Measurement of the Lifetime Difference in the $B_s^0$ System

We present a study of the decay $B_s^0 \rightarrow J/\psi \phi$. We obtain the $CP$-odd fraction in the final state at time zero, $R_L = 0.16 \pm 0.10 \text{(stat)} \pm 0.02 \text{(syst)}$, the average lifetime of the $(B_s^0, \bar{B}_s^0)$ system, $\tau(B_s^0) = 1.39^{+0.13}_{-0.16} \text{(stat)}^{+0.01}_{-0.02} \text{(syst)}$ ps, and the relative width difference between the heavy and light mass eigenstates, $\Delta \Gamma/\Gamma = (\Gamma_L - \Gamma_H)/\Gamma = 0.24^{+0.28}_{-0.38} \text{(stat)}^{+0.03}_{-0.04} \text{(syst)}$. With the additional constraint from the world...
Within the framework of the standard model (SM), the $B^0_\text{s}$ mesons are expected to mix in such a way that the mass and decay width differences between the heavy and light eigenstates, $\Delta M = M_H - M_L$ and $\Delta \Gamma = \Gamma_L - \Gamma_H$, are sizeable. The mixing phase $\delta \phi$ is predicted to be small and to a good approximation the two mass eigenstates are expected to be $CP$ eigenstates. New phenomena may alter $\delta \phi$, leading to a reduction of the observed $\Delta \Gamma$ compared to the SM prediction [1].

The decay $B^0_\text{s} \rightarrow J/\psi \phi$, proceeding through the quark process $b \rightarrow c\bar{c}ss$, gives rise to both $CP$-even and $CP$-odd final states. It is possible to separate the two $CP$ components of the decay $B^0_\text{s} \rightarrow J/\psi \phi$, and thus to measure the lifetime difference, through a simultaneous study of the time evolution and angular distributions of the decay products of the $J/\psi$ and $\phi$ mesons. The angular distribution of the decay $B^0_\text{s} \rightarrow J/\psi(\rightarrow \mu^+ \mu^-)\phi(\rightarrow K^+ K^-)$ involves three angles. Current statistics are such that the use of all three angles characterizing the final state is not yet beneficial. We use a variable particularly sensitive to separating the $CP$ states, the cosine of the transversity polar angle (called “transversity”), as defined below.

The analysis of data collected with the D0 detector [2] at the Fermilab Tevatron collider presented in this Letter is an extension of a recently published study [3] done under the single $B^0_\text{s}$ lifetime hypothesis. We perform an unbinned maximum likelihood fit to the data, including the $B^0_\text{s}$ candidate mass, lifetime, and transversity, in the decay sequence $B^0_\text{s} \rightarrow J/\psi \phi$, $J/\psi \rightarrow \mu^+ \mu^-$, $\phi \rightarrow K^+ K^-$. We extract three parameters characterizing the $B^0_\text{s}$ system and its decay: $\hat{\tau}(B^0_\text{s}) = 1/\Gamma$, where $\Gamma = (\Gamma_H + \Gamma_L)/2$; $\Delta \Gamma/\Gamma$; and $R_L$, the relative rate of the decay to the $CP$-odd states at time zero. The average lifetimes of $B^0_\text{s}$ and $B^0$, as defined above, are expected to be equal to within 1% [4], and their ratio is determined by also measuring the lifetime of $B^0$ in the similar decay topology of $J/\psi K^+$. The data were collected between June 2002 and August 2004. The sample is selected by requiring two reconstructed muons with a transverse momentum $p_T > 1.5$ GeV. Each muon is required to be detected as a track segment in at least one layer of the muon system and to be matched to a central track. One muon is required to have segments both inside and outside the toroid. We require the events to satisfy a muon trigger that does not include any cuts on the impact parameter. The sample corresponds to an integrated luminosity of approximately 450 pb$^{-1}$.

