{B}^{+}_{324} + B^{+}_{244}\tilde{B}^{+}_{314} - B^{+}_{344}\tilde{B}^{+}_{214}\right)$$

$$+\frac{1}{12}\left(B^{-}_{144}B^{-}_{324} + B^{-}_{244}B^{-}_{314} - B^{-}_{344}B^{-}_{214} - B^{-}_{144}B^{-}_{422}\right)\Big] + (2 \leftrightarrow 3) + (1 \leftrightarrow 3). \tag{23}$$

4 C-odd exchange

Evolution equation (16) for the C-odd exchange reads

$$\frac{\partial B^-_{123}}{\partial\eta} = \frac{\alpha_s 3}{4\pi^2}\int d\vec{z}_4 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2}\vec{z}_{42}^{\,2}}\Big[B^-_{423}+B^-_{143}-B^-_{123}$$

$$-B^-_{124}-B^-_{443}+B^-_{424}+B^-_{144}+\frac{1}{12}\left(B^+_{144}B^-_{324}+B^+_{244}B^-_{314}-B^+_{344}B^-_{214}\right)$$

$$+\frac{1}{12}\left(B^-_{144}B^+_{324}+B^-_{244}B^+_{314}-B^-_{344}B^+_{214}\right)\Big]+(2\leftrightarrow 3)+(1\leftrightarrow 3). \tag{24}$$

The linear part of this equation coincides with the result of [6], the authors of which proved it equivalent to Bartels-Kwiecinski-Praszalowicz (BKP) equation [11].

5 Connected contribution to the kernel

The connected part of the NLO kernel comes from the diagrams where all the three Wilson lines have nontrivial evolution. In the NLO evolution equation reads

$$\frac{\partial}{\partial\eta}\langle B^\eta_{123}\rangle = \langle K_{LO}\otimes B^\eta_{123}\rangle + \langle K_{NLO}\otimes B^\eta_{123}\rangle. \tag{25}$$

Here the brackets $\langle\rangle$ denote the calculation in the shock wave background. One can show that the connected contribution with 2 gluons intersecting the shockwave reads

$$\langle K^{conn}_{NLO}\otimes B^\eta_{123}\rangle|_{2g} = \frac{\alpha_s^2}{4\pi^4}\int d\vec{z}_0\int d\vec{z}_4\Big\{(U_2U_0^\dagger U_1)\cdot U_4\cdot(U_0U_4^\dagger U_3)$$

$$+(U_3U_4^\dagger U_0)\cdot U_4\cdot(U_1U_0^\dagger U_2)-\ (1\leftrightarrow 3, 0\leftrightarrow 4)\Big\}$$

$$\times\left[\frac{1}{2\vec{z}_{04}^{\,2}}\frac{(\vec{z}_{10}\vec{z}_{34})}{\vec{z}_{10}^{\,2}\vec{z}_{34}^{\,2}}+\frac{(\vec{z}_{10}\vec{z}_{40})}{\vec{z}_{10}^{\,2}\vec{z}_{40}^{\,2}}\frac{(\vec{z}_{24}\vec{z}_{34})}{\vec{z}_{24}^{\,2}\vec{z}_{34}^{\,2}}\right.$$

$$\left.+\frac{(\vec{z}_{04}\vec{z}_{34})}{\vec{z}_{04}^{\,2}\vec{z}_{34}^{\,2}}\frac{(\vec{z}_{10}\vec{z}_{20})}{\vec{z}_{10}^{\,2}\vec{z}_{20}^{\,2}}-\frac{(\vec{z}_{20}\vec{z}_{10})}{\vec{z}_{02}{}^2\vec{z}_{01}{}^2}\frac{(\vec{z}_{24}\vec{z}_{34})}{\vec{z}_{24}^{\,2}\vec{z}_{34}{}^2}\right]\ln\frac{\vec{z}_{02}^{\,2}}{\vec{z}_{24}^{\,2}}$$

$$+(2\leftrightarrow 1)+(2\leftrightarrow 3). \tag{26}$$

The connected contribution with one gluon intersecting the shockwave reads

$$\langle K^{conn}_{NLO}\otimes B^\eta_{123}\rangle|_{1g} = \frac{\alpha_s^2}{8\pi^3}\int d\vec{z}_0\left[\frac{(\vec{z}_{10}\vec{z}_{20})}{\vec{z}_{10}^{\,2}\vec{z}_{20}^{\,2}}-\frac{(\vec{z}_{30}\vec{z}_{20})}{\vec{z}_{30}^{\,2}\vec{z}_{20}^{\,2}}\right]\ln\frac{\vec{z}_{30}^{\,2}}{\vec{z}_{31}^{\,2}}\ln\frac{\vec{z}_{10}^{\,2}}{\vec{z}_{31}^{\,2}}$$

$$\times\left(B_{100}B_{320}-B_{300}B_{210}\right)+(2\leftrightarrow 1)+(2\leftrightarrow 3). \tag{27}$$

One can Fourier transform these contributions to the momentum space and compare the linearized result for the C-odd case to the $3\to 3$ contribution to the odderon kernel calculated within the BFKL approach [12]. They do not coincide. This means that there should be an equivalence transformation relating the kernels.

6 Summary and outlook

This talk gives a summary of the derivation of the LO evolution equation for the 3QWL operator and the result for the connected part of the NLO kernel for 3QWL. The calculation was done within Balitsky high energy operator expansion formalism [2, 10]. The details of the derivation and the calculation one can find in [8, 9].

The LO evolution equation in the C-odd case coincides with the known linear result [6] equivalent to the BKP equation [11]. As in [7], in the C-even case the Green function for the 3QWL may be rewritten as a sum of 3 dipole Green functions and a new 3-point Green function starting from the 4 gluon exchange.

The expressions for the connected linearized C-odd contribution to the kernel of the 3QWL operator in the momentum space are presented in [9]. They do not coincide with the connected $3 \to 3$ contribution to odderon kernel, obtained in [12] in the momentum representation. This fact indicates that as in the color dipole case [13], there should be an equivalence transformation connecting the whole kernels obtained in the high energy operator expansion formalism and in the formalism based on reggeization.

Moreover, the construction of a matrix element of a gauge invariant operator in the momentum representation from its Mobius form in the coordinate space consists of two steps [14]. First one does the Fourier transform and then adds to the result such terms that restore its gauge invariance but vanish after the convolution with the colorless impact factors. This procedure is to be applied to the whole NLO kernel for 3QWL evolution equation after its calculation.

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