Averaging of the inelastic cross-section measured by the CDF and the E811 experiments.

S. Klimenko, J. Konigsberg, T. Liss

1. Total and inelastic cross-sections

Both experiments measured the total cross-section using the luminosity independent method

$$\sigma_{tot} = 16\pi(hc)^2 \frac{b}{1+\rho^2} \frac{N_{el}}{N_{el} + N_{in}},$$

where N_{el} is the rate of elastic and N_{in} is the rate of inelastic $p\bar{p}$ scattering events. The slope b is defined as

$$b = \frac{1}{N_{el}} \frac{dN_{el}}{dt} |_{t \to 0}$$

and it is exactly the same for both experiments (see Table 1). For the slope the E811 is using the average of the CDF and the E710 measurements with the error dominated by the CDF measurement. Therefore comparing the CDF and E811 measurements, the b uncertainty should be excluded.

The inelastic cross-section is

$$\sigma_{in} = 16\pi(hc)^2 \frac{b}{1+\rho^2} \frac{N_{el} N_{in}}{(N_{el} + N_{in})^2}.$$

Introducing the ratio of the inelastic and elastic rates R, we can re-write the inelastic cross-section as

$$\sigma_{in} = 16\pi(hc)^2 \frac{b}{1+\rho^2} \frac{R}{(1+R)^2}.$$

2. Measured values

In both experiments the measured values are the number of elastic and inelastic events. Table 1 shows the slope b, the number of elastic and inelastic events and their ratio.

	CDF	E811
N_{el}	78691 ± 1463	508.1K ± 3.5K
N_{in}	240982 ± 2967	1799.5K ± 57.2K
R	3.062 ± 0.068	3.542 ± 0.113
b	16.98 ± 0.25	16.98 ± 0.22

Table 1. Input numbers for the inelastic cross-section.

For comparison of two experiments we should ignore the uncertainty of the slope b and compare the measured values of R only. The values of R and therefore all derived cross-sections disagree with the 3.6 standard deviation discrepancy.

To localize the source of disagreement lets look at how the inelastic rates were measured. The inelastic rate was measured as a sum of the double-arm rate N_2

(coincidence of two detectors measuring inelastic rates in the p and $\overline{p}$ directions) and the single-arm rate N_1. We could normalize all the rates by N_{el} :

$$x = \frac{N_2}{N_{el}},\ \ y = \frac{N_1}{N_{el}},\ \ R = x + y\ .$$

The measurement of the N_{el} and N_2 rates was similar in both cases and the N_2 measurement was the most straightforward one. The measurement of the rate N_1 was quite different.

The CDF estimated the rate of the single diffractive (SD) events by measuring the coincidence rate of the $\overline{p}$ elastic detector with the opposite inelastic detector (we will call it "the single diffractive rate"). After applying the selection cuts this rate had a little background, however it required a considerable acceptance and detection efficiency corrections to obtain the single diffractive rate. To avoid double counting, the double-arm single diffractive events were subtracted from the total number of the single diffractive events, which gives the estimation of the N_1 rate (32092±1503 events). The small contribution of the single-arm events from the non-diffractive processes (0.6%) was added to the N_2 rate as a simulation-calculated correction. Therefore the corrected number of the single-arm events is 33403±1520.

The non-diffractive correction could be model dependent. We could imagine some inelastic sub-process (for example, we could call it "double diffractive"), which gives small contribution both into the double-arm inelastic rate and the "single diffractive rate", but a considerable contribution into the single-arm rate. If such a sub-process exists, it may not be included into the inelastic rate measured by the CDF. However, it's not likely that the CDF has missed a considerable fraction of the inelastic events. First, the CDF measurement agrees with UA4 experiment at low energies. Second, according to Paolo, the CDF made a cross-check of the single diffractive events by measuring the single-arm rate.

The E811 measured the exclusive single-arm rate using the inelastic detectors, which should be quite efficient for the single diffractive events. However the background from losses was large (~93%). To obtain the 13% error quoted on the number of single-arm inelastic events, it required the measurement of the background with uncertainty better then 1%, which is a non-trivial task. The measurement of the single-arm rate was done during a special run with missing bunches. Therefore in order to use it in the analysis, in fact, the ratio of the single-arm and double-arm rates was measured: $r = 0.3220 \pm 0.0415$. A small correction to this number due to the final acceptance is δ=0.0107±0.006[1]. The total number of the single-arm events was estimated as $N_1 = N_2(r + \delta)$.

