



## Measurement of the CP-violation parameter of $B^0$ mixing and decay with 970 pb<sup>-1</sup> of $p\bar{p}$ data.

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We measure the dimuon charge asymmetry  $A$  in  $p\bar{p}$  collisions at a center of mass energy  $\sqrt{s} = 1960$  GeV. The data was recorded with the DØ detector and corresponds to an integrated luminosity  $\approx 970$  pb<sup>-1</sup>. Assuming that the asymmetry  $A$  (if any) is due to asymmetric  $B^0 \leftrightarrow \bar{B}^0$  mixing and decay, we extract the CP-violation parameter of  $B^0$  mixing and decay:

$$\frac{\Re(\epsilon_{B^0})}{1 + |\epsilon_{B^0}|^2} = -0.0011 \pm 0.0010(\text{stat}) \pm 0.0007(\text{syst}).$$

We also obtain the forward-backward asymmetry that quantifies the tendency (if any) of  $\mu^+$  to go in the proton direction and  $\mu^-$  to go in the anti-proton direction.

## I. INTRODUCTION

We measure the dimuon charge asymmetry

$$A = \frac{N^{++} - N^{--}}{N^{++} + N^{--}} \quad (1)$$

in  $p\bar{p}$  collisions at a center of mass energy  $\sqrt{s} = 1960$  GeV.  $N^{++}$  ( $N^{--}$ ) is the number of events with two positive (negative) muon candidates passing selection cuts. The data was recorded with the DØ detector between 2002 and 2005. The exposed integrated luminosity is  $\approx 970$  pb $^{-1}$ . Assuming that the asymmetry  $A$  (if any) is due to asymmetric  $B^0 \leftrightarrow \bar{B}^0$  mixing and decay, we extract the CP-violation parameter of  $B^0$  mixing and decay:[1, 2]

$$\frac{\Re(\epsilon_{B^0})}{1 + |\epsilon_{B^0}|^2} = \Im \left\{ \frac{\Gamma_{12}}{4M_{12}} \right\} = \frac{A_{B^0}}{4} \equiv f \cdot A. \quad (2)$$

$M_{12}$  ( $\Gamma_{12}$ ) is the real (imaginary) part of the transition matrix element of the Hamiltonian corresponding to ( $B^0, \bar{B}^0$ ) mixing and decay.[7]  $A_{B^0}$  is the dimuon charge asymmetry from direct-direct decays of  $B^0\bar{B}^0$  pairs. The dimuon charge asymmetry  $A$  in Equation (2) excludes events with a muon from  $K^\pm$ -decay. Equation (2) defines the factor  $f$  to be obtained below. As a sensitive cross check, we also measure the mean mixing probability  $\langle\chi\rangle$  of  $B \leftrightarrow \bar{B}$  hadrons (averaged over the mix of hadrons with a  $b$  quark).

The CP-violation parameter (2) is sensitive to several extensions of the Standard Model.[2, 3] Reference [2] concludes that “It is possible that the dilepton asymmetry could be one of the first indications of physics beyond the standard model”.

The DØ detector[4] has a superb muon system in Run II: large  $(\eta, \phi)$  coverage, good scintillator based triggering and cosmic ray rejection, low punch-through and precision tracking. A muon originating in a  $p\bar{p}$  collision traverses the silicon microstrip tracker and the scintillating fiber tracker in a 2 Tesla solenoidal magnetic field, the calorimeter, layer-A of the muon spectrometer, the magnetized iron toroid, and layers B and C. The muon is the particle with cleanest identification. The like-sign dimuon channel is particularly clean: few processes contribute to it and fewer still contribute to an asymmetry (if any). The polarities of the toroid and solenoid magnetic fields are reversed roughly every week so that the four solenoid-toroid polarity combinations are exposed to approximately the same integrated luminosity. This allows cancellation of first order detector effects. In conclusion, the DØ detector is well suited for this precision measurement.

