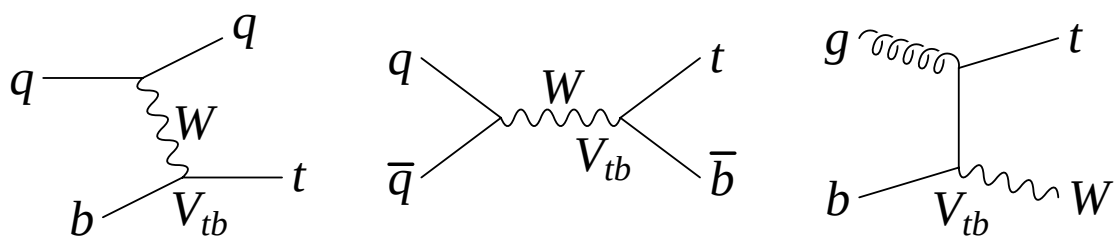
Fermilab Theory Seminar

THE DANGEROUS BEAUTY OF SINGLE-TOP-QUARK PRODUCTION



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In collaboration with B.W. Harris (*Robert Morris U.*), E. Laenen, L. Phaf (*NIKHEF*), S. Weinzierl (*U. di Parma*); P. Nadolsky (*SMU*); T. Stelzer, and S. Willenbrock (*UIUC*)

Outline

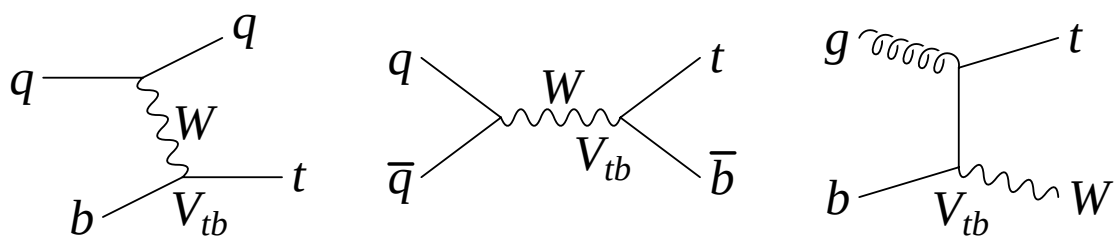
1. Why do we study single-top-quark production?
   - V_{tb}, production modes distinguish new physics
2. Inclusive cross sections
   - How W-gluon fusion becomes t-channel production by reordering perturbation theory.
     T. Stelzer, Z.S., S. Willenbrock, PRD 56, 5919 (1997)
3. How to discover the s-channel and t-channel modes at the Tevatron
   T. Stelzer, Z.S., S. Willenbrock, PRD 58, 094021 (1998)
   - Using a jet veto to find the signal
   - A word on parton distribution function uncertainties
     Z.S. and P. Nadolsky, eConf C010630, P511 (2002) [hep-ph/0111358], P510 (2002) [hep-ph/0110378]; Z.S., PRD 66, 075011 (2002)
4. New methods for fully differential NLO QCD calculations
   - Phase space slicing, massive dipole formalism
5. s- and t-channel NLO distributions
   - Effects of scale, m_t, jet definition
     B.W. Harris, E. Laenen, L. Phaf, Z.S., S. Weinzierl, PRD 66, 054024 (2002); IJMP A 16, Suppl. 1A, 379 (2001) [hep-ph/0102126].
6. Event generator problems, and a prescription to solve them
   Work in progress

(7. Effects of b-tagging, c mis-tagging on discovery and δV_{tb})

8. Summary

Single-Top-Quark Production

(Flagship Measurement for Run II and Beyond)



1. Single-top-quark production will be discovered at Run II.
   3σ evidence with ~ 130 pb^{-1}, 5σ discovery with ~ 370 pb^{-1}.

2. Measuring 1 and 2 b-tag exclusive states will give
   s-channel and t-channel cross sections separately.
   t-channel evidence (discovery) with ~ 370 pb^{-1}(~ 1 fb^{-1}).
   s-channel evidence (discovery) with ~ 730 pb^{-1}(~ 2 fb^{-1}).

3. New physics typically changes the s- and t-channel cross sections in different ways.

4. Will directly measure $|V_{tb}|$ by measuring the cross section in each channel. $\delta|V_{tb}| \sim 13.5$–15.5% with 2 fb^{-1}.

5. Can start to directly measure the polarization of the top quark at production with ~ 2 fb^{-1}.

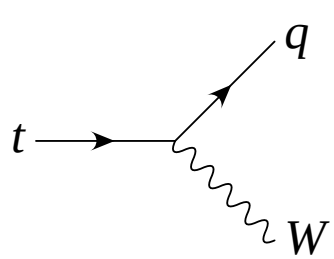
Weak Interaction

$$SU(2)_L \quad \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

When $SU(2)_L \times U(1)_Y$ breaks into $U(1)_{\mathrm{EM}}$ a mis-alignment between the weak interaction eigenstates (which see only the left-handed component of the quarks) and the mass eigenstates (which mix both left- and right-handed components) induces a slight modification to the simple $V - A$ term of the Lagrangian.

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \sum_{q=dsb} \bar{t}\gamma^\mu \frac{1}{2}(1-\gamma_5) V_{tq} q W_\mu^+$$

The CKM element V_{tq} differentiates the quarks from the charged leptons in the weak interactions.



$$-i\frac{g}{\sqrt{2}} V_{tq} \gamma^\mu \frac{1}{2}(1-\gamma_5)$$

We want to determine the structure of the W-t-b vertex.

Rare Decays

The partial width $\Gamma(t \to bW) = |V_{tb}|^2 \times 1.42$ GeV.

cf. K.G. Chetyrkin *et al.*, PRD 60, 114015 (1999); A. Denner and T. Sack, NPB 358, 46 (1991); R. Migneron and A. Soni, PRL 66, 3105 (1991).

Next most likely Standard Model decays are:

BR ($t \to sW$)	$\approx$	1.6×10^{-3} assuming $\lvert V_{ts}\rvert = 0.04$
BR ($t \to dW$)	$\approx$	1×10^{-4} assuming $\lvert V_{td}\rvert = 0.01$
BR ($t \to bWZ$)	$\approx$	10^{-6}–10^{-7}
BR ($t \to X$)	$<$	10^{-11}, X from a FCNC

Can we see these at the LHC? How would we know?

Any decay other than $t \to bW$ is a good sign of new physics.

Cabibbo-Kobayashi-Maskawa Matrix

There is a mis-alignment between the weak interaction eigenstates and the mass eigenstates of the quarks.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Assuming 3 generations and unitarity

$$= \begin{pmatrix} 0.9741 - 0.9756 & 0.219 - 0.226 & 0.002 - 0.005 \\ 0.219 - 0.226 & 0.9732 - 0.9748 & 0.038 - 0.044 \\ 0.004 - 0.014 & 0.037 - 0.044 & 0.9990 - 0.9993 \end{pmatrix}$$

Relaxing the assumption of 3 generations

$$= \begin{pmatrix} 0.9721 - 0.9747 & 0.215 - 0.224 & 0.002 - 0.005 & \dots \\ 0.209 - 0.227 & 0.966 - 0.976 & 0.038 - 0.044 & \dots \\ 0 - 0.09 & 0 - 0.12 & 0.08 - 0.9993 & \dots \\ \vdots & \vdots & \vdots & \end{pmatrix}$$

PDG, PRD 66, 010001 (2002)

What About V_{tb}

$$\frac{\mathrm{BR}(t\to Wb)}{\mathrm{BR}(t\to Wq)} = 0.94^{+0.31}_{-0.24} \quad \text{CDF PRL 86, 3233 (2001)}$$

$$\equiv \frac{|V_{tb}|^2}{|V_{td}|^2+|V_{ts}|^2+|V_{tb}|^2}$$

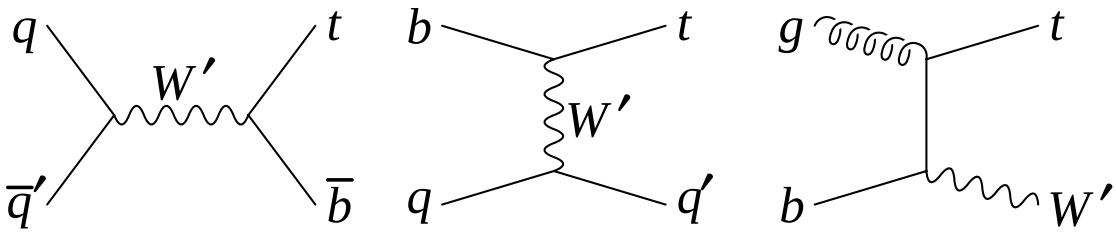
- Assuming exactly 3 generations, and no new physics, unitarity implies $|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$
  - $\Rightarrow |V_{tb}| = 0.97^{+0.16}_{-0.12}$ based on the CDF result
  - But we already know from unitarity and the CKM matrix $|V_{tb}| = 0.9990 - 0.9993$!
- With more than 3 generations, $|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 \neq 1$
  We only know $|V_{tb}| \gg |V_{td}|, |V_{ts}|$
- New physics can add to the branching fraction in the denominator, or lead to a fake signal,
  e.g. $(\widetilde{q} \to q'\widetilde{\chi}_1^+ \to q'W\widetilde{\chi}_1^0), \ldots$

There is no apparent way to measure V_{tb} in top-quark decays.

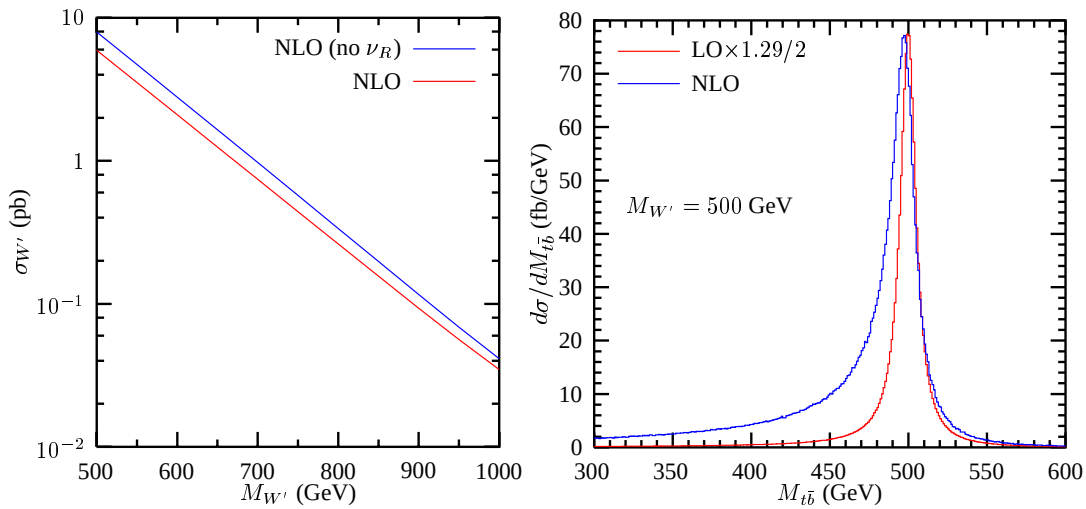
We need to look at single-top-quark production.

The search for new resonances

W' production



Only see in s-channel, $\sigma_t(W') \sim 1/M_{W'}^4$.

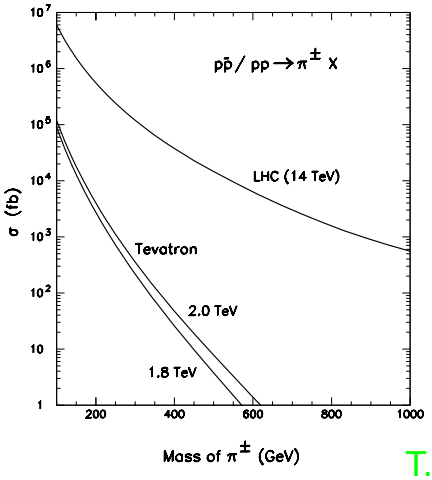


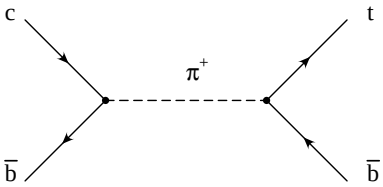
Z.S., PRD 66, 075011 (2002) [hep-ph/0207290].

CDF just set a bound of $M_{W'} > 536(566)$ GeV.hep-ex/0209030

The mass reach should extend to 800–900 GeV at Run II.

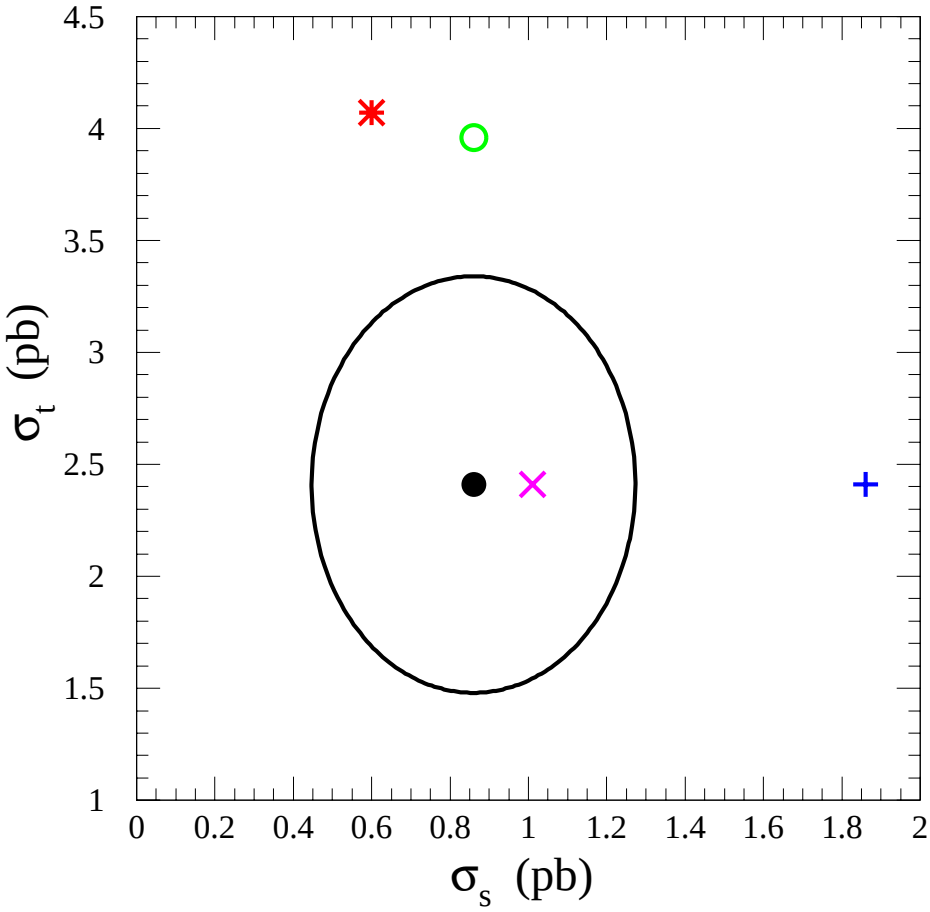
Charged scalars (top pions)





T. Tait, C.P. Yuan PRD 63, 014018 (01)

New physics effects in s-channel and t-channel



T. Tait, C.P. Yuan PRD 63, 014018 (01)

Small decrease in both cross sections if $V_{tb} < 1$.

New resonances affect the s-channel measurement.

- W', top pions π^+, top squarks $\tilde{t}, \not{R}_p$ SUSY

New t-q couplings mostly affect t-channel.

- Larger V_{ts} or V_{td} give PDF enhancement to σ_t.
- FCNC production modes from, e.g. Z-t-c, increase σ_t.

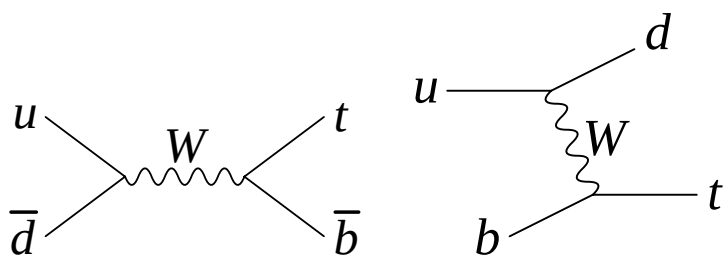
Measuring both production modes provides strong constraints on most new physics scenarios.

Single top quarks are polarized

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \sum_{q=dsb} \bar{t}\gamma^\mu \frac{1}{2}(1-\gamma_5) V_{tq} q W_\mu^+$$

The $V-A$ structure of the Lagrangian produces a 100% correlation between the direction of the d quark in the event and the spin s_t of the top quark.

M. Jeżabek, NPBPS 37B, 197 (94)



The large width of the top quark (~ 1.5 GeV) allows it to decay before it depolarizes ($\sim \lambda_{QCD}^2/m_t = 1$ MeV), or even hadronizes ($\sim \lambda_{QCD} = 300$ MeV).

A. Falk, M. Peskin, PRD 49, 3320 (94)

Measuring the polarization of the top quark

$$\frac{1}{\Gamma_{(t\to bl\nu)}}\frac{d\Gamma_{(t\to bl\nu)}}{d\cos\theta} = \frac{1}{2}(1+\frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}\cos\theta)$$

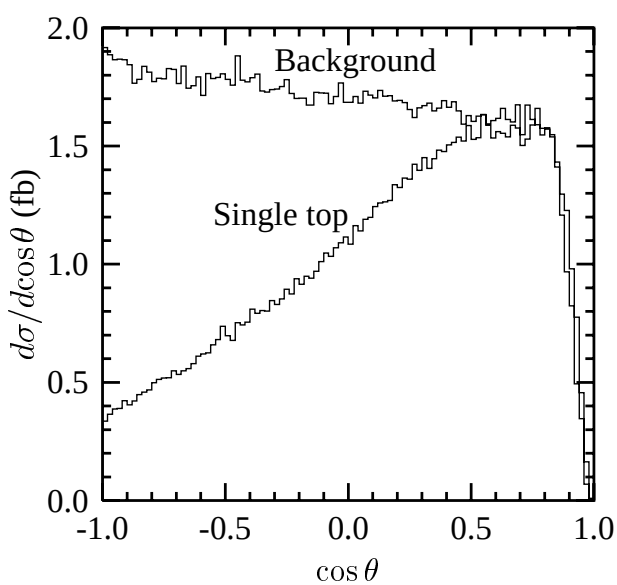
θ is the angle, in the top-quark rest frame, between the direction of the charged lepton and the spin of the top quark.

- At LO $\frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} = 0.96$ in anti-proton basis for s-channel. $\frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} = 0.91$ in spectator basis for t-channel.

  G. Mahlon, S. Parke, PRD 55, 7249 (97)

  Does this hold at NLO?

- The only analysis uses LO distributions in t-channel:

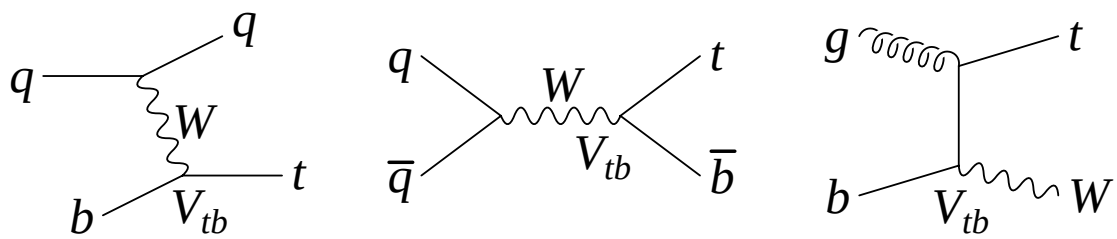


T. Stelzer, Z. S., S. Willenbrock PRD 58, 094021 (98)

$$A \equiv \frac{\sigma(-1<\cos\theta<-0.1)-\sigma(-0.1<\cos\theta<0.8)}{\sigma(-1<\cos\theta<-0.1)+\sigma(-0.1<\cos\theta<0.8)}$$

- At LO the asymmetry A is just visible (3σ) at Run II. At NLO?

Single-Top-Quark Production



Comparing the measured cross section for these processes to the theoretical prediction allows a direct extraction of $|V_{tb}|^2$.

Partially updated totally inclusive cross sections

	Tevatron, Run I	Tevatron, (2 TeV)	LHC	
$\sigma_t^{\rm NLO} =$	1.45 ± 0.09 pb	2.12 ± 0.14 pb	247 ± 12 pb	[1]
$\sigma_s^{\rm NLO} =$	0.75 ± 0.07 pb	0.92 ± 0.08 pb	10.7 ± 0.9 pb	[2]
$\sigma_{\rm assoc.}^{\rm LL} =$		0.093 ± 0.024 pb	56 ± 8 pb	[3]
$\sigma_{\rm Total} =$	2.2 ± 0.1 pb	3.13 ± 0.16 pb	313 ± 14 pb	

Total single-top-quark production rate $\sim \frac{1}{2}\, t\bar{t}$ rate at Tevatron.

[1] Based on† T. Stelzer, Z.S., S. Willenbrock, PRD 56, 5919 (1997)

[2] Based on† M. Smith, S. Willenbrock, PRD 54, 6696 (1996)

[3] T. Tait, PRD 61, 034001 (2000); A. Belyaev, E. Boos, PRD 63, 034012 (2001);

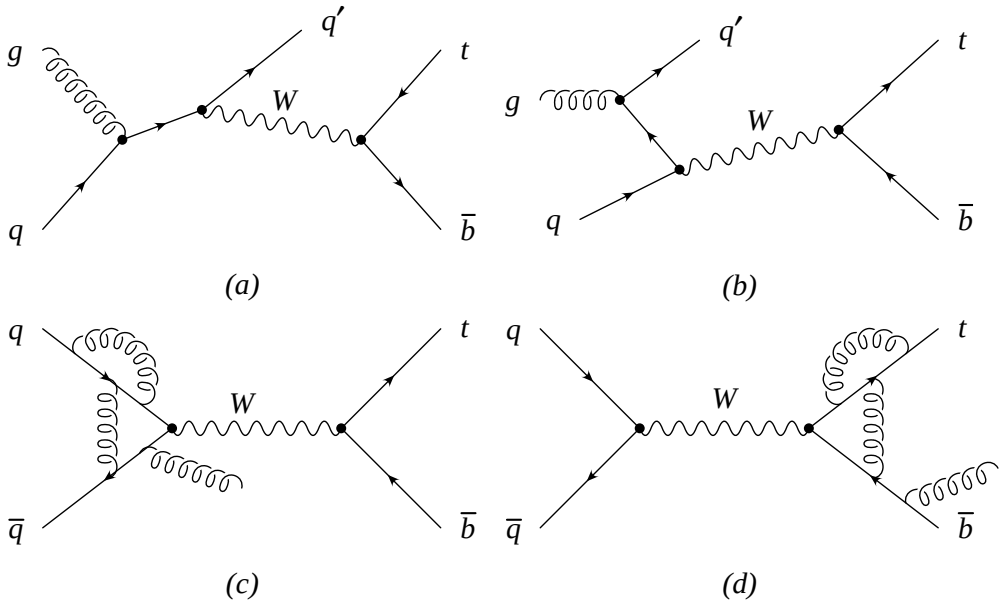
Needs NLO calculation

† Updated with CTEQ5M1 PDFs, all old CTEQ and MRS sets had bad b PDF.