Table 1 shows the *x* and *y* values measured by the CDF and E811 experiments

	CDF	E811
X	2.638 ± 0.058	2.657 ± 0.023
Y	0.424 ± 0.021	0.885 ± 0.115

Table 2. The *x* and *y* ratios measured by the CDF and the E811.

[1] The numbers are shown exactly as they appear in the article and C.Avila's talk.

The x values are in a very good agreement, but the y values disagree, which was pointed out by Paolo as the source of the CDF-E811 disagreement. It was interpreted as that the E811 has by factor of two more single diffractive events, possibly due to the error of the large background subtraction.

However, we can't do the direct comparison of the x and y values, because they have different expectation values. The CDF and E811 inelastic detectors had very different acceptances for the two-side events: $\varepsilon_2(CDF) \approx 98.7\%$, $\varepsilon_2(E811) = 88.85 \pm 2.0\%$. The E811 single-arm rate had a lot of non-diffractive events missed by the two-side inelastic trigger and the CDF N_1 rate was due to the single diffractive process only. Therefore in order to check if the E811 inelastic rates are consistent with the CDF rates we need to take into account the acceptance corrections. To make more intelligent comparison we need to estimate the non-diffractive and diffractive (e.g. SD) rates for both experiments. It's straightforward for the CDF and for the E811 the rates are

$$N_{nd} = N_2 / \varepsilon_2, \quad N_{sd} = N_2 (r + \delta - \frac{1-\varepsilon_2}{\varepsilon_2}).$$

There should be a few percent correction for the N_{sd}(E811) rate to account for the double-arm SD events, which we ignore at this moment. Table 3 shows the rates and their ratios to the elastic rate.

	CDF	E811
N_{nd}	203200 ± 2558	1519.7K ± 34.9K
N_{sd}	37782 ± 1770	279.8K ± 36.3K
N_{nd} / N_{el}	2.582±0.058	2.991±0.069
N_{sd} / N_{el}	0.480±0.029	0.551±0.072
N_{sd} / N_{nd}	0.186±0.009	0.184±0.024

Table 3. The x and y ratios measured by the CDF and the E811.

At this time we have a remarkable agreement between the CDF and the E811 for the ratio N_{sd} / N_{nd}. So both experiments see the same fraction of the single diffractive events. At the same time there is the discrepancy of 4.4 standard deviations between the ratios of the non-diffractive inelastic and elastic events. Similar ratio for the single diffractive rate is also greater for the E811, but the errors are large and the SD ratios are compatible. So, it is possible, the source of the CDF/E811 discrepancy is in the measurement of the elastic rates. Unfortunately, no detail documentation was found on the measurement of the elastic rate by the E811 experiment. Unless the documentation is provided and we find maybe more intelligent way, there are the following methods of averaging of the CDF and the E811 measurements.

3. Averaging of the CDF and E811 measurements.

Method A.

To find the mean value of the inelastic cross-section we should average the R measurements, which are not compatible. The PDG suggests the following algorithm:

- Find the average of two experiments using the standard approach: $\overline{R} = 3.19$.
- Find the average error using the standard approach: $\sigma_{\overline{R}} = 0.06$

- Calculate χ^2: 13.2
- Scale the error to get $\chi^2 = 1$: $\sigma_{\overline{R}} \to 0.058\sqrt{13.2} = 0.21$.

At the first approximation, ignoring the correlation of the slope b and $\overline{R}$, the inelastic cross-section relative error is

$$\delta^2 = \frac{\sigma_b^2}{b^2} + \frac{\sigma_{\overline{R}}^2}{\overline{R}^2} \cdot \left(\frac{1-\overline{R}}{1+\overline{R}}\right)^2 = (3.8\%)^2.$$

Finally the average inelastic cross-section is

$$\overline{\sigma}_{in} \cdot (1+\rho^2) = 60.4 \pm 2.3 \text{ mb},$$

which is 2.2% below the CDF measurement.

