Diffraction and scaling laws in high-energy QCD

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  the dipole picture in deep inelastic scattering (DIS)

- Geometric scaling

  - a consequence of the saturation regime of QCD
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- Diffusive scaling

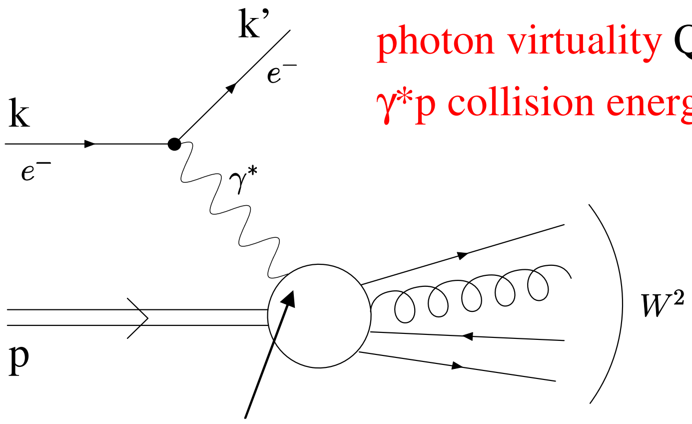
  - stochasticity in high-energy QCD
  - a new high-energy regime: the diffusive scaling regime
  - consequences for the total and diffractive cross-sections in DIS

The dipole picture of DIS

in the high-energy limit: $x \ll 1$

photon virtuality $Q^2 = -(k-k')^2 \gg \Lambda^2_{QCD}$

γ^*p collision energy $W^2 = (k-k'+p)^2$



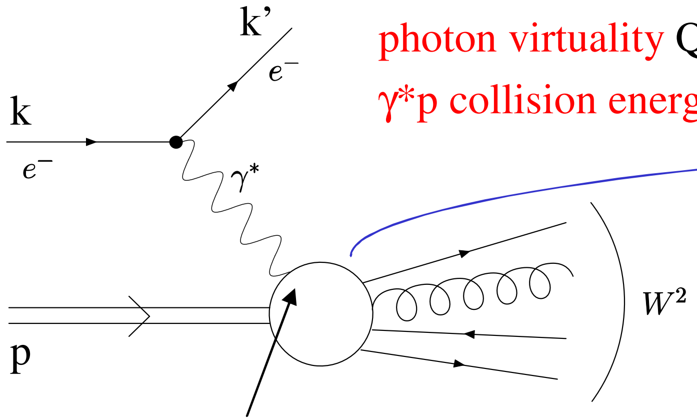
size resolution 1/Q

$$x = \frac{Q^2}{W^2 + Q^2}$$

$Y = \ln(1/x)$: the total rapidity

The dipole picture of DIS

in the high-energy limit: $x << 1$



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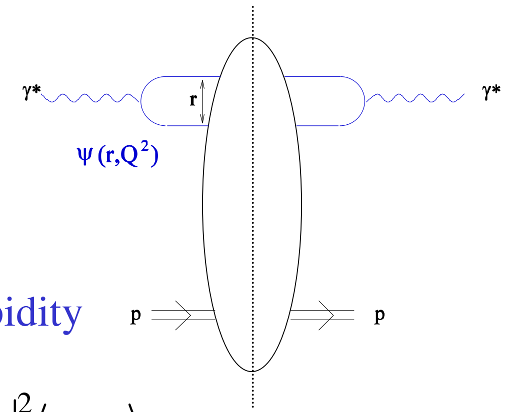

$$\frac{d\sigma_{DIS}}{d^2b} = 2\pi \int dr^2 \left|\psi(r, Q^2)\right|^2 \langle T(r) \rangle_Y$$

r : dipole size

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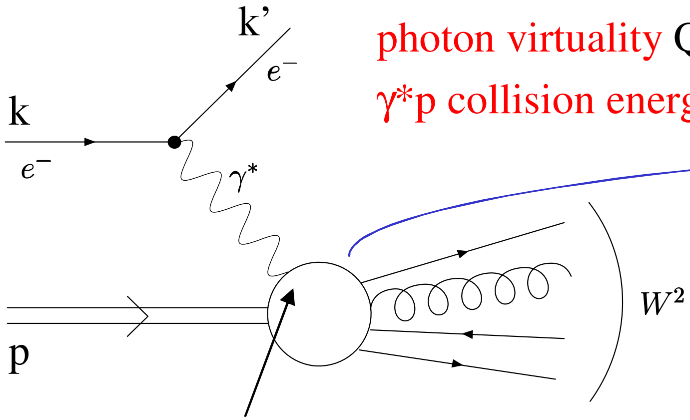

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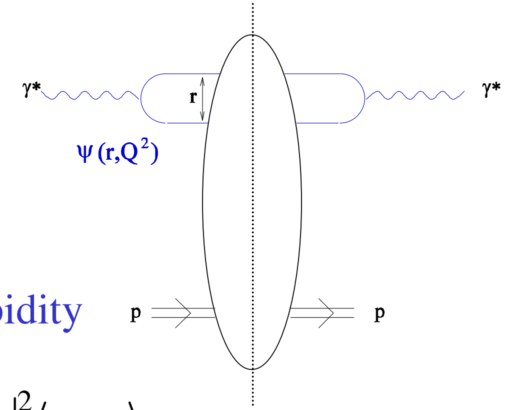

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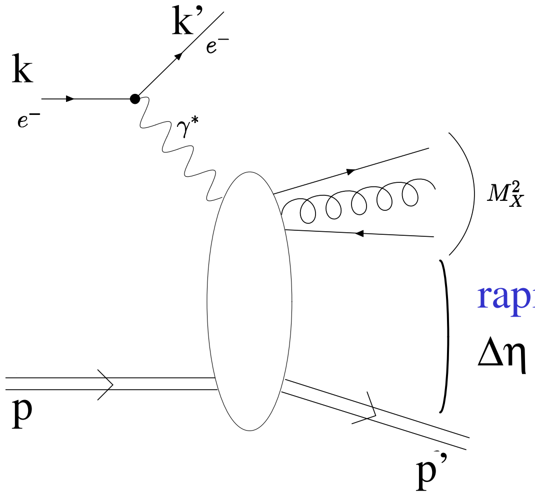
some events are diffractive

diffractive mass of the final state: $M_X{}^2 = (k-k'+p-p')^2$



$$\beta = \frac{Q^2}{M_X^2 + Q^2}$$

$$x_{pom} = x/\beta$$

rapidity gap:

$\Delta\eta = \ln(1/x_{pom})$

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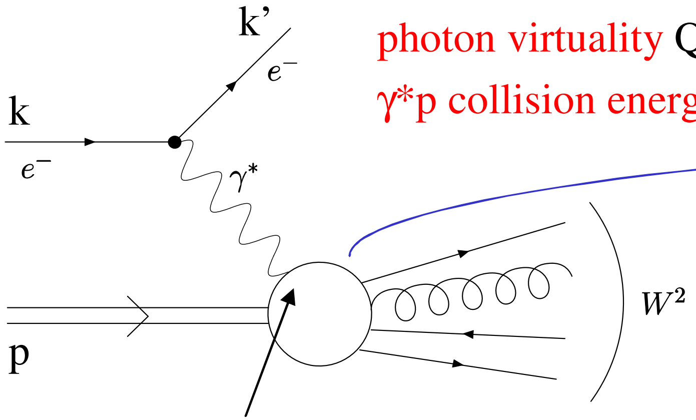

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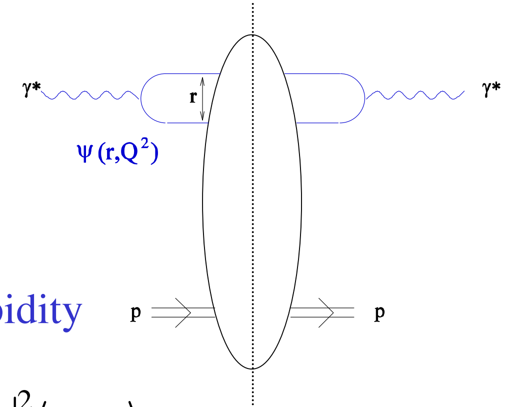

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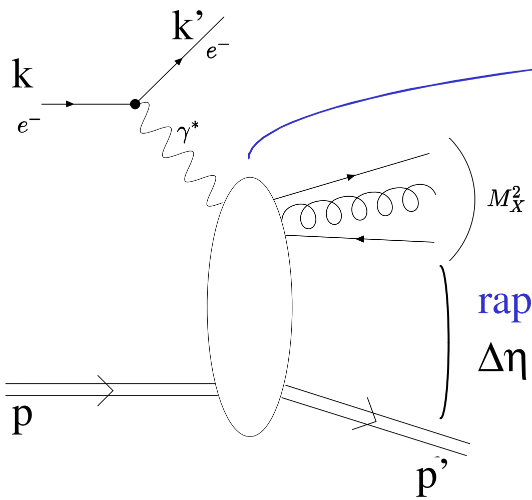

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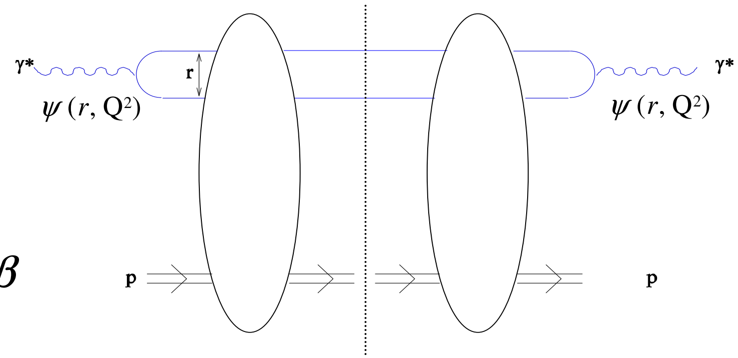
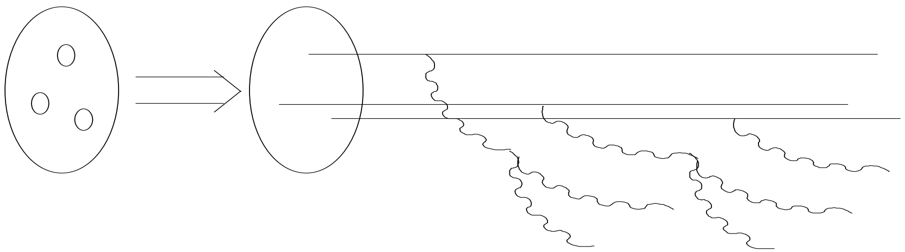


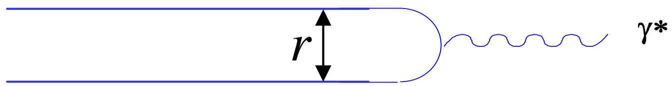
$$\frac{d\sigma_{DDIS}}{d^2b} = \pi \int dr^2 \left|\psi(r, Q^2)\right|^2 \langle T(r) \rangle_Y^2$$

Geometric scaling

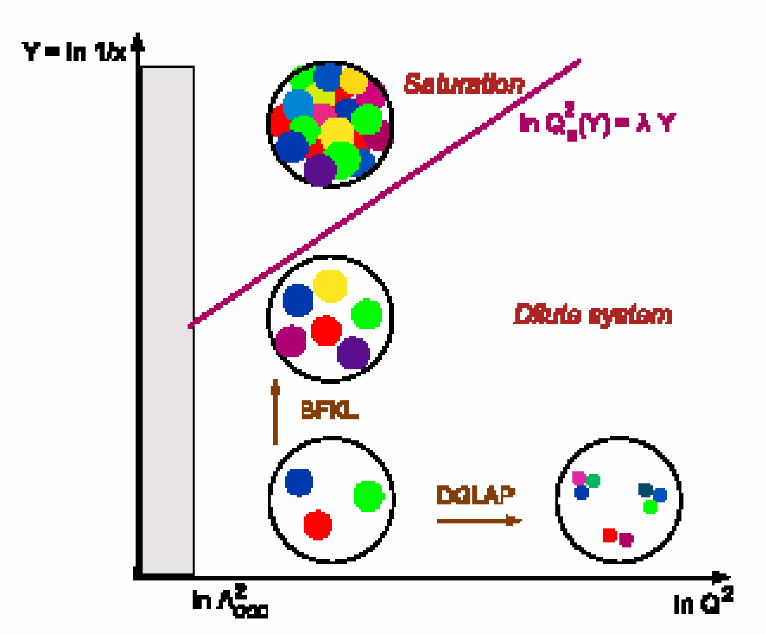
The dipole scattering amplitude $\langle T(r) \rangle_Y$

the dipole is probing small distances inside the proton: $r \sim 1/Q$





he sees the proton in the transverse plane:



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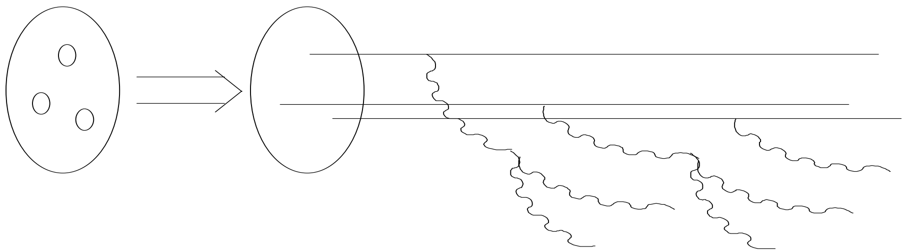

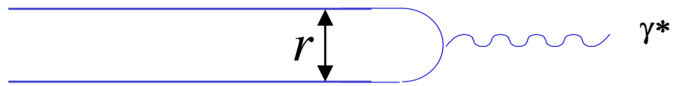

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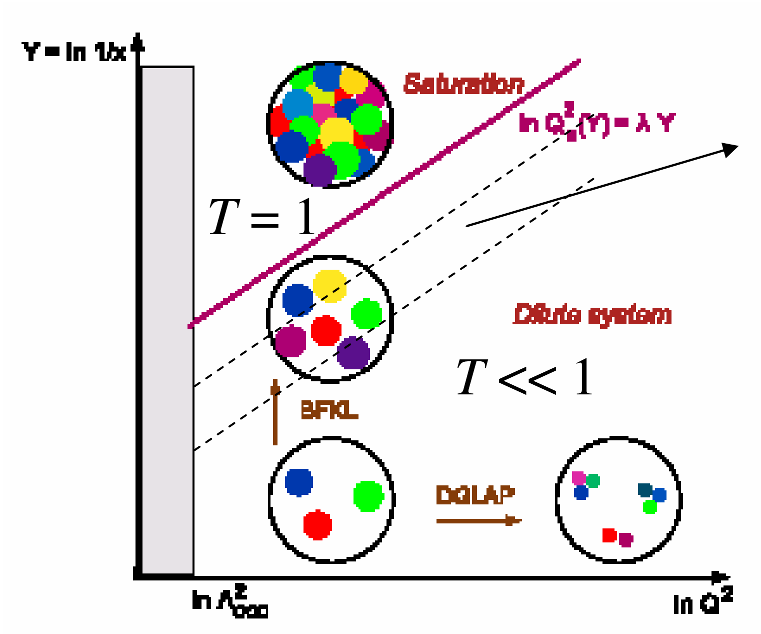
the physics is invariant along any line parallel to the saturation line



$$\langle T(r) \rangle_Y = T\left(r^2\, Q_S^2\,(Y)\right)$$

The dipole scattering amplitude $\langle T(r) \rangle_Y$

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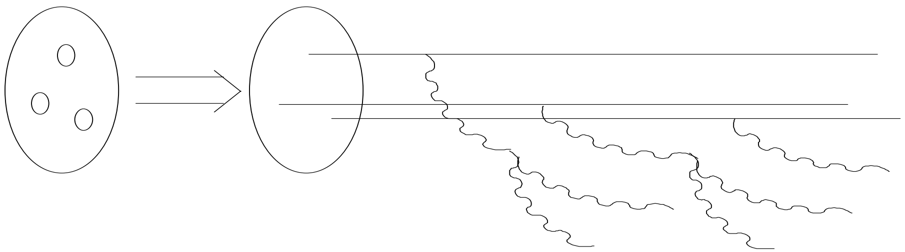

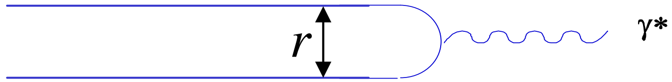

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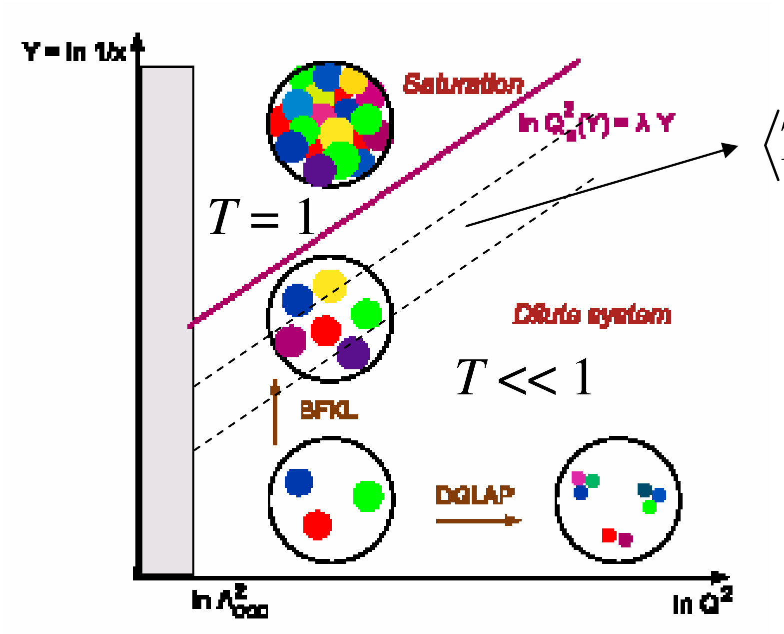

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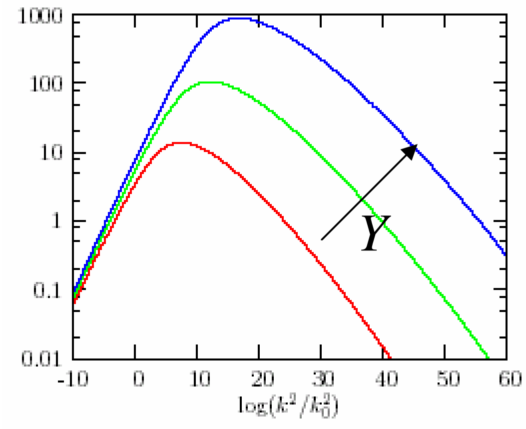
$$\langle T(r) \rangle_Y = T\left(r^2\, \mathrm{Q}_S^2\,(Y)\right)$$