CHANGE: $\delta\sigma_t$: $-15\%, -13\%, +1\%$, $\delta\sigma_s$: $+3\%, +4.5\%, +5\%$.

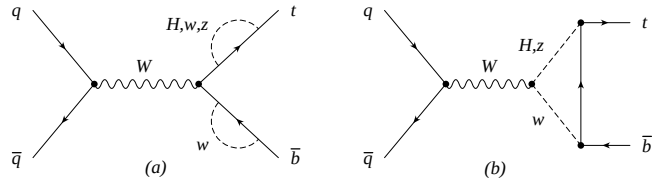
B.W. Harris, E. Laenen, L. Phaf, Z.S., S. Weinzierl, PRD 66, 054024 (2002)

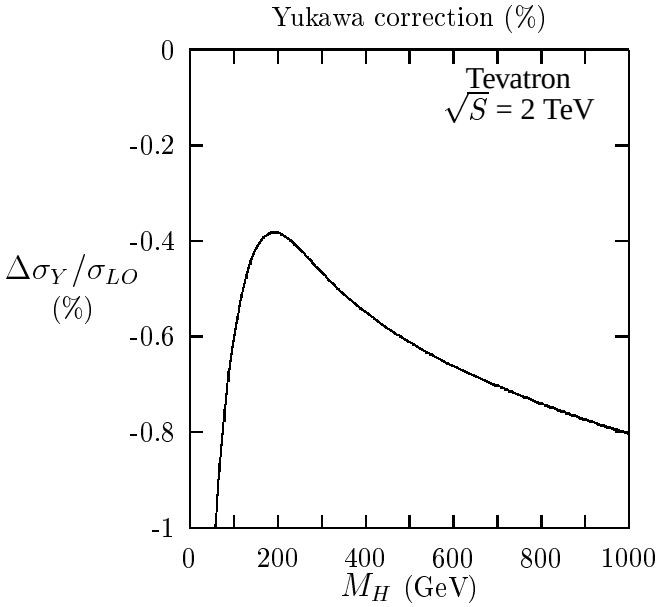
s-channel NLO correction



Corrections to the initial (a–c) and final (d) state are separable at NLO.

Yukawa corrections are small.



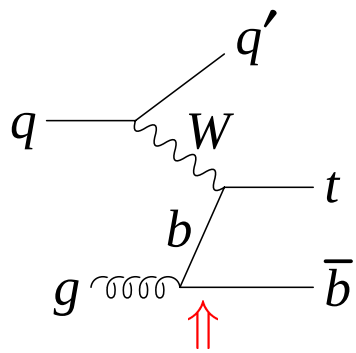


M. Smith, S. Willenbrock, PRD 54, 6696 (1996)

NLO W-gluon fusion (t-channel)

T. Stelzer, Z.S., S. Willenbrock, PRD 56, 5919 (1997)

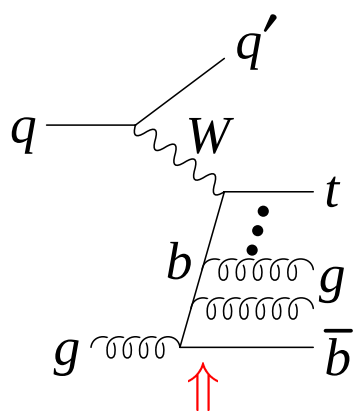
Leading Order



$$\sim \alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right) + \mathcal{O}(\alpha_s)$$

$m_t \approx 35 m_b$! So log is LARGE. $\alpha_s \ln \sim .7\text{-}.8$

Higher Orders



$$\sim \frac{1}{n!}\left[\alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right)\right]^n$$

LOOKS WORSE . . .

Reorder perturbation theory with heavy quark PDFs

A heavy-quark PDF is a mechanism to sum large logarithms due to (almost-)collinear singularities in gluon splitting.

The DGLAP equation:

$$\frac{dQ(\mu^2)}{d\ln(\mu^2)} \approx \frac{\alpha_s}{2\pi} P_{Qg} \otimes g + \cancel{\frac{\alpha_s}{2\pi} P_{QQ} \otimes Q} \quad Q \ll g$$

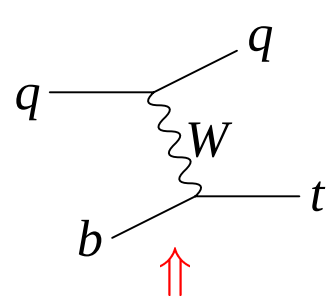
$$P_{Qg}(z) = \frac{1}{2}[z^2 + (1-z)^2] \ .$$

Solving the DGLAP equation for the heavy-quark Q PDF analytically [with $Q(x,\mu^2) = 0$ at $\mu = m_Q$]:

$$Q(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{m_Q^2}\right) \int_x^1 \frac{dz}{z} P_{Qg}(z) g\left(\frac{x}{z}, \mu^2\right)$$

Barnett, Haber, Soper, NPB 306, 697 (1988)
Olness, Tung, NPB 308, 813 (1988)
Aivazis, Collins, Olness, Tung PRD 50, 3102 (1994)

New Leading Order



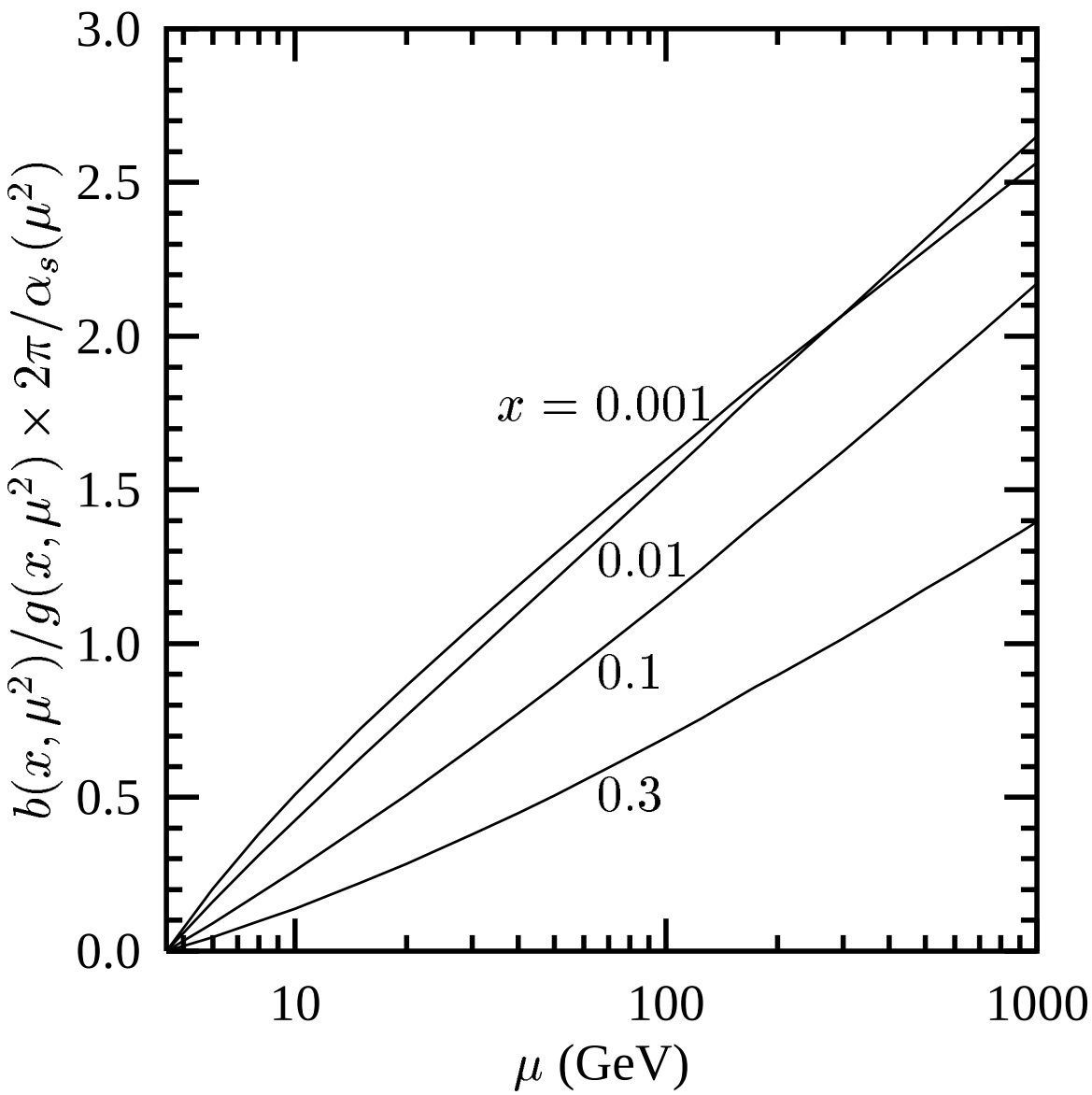
$$\Uparrow$$

$$b \sim \alpha_s \ln\left(\frac{\mu^2}{m_b^2}\right) \times g$$

b PDF vs. μ

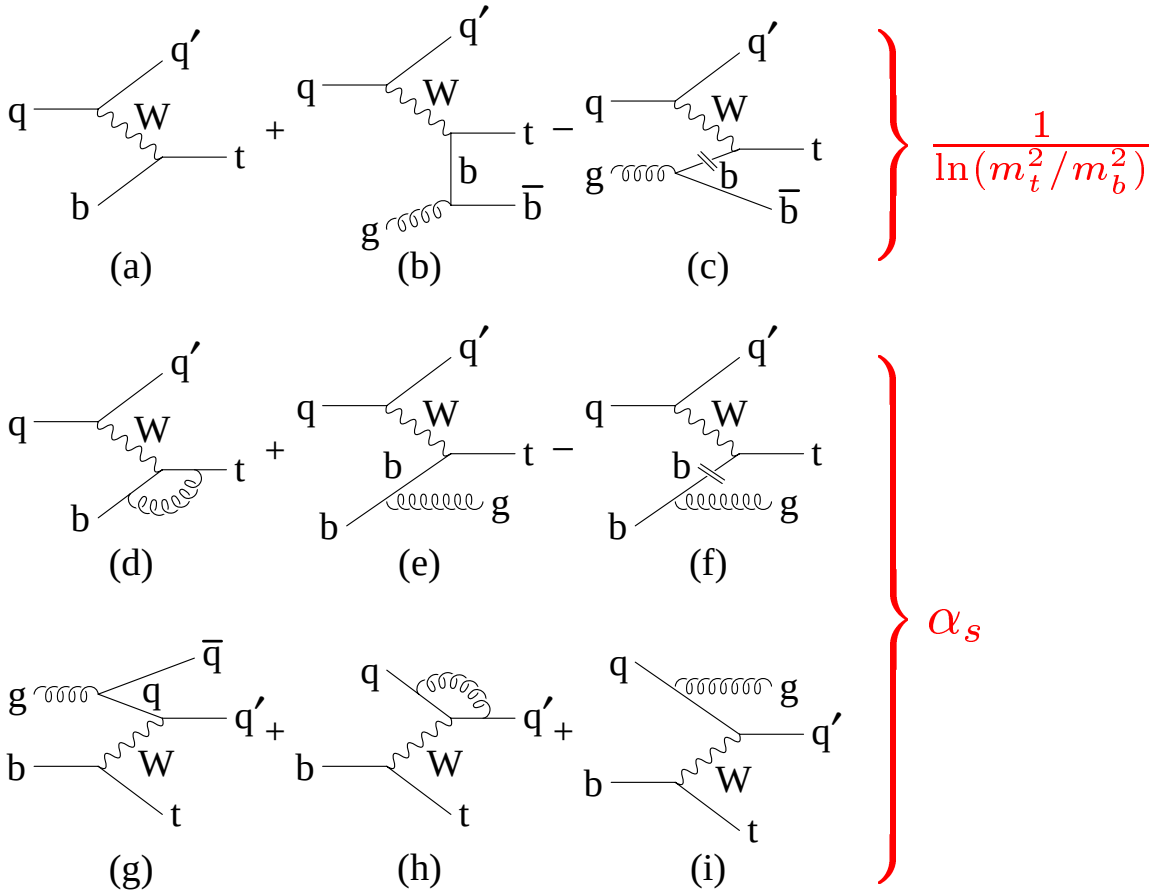
Using NLO CTEQ5M1 PDFs,

$$b(x,\mu^2) \propto [\alpha_s(\mu^2)/2\pi]\ln(\mu^2/m_b^2)g(x,\mu^2)$$



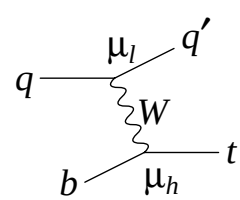
Updated Stelzer, Sullivan, Willenbrock, PRD 56, 5919 (1997)

2-to-2 t-channel NLO correction (DDIS)

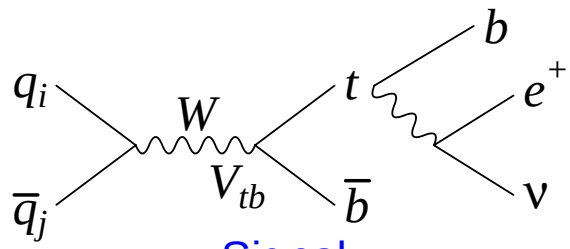


Gluon splitting to top-pair diagrams are not shown.

- Corrections to the “heavy-quark” (b–f) and “light-quark” (g–i) lines are factorizable at NLO.
- The NLO analytic formulae look like Double Deep Inelastic Scattering (DDIS) with scales explicitly entering in the
  - “light-quark” line as $\ln\left(\frac{\mu_l^2}{Q^2}\right)$ as in DIS,
  - “heavy-quark” line as $\ln\left(\frac{\mu_h^2}{Q^2+m_t^2}\right)$ as in DIS with a massive quark.



s-channel Production

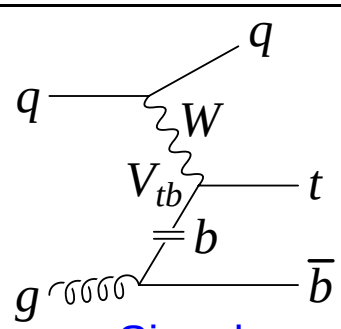


Signal

2 b-tags, 1 electron/muon, missing energy

	Detector$+b$-tag (Jet Veto$+M_{b\ell\nu}$)	
$\overline{b} + t \to \overline{b}\, b\, \ell\, \nu$	$\sim 30(20)$ events/fb^{-1}	s-channel
	$\sim 20(3)$ events/fb^{-1}	t-channel
	$\sim 100(20)$ events/fb^{-1}	$Wb\overline{b}$
	$\sim 500(8)$ events/fb^{-1}	$t\overline{t}$

t-channel Production

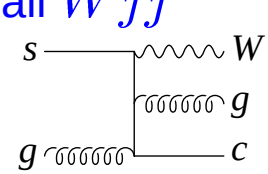


Signal

1 b-tag, 1 forward jet, 1 electron/muon, missing energy

$t + j \to b\, \ell\, \nu\, j$	$\sim 115(66)$ events/fb^{-1}	t-channel
	$\sim 50(25)$ events/fb^{-1}	s-channel
	$\sim 840(25)$events/fb^{-1}	$t\overline{t}$
	$\sim 750(150)$ events/fb^{-1}	all Wjj

Biggest contribution is from c/j faking a b-tag



Recommended Search Strategies

Fast track to discovery!

Jet Veto

- Require 2 jets, 1 lepton, missing energy, at least 1 b-tag.
- If there are any additional "hard" jets, reject the event. "hard" is a loose term, $p_{Tj} >$ 15-25 GeV, $|\eta_j| <$ 2.5-4.
- Add top-quark mass reconstruction to kill Wjj.

3 successive measurements

1. $jb\ell\not{E}_T$ or $bb\ell\not{E}_T$; j may or may not be b-tagged.

   $\Rightarrow$ All single-top modes ($s + t+$ associated).

2. $jb\ell\not{E}_T$; If j is b-tagged, reject the event.

   $\Rightarrow$ t-channel exclusively

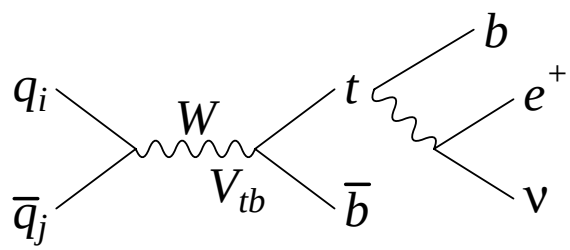
3. $bb\ell\not{E}_T$; Must have 2 b-tags.

   $\Rightarrow$ s-channel exclusively

H_T Search

- $H_T \equiv \sum_i E_{Tj_i} + E_{T\ell} + \not{E}_T$; Sum all "observed" objects, just remove "jet veto" step above.
- Should be similar to Jet Veto for summing over modes.
  - Main use is reducing $t\bar{t}$ — just like "jet veto."
- Difficult to estimate theoretical uncertainty.
  - 10-45 GeV shifts at NLO.
- Should be used as a complement to the "jet veto" search.

s-channel Production



DØ and CDF have Run I limits of 17 pb and 18 pb, respectively.

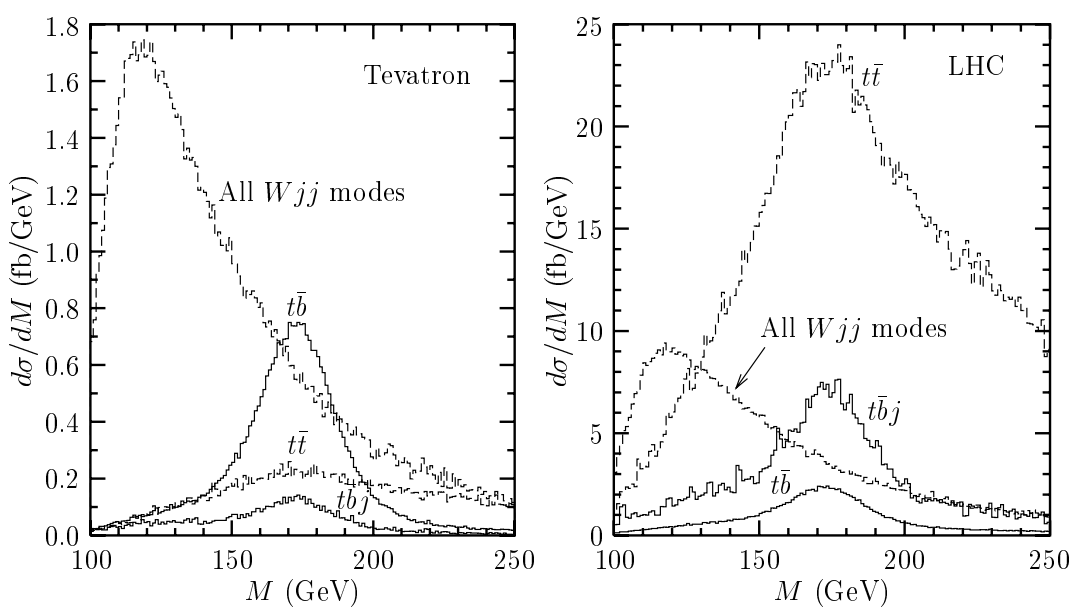
DØ, PLB 517, 282 (01); CDF, PRD 65, 091102 (02)

Advantages

- Parton luminosity ($u\bar{d} \to W$) is well understood $\sim 4\%$.
- Total theoretical error is low $\sim 10\%$. (dominated by m_t)
- V_{tb} can be measured to about 15.5% at Run II, and is statistically dominated. (15 fb^{-1} $\to 5\%$ stat., 9% total)

Disadvantages

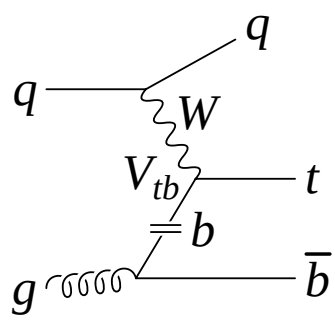
- Small cross section means V_{tb} measurement needs a lot of integrated luminosity to be competitive.
- Considerable irreducible background ($t\bar{t}$) at LHC.



T. Stelzer, Z. S., S. Willenbrock PRD 58, 094021 (98)

NLO K-factors and LO distributions were used. Is this OK?

t-channel Production



$\sigma(p_{T\bar{b}}<20\mathrm{GeV})=1.07,1.54,160$ pb $\pm 15\%$ at Run I, II, LHC

D$\emptyset$ and CDF have Run I limits of 22 pb and 13 pb, respectively.

D$\emptyset$, PLB 517, 282 (01); CDF, PRD 65, 091102 (02)

Advantages

- Large cross section at Tevatron, much larger at LHC.
- Run IIA can measure this cross section to about 17%, LHC is limited by systematics.
- V_{tb} can be measured to about 13% at Run IIA, 9% at LHC.

Disadvantage

Extraction of V_{tb} is limited by theoretical uncertainty in NLO cross section after cuts.

– 15% uncertainty due to inadequate modeling of $p_{T\bar{b}}$.

– $15\%^{\dagger}$ uncertainty from q–g luminosity hidden in b PDF.

Solutions?

– Can q–g luminosity be better constrained? w/ Zj/Wj?

– Model $p_{T\bar{b}}$ with fully differential NLO cross section. $\Rightarrow \pm 6\%$

† Calculated in Z.S., P. Nadolsky, eConf C010630, P511 (2002) [hep-ph/0111358]
This will improve soon. . .

Heavy-quark PDFs and uncertainties

$$b_{LO}(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{m_b^2}\right) \int_x^1 \frac{dz}{z} P_{bg}\left(\frac{x}{z}\right) g\left(z,\mu^2\right)$$

Even at NLO, $b(x,\mu^2) \propto [\alpha_s(\mu^2)/2\pi] \ln(\mu^2/m_b^2) g(x,\mu^2)$, and it was assumed that $\delta b/b$ and $\delta g/g$ were the same 10%.

[$\rightarrow$ Fig.]

We check this assumption using the 41 PDFs of CTEQ6. Since the minimum (z_i^0) of the PDF fit is not the minimum of the observable, we define the Modified Tolerance Method as an improved measure of the observable O uncertainty:

$$\delta O_+ = \sqrt{\sum_{i=1}^{20} \left(\max[\,O(z_i^0+t)-O(z_i^0), O(z_i^0-t)-O(z_i^0), 0]\right)^2}$$

$$\delta O_- = \sqrt{\sum_{i=1}^{20} \left(\max[\,O(z_i^0)-O(z_i^0+t), O(z_i^0)-O(z_i^0-t), 0]\right)^2}$$

Z.S., PRD 66, 075011 (2002);
Z.S., P. Nadolsky, eConf C010630, P511 (2002);
eConf C010630, P510 (2002)

We sum the maximal deviations to find the asymmetric error.