The outline of the paper is as follows. In Section II we describe the event selection. Physics and detector asymmetries are studied in Section III. The processes contributing to the asymmetry  $A$  are presented in Section IV, and their weights are summarized in Section V. The breakdown of systematic errors of  $A$  are discussed in Section VI. Cross-checks are listed in Section VII. Final results are summarized in Section VIII.

## II. EVENT SELECTION

We find that the DØ detector, operating with a given toroid and solenoid polarity, introduces an apparent dimuon charge asymmetry less than 0.006 (to be discussed in Section III). This detector effect changes sign when the toroid and solenoid polarities are reversed (since the exact same track that is called “positive” with one polarity is called “negative” with the opposite polarity). So, to cancel detector effects to first order, we always consider data sets which have equal integrated luminosities for each toroid-solenoid magnet polarity (or weight the events appropriately). Events with equal (opposite) toroid and solenoid polarities are studied separately so that the cancellation is possible.

Our final cuts require that each local muon candidate have a matching track found in the central tracking system. To reduce punch-through of hadrons we only consider muons that traverse the iron toroid. To get through the toroid with momentum  $\gtrsim 0.2$  GeV/c, we require  $p_T > 4.2$  GeV/c or  $|p_z| > 6.4$  GeV/c, where  $p_T$  is the momentum transverse to the beam, and  $p_z$  is the component of the momentum in the direction of the proton beam. We require at least 2 wire chamber hits in the A layer and at least 3 hits in layers B or C. We require local and global track fits with good  $\chi^2$ . To reduce cosmic ray background we require at least 1 scintillator hit associated with the muon to be within a time window  $|\Delta t| < 5$ ns. To reduce muons from  $K^\pm$  and  $\pi^\pm$ -decay we require  $p_T > 3.0$  GeV/c. The track is required to have a distance of closest approach to the beam less than 0.3cm. We use the full pseudo-rapidity range  $|\eta| < 2.2$ . We require that both muon candidates pass within 2.0 cm in the direction along the beam line at the point of closest approach to the beam. To further reduce cosmic rays and repeated reconstructions of the same track (with different hits), we require the 3-dimensional opening angle between the muons to be between  $10^0$  and  $170^0$ . We also require that the two muons have different A-layer position (by at least 5cm), different local momentum (by at least

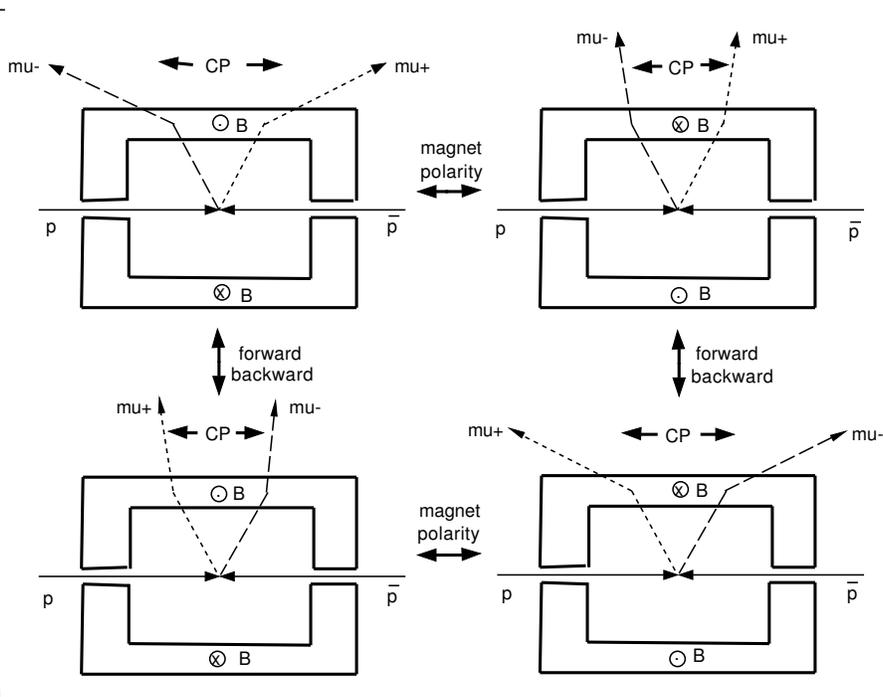


FIG. 1: Schematic drawing of the magnetized iron toroids of the DØ detector, and muon tracks related by toroid polarity reversal, CP conjugation and forward-backward reflection. The  $p$  and  $\bar{p}$  beams are unpolarized.