Method B.

Now lets take into account the correlation between the slope b and the ratios R. As Heidi mentioned[2], the elastic rate is the raw n_{el} rate measured in each experiment divided by the "acceptance"

$$N_{el} = n_{el} / \left(\exp(-bt_{\min}) - \exp(-bt_{\max})\right),$$

where $(t_{\min}, t_{\max})$ is a range of t used by the CDF ($0.04 < t < 0.29$) and E811 ($0.0045 < t < 0.036$). Both measurements depend on the slope b and, in fact, they are anti-correlated. Namely, if we increase b by one standard deviation (1.5%), the CDF value of R increases by ~1% and the E811 value decreases by ~1%. The covariance matrix *cov(R$_i$,R$_j$)* is

$$C = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\alpha \\ \sigma_1\sigma_2\alpha & \sigma_2^2 \end{bmatrix},$$

where $\sigma_1(\sigma_2)$ is the standard deviation of the ratio $R_1(R_2)$ for the CDF (E811) measurement and the coefficient α is estimated to be –0.09. The average value of the ratio R is

$$\overline{R} = fR_1 + (1-f)R_2,$$

where the weight f

$$f = \frac{\sigma_2^2 - \alpha\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\alpha\sigma_1\sigma_2}$$

can be found by minimization of the variance of $\overline{R}$

$$\text{var}(\overline{R}) = FCF^T, \quad F = (f, 1-f).$$

Assuming that the standard deviations σ_1, σ_2 are given by the errors listed in Table 1[3], the average value of R is

$$\overline{R} = 3.20 \pm 0.06,$$

which is very close to the number obtained by the method A. Calculating the χ^2

$$\chi^2 = \sum_{i,j} \frac{(R_i - \overline{R})}{\sigma_i} \cdot C_{ij}^{-1} \cdot \frac{(R_j - \overline{R})}{\sigma_j}; (i,j = 1,2)$$

[2] For details see Heidi's note.

[3] Actually the E811 error is underestimated, because the error of the slope b is ignored in the R ratio.

and applying the same procedure for the scaling of the error of $\overline{R}$, the inelastic cross-section is

$$\overline{\sigma}_{in} \cdot (1+\rho^2) = 60.3 \pm 2.2 \text{ mb}.$$

Method C.

The averaging procedure described above can be applied to the inelastic and elastic cross-sections measured by the CDF and E811. Using the functional dependences of the inelastic and elastic cross-sections on the slope *b* and the ratio *R*, lets, first, derive the mean value and the error for each experiment and compare with quoted numbers (see Table 4).

	CDF	E811
Quoted σ_{tot}*, mb*	80.03 ± 2.25	71.71 ± 2.02
Derived σ_{tot}*, mb*	80.03 ± 2.17	71.70 ± 1.90
Quoted σ_{in}*, mb*	60.33 ± 1.40	55.92 ± 1.19
Derived σ_{in}*, mb*	60.32 ± 1.34	55.90 ± 1.15

Table 4. Comparison of the values of the cross-sections and their errors quoted by the CDF $(\rho = 0.15)$ and E811 $(\rho = 0.145)$ and derived in this note.

The derived errors are slightly smaller, however they all are compatible with the quoted errors. It means that the errors are mainly determined by the errors of the slope *b* and the ratio *R*.

Note, that the CDF and E811 errors are approximately the same. Therefore the averaging procedure described above yields the weight f=0.5, independent on the

The averaged cross-sections with their inflated errors are

$$\overline{\sigma}_{tot} \cdot (1+\rho^2) = 76.8 \pm 4.7 \text{ mb},$$

$$\overline{\sigma}_{in} \cdot (1+\rho^2) = 58.8 \pm 2.7 \text{ mb}.$$

4. Conclusion

Table 5 shows the average cross-sections for all three methods.

	$\overline{\sigma}_{in} \cdot (1+\rho^2)$	$\overline{\sigma}_{tot} \cdot (1+\rho^2)$
Method A	60.4 ± 2.3	79.3 ± 4.2
Method B	60.3 ± 2.2	79.1 ± 4.0
Method C	58.8 ± 2.7	76.8 ± 4.7

Table 5. Average cross-sections.