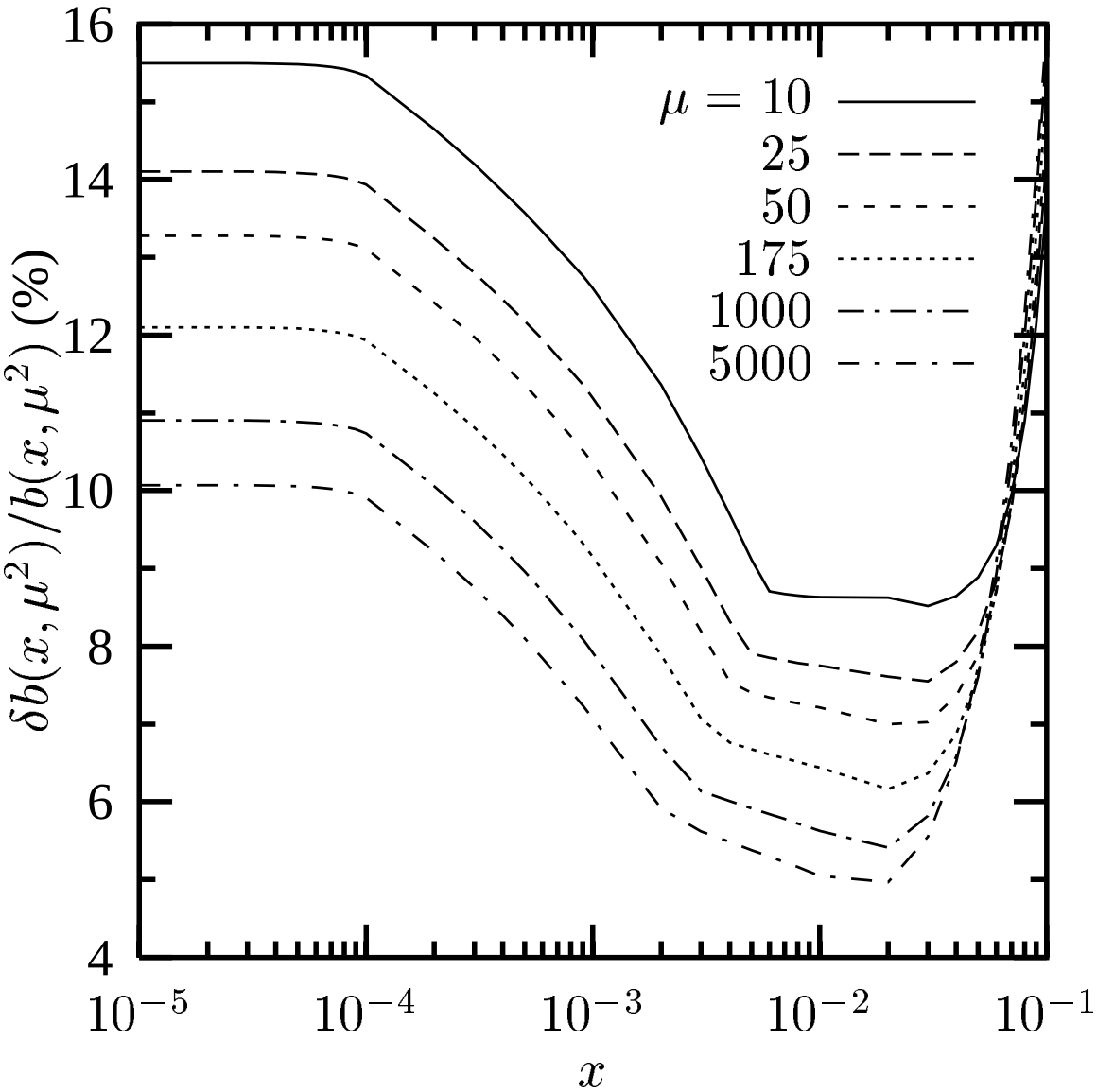
The result for t-channel at 2.0 TeV is $2.12^{+0.32}_{-0.29}$ pb or $^{+15}_{-14}\%$.

Z.S., P. Nadolsky, eConf C010630, P511 (2002)

The s-channel PDF uncertainty is $\pm 4\%$.

Note that pre-CTEQ6 PDFs were used at Snowmass.
The t-channel uncertainty will come down soon. . .

Uncertainty of b PDF vs. x



Z.S., P. Nadolsky, eConf C010630, P511 (2002)

At the Tevatron, $x \sim m_t/\sqrt{S} \sim 0.1 \Rightarrow \delta\sigma_t(\delta b) \approx 15\%$.

At the LHC, $x \sim m_t/\sqrt{S} \sim 0.01 \Rightarrow \delta\sigma_t(\delta b) \approx 6.5\%$.

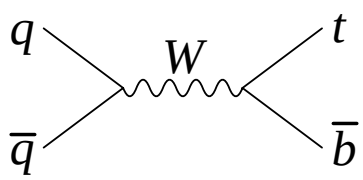
The LHC probes the best measured region of the PDFs!

b uncertainty $\approx 1.5 \times g$ uncertainty for same x, Q^2.

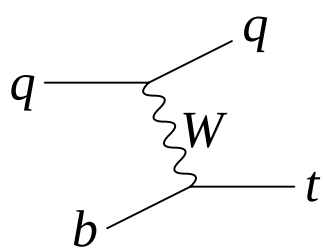
Fully Differential NLO QCD Calculations

1. Calculate NLO cross section keeping arbitrary 4-momenta and spins of all final state particles.

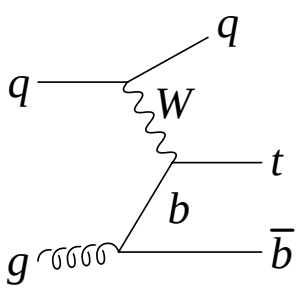
   (a) s-channel (LO: $t + \bar{b}$, NLO: $t + \bar{b} + 1$ jet)



   (b) 2-to-2 t-channel (LO: $t + 1$ jet, NLO: $t + 2$ jets, $t + \bar{b} + j$)



   (c) 2-to-3 t-channel (Wg fusion) (LO: $t + \bar{b} + 1$ jet, NLO: $t + \bar{b} + 2$ jets)



2. Add spin-dependent top-quark decays to see polarization.

3. Interface to event generator. Avoid double-counting?

NOTE: Every b or c in the final state may be b-tagged. Hence, we must keep track these.

Fully Differential NLO Techniques

There are a few modern techniques that allow us to calculate the fully differential QCD corrections. These provide the 4-vectors and corresponding weights of the final-state configuration. The final answers must be independent of the method, so we use more than one as a cross check.

- Phase space slicing method with 2 cutoffs.

  L.J. Bergmann, Ph.D. Thesis, FSU (89)
  cf. H. Baer, J. Ohnemus, J.F. Owens, PRD 40, 2844 (89)
  B.W. Harris, J.F. Owens, PRD 65, 094032 (2002)

- Phase space slicing method with 1 cutoff.

  W.T. Giele, E.W.N. Glover, PRD 46, 1980 (92)
  cf. W.T. Giele, E.W.N. Glover, D.A. Kosower, NPB 403, 633 (93)
  E. Laenen, S. Keller, PRD 59, 114004 (99)

- Massive dipole formalism (a subtraction method) coupled with helicity-spinor calculation.

  cf. L. Phaf, S. Weinzierl JHEP 0104, 006 (01)
  S. Catani, S. Dittmaier, M. Seymour, Z. Trocsanyi, NPB 627,189 (02)

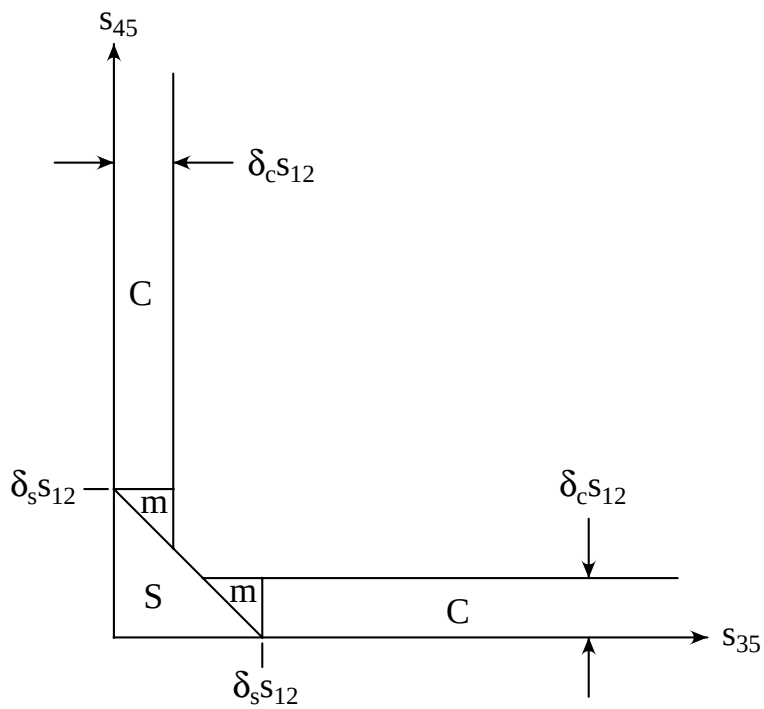
We use different renormalization schemes and methods of treating γ_5 to check consistency ("naive γ_5" and dimensional regularization, t'Hooft-Veltman scheme, modified dimensional reduction).

Phase Space Slicing Method (2 cutoffs)

- Phase space is divided into soft, hard-collinear, and hard-non-collinear regions using two parameters: δ_s, δ_c.
  - collinear: any invariant $\hat{s}_{34}, t_{13}, \hat{s}_{35}, \ldots < \delta_c \hat{s}$; soft: $E_g \leq \delta_s \sqrt{\hat{s}}/2$; both are integrated out analytically. [$\rightarrow$ Fig.]
  - Hard-non-collinear region (i.e. 2-to-3, all particles well separated and non-soft) is integrated numerically.
  - After adding virtual and mass factorization terms, all poles cancel.
- Cutoff dependence cancels in any IR-safe observable at histogramming stage. [$\rightarrow$ Fig.]
- Any jet finding algorithm may be implemented (fixed-cone, k_T, ...). Results here use k_T algorithm with $R = 1$ (roughly equivalent to $R_{\rm cone} = 0.74$). Can study cone-dependence. [see later]
- Experimental cuts may be applied.

Phase Space Slicing Method (2 cutoffs)

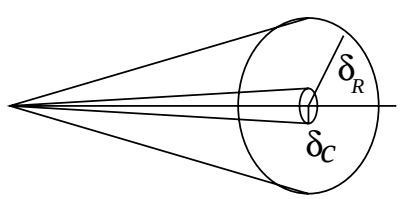
Phase space plane (s_{35}, s_{45})



The triangles marked m give vanishing contribution for $\delta_c \ll \delta_s$.

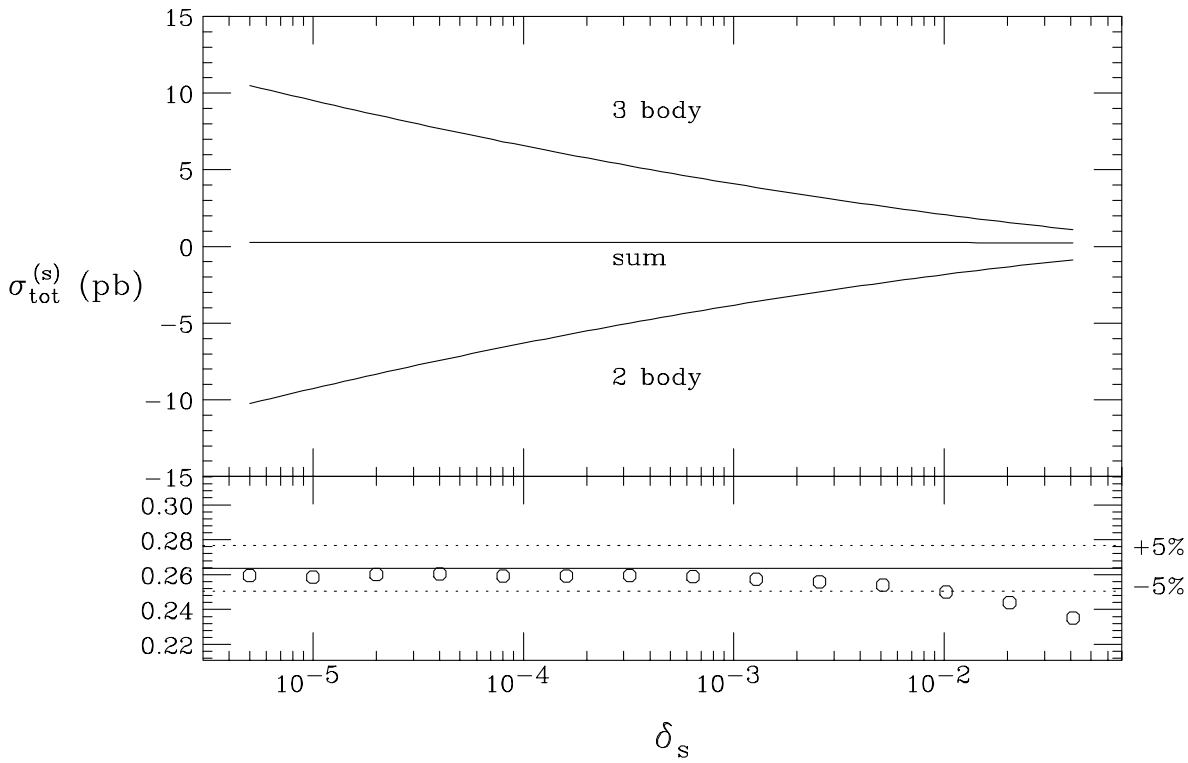
B.W. Harris, J.F. Owens, PRD 65, 094032 (2002)

Physically you can think of phase space slicing as forming a "pre-jet" that is much smaller than your final jet of radius R. ($\delta_c \ll \delta_R$)



The essential challenge of NLO differential calculations is dealing with final-state soft or collinear divergences.

Cut-off dependence of s-channel NLO correction

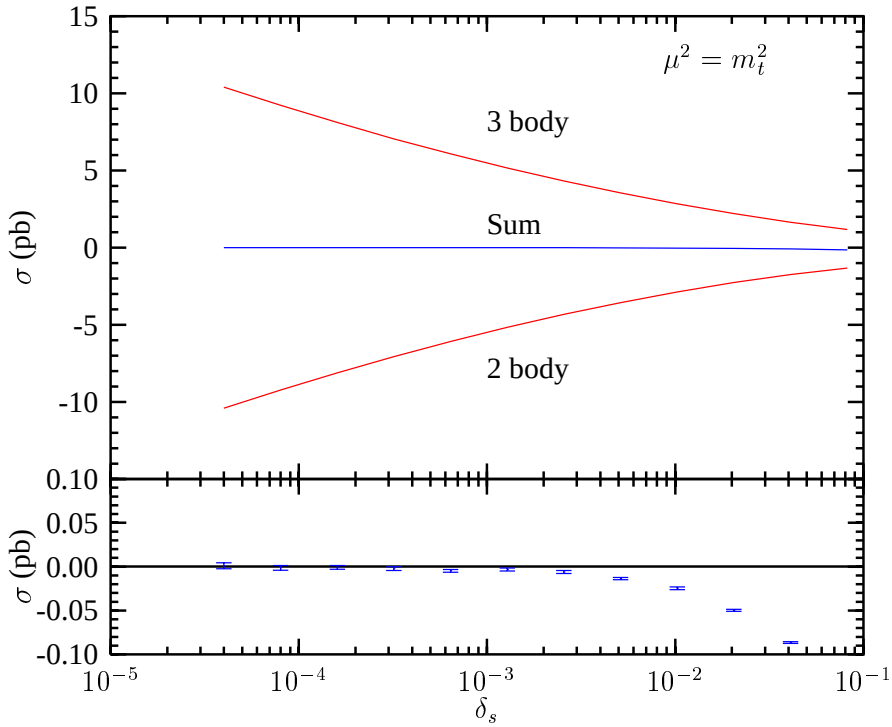


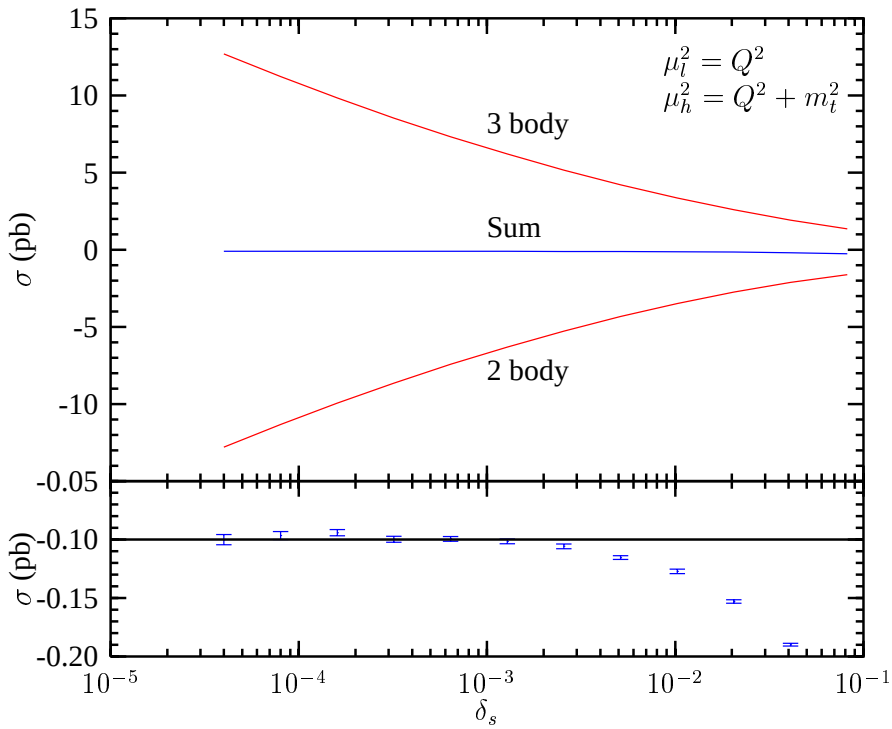
Here $\delta_c = \delta_s/300$, using CTEQ4M PDFs, $\mu = m_t$.

The 2-to-2 and 2-to-3 components of the correction each depend logarithmically on the cutoffs, but the sum is independent of δ_c and δ_s. So take δ_c and δ_s to 0.

Cut-off dependence of t-channel NLO correction

Here $\delta_c = \delta_s/300$, using CTEQ5M1 PDFs.





The 2-to-2 and 2-to-3 components of the correction each depend logarithmically on the cutoffs, but the sum depends only linearly on δ_c and δ_s. So take δ_c and δ_s to 0.

Massive Dipole Formalism (subtraction)

$$\begin{aligned}\sigma_{NLO} &= \int_{n+1} d\sigma^{\mathrm{Real}} + \int_{n} d\sigma^{\mathrm{Virtual}} \\ &= \int_{n+1} \left(d\sigma^{R} - d\sigma^{A}\right) + \int_{n} \left(d\sigma^{V} + \int_{1} d\sigma^{A}\right)\end{aligned}$$

- $d\sigma^A$ is a sum of color-ordered dipole terms.
  - $d\sigma^A$ must have the same point-wise singular behavior in D dimensions as $d\sigma^R$.
    $\Rightarrow$ $d\sigma^A$ is a local counterterm for $d\sigma^R$.
  - $\int_1 d\sigma^A$ is analytic in D dimensions, and reproduces the soft and collinear divergences of $d\sigma^V$.
- Some advantages over Phase Space Slicing are:
  - You can easily project out spin eigenstates.
    $\Rightarrow$ We can explicitly test effectiveness of different spin bases at NLO after cuts.
  - Event generators use color-ordered matrix elements.
- Both methods have some contribution to n-body final states from $n+1$ phase-space. Hence, you must do 2 separate integrations.

Updated Cross Sections

We check that our codes reproduce the previously calculated total cross sections. We then update the cross sections for Tevatron and LHC using CTEQ5M1 PDFs.

Updated totally inclusive cross sections

	Tevatron, Run I	Tevatron, (1.96 TeV)	LHC	
$\sigma_t^{\rm NLO} =$	1.45 ± 0.08 pb	1.98 ± 0.13 pb	247 ± 12 pb	[1]
$\sigma_s^{\rm NLO} =$	0.75 ± 0.07 pb	0.88 ± 0.09 pb	10.7 ± 0.9 pb	[2]

B.W. Harris, E. Laenen, L. Phaf, Z.S., S. Weinzierl, PRD 66, 054024 (2002)

[1] The scales are $\mu_q^2 = Q^2$ of the W, and $\mu_h^2 = Q^2 + m_t^2$. Using $\mu^2 = m_t^2$, $\sigma_t^{\rm NLO} = 1.92 \pm 0.12$ pb at Run II.
Not included above is a $\pm 15\%$ uncertainty due to q–g luminosity.

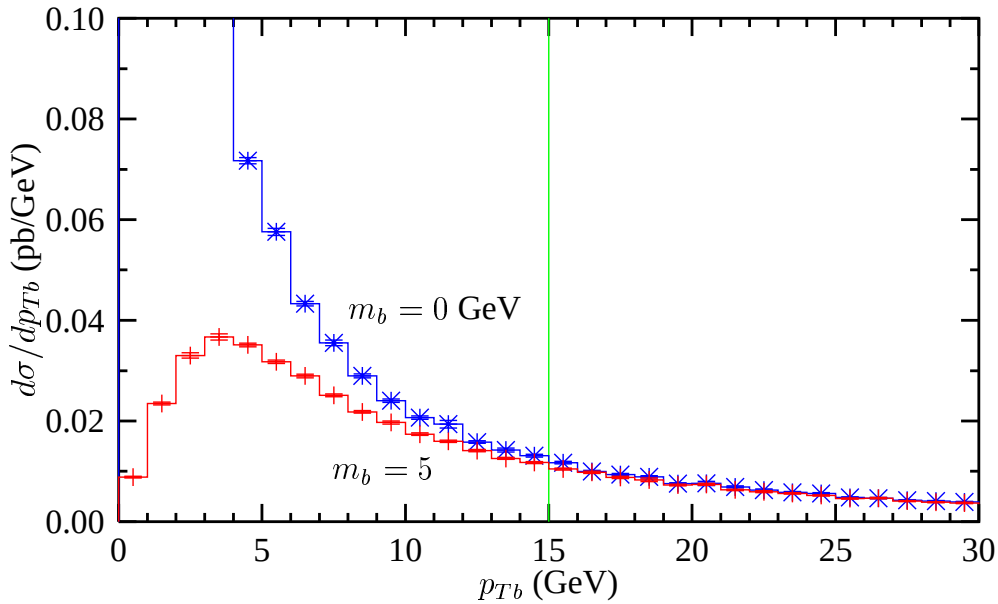
Z.S., P. Nadolsky, eConf C010630, P511 (2002) [hep-ph/0111358]

[2] The scale is the invariant mass of the top and bottom quark ($\mu = M_{tb}$). This is equivalent to q^2 in the 2-to-2 processes, but we cannot find q^2 in the 2-to-3 process. Hence, the scale uncertainty is $\pm 5.7\%$ instead of $\pm 4\%$. Top mass uncertainty is $\pm \delta m_t$(GeV)$\times 2.5\%$. Not included above is the u–$\bar{d}$, $\bar{u}$–d luminosity uncertainty of $\pm 4\%$.