0.2 GeV/c), and different central track momentum (by at least 0.2 GeV/c). To avoid a bias due to miss-matched central tracks (which are charge-asymmetric due to showers) we use the local muon charge instead of the matching central track charge. To minimize the statistical error we use all triggers (but studies requiring a dimuon trigger from a list were also done, and no significant difference was observed or expected). We apply a cut  $p_T < 15.0$  GeV/c to reduce the number of muons reconstructed with wrong sign, and to reduce the background from  $W^\pm$  and Z-decay.

### III. ASYMMETRIES

In this Section we study detector and physics effects that may alter the observed charge asymmetry and therefore contribute to corrections and systematic errors. The muon detector is shown schematically in Figure 1. Let  $n_\alpha^{\beta\gamma}$  be the number of muons passing cuts with charge  $\alpha = \pm 1$ , toroid polarity  $\beta = \pm 1$ , and  $\gamma = +1$  if  $\eta > 0$  and  $\gamma = -1$  if  $\eta < 0$ . We model the physics and the detector as follows:

$$n_\alpha^{\beta\gamma} \equiv \frac{1}{4} N \epsilon^\beta (1 + \alpha A) (1 + \alpha \gamma A_{fb}) (1 + \gamma A_{det}) (1 + \alpha \beta \gamma A_{ro}) \times (1 + \beta \gamma A_{\beta\gamma}) (1 + \alpha \beta A_{\alpha\beta}). \quad (3)$$

$N$  is the number of muons passing cuts, and  $\epsilon^\beta$  is the fraction of integrated luminosity with toroid polarity  $\beta$  ( $\epsilon^+ + \epsilon^- = 1$ ). Equation (3) defines six asymmetries.  $A$  is the dimuon charge asymmetry,  $A_{fb}$  is the forward-backward asymmetry (that quantifies the tendency of  $\mu^+$  to go in the proton direction and  $\mu^-$  to go in the anti-proton direction),  $A_{det}$  measures the north-south asymmetry of the detector, and  $A_{ro}$  is the range-out asymmetry (that quantifies the change in acceptance and range-out of muon tracks that bend toward, or away from, the iron toroid magnet).  $A_{\alpha\beta}$  is a detector asymmetry between tracks bending north and tracks bending south.  $A$  and  $A_{fb}$  are physics asymmetries that we want to measure, and  $A_{det}$ ,  $A_{ro}$  and  $A_{\alpha\beta}$  are detector asymmetries.  $A_{\beta\gamma}$  is an asymmetry that is different from zero only if the physics asymmetries are different from zero (and is measured to be compatible with zero). If the selection cuts require two like-sign muons we use capital  $A$  for the asymmetries. If the cuts only require single muons we use lower case  $a$ . The model (3) fits 8 numbers  $n_\alpha^{\beta\gamma}$  with 8 parameters ( $N$ ,  $\epsilon^+$ , and 6 asymmetries). There is a solution for small asymmetries because the determinant of the linear terms is different from zero. In Tables I and II we show the numbers  $n_\alpha^{\beta\gamma}$  for our standard cuts. The measured asymmetries are presented in Table III.