To select the $B^0_\text{s}$ candidate sample, we apply the following kinematic and quality cuts. Minimum values of momenta in the transverse plane for $B^0_\text{s}$, $\phi$, and $K$ mesons are set at 6.0 GeV, 1.5 GeV, and 0.7 GeV, respectively. $J/\psi$ candidates are accepted if the invariant mass is in the range 2.90–3.25 GeV. Successful candidates are constrained to the average reconstructed $J/\psi$ mass of 3.072 GeV. Decay products of the $\phi$ candidates are required to satisfy a fit to a common vertex and to have an invariant mass in the range 1.01–1.03 GeV. We require the $(J/\psi, \phi)$ pair to be consistent with coming from a common vertex, and to have an invariant mass in the range 5.0–5.8 GeV. In case of multiple $\phi$ meson candidates, we select the one with the highest transverse momentum. Monte Carlo (MC) studies show that the $p_T$ spectrum of the $\phi$ mesons coming from $B^0_\text{s}$ decay is harder than the spectrum of a pair of random tracks from hadronization. We define the signed decay length of a $B^0_\text{s}$ meson $L_{xy}^B$ as the vector pointing from the primary vertex to the decay vertex projected on the $B^0_\text{s}$ transverse momentum. To reconstruct the primary vertex, we select tracks with $p_T > 0.3$ GeV that are not used as decay products of the $B_s$ candidate and apply a constraint to the average beam spot position. The proper decay length, $c\tau$, is defined by the relation $c\tau = L_{xy}^B M_{B^0_\text{s}}/p_T$ where $M_{B^0_\text{s}}$ is the world average mass of the $B^0_\text{s}$ meson [5]. The distribution of the proper decay length uncertainty $\sigma(c\tau)$ of $B^0_\text{s}$ mesons peaks around 25 $\mu$m. We accept events with $\sigma(c\tau) < 60$ $\mu$m. There are 9699 events satisfying the above cuts.

The resulting invariant mass distribution of the $(J/\psi, \phi)$ system is shown in Fig. 1. The curves are projections of the

![FIG. 1 (color online). The invariant mass distribution of the $(J/\psi, \phi)$ system for $B^0_\text{s}$ candidates. The curves are projections of the maximum likelihood fit (see text).](image-url)
maximum likelihood fit, described below. The fit assigns 513 ± 33 events to $B^0_d$ decay.

Using the same data sample and analogous kinematic and quality cuts, we find 1913 events of the decay sequence $B^0 \rightarrow J/\psi K^*, J/\psi \rightarrow \mu^+ \mu^-, K^* \rightarrow K^\pm \pi^\mp$, and update the $B^0$ lifetime measurement reported in Ref. [4] with larger statistics.

We perform a simultaneous unbinned maximum likelihood fit to the proper decay length, transversity, and mass. The likelihood function $L$ is given by

$$L = \prod_i [f_{\text{sig}} \mathcal{F}_{\text{sig}}^i + (1 - f_{\text{sig}}) \mathcal{F}_{\text{bck}}^i],$$

where $N$ is the total number of events, $\mathcal{F}_{\text{sig}}^i$ ($\mathcal{F}_{\text{bck}}^i$) is the product of the mass, proper decay length, and the transversity probability density functions for the signal (background), and $f_{\text{sig}}$ is the fraction of signal in the sample. The background is divided into two categories, based on their origin and lifetime characteristics. "Prompt" background is due to directly produced $J/\psi$ mesons accompanied by random tracks arising from hadronization. This background is distinguished from "nonprompt" background, where the $J/\psi$ meson is a product of a $B$-hadron decay while the tracks forming the $\phi$ candidate emanate from a multibody decay of the same $B$ hadron or from hadronization. We allow for independent parameters for the two background components in mass, lifetime, and transversity. There are 19 free parameters in the fit.