the unintegrated gluon distribution

$$\Phi(k, Y)$$

is peaked around

$k \sim \mathrm{Q}_S(Y)$



The geometric scaling of $\sigma_{DIS}(x, Q^2)$

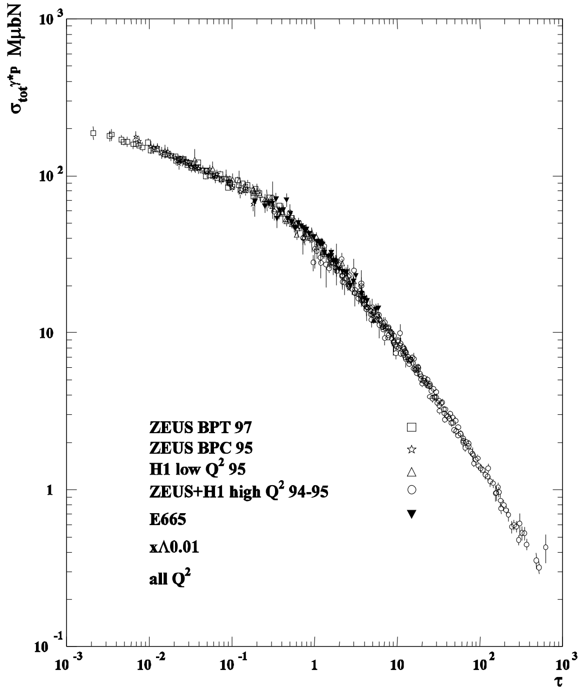
$$\langle T(r) \rangle_Y = T\left(r^2\, Q_S^2(Y)\right) \qquad Q_s^2(Y) = Q_0^2\, e^{\lambda Y}$$

$$\Downarrow$$

$$\sigma_{DIS}(x, Q^2) = \sigma_{DIS}(\tau \equiv Q^2/Q_s^2(x))$$

this is seen in the data with $\lambda \approx 0.3$

Stasto, Golec-Biernat and Kwiecinski (2001)



update

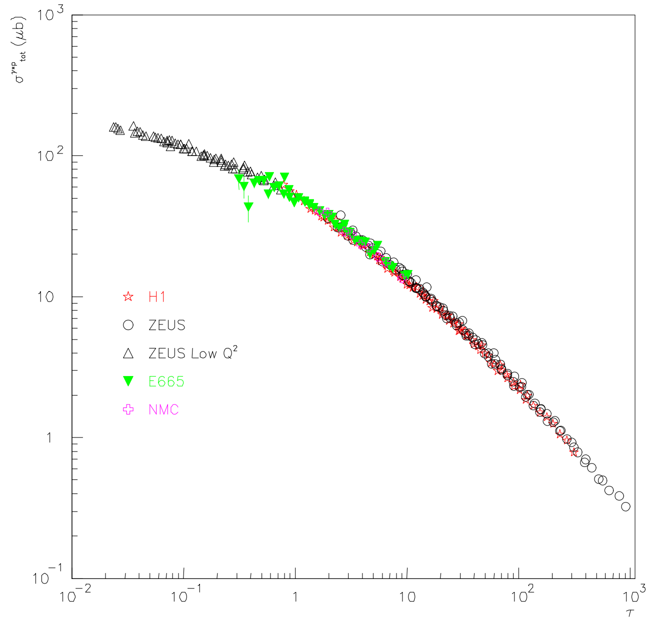
saturation models fit well F_2 data

Golec-Biernat and Wüsthoff (1999)

Bartels, Golec-Biernat and Kowalski (2002)

Iancu, Itakura and Munier (2003)

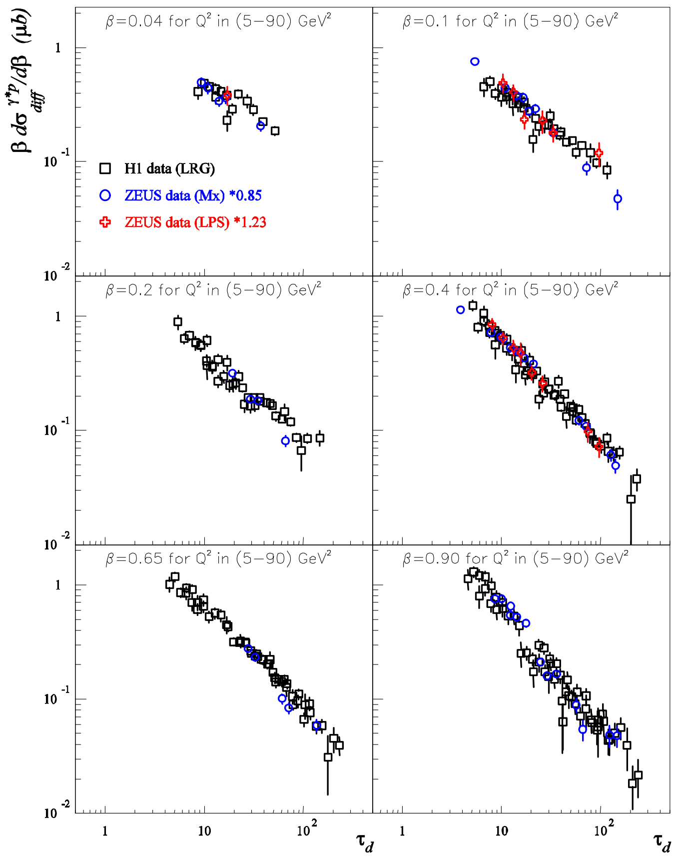
and they give predictions which describe accurately a number of observables at HERA (F_2^D, F_L, DVCS, vector mesons) and RHIC (nuclear modification factor in d-Au)



Geometric scaling in diffraction

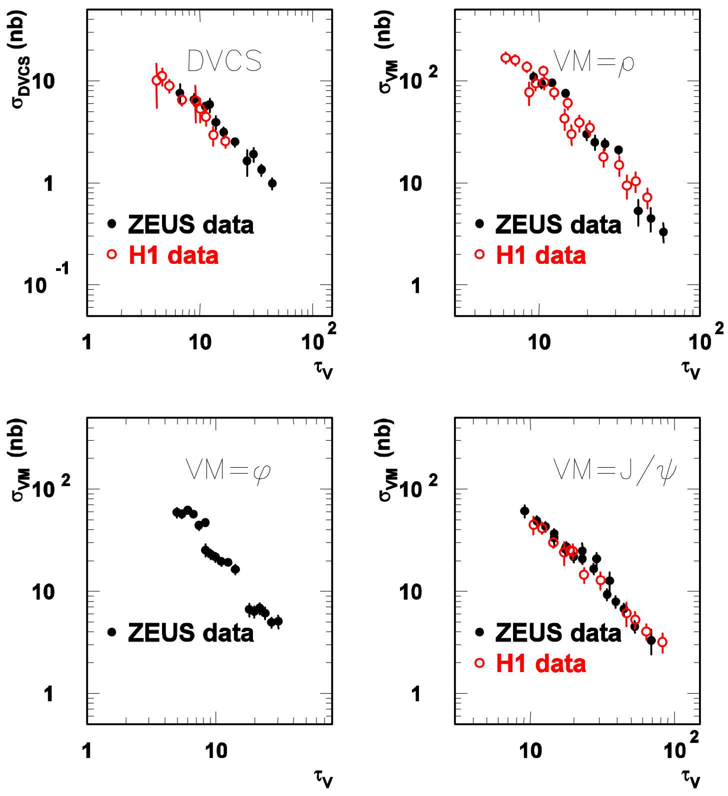
$$\langle T(r)\rangle_Y = T\left(r^2\, Q_S^2\,(Y)\right) \quad \Rightarrow \quad \sigma_{DDIS}(\beta, x_{pom}, Q^2) = \sigma_{DDIS}(\beta, \tau_d \equiv Q^2/Q_s^2(x_{pom}))$$