From Equation (3) we obtain (up to second order terms) the charge asymmetry  $A$ :

$$\frac{(n_+^{++} + n_+^{+-} - n_-^{++} - n_-^{+-}) + e(n_+^{-+} + n_+^{--} - n_-^{-+} - n_-^{--})}{(n_+^{++} + n_+^{+-} + n_-^{++} + n_-^{+-}) + e(n_+^{-+} + n_+^{--} + n_-^{-+} + n_-^{--})} = A + A_{fb} A_{det}. \quad (4)$$

charge $\alpha$	toroid polarity $\beta$	$-2.2 < \eta < 0.0$ $\gamma = -1$	$0.0 < \eta < 2.2$ $\gamma = +1$
+1	+1	406531	370947
-1	+1	383937	391791
+1	-1	372660	381309
-1	-1	394481	359216

TABLE I: Numbers  $n_{\alpha}^{\beta\gamma}$  of muons passing standard single and dimuon cuts with charge  $\alpha$ , toroid magnet polarity  $\beta$ , and pseudo-rapidity  $< 0$  ( $\gamma = -1$ ) or  $> 0$  ( $\gamma = +1$ ). Two entries per event. Opposite solenoid and toroid polarities. In total, 177950 positive-positive, 176939 negative-negative, and 1175547 positive-negative dimuon events. These data have no mass cuts applied and include all triggers used in this analysis.

charge $\alpha$	toroid polarity $\beta$	$-2.2 < \eta < 0.0$ $\gamma = -1$	$0.0 < \eta < 2.2$ $\gamma = +1$
+1	+1	336283	305358
-1	+1	317106	324579
+1	-1	344582	355747
-1	-1	365888	334327

TABLE II: Same as Table I, but equal solenoid and toroid polarities. In total, 156183 positive-positive, 156148 negative-negative, and 1029604 positive-negative dimuon events.

$e \equiv \epsilon^+/\epsilon^-$  is determined counting single muons for the single muon asymmetries, or dimuons for the dimuon analysis. This procedure introduces no bias since we count all muons or dimuons regardless of the charges of each muon. The left hand side is the measured asymmetry,  $A$  is the corrected asymmetry, and  $-A_{fb}A_{det}$  is the correction due to the forward-backward **and** detector asymmetries. We do not apply this correction because it turns out to be negligible and compatible with zero, see Table III. We use the correction to estimate the corresponding systematic error. We can understand the last term in (4): if positive (negative) muons prefer to go in the proton (antiproton) direction **and** the detector is north-south asymmetric, then we obtain an apparent charge asymmetry. The forward-backward asymmetry  $A_{fb}$  is:

$$\frac{(n_+^{++} + n_-^{+-} - n_+^{+-} - n_-^{++}) + e(n_+^{--} + n_-^{+-} - n_+^{+-} - n_-^{--})}{(n_+^{++} + n_-^{+-} + n_+^{+-} + n_-^{++}) + e(n_+^{--} + n_-^{+-} + n_+^{+-} + n_-^{--})} = A_{fb} + AA_{det}. \quad (5)$$

We have repeated the study of detector asymmetries for the central ( $|\eta| < 0.95$ ) and forward ( $0.95 < |\eta| < 2.2$ ) muon systems separately. To show the extent of the detector effects **before** averaging over magnetic field polarities, we include in Table III the observed single muon charge asymmetries  $A^+$  and  $A^-$  corresponding respectively to toroid polarity  $\beta = +1$  or  $-1$  for events passing dimuon cuts. We note that the detector effects on  $A^+$  and  $A^-$  **before** averaging over magnetic field polarities are less than  $\approx 0.002$ , which corresponds to a dimuon charge asymmetry less than  $\approx 0.006$ . Also shown in Table III is the dimuon asymmetry for “flavor creation” (defined as  $\Delta\alpha \geq 90^\circ$ ) and “flavor excitation” (defined as  $\Delta\alpha < 90^\circ$ ), where  $\Delta\alpha$  is the 3-dimensional angle between the two muons. “Flavor creation” corresponds to the  $b$  and  $\bar{b}$  quarks in opposite jets, while “flavor excitation” corresponds generally to a  $b\bar{b}$  pair produced in the hadronization of one parton. The accepted cross sections for “flavor creation” and “flavor excitation” are nearly equal. In Table III we also show the ratio  $R = (N^{++} + N^{--})/N^{+-}$  of like-sign to opposite-sign dimuon events in the mass window 5.0 to 8.7 GeV/ $c^2$  or 11.5 to 30 GeV/ $c^2$ . This ratio is used for mixing studies. These mass windows reduce backgrounds from same-side direct-sequential muon pairs (process  $P_4$  in Table IV),  $J/\psi$  and  $\Upsilon$  and their resonances.