The methods A and B are based on the averaging of the R ratio. Since the R measurements disagree for the CDF and E811, the error of the average value of R should be inflated. In this way we ignore the accurate error analysis done by both experiments and, in order to calculate the final cross-sections, we don't care about the correlation of the errors of b and R anymore. The method B is a more accurate version of the method A and gives almost the same result.

With the method C we average the quoted cross-sections itself. It seems to be less straightforward then the methods A and B. First, one needs to estimate the covariance

$$\mathrm{cov}(\sigma_{in}^{CDF},\sigma_{in}^{E811}) \approx 0.41,\ \mathrm{cov}(\sigma_{tot}^{CDF},\sigma_{tot}^{E811}) \approx 0.24.$$

They are large compare to the covariance $\mathrm{cov}(R_1,R_2) \approx -0.09$, which can be neglected as the comparison of the methods A and B shows. Second, it does not show the actual level of disagreement between the CDF and E811 measurements. For example, the χ^2 values calculated for different measurements are

$$\chi_R^2 = 12.0,\ \chi_{tot}^2 = 8.6,\ \chi_{in}^2 = 6.6$$

and the measurements agree with the confidence level of

$$CL_R < 0.1\%,\ CL_{tot} \approx 0.3\%,\ CL_{in} \approx 1.0\%$$

respectively. In fact, we can not even say that there is a disagreement for the inelastic cross-section.

Therefore I suggest the method A for the averaging of the CDF and E811 measurements. Using the ρ value of 0.135 the average inelastic cross-section is

$$\overline{\sigma}_{in} = 59.3 \pm 2.3.$$

5. How the CDF luminosity is affected.

The CDF luminosity is derived from the rate of the inelastic $p\bar{p}$ events measured with the luminosity monitor (CLC), the CLC acceptance and the inelastic cross-section

$$L = \frac{R_{p\bar{p}}}{\sigma_{in}\varepsilon_{clc}}$$

with the systematic errors of 1.8%, 4.0% and 3.8% respectively. The total systematic error on the luminosity is 5.8%.

Also the new value of the inelastic cross-section will shift the mean value of the luminosity. The inelastic processes can be divided on three groups: a) hard-core, b) single diffractive, c) double diffractive. The single diffractive cross-section was measured by the CDF and E710 experiments: 9.46 ± 0.44 mb (CDF), 8.1 ± 1.7 mb, 11.7 ± 2.3 mb (E710). Since the average single diffractive cross-section is not much different from the CDF measurement, I will use the CDF number. The double diffractive cross-section was "measured" by CDF (PRL 87, 2001) and according to Mary Convery it is 7 ± 2 mb. The rest of the inelastic cross-section ($43.95 = 60.41 - 9.46 - 7.0$), where the inelastic cross-section is given for ρ=0.135, is the hard-core cross-section. All the numbers are given for the center of mass energy of 1.8 TeV.

At 1.96 TeV the CLC group used the following values of the inelastic cross-sections from the MBR generator: $\sigma_{hc} = 44.4$ mb, $\sigma_{sd} = 10.3$, $\sigma_{dd} = 7.0$. Therefore the total inelastic cross-section of $\sigma_{in} = 61.7$ mb at 1.96 TeV was used for the luminosity estimation.

The CLC acceptance was estimated to be 60.2±2.4% assuming the relative fraction of the inelastic processes 44.4/10.3/7.0. The CLC acceptances for each process were estimated to be $\varepsilon_{hc} = 79.2\%$, $\varepsilon_{sd} = 19.0\%$, $\varepsilon_{dd} = 6.5\%$. Given the relative fraction of

43.95/9.46/7.0 at 1.8 TeV, the CLC acceptance is 60.8%, which is very close to the number above. If the Tevatron energy would be 1.8 TeV, the luminosity correction is

$$\frac{\delta L}{L} = 1 - \frac{\sigma_{in}(CDF)}{\overline{\sigma}_{in}} = 1 - \frac{60.41}{59.3} = 0.019 .$$

Assuming that the extrapolation for the inelastic cross-section from 60.41mb (1.8TeV) to 61.7 mb (1.96TeV) is correct, the same correction of +1.9% should be applied to the CLC luminosity at 1.96TeV.