C.M. and L. Schoeffel, hep-ph/0606079



scaling also for vector meson production :

$$\sigma_{VM}(x, Q^2) = \sigma_{VM}(\tau_V \equiv (Q^2 + M_V^2)/Q_s^2(x))$$

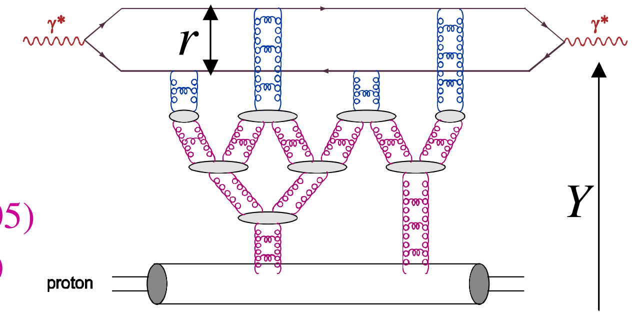


Diffusive scaling

The dipole scattering amplitude $\langle T(r) \rangle_Y$

Pomeron loops $\Leftrightarrow$ stochasticity in the evolution

One obtains the physical amplitude $\langle T(r) \rangle_Y$ from an event-by-event dipole amplitude $T_Y(r)$ which obeys a Langevin equation



Mueller and Shoshi (2004)
Iancu, Mueller and Munier (2005)
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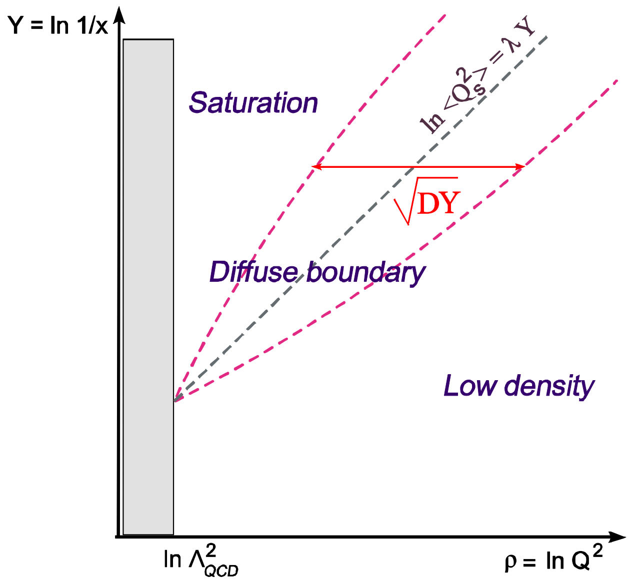

λ : average speed
D : diffusion coefficient

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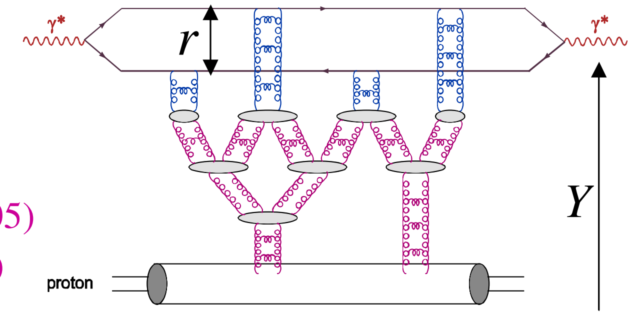

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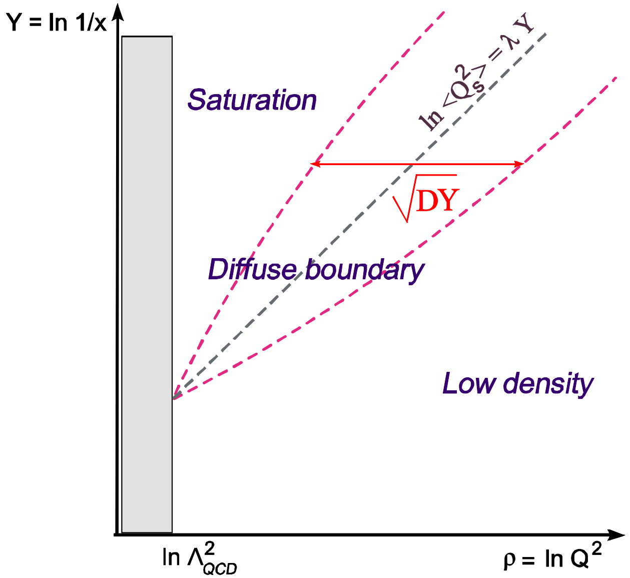

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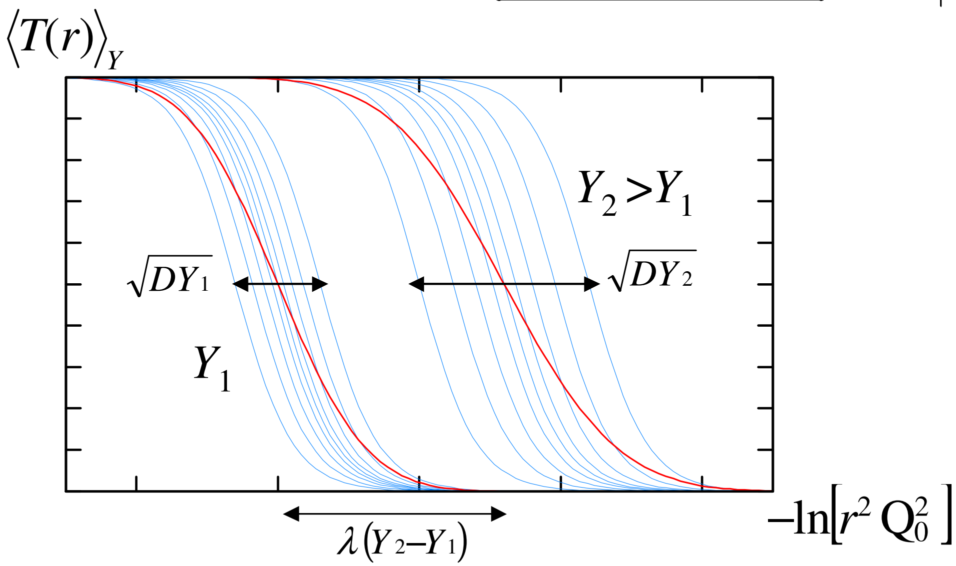