#### IV. DIMUON PROCESSES

In this Section we obtain the factor  $f$  defined in Equation (2). We consider the processes listed in Table IV. Examples of “other” processes are dimuons with the following parents:  $B^\pm$  and  $\pi^\pm$ ,  $\bar{B}^0$  and  $\tau^\pm$ ,  $Bs^0$  and  $J/\psi$ ,  $B^0$  and  $\tau^\pm$ ,  $\bar{B}^0$  and  $J/\psi$ ,  $B^\pm$  and  $\tau^\pm$ ,  $b$  and unrelated  $c$ . Let  $\chi_d$  be the probability that a  $\bar{B}^0(b\bar{d})$  meson mixes and decays as a  $B^0(db)$ . Similarly,  $\bar{\chi}_d$  is the probability that a  $B^0$  meson mixes and decays as a  $\bar{B}^0$ . In this article we consider the

cuts trigger torpol*solpol	standard all -1	standard all 1
$\text{pb}^{-1}$	$\approx 970$ total	
$e$	1.0302	0.9163
$a$ (all)	$0.0008 \pm 0.0004$	$-0.0005 \pm 0.0005$
$a_{fb}$ (all)	$0.0021 \pm 0.0004$	$0.0015 \pm 0.0005$
$a_{det}$ (all)	$-0.0066 \pm 0.0004$	$-0.0041 \pm 0.0005$
$a_{ro}$ (all)	$-0.0282 \pm 0.0004$	$-0.0311 \pm 0.0005$
$a$ (c)	$-0.0007 \pm 0.0007$	$-0.0031 \pm 0.0008$
$a_{fb}$ (c)	$0.0002 \pm 0.0007$	$-0.0003 \pm 0.0008$
$a_{det}$ (c)	$-0.0072 \pm 0.0007$	$-0.0073 \pm 0.0008$
$a_{ro}$ (c)	$-0.0888 \pm 0.0007$	$-0.0914 \pm 0.0008$
$a$ (f or b)	$0.0017 \pm 0.0006$	$0.0013 \pm 0.0006$
$a$ (f)	$0.0051 \pm 0.0008$	$0.0040 \pm 0.0009$
$a$ (b)	$-0.0016 \pm 0.0008$	$-0.0014 \pm 0.0009$
$a_{fb}$ (fb)	$0.0033 \pm 0.0006$	$0.0027 \pm 0.0006$
$a_{det}$ (fb)	$-0.0061 \pm 0.0006$	$-0.0019 \pm 0.0006$
$a_{ro}$ (fb)	$0.0129 \pm 0.0006$	$0.0099 \pm 0.0006$
$A_{fb}$ (all)	$0.0006 \pm 0.0006$	$0.0003 \pm 0.0006$
$A_{det}$ (all)	$-0.0178 \pm 0.0006$	$-0.0164 \pm 0.0006$
$A_{ro}$ (all)	$-0.0285 \pm 0.0006$	$-0.0302 \pm 0.0006$
$A_{\alpha\beta}$ (all)	$-0.0068 \pm 0.0006$	$-0.0075 \pm 0.0006$
$A_{\beta\gamma}$ (all)	$-0.0001 \pm 0.0006$	$-0.0019 \pm 0.0006$
$R_{mix}$	$0.4799 \pm 0.0012$	$0.4800 \pm 0.0013$
$A^+$ (all dimu)	$0.0011 \pm 0.0008$	$-0.0000 \pm 0.0009$
$A^-$ (all dimu)	$0.0002 \pm 0.0008$	$0.0001 \pm 0.0009$
$A_{fc}$ (all)	$0.0021 \pm 0.0024$	$-0.0014 \pm 0.0026$
$A_{fe}$ (all)	$0.0036 \pm 0.0024$	$0.0016 \pm 0.0026$
$A$ (cc)	$0.0059 \pm 0.0040$	$0.0007 \pm 0.0043$
$A$ (f or b)	$0.0040 \pm 0.0029$	$0.0025 \pm 0.0032$
$A$ (all)	$0.0028 \pm 0.0017$	$0.0001 \pm 0.0018$