For the signal mass distribution, we use a sum of two Gaussian functions with a fixed ratio of widths and normalizations, obtained in a fit to the signal-dominated subset satisfying $ct/\sigma(ct) > 5$. We allow for two free parameters, the common mean value and the width of the narrow component. The lifetime distribution of the signal is parametrized by an exponential convoluted with a Gaussian function with the width taken from the event-by-event estimate of $\sigma(ct)$. To allow for the possibility of the lifetime uncertainty to be systematically underestimated, we introduce a free scale factor.

The transversity distribution of the signal is determined in the following way. The time-dependent three-angle distribution for the decay of untagged $B^0_d$ mesons, i.e., summed over $B^0_s$ and $B^0$, expressed in terms of the linear polarization amplitudes $|A_i(t)|$ and their relative phases $\delta_i$ is [6]

$$d^3 \Gamma(t) d\cos\theta d\varphi d\cos\psi \propto 2|A_0(0)|^2 e^{-\Gamma_{t'} t} \cos^2 \psi (1 - \sin^2 \theta \cos^2 \varphi) + \sin^2 \psi |A_\parallel(0)|^2 e^{-\Gamma_{t'} t} (1 - \sin^2 \theta \sin^2 \varphi)$$

$$+ |A_\perp(0)|^2 e^{-\Gamma_{t'} t} \sin^2 \theta + \frac{1}{\sqrt{2}} \sin 2\psi |A_0(0)||A_\parallel(0)| \cos(\delta_2 - \delta_1) e^{-\Gamma_{t'} t} \sin^2 \theta \sin 2\varphi$$

$$+ \left\{ \frac{1}{\sqrt{2}} |A_0(0)||A_\perp(0)| \cos \delta_2 \sin 2\psi \sin 2\theta \cos \varphi - |A_\parallel(0)||A_\perp(0)| \cos \delta_1 \sin^2 \psi \sin 2\theta \sin \varphi \right\} \times \frac{1}{2} (e^{-\Gamma_{t'} t} - e^{-\Gamma_{t''} t}) \delta_\phi.$$
and the corresponding value of the likelihood, are listed in Table I. We verify the procedure by performing fits on a sample of approximately 50 000 MC events passed through the full chain of detector simulation, event reconstruction, and maximum likelihood fitting. We see no bias in the event reconstruction or in the fitting procedure. The fits reproduce the inputs \((c\tau = 439 \mu \text{m}, \Delta \Gamma / \Gamma = 0,\) and a range of \(R_\perp\) between 0 and 1) correctly within the statistical precision of 2 \(\mu \text{m}\) for \(c\tau, 0.01\) for \(R_\perp,\) and 0.025 for \(\Delta \Gamma / \Gamma\). We test the sensitivity of the results to the parameterization of the signal and background mass distributions by varying the parameters of the two-Gaussian function. To test the sensitivity of the results to the background model, we add a quadratic term in the background mass distribution. We find a non-negligible effect from the extra term in the nonprompt background on \(c\tau\) and \(\Delta \Gamma / \Gamma\). We have also tested the sensitivity of the results to the assumption that the lifetime and the transversity distributions of background are independent of mass. The effect of the uncertainty of the detector alignment on the lifetime measurement was estimated in Ref. [3]. The effects of systematic uncertainties are listed in Table II.

We also conduct a test with an ensemble of 1000 pseudoexperiments with similar statistical sensitivity, chosen from the distribution described by Eq. (1), with the same parameters as obtained in this analysis. Both the spread of uncertainties and of the central values of the fit parameters are in good agreement with the results reported here. Our results are consistent with the CDF Collaboration results [9], also shown in Fig. 4.