NOTE: Running the Tevatron at 1.96 instead of 2.0 TeV causes a 7% loss in t-channel, and a 4% loss in s-channel.
A similar loss occurs in ALL high mass final states!

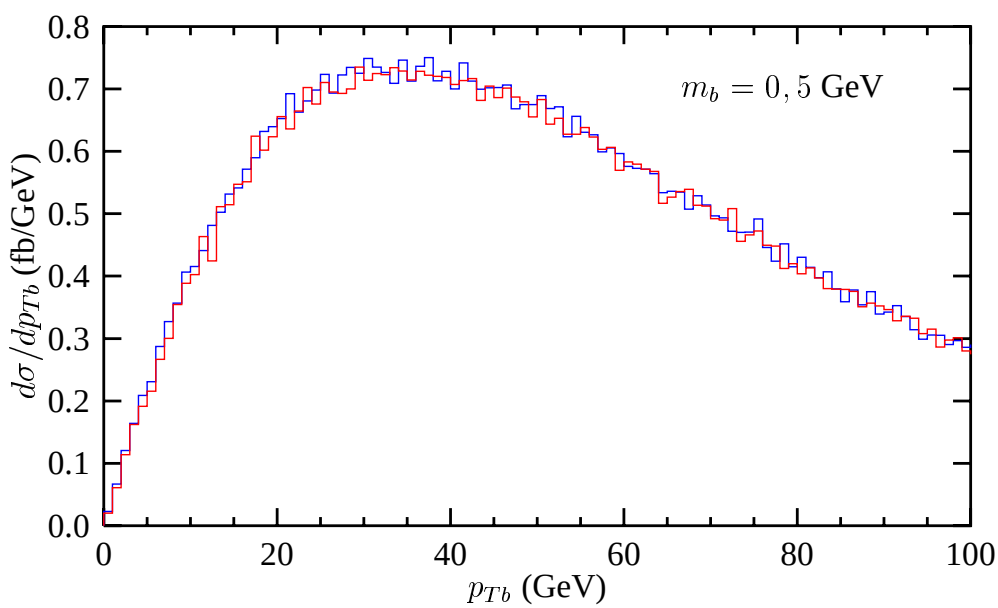
Effect of mass on $\bar{b}$ transverse momentum at LO

t-channel $\left(Wg \to t\bar{b}j \to (bl\nu)\bar{b}j\right)$



There is typically a cut of $p_{Tb} > 15$–20 GeV to tag the b.

s-channel $\left(q\bar{q} \to t\bar{b} \to (bl\nu)\bar{b}\right)$



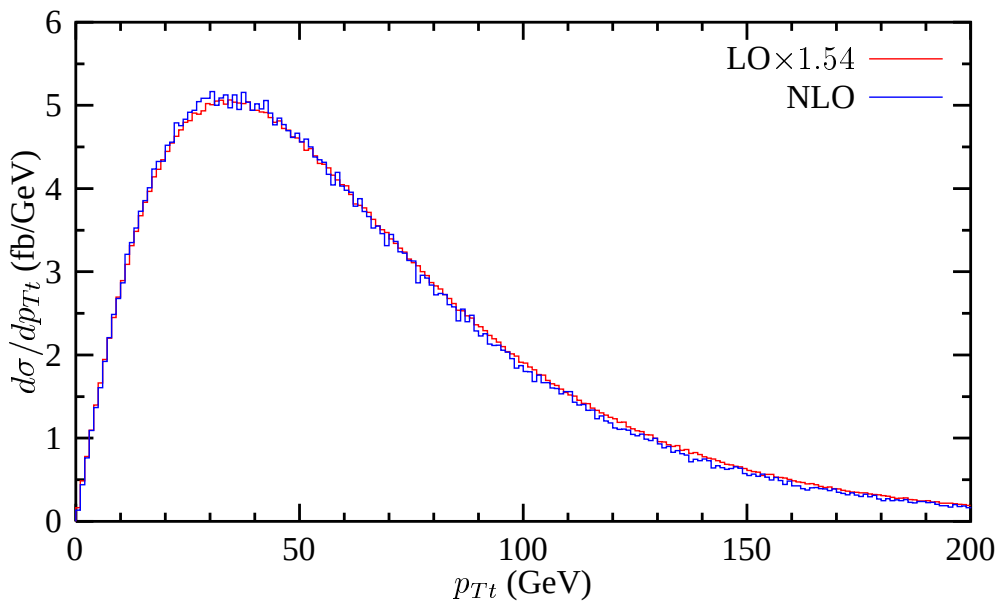
At LO the b is recoiling against the massive top quark.

t (no $\bar{t}$) distributions in s-channel at Run II

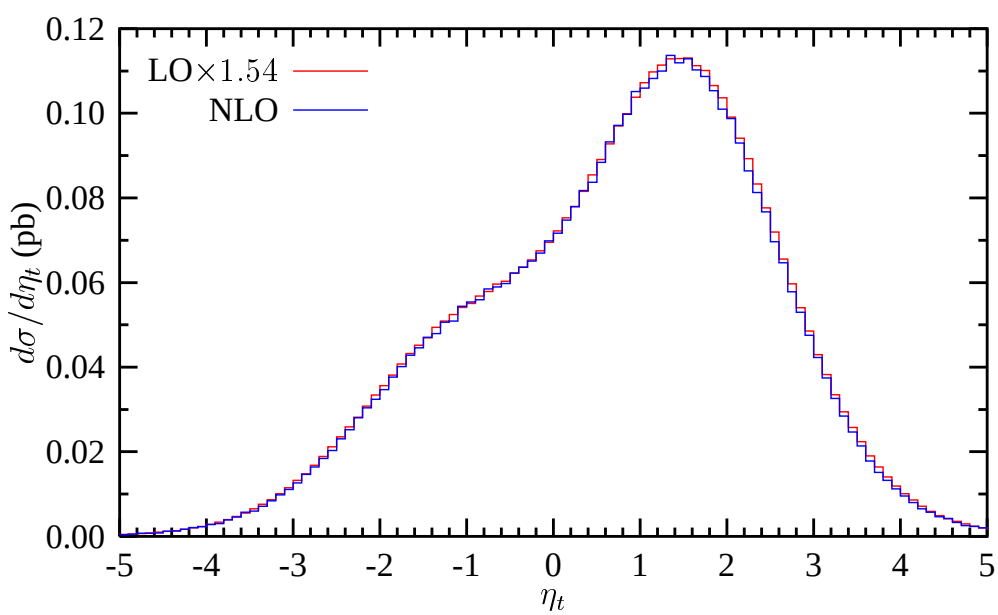
LO: CTEQ5L, 1-loop α_s, $\mu = M_{t\bar{b}}$, $m_t = 175$ GeV, $m_b = 0$.

NLO: CTEQ5M1, 2-loop α_s, k_T with $R = 1.0$.

Shape of top p_{Tt} is unchanged.



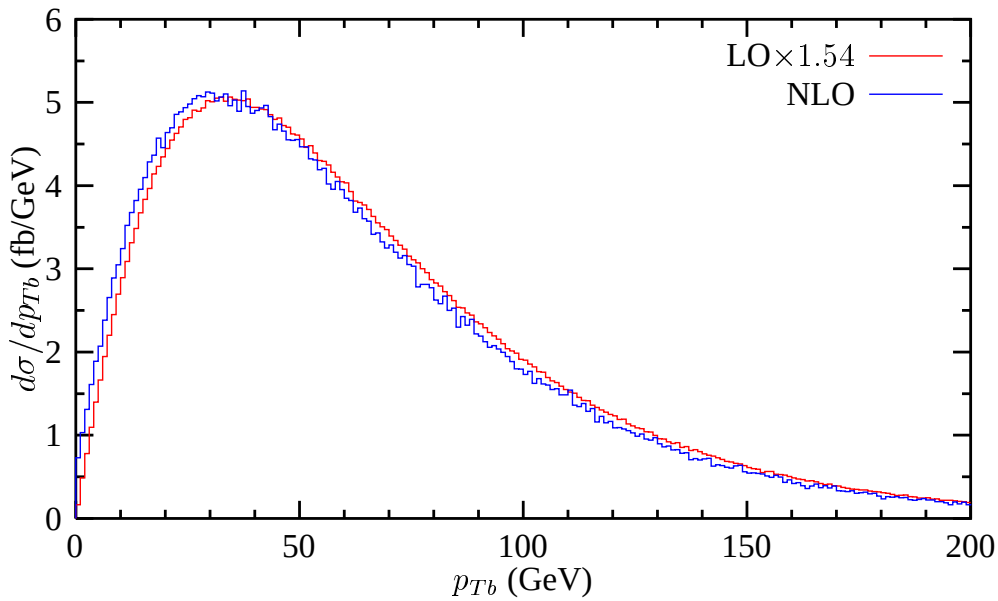
Shape of top pseudo-rapidity η_t is unchanged.



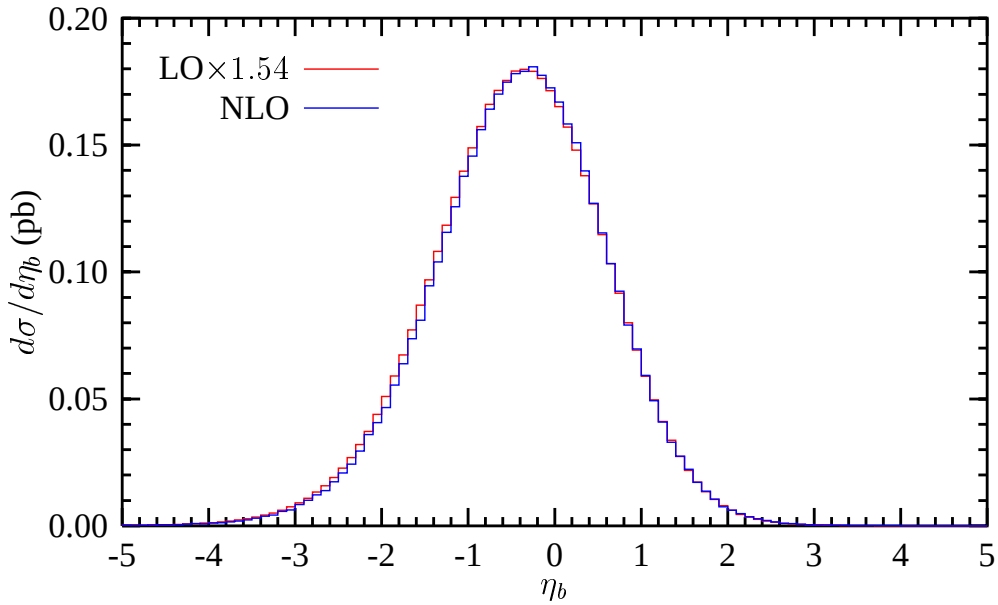
b jet transverse momentum in s-channel

At NLO the total cross section changes by $< 1\%$ for $m_b \neq 0$.

Shape of bottom-jet p_{Tb} is unchanged.



Shape of bottom-jet pseudo-rapidity η_t is unchanged.

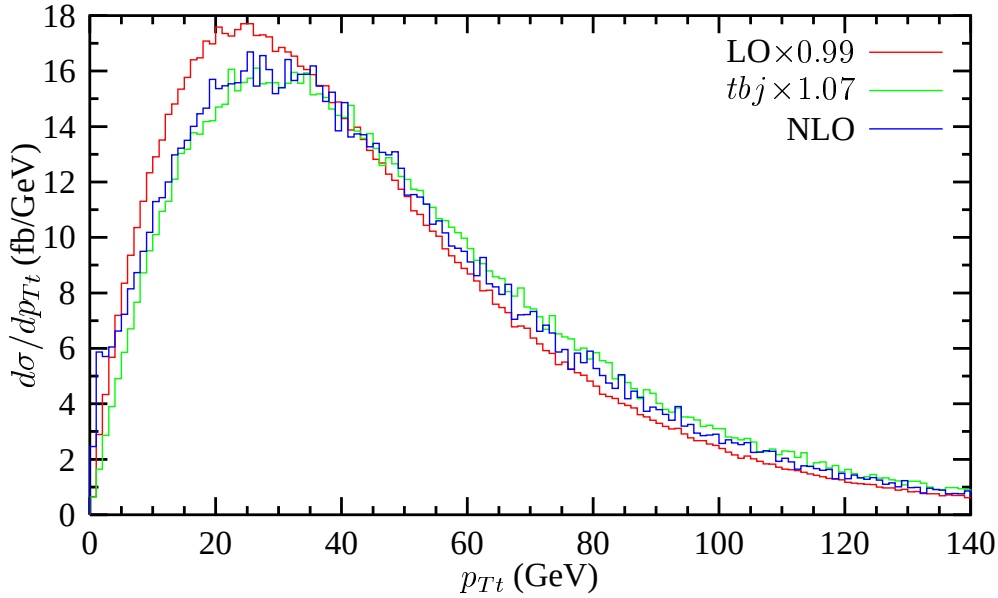


The bottom distribution is approximated well by LO calculations.

How do full NLO distributions in t-channel compare to LO $t\bar{b}j$?

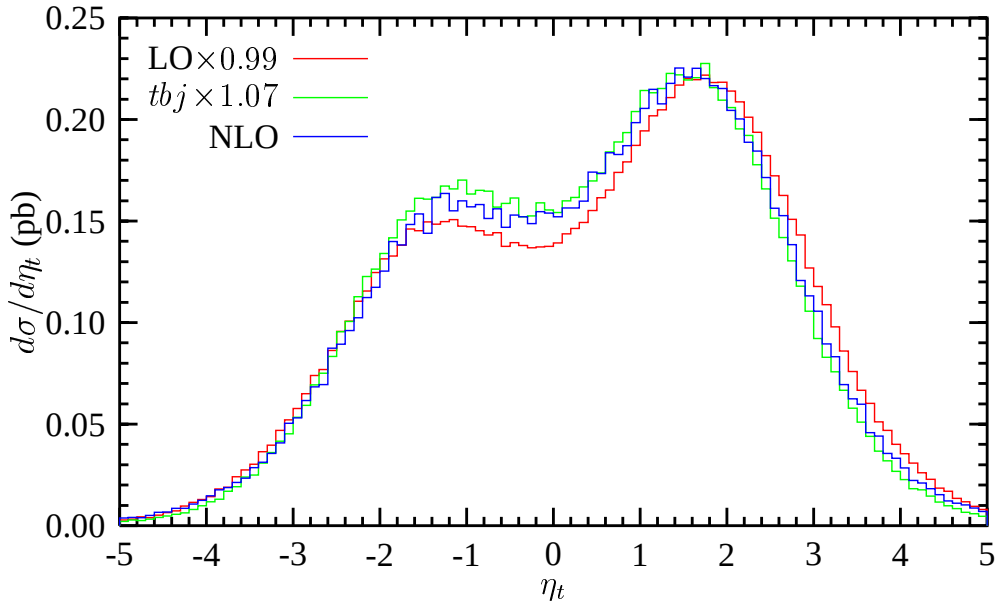
Using Q^2 for light-quark lines. For heavy-quark lines using $Q^2 + m_t^2$ in LO/NLO and, $M_{Tb}^2 = m_b^2 + p_{Tb}^2$ in $t\bar{b}j$.

NLO shape of top-quark p_{Tt} is well modeled by $t\bar{b}j$.



Top is slightly harder at NLO than at LO.

NLO shape of top pseudo-rapidity η_t is well modeled by $t\bar{b}j$.

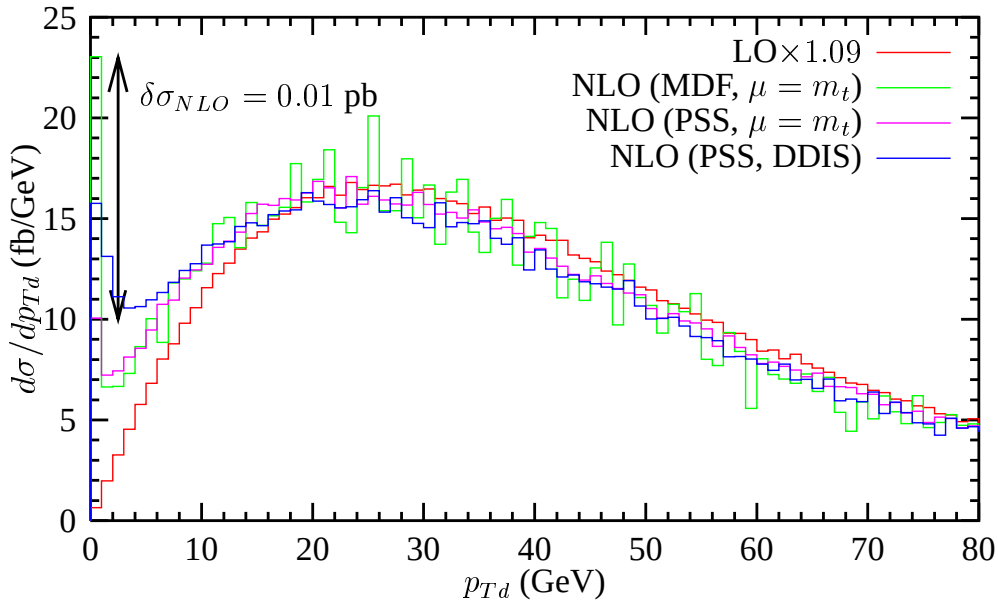


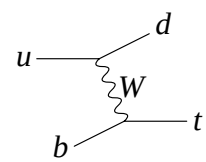
Top is slightly more central at NLO than at LO.

Jet transverse momentum in t-channel

At LO, a d-quark recoils against the top quark

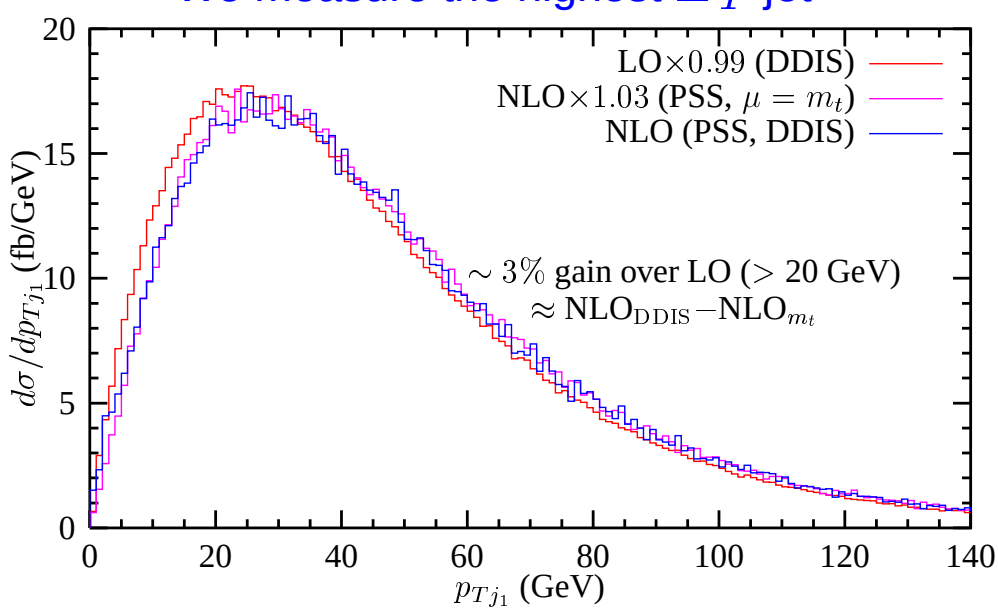
NLO "d-jet" (no cuts)





- Perturbation theory it not terribly stable at low p_T.
- Entire difference between MDF and PSS is in first bin.
- This is not what we want. Be careful what you ask for!

We measure the highest E_T jet

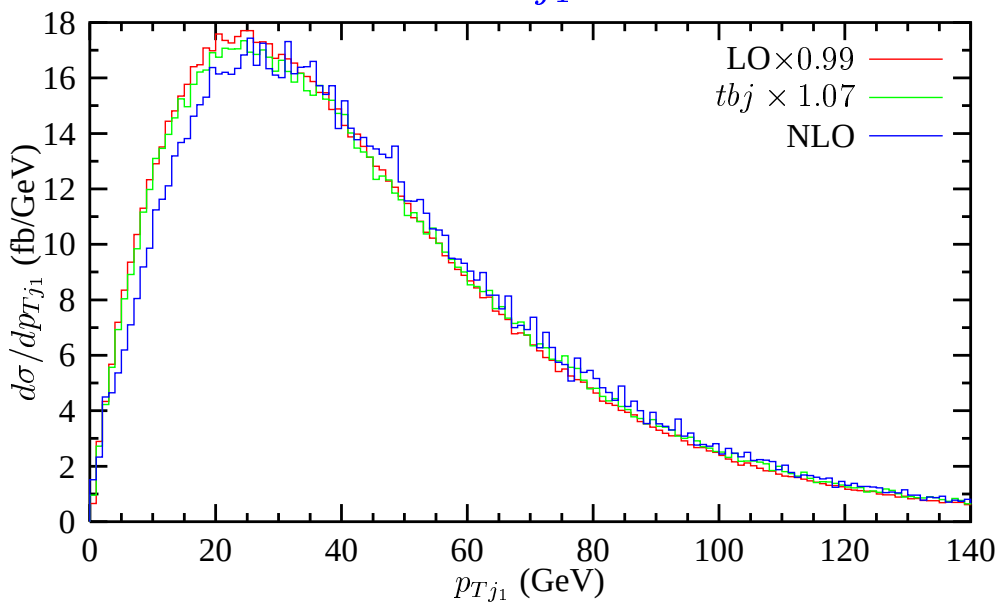


The highest E_T jet recoils against the top. The measurable change in shape is comparable to the scale uncertainty.

Is t-channel really Double-DIS?