TABLE III: Asymmetries described in Section III are shown for central (c), forward (f), backward (b), and all muons. Central has  $|\eta| < 0.95$ . Forward (+ backward) has  $0.95 < |\eta| < 2.2$ .  $e$  is the ratio of dimuons passing cuts for toroid polarity  $\beta = +1$  over the corresponding number for toroid polarity  $-1$ . All errors are statistical. The average for torpol=solpol = 1 and -1 is  $A = 0.0015 \pm 0.0012$  (all).

process	weight	$N^{++}$	$N^{--}$	$N^{+-}$
$b \rightarrow \mu^-, \bar{b} \rightarrow \mu^+$	$P_1 \equiv 1$	$\chi(1 - \bar{\chi})$	$(1 - \chi)\bar{\chi}$	$1 - \xi$
$b \rightarrow \mu^-, \bar{b} \rightarrow \bar{c} \rightarrow \mu^-$	$P_2$	$\frac{1}{2}(1 - \xi)$	$\frac{1}{2}(1 - \xi)$	$\xi$
$b \rightarrow c \rightarrow \mu^+, \bar{b} \rightarrow \bar{c} \rightarrow \mu^-$	$P_3$	$(1 - \chi)\bar{\chi}$	$\chi(1 - \bar{\chi})$	$1 - \xi$
$b \rightarrow \mu^- c \rightarrow \mu^+$	$P_4$	0	0	1
$c \rightarrow \mu^+, \bar{c} \rightarrow \mu^-$	$P_5$	0	0	1
Drell-Yan, $J/\psi$ , $\Upsilon$	$P_6$	0	0	1
dimuon cosmic rays	$P_7$	$\approx 0.14$	$\approx 0.14$	$\approx 0.72$
$\mu + K^\pm$ decay	$P_8$	$0.25 \cdot (1 + a + \alpha)$	$0.25 \cdot (1 - a - \alpha)$	0.5
$\mu +$ cosmic	$P_9$	$0.25 \cdot (1 + \alpha)$	$0.25 \cdot (1 - \alpha)$	$\approx 0.5$
$\mu +$ punch-through	$P_{10}$	$0.25 \cdot (1 + \alpha)$	$0.25 \cdot (1 - \alpha)$	0.5
$\mu +$ combinatoric	$P_{11}$	$0.25 \cdot (1 + \alpha)$	$0.25 \cdot (1 - \alpha)$	0.5
other	$P_{12}$	$0.25 \cdot (1 + \alpha)$	$0.25 \cdot (1 - \alpha)$	0.5
dimuon w. wrong sign	$P_{13}$	0.345	0.345	0.31

TABLE IV: Processes contributing to dimuon events. Each row includes processes related by CP conjugation and  $b \leftrightarrow \bar{b}$  mixing. The weights are normalized to direct-direct  $b\bar{b}$  decay  $P_1 \equiv 1$ .  $\xi \equiv \chi + \bar{\chi} - 2\chi\bar{\chi}$ .  $\chi = f_d\chi_d + f_s\chi_s$  is the probability that  $b$  quarks mix and decay as  $\bar{b}$ .  $\bar{\chi} = f_d\bar{\chi}_d + f_s\bar{\chi}_s$  is the probability that  $\bar{b}$  anti-quarks mix and decay as  $b$ . The fraction of prompt muons from  $b$ -decay is  $\approx 0.6$ .<sup>[5]</sup>  $\alpha \equiv 0.6 \cdot (\chi - \bar{\chi})$ .  $a = 0.026 \pm 0.005$  is the charge asymmetry of  $K^\pm$ -decay, see the text.