\(B_s^0\) lifetime measurements from semileptonic (flavor-specific) data provide an independent constraint on the average lifetime and lifetime difference in the \(B_s^0\) system. The world average [5] \(B_s^0\) lifetime is \(\tau_{fs} = 1 / \Gamma_{fs} = 1.442 \pm 0.066\) ps. This result is based on single-exponential fits in the flavor-specific decay channels, which determine the following relation [10] (shown in Fig. 4) of \(\Gamma\) and \(\Delta \Gamma / \Gamma\): \(\Gamma_{fs} = \Gamma - (\Delta \Gamma)^2 / 2 \Gamma + O(\Delta \Gamma)^3 / \Gamma^2\). Applying the above constraint to our measurement, we obtain \(\tau(B_s^0) = 1.39 \pm 0.06\) ps and \(\Delta \Gamma / \Gamma = 0.25^{+0.14}_{-0.15}\). This result is consistent with the SM expectation [11] of 0.12 \pm 0.05.

\[5.26 < M(B_s^0) < 5.42 \text{ GeV}\]
\(B_s^0 \rightarrow J/\psi \phi\)

FIG. 2 (color online). The proper decay length \(c\tau\) of the \(B_s^0\) candidates in the signal mass region. The curves show the signal contribution, dashed (red online); the background, lower solid line (green online); and total, upper solid line (black) in the signal mass region.

FIG. 3 (color online). The distribution of the cosine of the transversity polar angle in the signal mass region, for nonprompt events, with the results of the maximum likelihood fit overlaid.

FIG. 4 (color online). The D0 default one-\(\sigma\) (stat) contour \([\delta \ln(L) = 0.5]\) compared to a one-\(\sigma\) band for the world average (WA) measurement based on flavor-specific decays, \(\tau_{fs} = 1.442 \pm 0.066\) ps. A simultaneous fit to our data and the WA gives a one-\(\sigma\) range \(\bar{\tau}(B_s^0) = 1.39 \pm 0.06\) ps and \(\Delta \Gamma / \Gamma = 0.25^{+0.14}_{-0.15}\). The SM theoretical prediction is shown as the horizontal band.

We also conduct a test with an ensemble of 1000 pseudoexperiments with similar statistical sensitivity, chosen from the distribution described by Eq. (1), with the same parameters as obtained in this analysis. Both the spread of uncertainties and of the central values of the fit parameters are in good agreement with the results reported here. Our results are consistent with the CDF Collaboration results [9], also shown in Fig. 4.

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TABLE I. Fit results for $\Delta \Gamma / \Gamma$ at fixed values of $\tilde{\tau}(B^0_s)$. For each assumed value of $\tilde{\tau}(B^0_s)$, the likelihood as function of $\Delta \Gamma / \Gamma$ is symmetric and parabolic.

<table>
<thead>
<tr>
<th>$\tilde{\tau}(B^0_s)$ (ps)</th>
<th>$\Delta \Gamma / \Gamma$</th>
<th>$\Delta \ln(L)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.23</td>
<td>$-0.13 \pm 0.15$</td>
<td>0.51</td>
</tr>
<tr>
<td>1.27</td>
<td>$-0.03 \pm 0.17$</td>
<td>0.51</td>
</tr>
<tr>
<td>1.31</td>
<td>0.07 $\pm 0.19$</td>
<td>0.17</td>
</tr>
<tr>
<td>1.35</td>
<td>0.16 $\pm 0.21$</td>
<td>0.04</td>
</tr>
<tr>
<td>1.39</td>
<td>0.24 $\pm 0.20$</td>
<td>0.0</td>
</tr>
<tr>
<td>1.43</td>
<td>0.31 $\pm 0.19$</td>
<td>0.06</td>
</tr>
<tr>
<td>1.47</td>
<td>0.37 $\pm 0.18$</td>
<td>0.20</td>
</tr>
<tr>
<td>1.51</td>
<td>0.43 $\pm 0.18$</td>
<td>0.42</td>
</tr>
<tr>
<td>1.55</td>
<td>0.48 $\pm 0.18$</td>
<td>0.69</td>
</tr>
</tbody>
</table>

All the results presented above are obtained under a tacit assumption that the $C^P$-violating phase is negligible, as predicted by the SM ($\delta \phi = \delta \phi_{\text{CKM}} = -0.03$). Future improvements on the measurement of $\Delta \Gamma / \Gamma$ may exclude models predicting large deviations of $\delta \phi$ from the SM value.