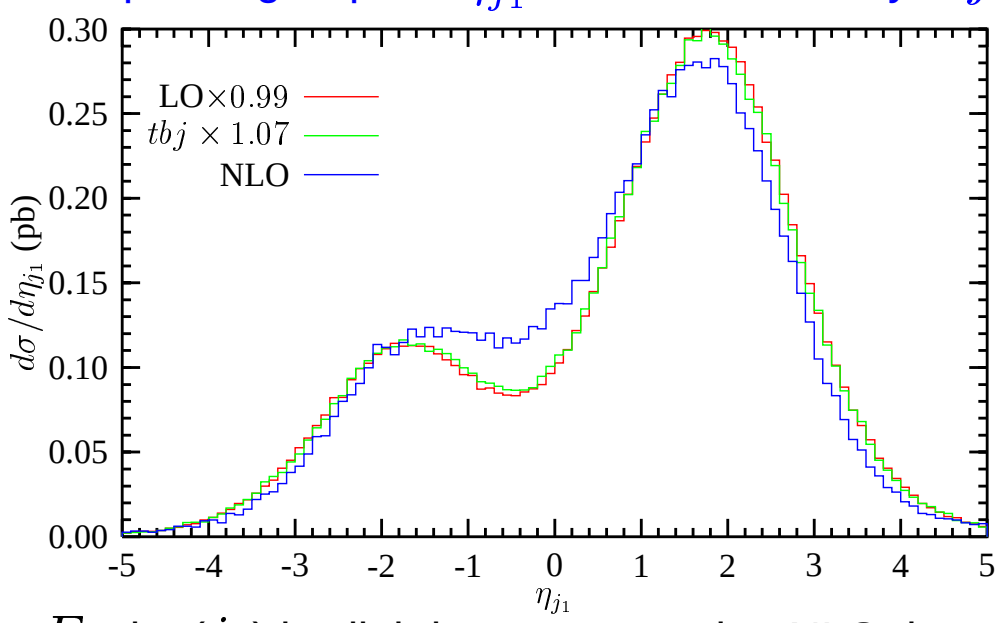
If this is Double DIS (DDIS), then the light-quark jet in $t\bar{b}j$ should look just like it does in LO (2-to-2).

LO shape of light-quark jet p_{Tj_1} is well modeled by $t\bar{b}j$.



Highest E_T jet (j_1) is slightly harder at NLO than at LO.

LO shape of light-quark η_{j_1} is well modeled by $t\bar{b}j$.

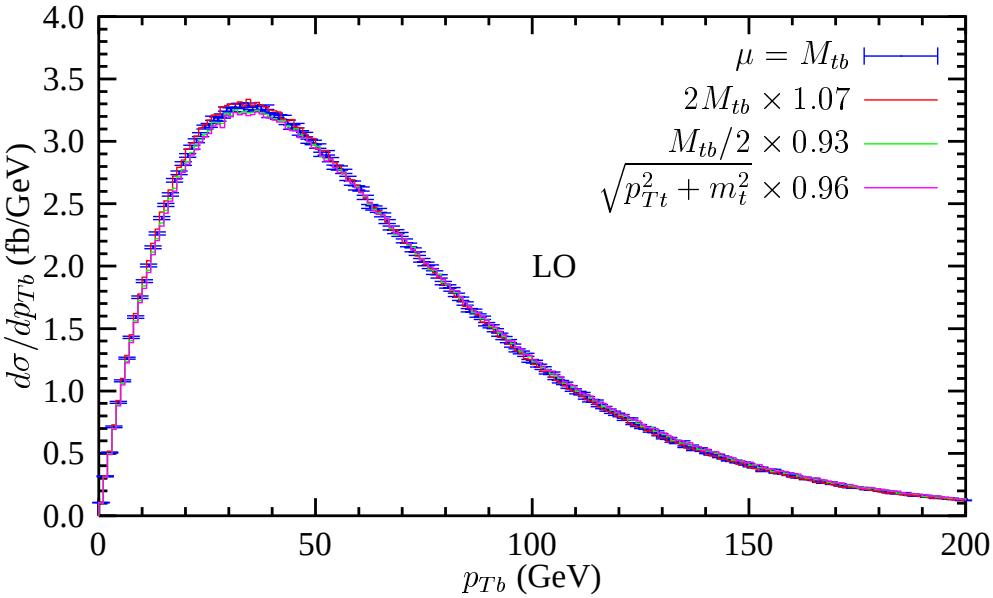


Highest E_T jet (j_1) is slightly more central at NLO than at LO.

This is expected since j_1 takes most of the recoil of the top quark.

Scale (μ) dependence on b jet transverse momentum

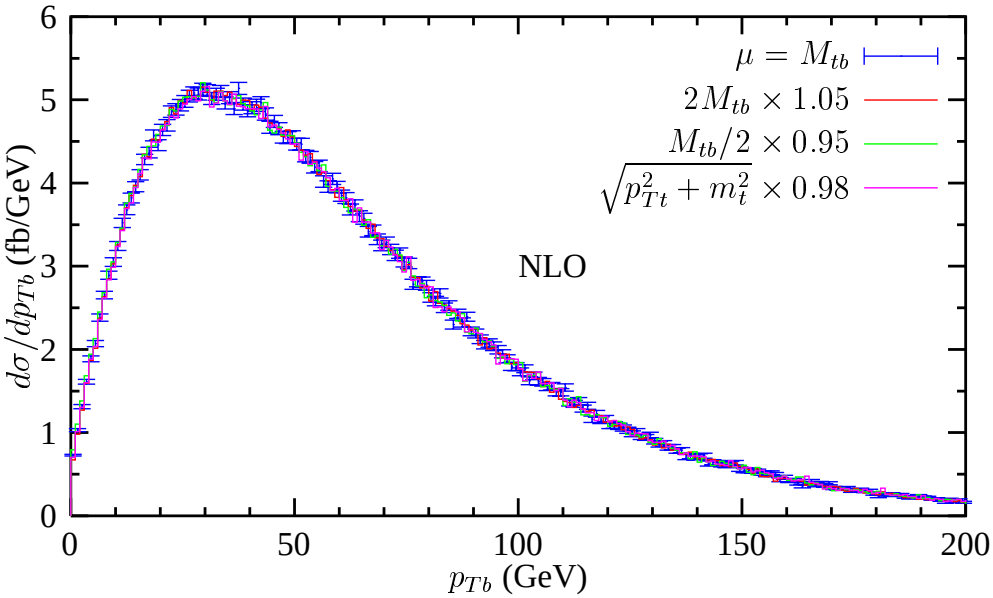
LO



Varying scale by a factor of 2 gives a 7.8% uncertainty at LO.

Shape is not affected. M_{tb} is $t\bar{b}$ invariant mass.

NLO



Varying scale by a factor of 2 gives a 5.7% uncertainty at NLO.

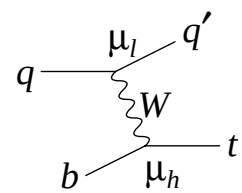
Shape is not affected. $\mu = m_t$ is not shown, but it is nearly identical to $\mu = \sqrt{p_{Tt}^2 + m_t^2}$, the transverse mass scale.

Scale (μ) dependence of the t-channel jets and top

- The shapes of the p_T and η distributions do not change if you vary the scales. Only the normalization changes.
- If you vary the 2 independent scales at the same time you underestimate the uncertainty.

Note: The scales used in Double Deep Inelastic Scattering are

$$\mu_l = Q^2, \mu_h = Q^2 + m_t^2.$$

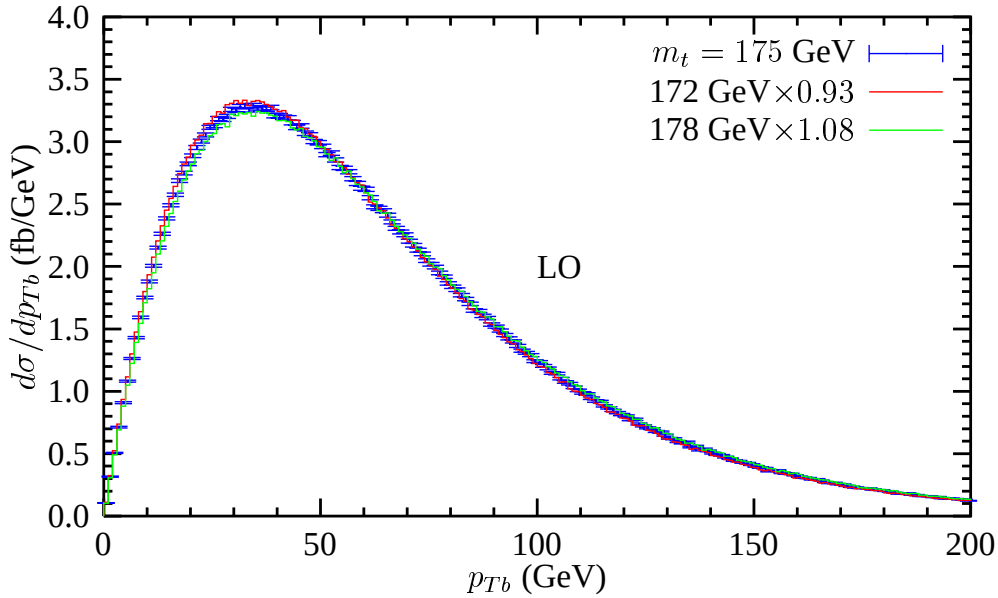


$\mu/2 - 2\mu$	LO$_t$ (m_t)	NLO$_t$ (m_t)	LO$_t$ (DDIS)	NLO$_t$ (DDIS)
fixed	0.95 pb	1.03 pb	1.07 pb	1.06 pb
μ_l & μ_h	$\pm 1\%$	$\pm 2.5\%$	$^{+0.1}_{-2}\%$	$\pm 3.5\%$
μ_h	$^{-7.5}_{+5.5}\%$	$^{-3.5}_{+4}\%$	$^{-7.2}_{+5.2}\%$	$^{-3}_{+4}\%$
μ_l	$^{+6.7}_{-5.8}\%$	$\pm 1\%$	$^{+8}_{-6.8}\%$	$\pm 0.6\%$

- Summing the independent variations in quadrature predicts $\sim \pm 11\%$ uncertainty at LO (consistent with the results).
- At NLO we get $\sim \pm 4\%$ uncertainty due to scale variation.

m_t dependence on shape of b jet transverse momentum

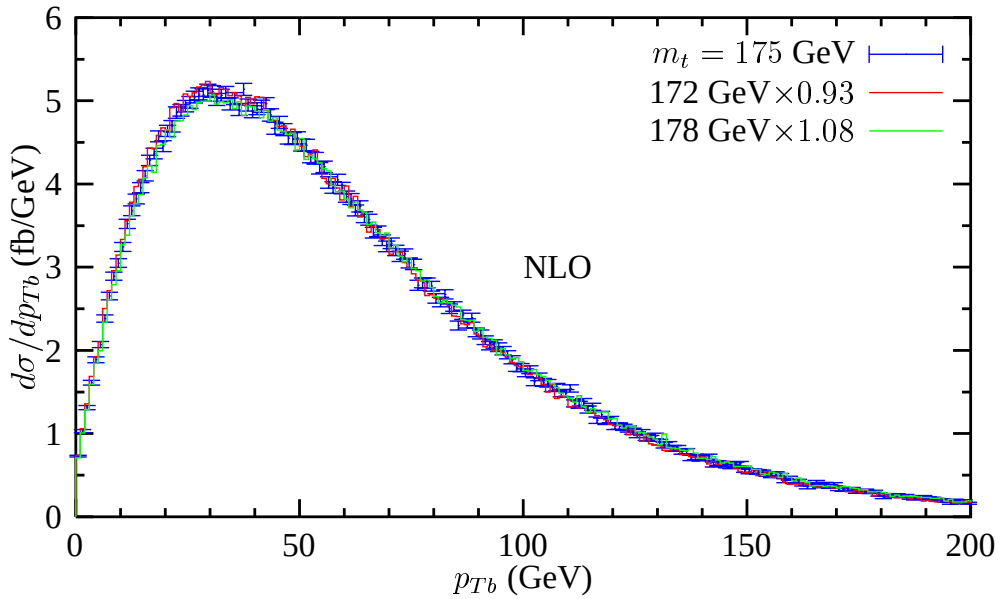
LO



Varying mass 3 GeV gives an 8% uncertainty at LO.

Shape is not affected.

NLO

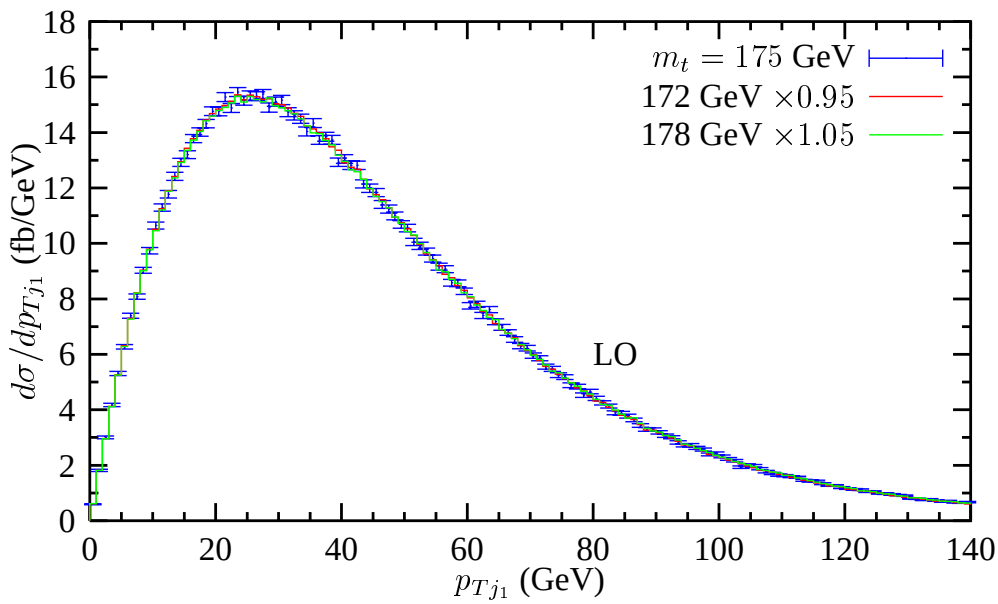


Varying mass 3 GeV gives an 8% uncertainty at NLO.

Shape is not affected. Varying $m_t = 175 \pm 2$ GeV gives a 5% uncertainty at both LO and NLO.

m_t dependence on shape of jet transverse momenta

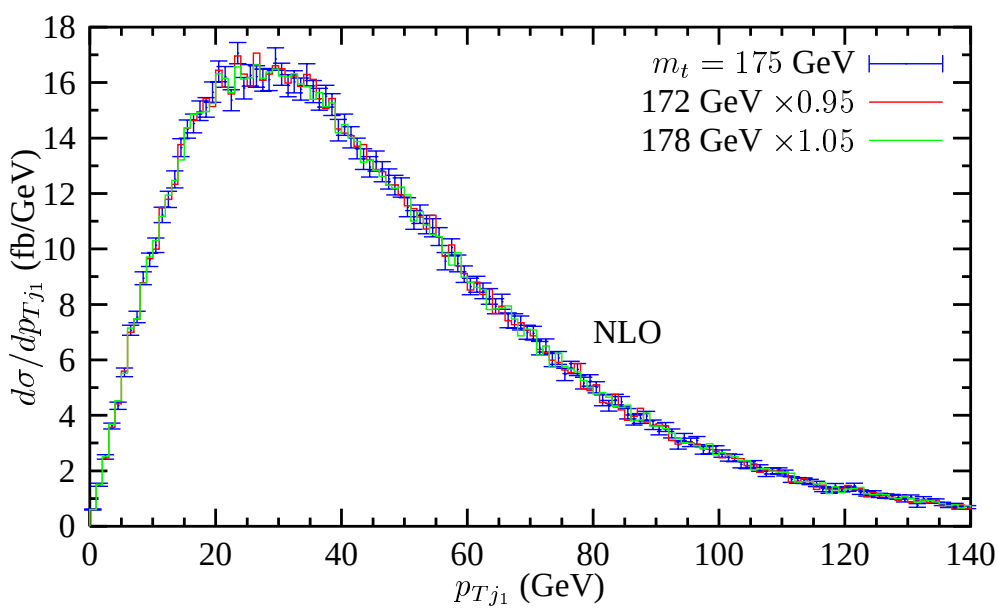
LO light-quark jet



Varying mass 3 GeV gives a 5% uncertainty at LO.

Shape is not affected.

NLO highest E_T jet



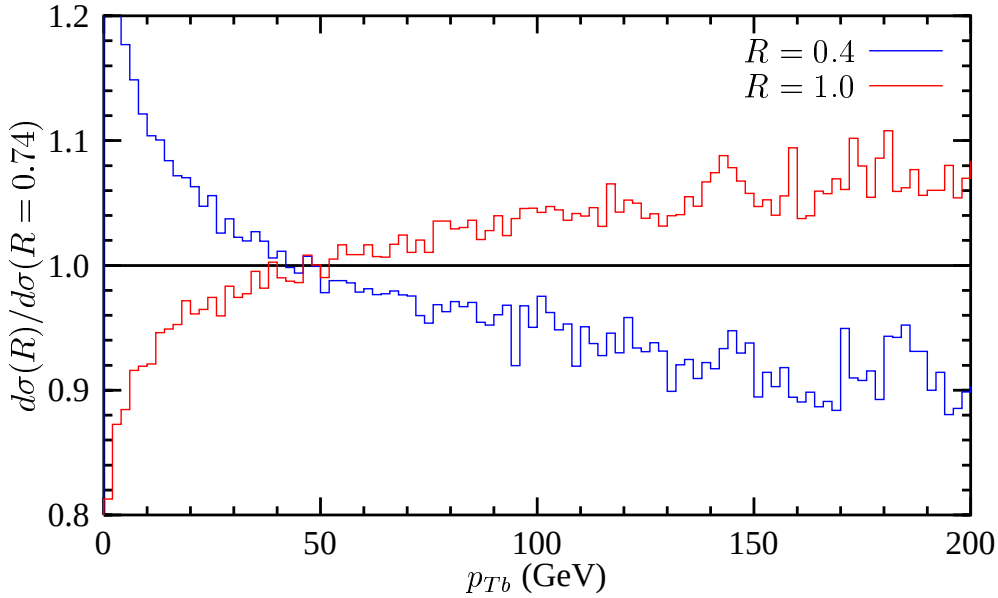
Varying mass 3 GeV gives a 5% uncertainty at NLO.

Shape is not affected. Varying $m_t = 175 \pm 2$ GeV gives a 3% uncertainty at both LO and NLO.

Cone-size dependence on b jet distributions

You can study the effect of the cone size used in the k_T algorithm on the reconstructed p_T and η of the b.

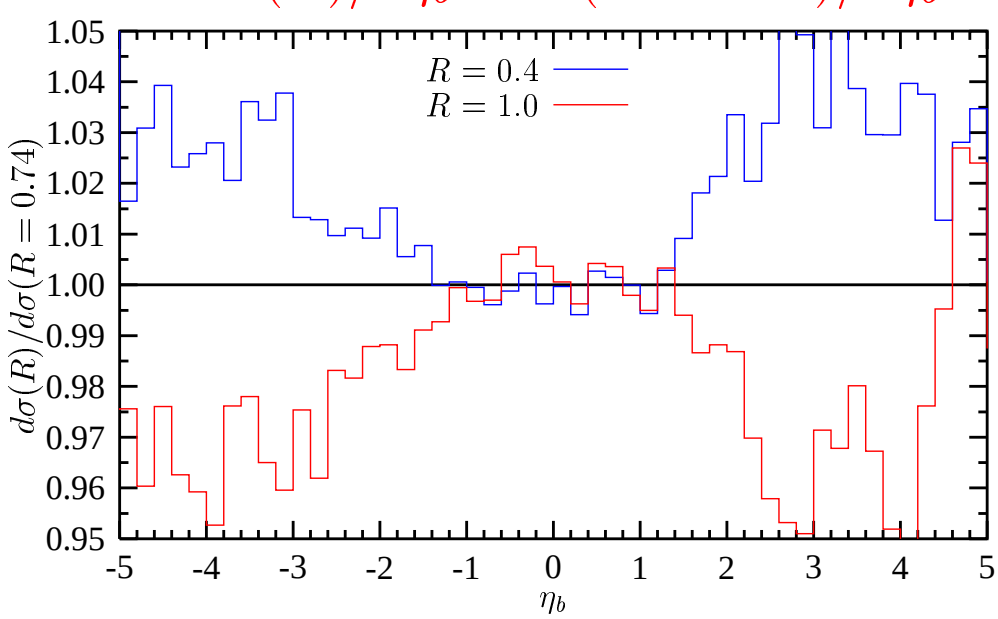
Ratio of $d\sigma(R)/dp_{Tb}$ to $d\sigma(R=0.74)/dp_{Tb}$



Note: R refers to the "fixed-cone" equivalent, $R_{\mathrm{stnd.}} = 0.7$.

For "reasonable" values of R the variation is $< 10\%$, but should be checked in any given analysis.

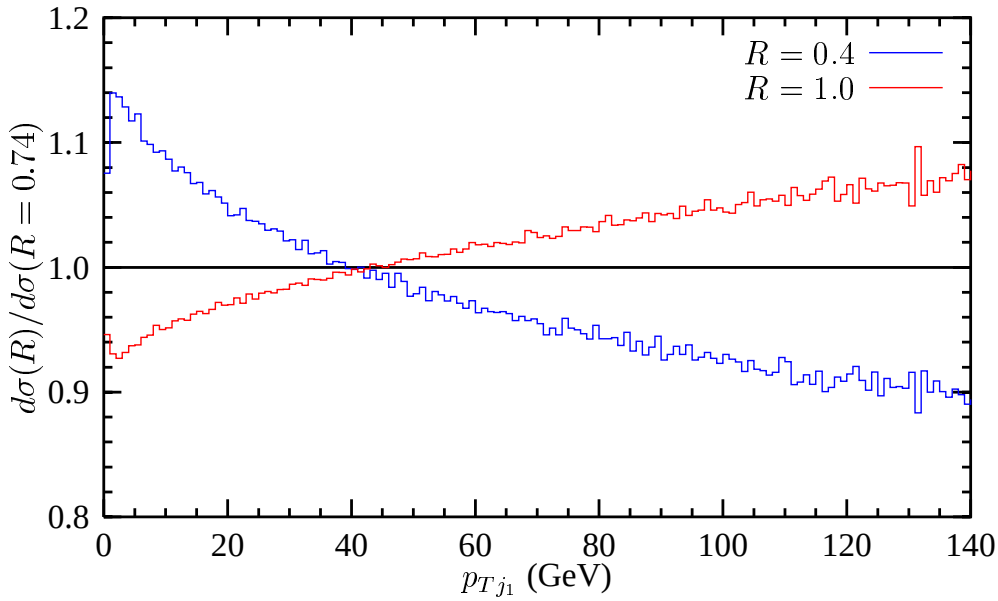
Ratio of $d\sigma(R)/d\eta_b$ to $d\sigma(R=0.74)/d\eta_b$



Cone-size dependence on jet distributions

You can study the effect of the cone size used in the k_T algorithm on the reconstructed p_T and η of the jet.