$P_1$	$\equiv 1$
$P_2$	$0.116 \pm 0.055$
$P_3$	$0.003 \pm 0.003$
$P_4$	$0.093 \pm 0.049$
$P_5$	$0.070 \pm 0.042$
$P_6$	$0.023 \pm 0.023$
$P_7$	$0.003 \pm 0.003$
$P_8$	$0.098 \pm 0.014$
$P_9$	$0.0001 \pm 0.0001$
$P_{10}$	$0.001 \pm 0.001$
$P_{11}$	$0.0002 \pm 0.0002$
$P_{12}$	$0.163 \pm 0.066$
$P_{13}$	$0.0005 \pm 0.0005$

TABLE V: Weights of dimuon processes for standard cuts (obtained as described in the text). Note that 63% of dimuons are from direct-direct  $b\bar{b}$ -decay.

possibility that  $\chi_d \neq \bar{\chi}_d$ . The probability that a  $b$ -quark mixes and decays as a  $\bar{b}$  is

$$\chi = f_d \frac{\beta_d}{\langle\beta\rangle} \chi_d + f_s \frac{\beta_s}{\langle\beta\rangle} \chi_s \quad (6)$$

where  $f_d$  and  $f_s$  are the fractions of  $b$ -hadrons that are produced as  $B^0$  or  $\bar{B}^0$  and  $B_s^0$  or  $\bar{B}_s^0$  respectively, and  $\beta_d$ ,  $\beta_s$  and  $\langle\beta\rangle$  are the branching fractions for  $B^0$ ,  $B_s^0$  and the  $b$ -hadron admixture respectively decaying to  $\mu X$  with  $\mu$  passing cuts. Similarly

$$\bar{\chi} = f_d \frac{\beta_d}{\langle\beta\rangle} \bar{\chi}_d + f_s \frac{\beta_s}{\langle\beta\rangle} \chi_s \quad (7)$$

is the probability that a  $\bar{b}$  mixes and decays as a  $b$ . From [1] we take  $f_d = 0.397 \pm 0.010$ ,  $f_s = 0.107 \pm 0.011$ ,  $\frac{1}{2}(\chi_d + \bar{\chi}_d) = 0.186 \pm 0.004$ , and  $\chi_s > 0.49883$ . We take  $\beta_d = \beta_s = \langle\beta\rangle$ . To abbreviate we define  $\xi \equiv \chi + \bar{\chi} - 2\chi\bar{\chi}$ . From Table IV we obtain the dimuon charge asymmetry  $A$  after correcting for asymmetric kaon decay, (*i.e.* after subtracting a term  $0.5aP_8$  in the numerator),

$$A = \frac{(\chi - \bar{\chi})(P_1 - P_3 + 0.3P'_8)}{\xi(P_1 + P_3) + (1 - \xi)P_2 + 0.28 \cdot P_7 + 0.5 \cdot P'_8 + 0.69 \cdot P_{13}}, \quad (8)$$

and the factor  $f$  in Equation (2):

$$f = \frac{\xi(P_1 + P_3) + (1 - \xi)P_2 + 0.28 \cdot P_7 + 0.5 \cdot P'_8 + 0.69 \cdot P_{13}}{4f_d\xi_d(P_1 - P_3 + 0.3P'_8)}, \quad (9)$$

where  $P'_8 \equiv P_8 + P_9 + P_{10} + P_{11} + P_{12}$ .