In summary, we have measured the $C^P$-odd fraction for the decay $B^0_s \to J/\psi \phi$, and the correlated parameters of the average lifetime of the ($B^0_s, B^0_s$) system $\bar{\tau}(B^0_s) = 1/\Gamma$, and the relative width difference $\Delta \Gamma / \Gamma$, or, equivalently, the mean lifetimes of the light and heavy $B^0_s$ eigenstates, respectively. We obtain

$$R_\perp = 0.16 \pm 0.10(\text{stat}) \pm 0.02(\text{syst}),$$
$$\Delta \Gamma / \Gamma = 0.24^{+0.28}_{-0.38}(\text{stat})^{+0.03}_{-0.04}(\text{syst}),$$
$$\tilde{\tau}(B^0_s) = 1.39^{+0.13}_{-0.16}(\text{stat})^{+0.01}_{-0.02}(\text{syst}) \, \text{ps},$$
$$\tau_L = 1.24^{+0.14}_{-0.11}(\text{stat})^{+0.01}_{-0.02}(\text{syst}) \, \text{ps},$$
$$\tau_H = 1.58^{+0.39}_{-0.42}(\text{stat})^{+0.01}_{-0.02}(\text{syst}) \, \text{ps}.$$

We have updated the measurement of the mean lifetime of the $B^0$ meson with doubled statistics. With the systematic uncertainty estimated in Ref. [3], the updated measurement is

$$\tau(B^0) = 1.530 \pm 0.043(\text{stat}) \pm 0.023(\text{syst}) \, \text{ps}.$$  

For the ratio of the average $B^0_s$ lifetime to the $B^0$ lifetime, we obtain

$$\frac{\tilde{\tau}(B^0_s)}{\tau(B^0)} = 0.91 \pm 0.09(\text{stat}) \pm 0.003(\text{syst}).$$

TABLE II. Sources of systematic uncertainty. The numbers reflect the variation of the fitted central values associated with the one-$\sigma$ variation of the corresponding external input parameters. The second item includes contributions from the variation of the acceptance as a function of $\varphi$ and $\phi$, as well as from a one-$\sigma$ variation of the quantity $\left| A_0(0) \right|^2 - \left| A_1(0) \right|^2$.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\epsilon \tilde{\tau}(B^0_s)$, $\mu \text{m}$</th>
<th>$\Delta \Gamma / \Gamma$</th>
<th>$R_\perp$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptance vs $\cos \theta$</td>
<td>$\pm 0.6$</td>
<td>$\pm 0.001$</td>
<td>$\pm 0.005$</td>
</tr>
<tr>
<td>Integration over $\varphi$, $\phi$</td>
<td>$\pm 0.2$</td>
<td>$\pm 0.001$</td>
<td>$\pm 0.002$</td>
</tr>
<tr>
<td>Procedure test</td>
<td>$\pm 0.2$</td>
<td>$\pm 0.025$</td>
<td>$\pm 0.01$</td>
</tr>
<tr>
<td>Momentum scale</td>
<td>$-3.0$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>Signal mass model</td>
<td>$1.0 \pm 0.009, -0.017 \pm 0.007$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Background mass model</td>
<td>$-3.5 \pm 0.02$</td>
<td>$-0.002$</td>
<td></td>
</tr>
<tr>
<td>Detector alignment</td>
<td>$\pm 2.0$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>Background model</td>
<td>$\pm 0.5$</td>
<td>$\pm 0.016$</td>
<td>$\pm 0.005$</td>
</tr>
<tr>
<td>Total</td>
<td>$-5.6, +3.1$</td>
<td>$-0.04, +0.03$</td>
<td>$\pm 0.02$</td>
</tr>
</tbody>
</table>

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