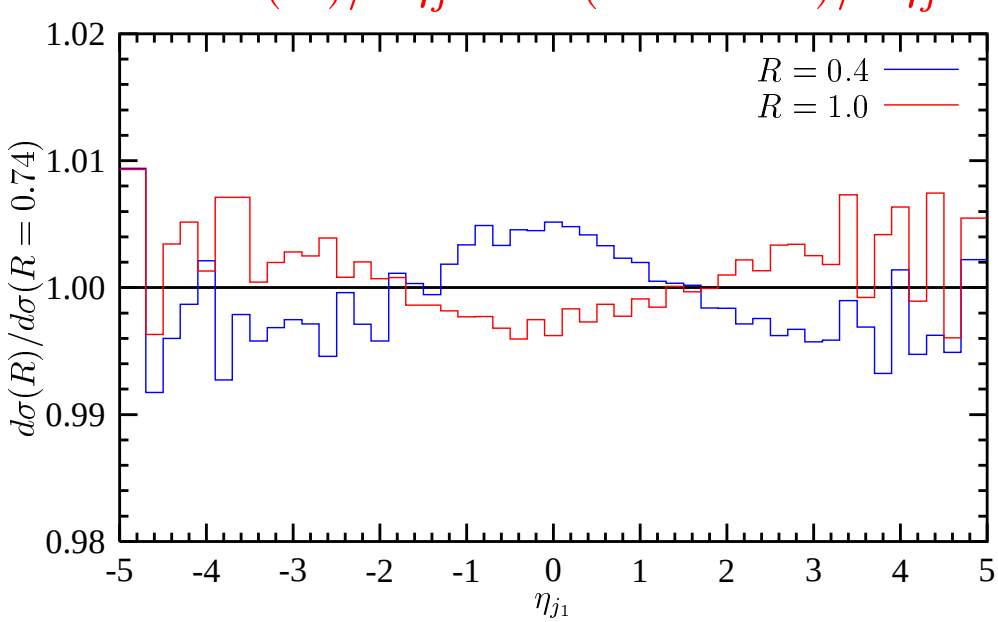
Ratio of $d\sigma(R)/dp_{Tj}$ to $d\sigma(R=0.74)/dp_{Tj}$



Note: R refers to the "fixed-cone" equivalent, $R_{\mathrm{stnd.}} = 0.7$.

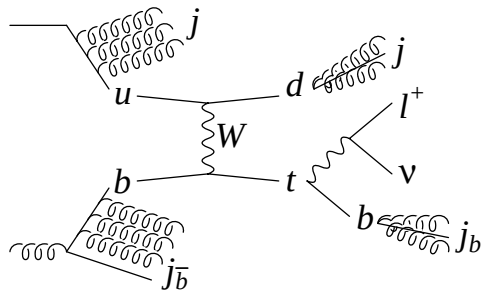
For "reasonable" values of R the variation is $< 10\%$, but should be checked in any given analysis.

Ratio of $d\sigma(R)/d\eta_j$ to $d\sigma(R=0.74)/d\eta_j$



Using event generators (PYTHIA/HERWIG)

How do PYTHIA/HERWIG generate t-channel single-top?



Initial-state radiation (ISR) is generated by backward evolution of angular-ordered showers.

$\Rightarrow$ The jet containing the extra $\bar{b}$ comes from soft ISR.

Event generators completely underestimate (by factors of 2 or more) additional hard radiation. [$\rightarrow$ Figs.]

Our jet veto cuts on this hard radiation. We need something else.

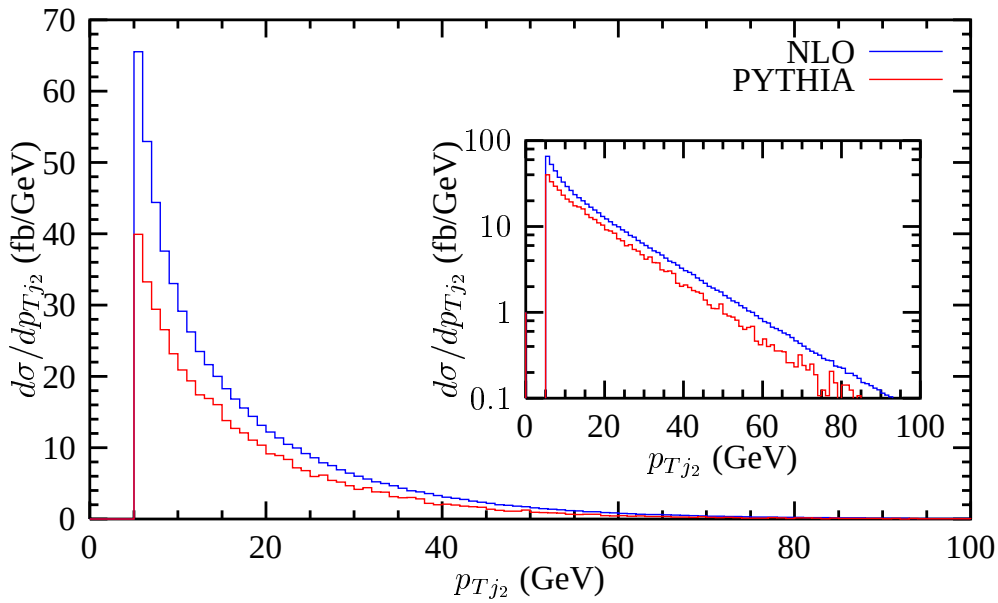
Warnings

- All processes suffer from this. Be suspect of off-shell event generator predictions for signals and backgrounds!
- Neural network calculations use these event generators to estimate regions away from the poles. This can lead to exponential errors.

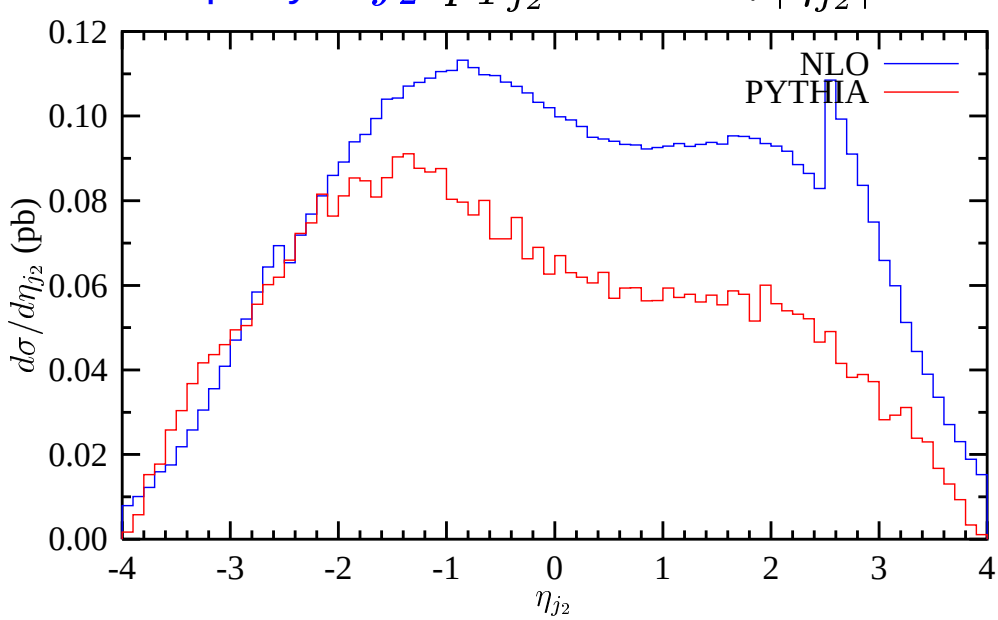
Event generators (PYTHIA/HERWIG) fail

Consider the shape of the 2nd-highest E_T jet j_2 ($t+ \geq 2j$)

Transverse momentum of j_2 $p_{Tj_2} > 5$ GeV, $|\eta_{j_2}| < 4$



Pseudorapidity of j_2 $p_{Tj_2} > 5$ GeV, $|\eta_{j_2}| < 4$



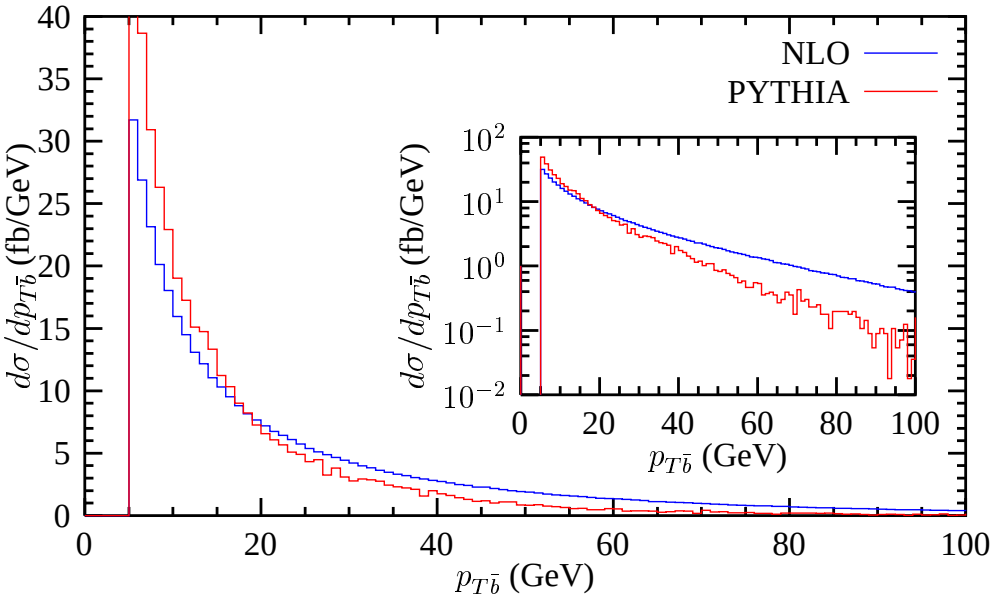
PYTHIA gets shape of the transverse momentum of the 2nd-highest E_T jet, but the pseudorapidity is poor.

Hidden in this distribution is the fact that the flavor content is wrong. . .

Event generators (PYTHIA/HERWIG) fail

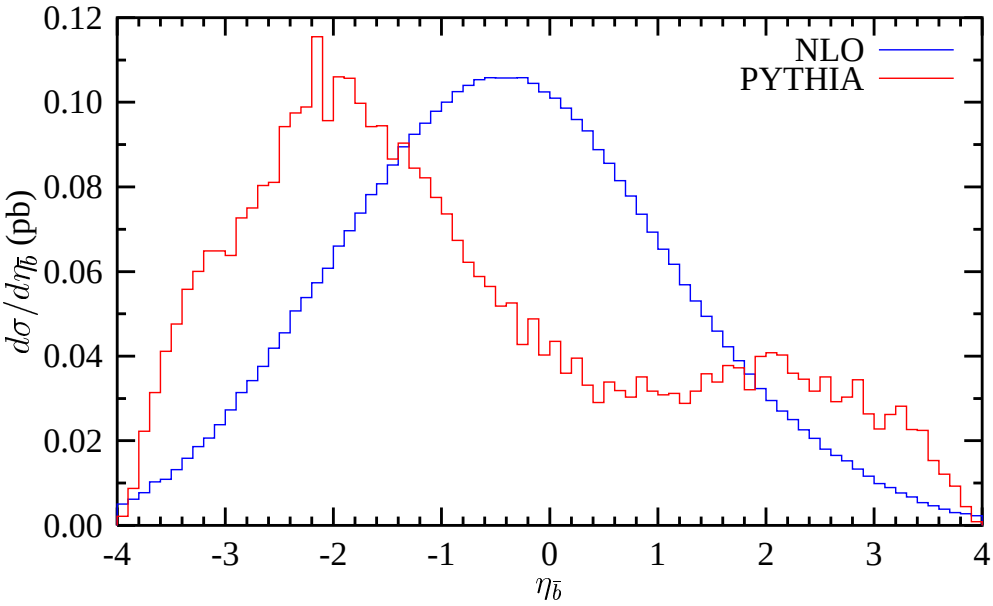
Compare the $\bar{b}$-jet from PYTHIA and NLO.

Transverse momentum of $\bar{b}$-jet $p_{T j_{\bar{b}}} > 5$ GeV, $|\eta_{j_{\bar{b}}}| < 4$



PYTHIA gets the wrong shape.

Pseudorapidity of $\bar{b}$-jet $p_{T j_{\bar{b}}} > 5$ GeV, $|\eta_{j_{\bar{b}}}| < 4$



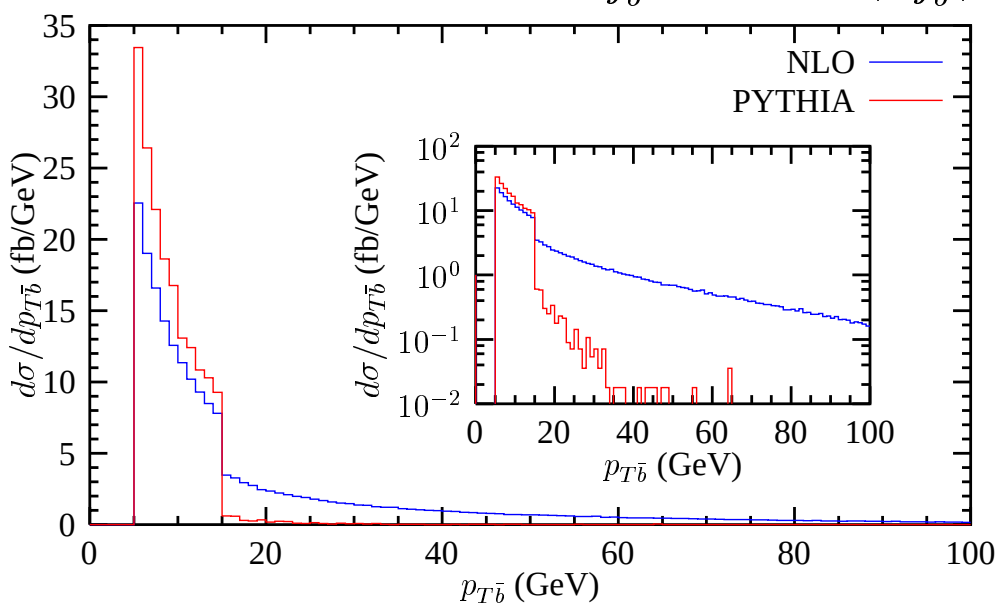
Again PYTHIA gets the wrong shape.

The angular-ordered distributions are just crazy!

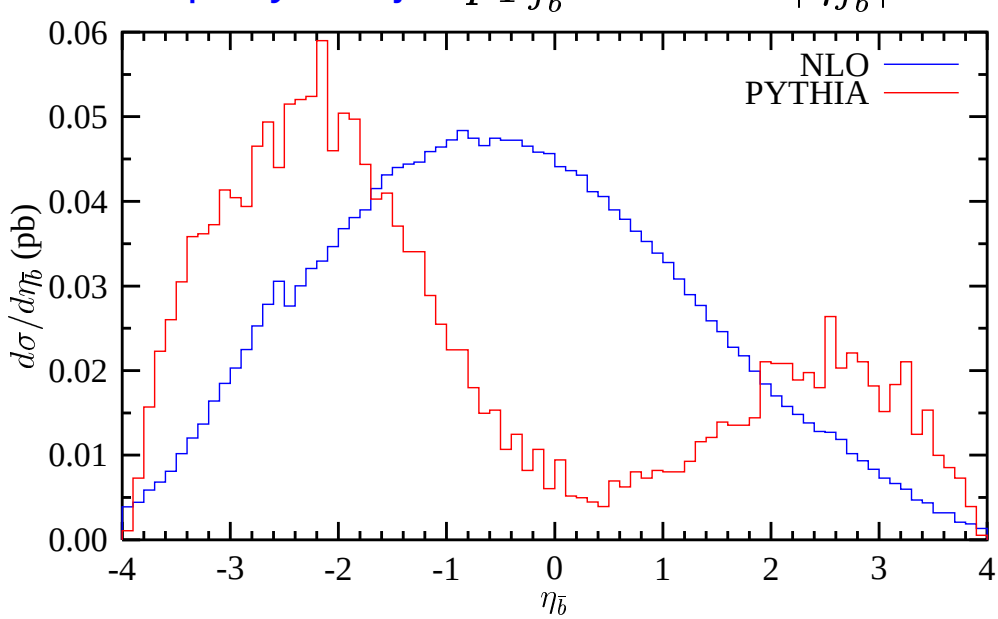
Event generators (PYTHIA/HERWIG) fail

Consider the shape of the b-jet after the "jet veto." $(t + 1j)$

Transverse momentum of $\bar{b}$-jet $p_{Tj_{\bar{b}}} > 5$ GeV, $|\eta_{j_{\bar{b}}}| < 4$



Pseudorapidity of $\bar{b}$-jet $p_{Tj_{\bar{b}}} > 5$ GeV, $|\eta_{j_{\bar{b}}}| < 4$



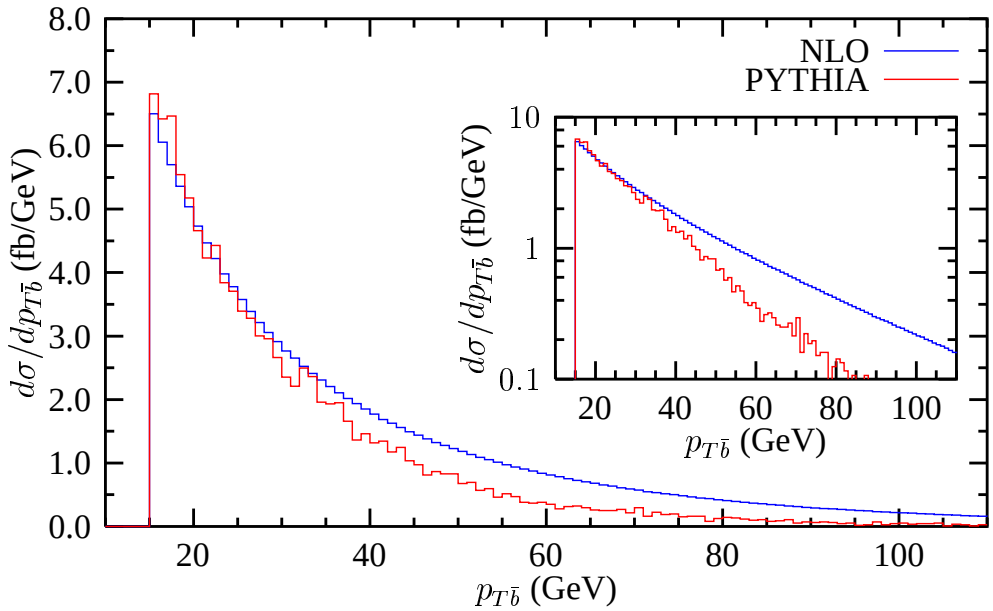
PYTHIA completely misses the times when the $\bar{b}$ jet is the highest E_T jet (after the b-jet from the top decay).

PYTHIA underestimates the t-channel contribution to the 2 b-tag sample by at least a factor of 2!

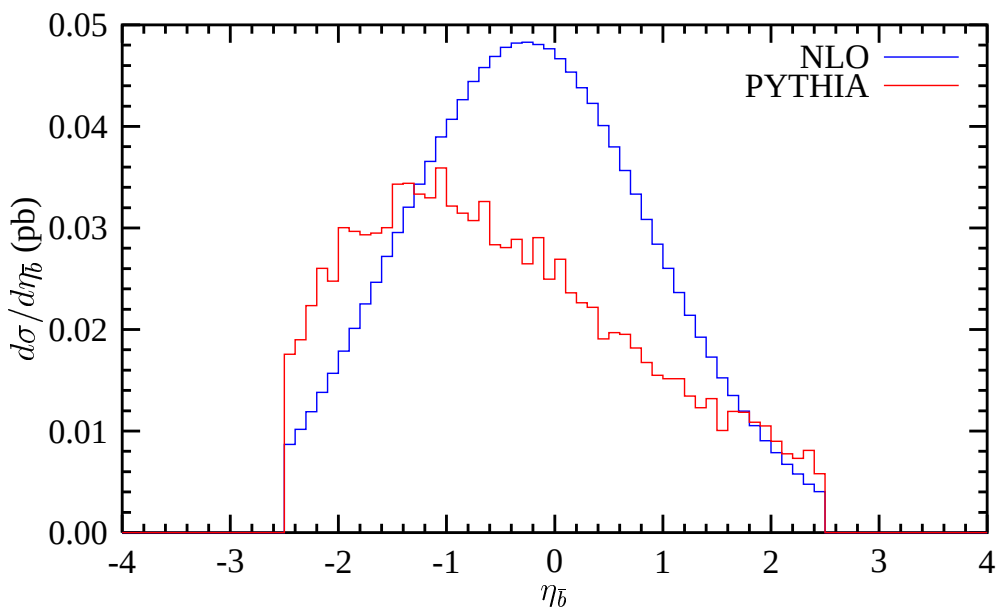
Event generators (PYTHIA/HERWIG) fail

What happens when there are 2 high-E_T jets? ($t + 2j$)

Transverse momentum of $\bar{b}$-jet $p_{T j_{\bar{b}}} > 15$ GeV, $|\eta_{j_{\bar{b}}}| < 2.5$



Pseudorapidity of $\bar{b}$-jet $p_{T j_{\bar{b}}} > 15$ GeV, $|\eta_{j_{\bar{b}}}| < 2.5$



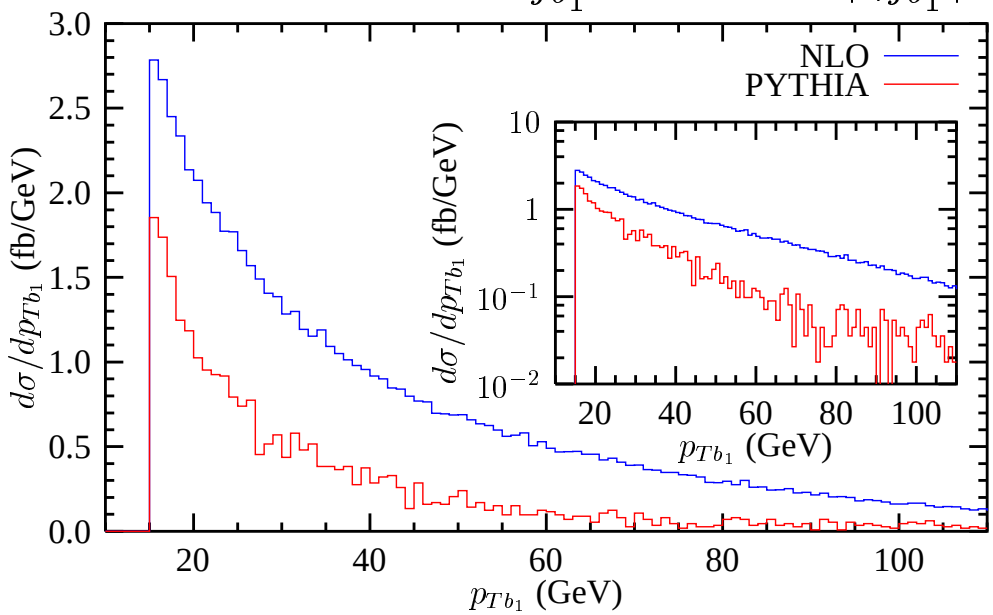
Even at larger transverse momenta, PYTHIA predicts b-jets that are both too soft and too far forward.