Blue curves: different realizations of $T_Y(r)$

Red curve: the physical amplitude $\langle T(r) \rangle_Y$

A new scaling law

Properties of the dipole amplitude $\langle T(r) \rangle_Y$ have been obtained by exploiting the similarities between the QCD equation and the s-FKPP equation well-known in statistical physics

The saturation scale is a stochastic variable distributed according to a Gaussian probability law:

$$P(\ln Q_S^2) = \frac{1}{\sqrt{\pi D Y}} \exp\left(- \frac{\ln^2(Q_S^2 / \overline{Q}_S^2)}{DY} \right) \qquad \left(\text{for } \left| \ln\left[Q_S^2 / \overline{Q}_S^2\right] \right| << DY \right)$$

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If $DY >> 1$, the diffusion is important and $\langle T(r)\rangle_Y = T\left(-\ln\left[r^2\,\bar{Q}_S^2(Y)\right]\big/\sqrt{DY}\right)$

we even know the functional form for $-\ln\left[r^2\bar{Q}_S^2\right] << DY$:

$$\langle T(r)\rangle_Y = \frac{1}{2}\, Erfc\left(-\ln\left[r^2\,\bar{Q}_S^2(Y)\right]\big/\sqrt{DY}\right)$$

Iancu and Triantafyllopoulos (2005), C.M., R. Peschanski and G. Soyez, *Phys. Rev.* **D73** (2006) 114005

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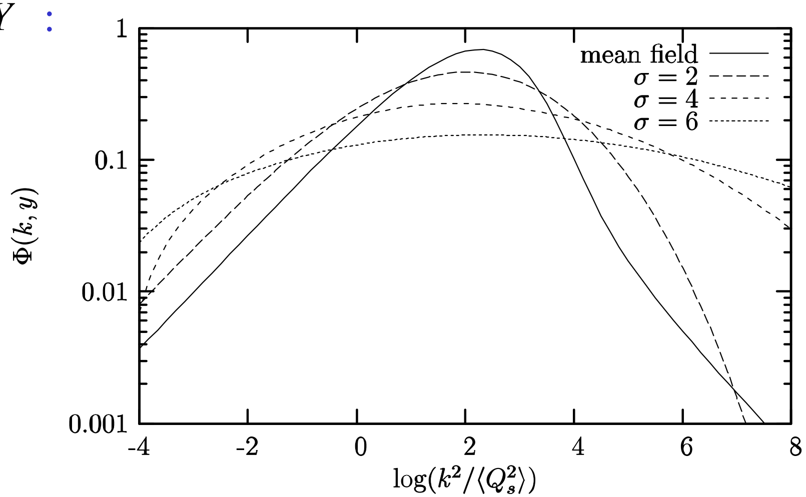
The unintegrated gluon distribution:

$$\Phi(k, Y) \approx \frac{1}{\sqrt{DY}} \exp\left(- \frac{\ln^2(k^2/\bar{Q}_s^2)}{DY} \right)$$



E. Iancu, C.M. and G. Soyez, hep-ph/0605174

Consequences for the observables

Y. Hatta, E. Iancu, C.M., G. Soyez and D. Triantafyllopoulos, *Nucl. Phys.* **A773** (2006) 95

$$\frac{d\sigma_{DIS}}{d^2b} = 2\pi \int dr^2 \left|\psi\left(r, \mathrm{Q}^2\right)\right|^2 \left\langle T(r) \right\rangle_Y \qquad \frac{d\sigma_{DDIS}}{d^2b} = \pi \int dr^2 \left|\psi\left(r, \mathrm{Q}^2\right)\right|^2 \left\langle T(r) \right\rangle_Y^2$$

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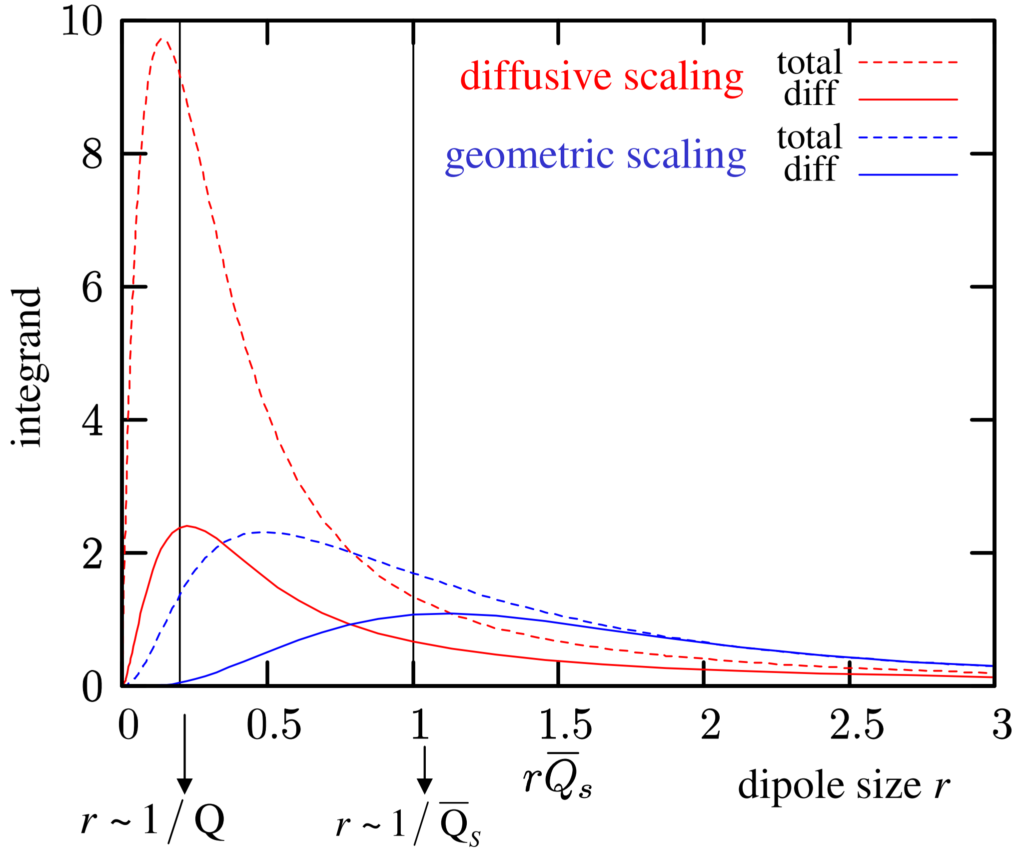
$Q^2 = 100\,\overline{Q}_s^2$



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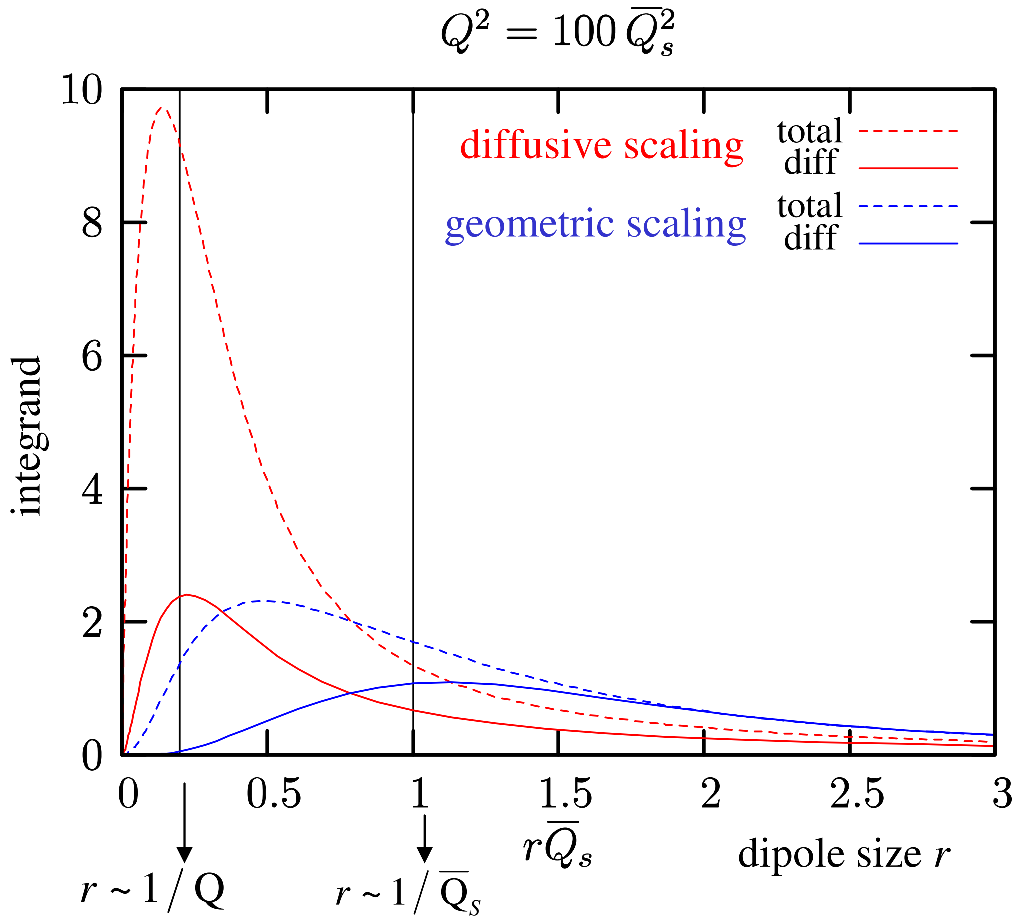
geometric scaling regime:

σ_{DIS} dominated by relatively hard sizes

$$1/Q < r < 1/\overline{Q}_S$$

σ_{DDIS} dominated by semi-hard sizes

$$r \sim 1/\overline{Q}_S$$



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geometric scaling regime:

σ_{DIS} dominated by relatively hard sizes

$$1/Q < r < 1/\overline{Q}_S$$

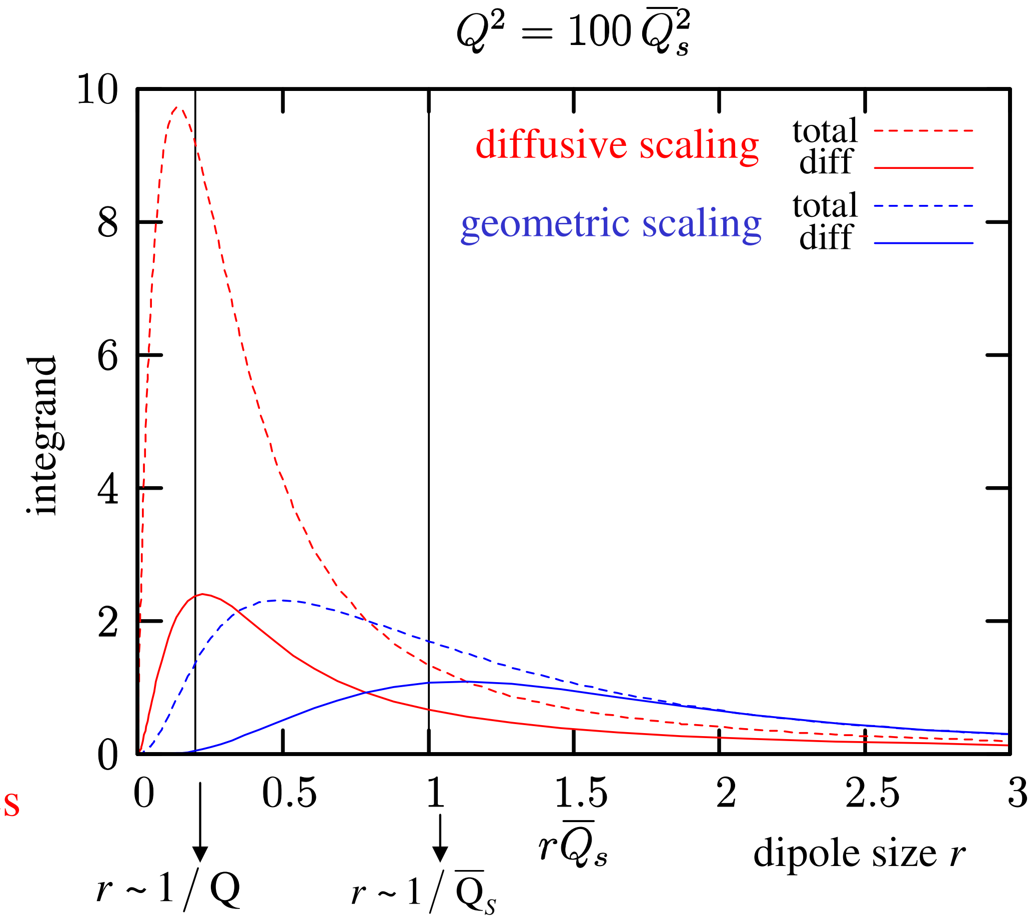
σ_{DDIS} dominated by semi-hard sizes

$$r \sim 1/\overline{Q}_S$$

diffusive scaling regime:

both σ_{DIS} and σ_{DDIS} are dominated by hard sizes $r \sim 1/Q$

yet saturation is the relevant physics



Some analytic estimates

Analytical estimates for $\sigma_{DIS}(x, Q^2)$ in the diffusive scaling regime:

valid for $\sqrt{D\ln(1/x)} << \ln(Q^2/\bar{Q}_S^2) << D\ln(1/x)$

$$\frac{d\sigma_{DIS}}{d^2b} = F\sqrt{\pi D\ln(1/x)}\,\frac{\exp(-Z^2)}{Z^2} \qquad \text{with} \qquad F = \frac{N_c\alpha_{em}}{12\pi^2}\sum_f e_f^2 \qquad \text{and} \qquad Z \equiv \frac{\ln(Q^2/\bar{Q}_S^2)}{\sqrt{D\ln(1/x)}}$$