This leads to dangerous underestimates of SM backgrounds . . .

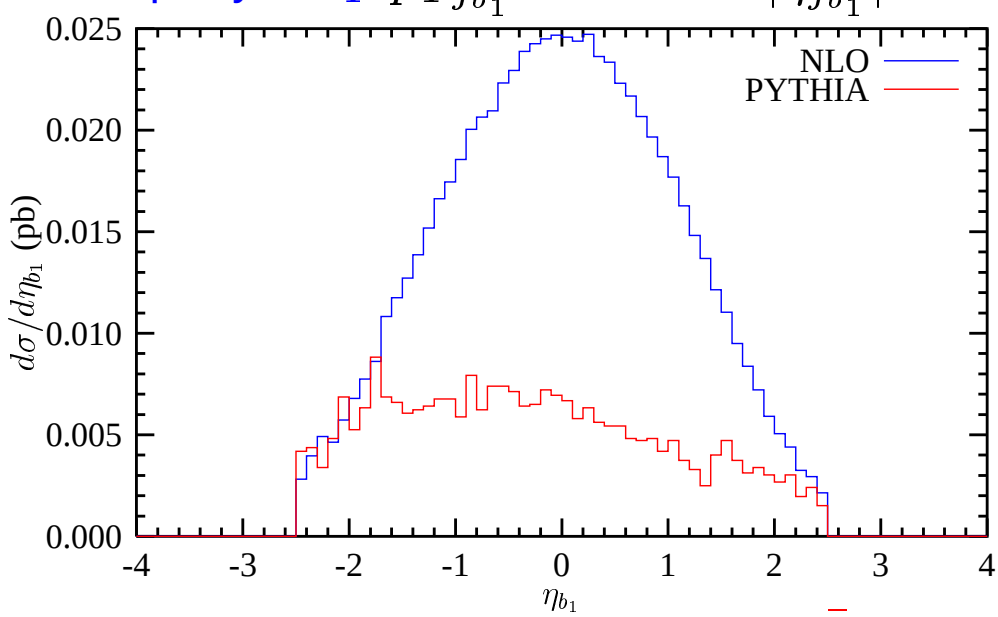
Event generators (PYTHIA/HERWIG) fail

What happens when the highest-E_T jet is the $\bar{b}$-jet? $(t+b)$

Transverse momentum of b_1 $p_{Tj_{b_1}} > 15$ GeV, $|\eta_{j_{b_1}}| < 2.5$



Pseudorapidity of b_1 $p_{Tj_{b_1}} > 15$ GeV, $|\eta_{j_{b_1}}| < 2.5$



PYTHIA completely misses the times when the $\bar{b}$ jet is the highest E_T jet (after the b-jet from the top decay).

PYTHIA underestimates the t-channel contribution to the 2 b-tag sample by at least a factor of 2!

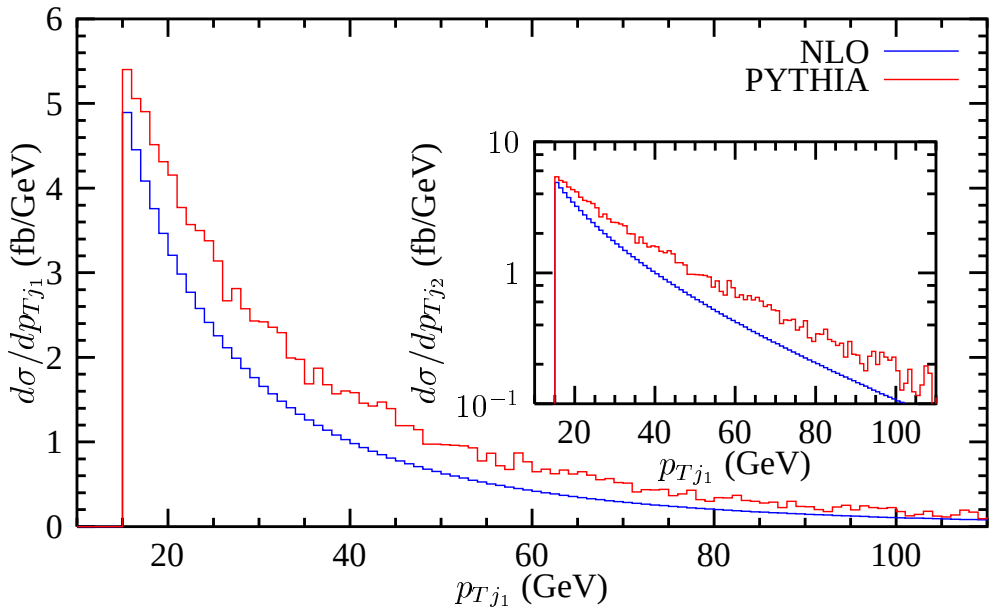
This makes it more difficult to distinguish s- and t-channels!

Event generators (PYTHIA) in s-channel

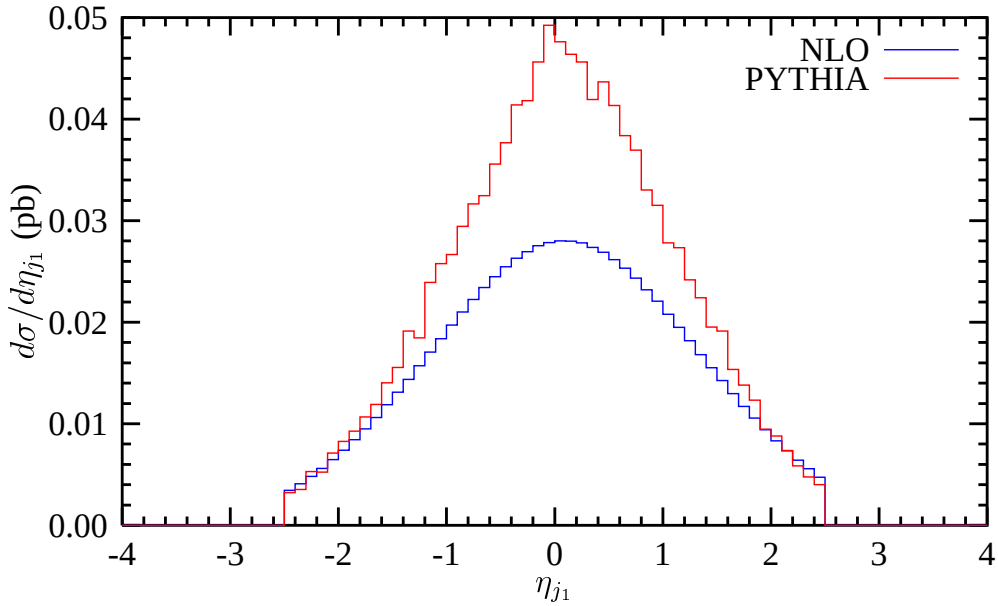
Shapes are pretty good, but large tuning is needed.

j_1 is the highest-E_T jet in addition to the $\bar{b}$-jet.

Transverse momentum of j_1 $p_{Tj_1} > 15$ GeV, $|\eta_{j_1}| < 2.5$



Pseudorapidity of j_1 $p_{Tj_1} > 15$ GeV, $|\eta_{j_1}| < 2.5$



PYTHIA produces 40% too much additional radiation by default.

Since the shapes are OK, this might be tuned away with care.

Putting it all together

An optimal matching prescription

t-channel

1. Generate $t+j$, $t+j+j$, $t+j+b$ using MadEvent/CompHEP and the DDIS scales ($\mu_l = Q^2$, $\mu_h = Q^2 + m_t^2$).

2. Feed the events into PYTHIA/HERWIG and shower them.

3. Create 4 samples with a given jet definition, e.g. cones with $\Delta R = 0.7$, or k_T with $\Delta R = 1$, and minimal cuts:

   – $t+j$, $t+b$, $t+j+j$, $t+j+b$

   – Remember that comparisons are made at generator-level, i.e. apply jet reconstruction to $\pi^{\pm}$s, γs, $e^{\pm}$ or $\mu^{\pm}$ inside jets, etc. in the event record.

4. Normalize each sample to the NLO prediction after cuts, and with the same jet definition.

   – NLO jets must be E_T ordered.

5. Finally, feed into the detector simulations.

s-channel

- Tune the event generators to the correct amount of radiation.

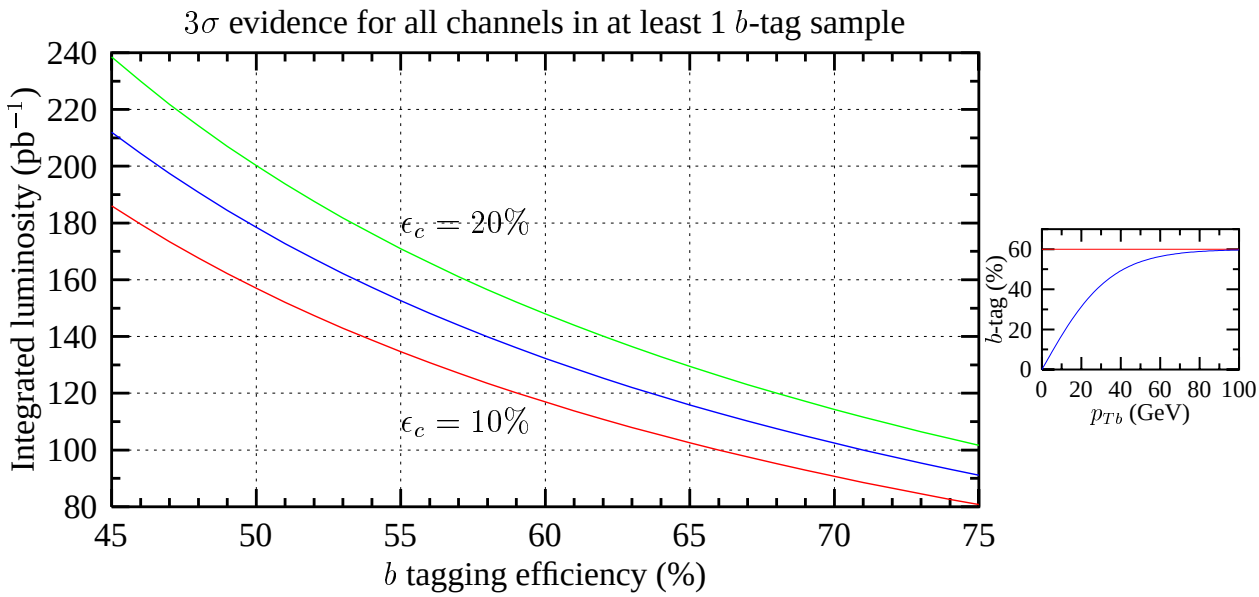
  – Shapes will be pretty good, so just scale to the cross section after cuts for the same jet definition.

- or Feed matrix elements out of MadEvent/CompHEP into PYTHIA/HERWIG for more fine control.

Integrated luminosity needed to see single-top

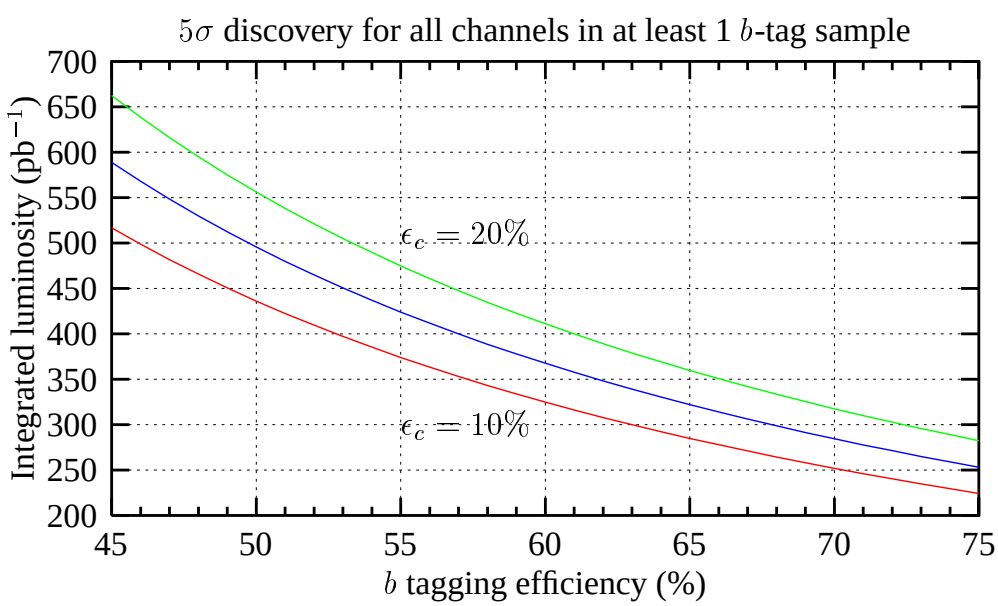
First look in the at least 1 b-tag sample. $(bj\ell\not{E}_T + bb\ell\not{E}_T)$

Apply a jet veto: i.e. do not allow more than 2 high E_T jets.



130 pb^{-1} for 3σ evidence of single top $(t + s + Wt)$!

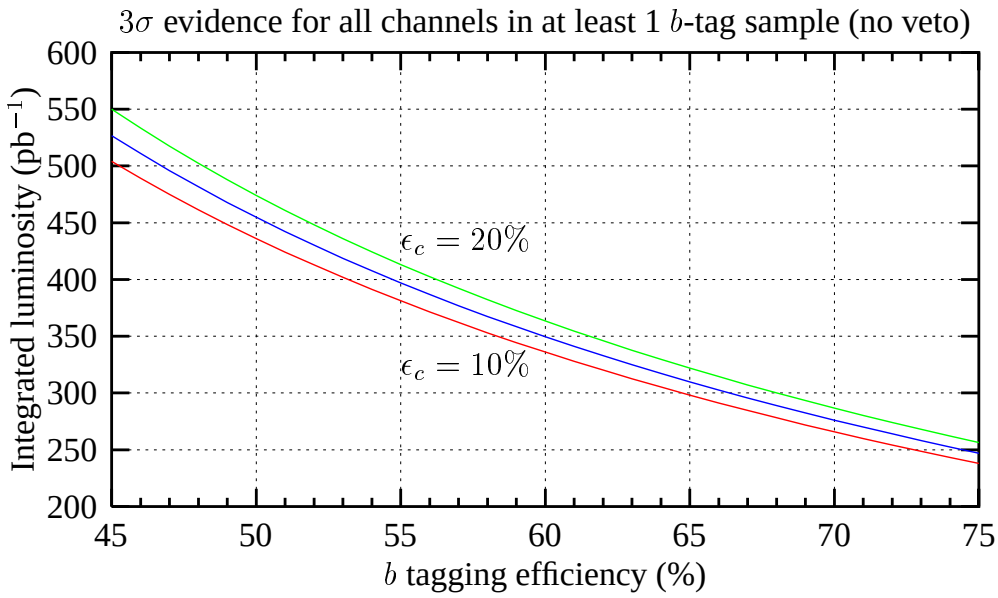
Good b-tagging efficiency is vital. A lower c-mistag rate helps.



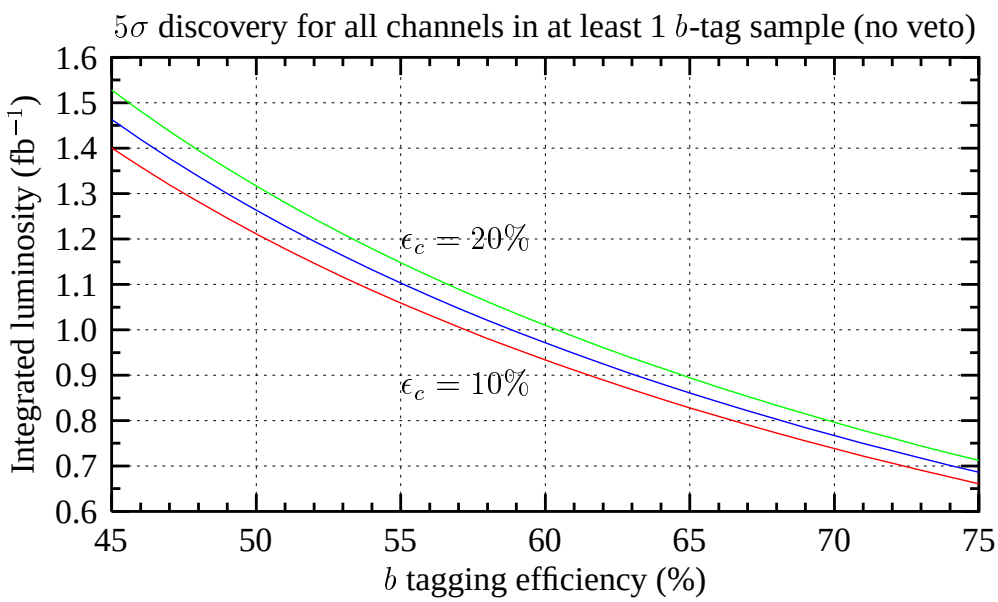
370 pb^{-1} for 5σ discovery of single top $(t + s + Wt)$!

Integrated luminosity needed with no veto

Look in the at least 1 b-tag sample. $(bj\ell\not{E}_T + bb\ell\not{E}_T) + X$



350 pb^{-1} for 3σ evidence of single top $(t + s + Wt)$!



975 pb^{-1} for 5σ discovery of single top $(t + s + Wt)$!

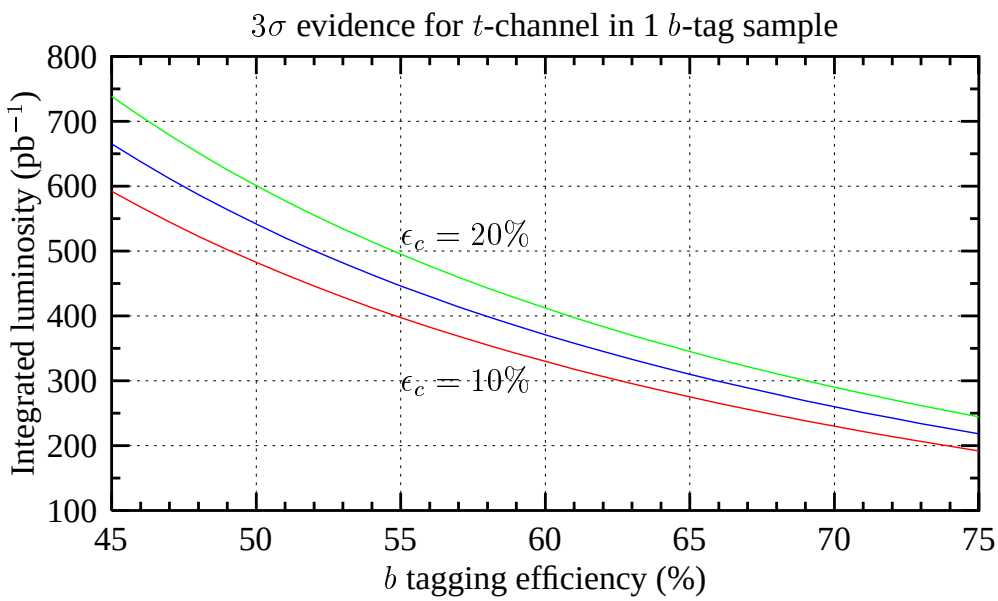
We can still do it, but we need 3 times the data to see single-top without the jet veto.

Without the jet veto we cannot distinguish s- and t-channels.

Integrated luminosity needed to see t-channel

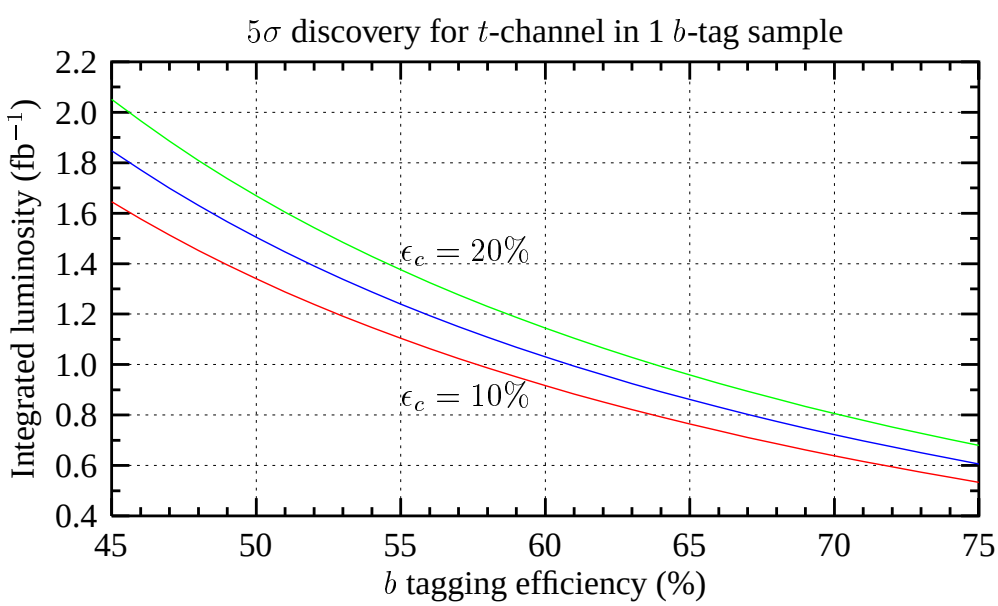
Look in the exactly 1 b-tag sample. ($bj\ell\not{E}_T, j \neq b$)

Apply a jet veto: i.e. do not allow more than 2 high E_T jets.



370 pb^{-1} for 3σ evidence of t-channel single top!