And for the diffractive cross-section $\displaystyle \frac{d\sigma_{DDIS}}{d^2b} = \frac{F}{4}\sqrt{D\ln(1/x)}\,\frac{\exp(-2Z^2)}{Z^3}$

- the cross-sections (total and diffractive) are dominated by small dipole sizes $r \sim 1/Q$
- there is no Pomeron (power-like) increase
- the diffractive cross-section is dominated by the scattering of the quark-antiquark component

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Analytical estimates for $\sigma_{DIS}(x, Q^2)$ in the diffusive scaling regime:

valid for $\sqrt{D\ln(1/x)} << \ln(Q^2/\bar{Q}_S^2) << D\ln(1/x)$

$$\frac{d\sigma_{DIS}}{d^2b} = F\sqrt{\pi D\ln(1/x)}\,\frac{\exp(-Z^2)}{Z^2} \qquad \text{with} \qquad F = \frac{N_c\alpha_{em}}{12\pi^2}\sum_f e_f^2 \qquad \text{and} \qquad Z \equiv \frac{\ln(Q^2/\bar{Q}_S^2)}{\sqrt{D\ln(1/x)}}$$

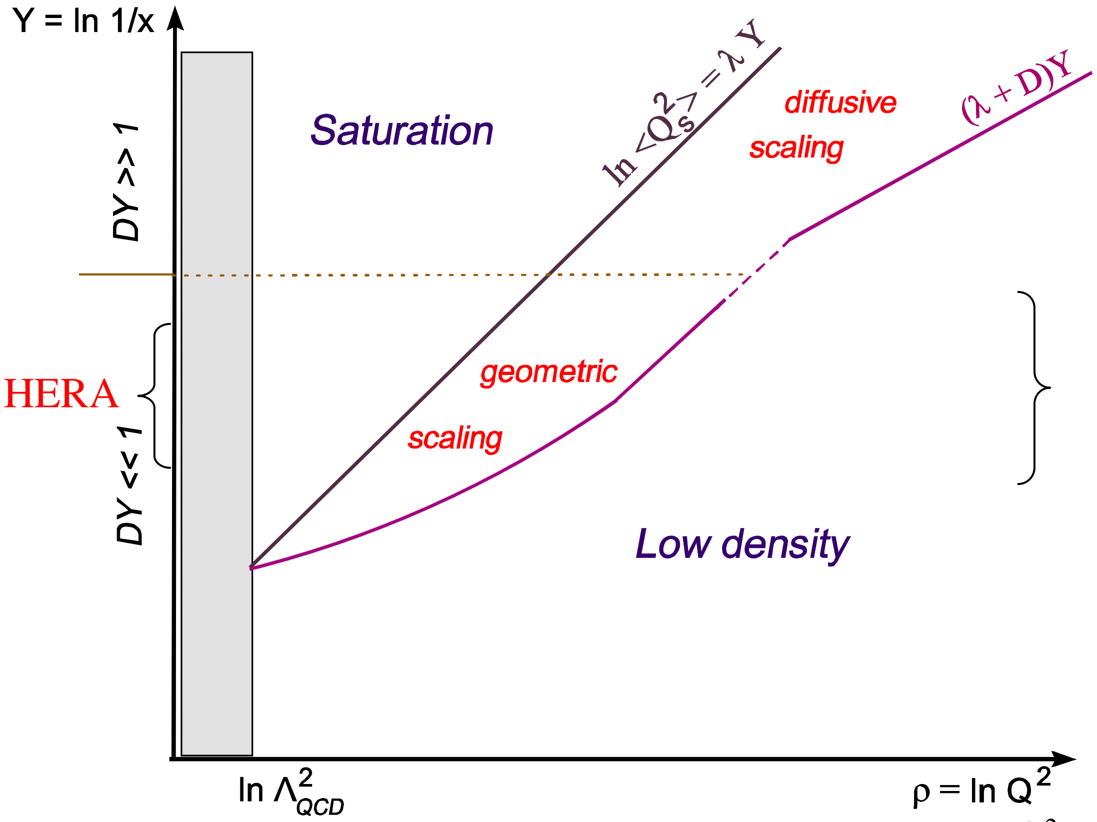
And for the diffractive cross-section $\displaystyle \frac{d\sigma_{DDIS}}{d^2b} = \frac{F}{4}\sqrt{D\ln(1/x)}\,\frac{\exp(-2Z^2)}{Z^3}$

- the cross-sections (total and diffractive) are dominated by small dipole sizes $r \sim 1/Q$
- there is no Pomeron (power-like) increase
- the diffractive cross-section is dominated by the scattering of the quark-antiquark component

In the diffusive scaling regime (up to momenta Q^2 much bigger than the saturation scale $\bar{Q}_s^2(x)$):

- cross-sections are dominated by rare events, in which the photon hits a black spot, that he sees dense (at saturation) at the scale Q^2
- saturation is the relevant physics
- the features expected when $Q^2 < \bar{Q}_s^2(x)$ are extended up to much higher Q^2

Conclusions



$Q^2 \sim 1/r^2$

at higher energies, a new scaling law: diffusive scaling

$$\langle T(r)\rangle_Y = T\left(-\ln\left[r^2\, \overline{Q}_S^2(Y)\right]/\sqrt{DY}\right)$$

within the LHC energy range?

an intermediate energy regime: geometric scaling

$$\langle T(r)\rangle_Y = T\left(r^2\, \overline{Q}_S^2(Y)\right)$$

$$\bar{Q}_s^2(Y) = Q_0^2\, e^{\lambda Y}$$

it seems that HERA is probing the geometric scaling regime

In the diffusive scaling regime, saturation is the relevant physics up to momenta much higher than the saturation scale