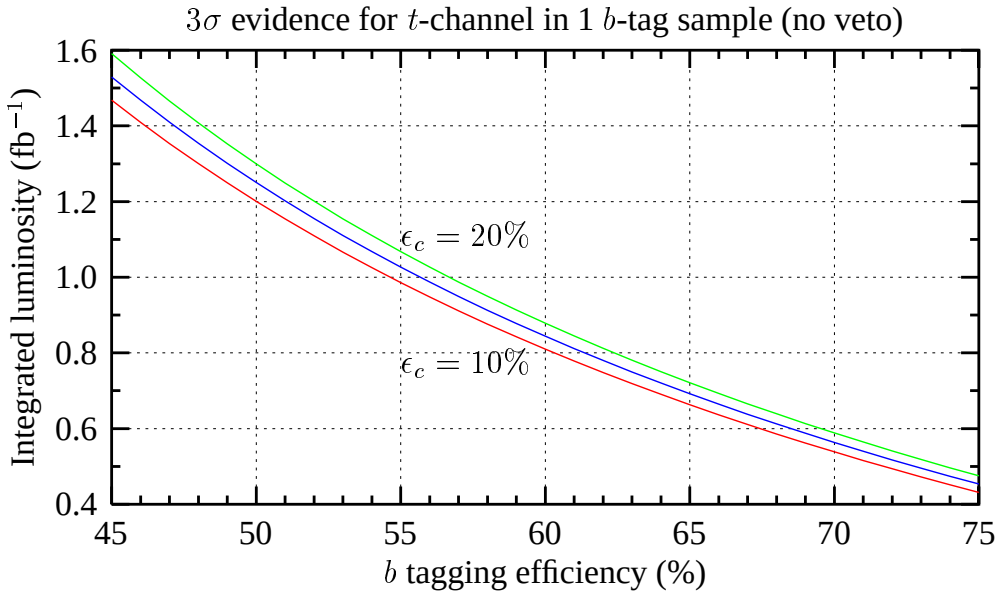
Good b-tagging efficiency is vital. A lower c-mistag rate helps.



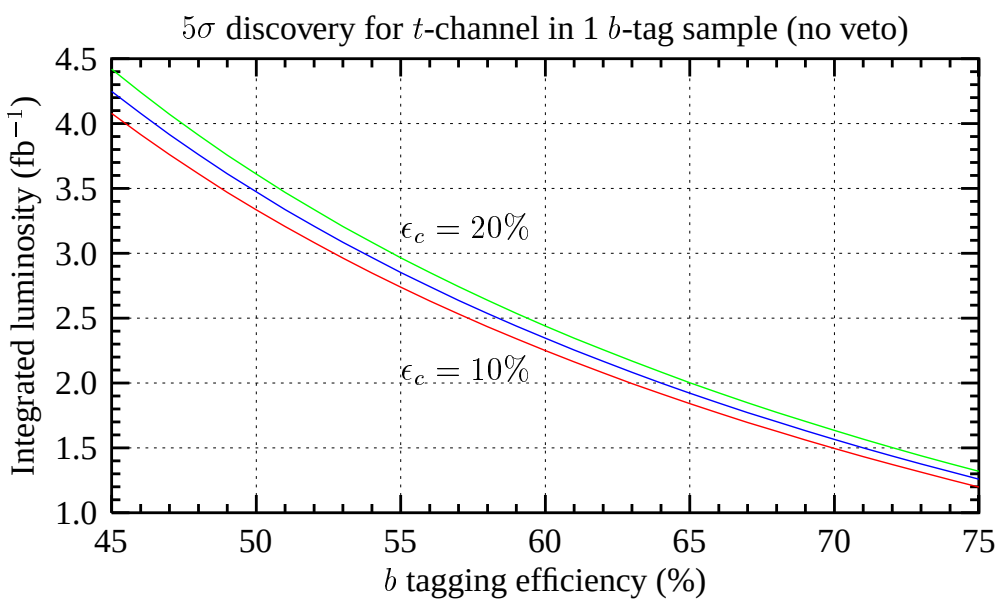
1 fb^{-1} for 5σ discovery of t-channel single top!

Integrated luminosity needed to see t-channel (no veto)

Look in the exactly 1 b-tag sample. $(bj\ell\not{E}_T, j \neq b) + X$



850 pb^{-1} for 3σ evidence of t-channel single top!



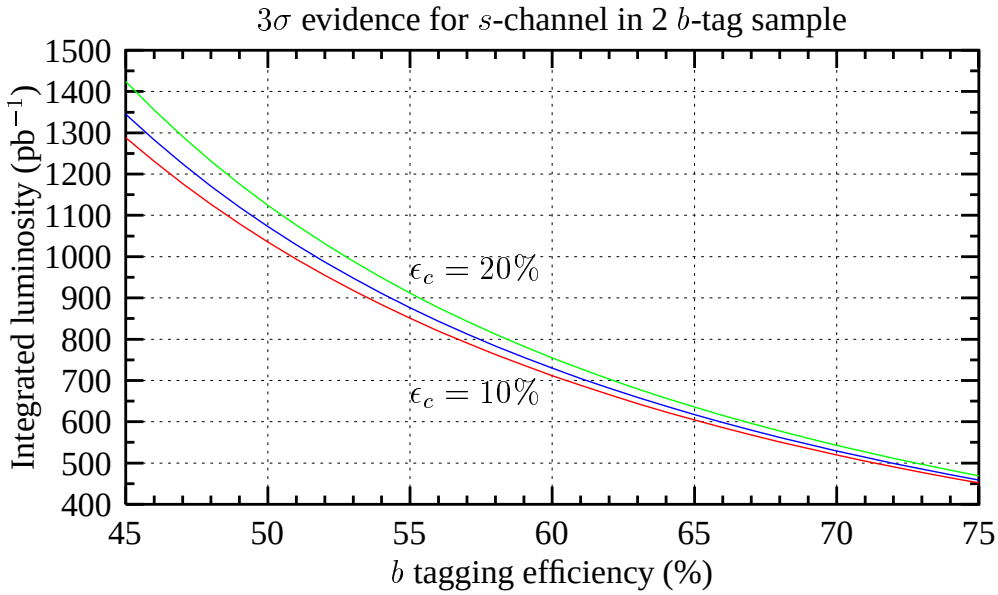
2.3 fb^{-1} and great b-tagging needed for 5σ discovery.

This is out of reach for Run II. We need the jet veto.

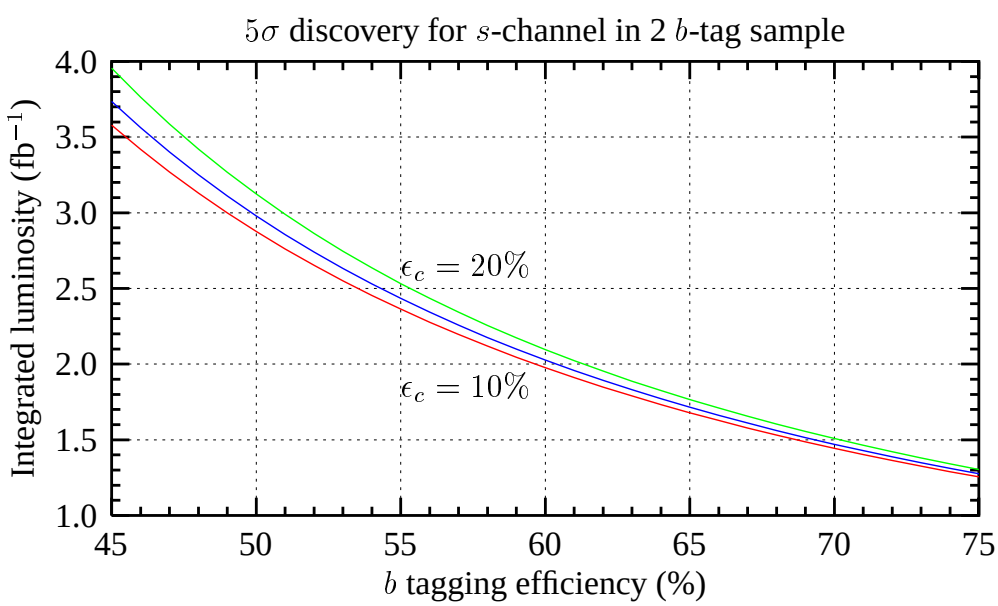
Integrated luminosity needed to see s-channel

Look in the exactly 2 b-tags sample. ($bb\ell\not{E}_T$)

Apply a jet veto: i.e. do not allow more than 2 high E_T jets.



730 pb^{-1} for 3σ evidence of s-channel single top!

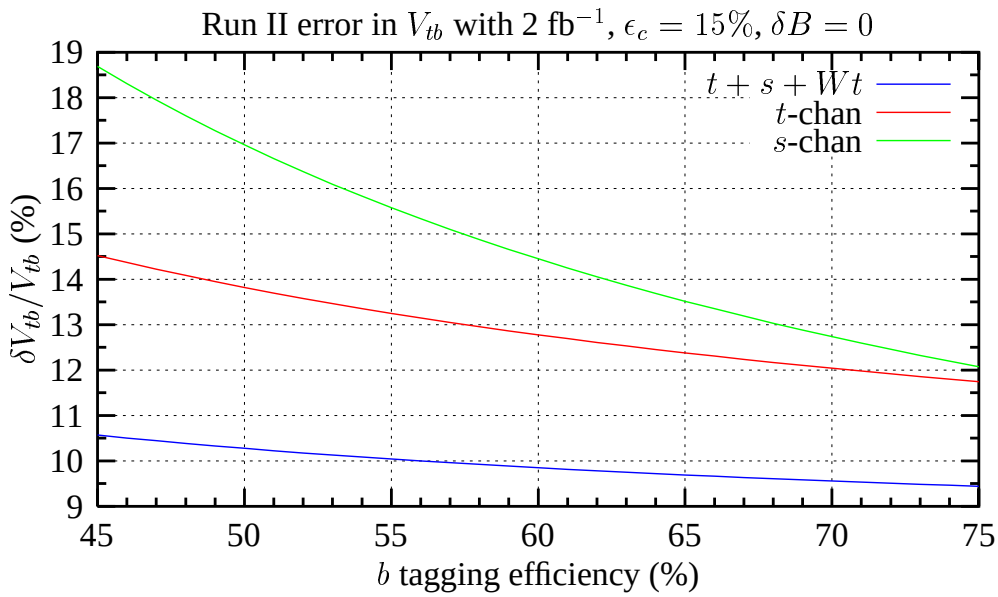


2 fb^{-1} for 5σ discovery of s-channel single top!

Good b-tagging efficiency is really vital. c backgrounds are small, so we can sacrifice b-purity for acceptance.

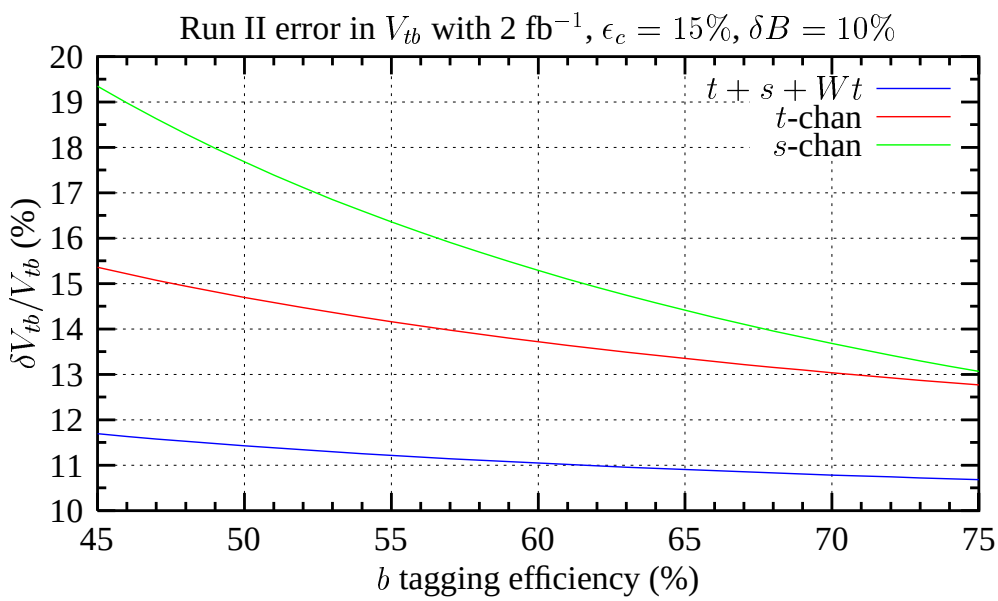
s-channel is completely statistics limited.

Error in extraction of V_{tb} with 2 fb^{-1}



Combined channels are nearly systematics and theory limited.

Including the background uncertainty



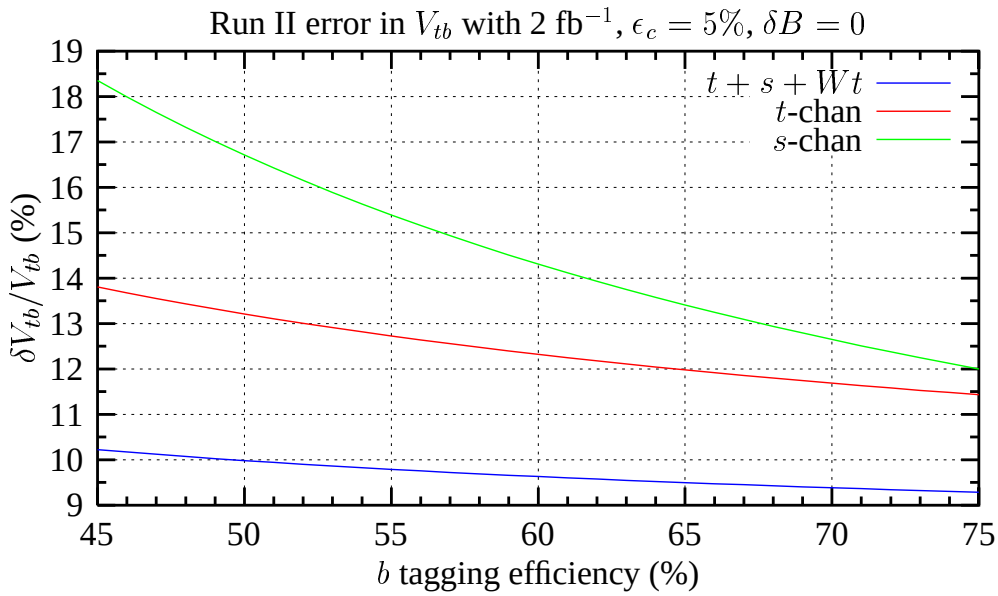
The background uncertainty is a small effect.

V_{tb} can be determined to $\pm 11\%$ in combined sample.

V_{tb} can be determined in t (s)-channel to $\pm 13.5\%$ ($\pm 15.5\%$).

Both s- and t-channel are statistics limited.

Error in extraction of V_{tb} if c mistag rate is reduced

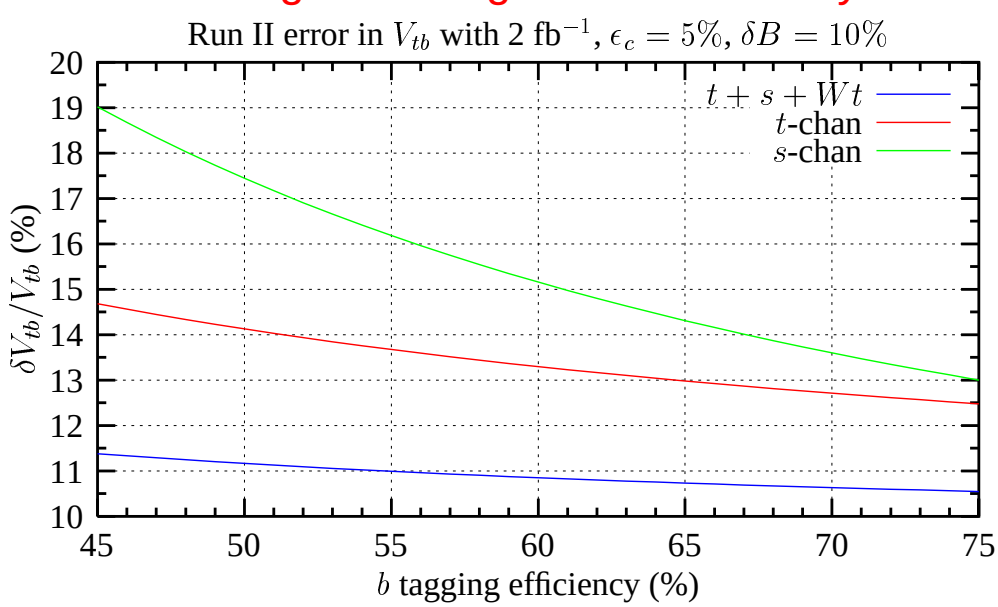


t-channel: background from Wcj is significantly reduced.

Measurement of V_{tb} improved by 1%.

Other systematic and theoretical errors are more important.

Including the background uncertainty



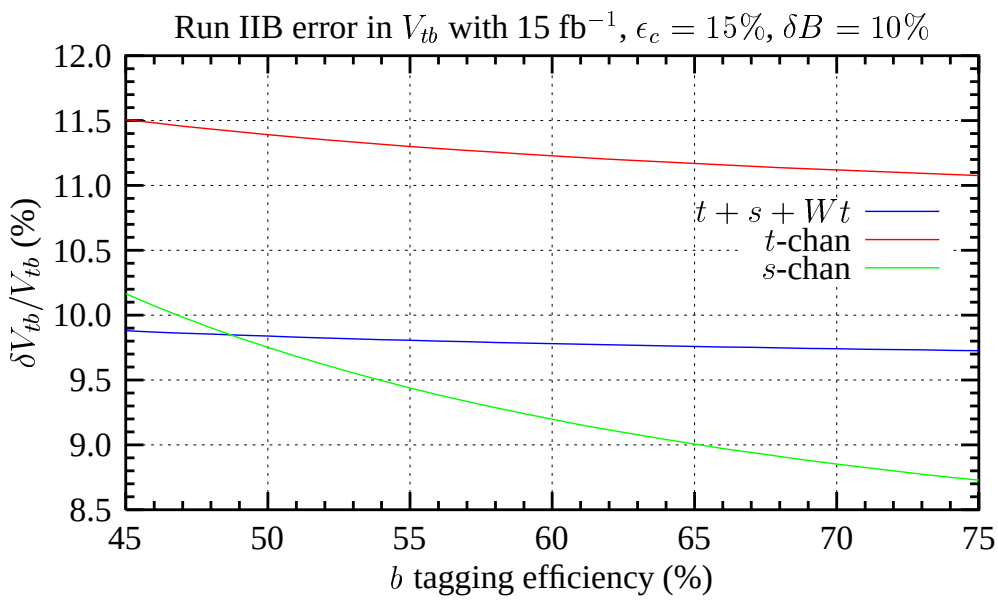
V_{tb} can be determined to $\pm 11\%$ in combined sample.

V_{tb} can be determined in t-channel to $\pm 13.5\%$.

V_{tb} can be determined in s-channel to $\pm 15\%$. $\rightarrow$ Little change!

Error in extraction of V_{tb} with 15 fb^{-1}

Including the background uncertainty



The background uncertainty is a 1% effect.

Combined and t-channels are completely theory limited.

s-channel is still somewhat statistics limited.

V_{tb} can be determined in s-channel to $\sim 9\%$.

V_{tb} can be determined in combined channels to $\sim 10\%$.

V_{tb} can be determined in t-channel to $\sim 11\%$.

Right now s-channel is the way to go if we get high luminosity.

New PDF uncertainties will bring the t-channel extraction error down to $\sim 9\%$ — competitive with s-channel.

Improvements in background estimation and top-quark mass could reduce both to $\sim 7\%$.

We have seen the beauty of single top quarks

- To measure V_{tb} or new physics we want to look for both s- and t-channel single-top-quark production.
- NLO QCD corrections to t- and s-channel total rates exist.
  - For t-channel we reorder perturbation theory $\Rightarrow$ b PDFs.
  - There are 2 types of corrections: α_s and $1/\ln(m_t^2/m_b^2)$.
  - t-channel looks like Double DIS.
- Cross sections are updated with CTEQ5M1 PDFs:

	Tevatron, Run I	Tevatron, Run II	LHC
$\sigma_t^{\rm NLO} =$	1.45 ± 0.08 pb	1.98 ± 0.13 pb	247 ± 12 pb
$\sigma_s^{\rm NLO} =$	0.75 ± 0.07 pb	0.88 ± 0.09 pb	10.7 ± 0.9 pb

  - Honest PDF uncertainties are now calculated: add $\pm 15\%$ in t-channel, $\pm 4\%$ in s-channel.
  - A 1.96 TeV Tevatron will produce 4–10% fewer events in all high mass channels than predicted for 2.0 TeV.
- Fully differential NLO QCD corrections to (2-to-2) t- and s-channel are now published. PRD 66, 054024 (2002)
  - 2 phase space slicing methods and the massive dipole formalism are available.
  - Experimental cuts may be varied.
  - Various jet definitions are implemented.
  - Any IR-safe variable can be predicted at NLO.
  - Uncertainty from modeling $p_{T\bar{b}}$ reduced to $\sim 6\%$.

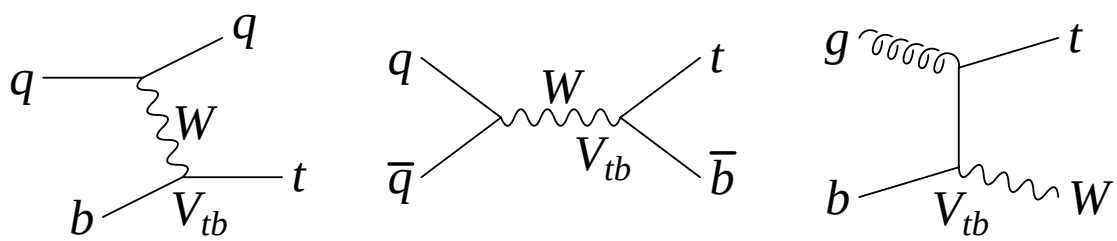
The beauty of single top quarks to come

- Comparisons to PYTHIA/HERWIG are almost complete.
  - Matching to NLO will give an accurate prediction.
- We can test spin-bases at NLO and after cuts.
  - Expect some degradation in the asymmetries.
- What about adding in top quark decay?
  - Top quark is very narrow, so effects should be small, but non-trivial.
- Are fully differential NLO QCD corrections to $t + 2$ jets needed?
  - The Tevatron is statistics limited, but maybe at LHC?
  - Involves new challenges: new 5-point integrals, new interferences, new scales, new fun!

Understanding single-top-quark production leads to exciting new horizons.

Single-Top-Quark Production

(Flagship Measurement for Run II and Beyond)



1. Single-top-quark production will be discovered at Run II.
   3σ evidence with ~ 130 pb^{-1}, 5σ discovery with ~ 370 pb^{-1}.

2. Measuring 1 and 2 b-tag exclusive states will give
   s-channel and t-channel cross sections separately.
   t-channel evidence (discovery) with ~ 370 pb^{-1}(~ 1 fb^{-1}).
   s-channel evidence (discovery) with ~ 730 pb^{-1}(~ 2 fb^{-1}).

3. New physics typically changes the s- and t-channel cross sections in different ways.

4. Will directly measure $|V_{tb}|$ by measuring the cross section in each channel. $\delta|V_{tb}| \sim 13.5$–15.5% with 2 fb^{-1}.

5. Can start to directly measure the polarization of the top quark at production with ~ 2 fb^{-1}.