## V. WEIGHTS $P_i$ OF DIMUON PROCESSES

The weights  $P_2 - P_{13}$ , normalized to direct-direct  $b\bar{b}$  decay  $P_1 \equiv 1$ , are summarized in Table V. The weights  $P_2$ ,  $P_4$ ,  $P_5$ ,  $P_6$  and  $P_{12}$  were obtained from Monte Carlo with full detector simulation (based on the Geant program[6]) and event reconstruction and selection. A cross-check for weight  $P_2$  is the measurement of the average mixing probability of  $B$ -hadrons to be described below. Weights  $P_4$ ,  $P_5$  and  $P_6$  do not contribute like-sign dimuons, and so do not enter into the measurement of the CP violation parameter. Weight  $P_3$  was obtained from  $P_3 \approx P_2^2/(4P_1)$ . Weight  $P_7$  was obtained by two methods: (i) from the data of a cosmic ray run, and (ii) extrapolating the out-of-time muon background (as measured by the scintillators) into the acceptance window of  $\pm 5$ ns. Weight  $P_9$  was obtained using the data of the cosmic ray run. Weight  $P_{10}$  was obtained by counting the number of tracks that had enough momentum to traverse the calorimeter and iron toroid, and multiplying by the probability  $\exp(-14)$  that they do not interact (the calorimeter has  $\approx 7$  nuclear interaction lengths, and the iron toroid has  $\approx 7$  nuclear interaction lengths). Weight  $P_{11}$  was estimated by relaxing the number of required wire hits. Weight  $P_{13}$  was estimated using the measured resolution of the local muon spectrometer. In our data set, passing standard single and dimuon cuts, we expect  $\approx 1$  dimuon event from  $Z$ -decay, and  $< 1$  events from prompt muons plus  $W^\pm$ -decay.

Source of error	$\Delta A$
detector	0.00015
$e = \epsilon^+/\epsilon^-$	0.00018
prompt $\mu + K^\pm$ decay	0.00068
dimuon cosmic rays	0.00010
prompt $\mu +$ cosmic $\mu$	0.00001
wrong charge sign	0.00015
punch-through	0.00001
Total	0.00074

TABLE VI: Systematic errors of the dimuon charge asymmetry  $A$  for standard cuts.

Let us consider the weight  $P_8$  in some detail. This weight corresponds to prompt muons from  $b$  or  $c$  or  $s$ -decay plus  $K^\pm$ -decay. This is an important background because kaon decay is charge asymmetric, and dominates the systematic error of the measurement of the CP violation parameter. The inelastic interaction length of  $K^+$  in the calorimeter is longer than the inelastic interaction length of  $K^-$ . This difference is due to the existence of hyperons  $Y$  (strangeness -1 baryons:  $\Lambda$ ,  $\Sigma$ ,  $Y^*$ ). Reactions  $K^- N \rightarrow Y \pi$  have no  $K^+ N$  analog. Therefore  $K^+$  has more time to decay than  $K^-$ . The result is a charge asymmetry from  $K^\pm$ -decay. The single muon charge asymmetry from  $K^\pm$ -decay is obtained from the inelastic cross section for  $K^-d$  and  $K^+d[1]$  and the geometry and materials of the  $D\mathcal{O}$  detector:  $a \equiv (n^+ - n^-)/(n^+ + n^-) = 0.026 \pm 0.005$ .

We measure the weight  $P_8$  directly from data using the exclusive decay  $B^0 \rightarrow D^*(2010)^- \mu^+ \nu_\mu$ ,  $D^*(2010)^- \rightarrow \bar{D}^0 \pi^-$ ,  $\bar{D}^0 \rightarrow K^+ \pi^-$ , and its charge conjugate. We apply standard single and dimuon cuts, and count events with a muon matching the kaon track. The result is  $P_8 = 0.098 \pm 0.014$ . From studies with this exclusive decay, we learn that the global  $\chi^2$ -cut is not very effective in reducing  $K^\pm$ -decay kinks (for the high momentum muons passing cuts). Therefore we must correct  $A$  for  $K^\pm$ -decay as discussed in Section VI. Two complementary estimates of  $P_8$  were as follows. (i) We use the following trick to estimate this background from data. Instead of  $K^+ \rightarrow \mu^+ \nu$ , we study  $K_s \rightarrow \pi^+ \pi^-$ . The  $K^+$  and  $K_s$  have different lifetimes, so we only consider  $K_s$ -decays in a volume down-scaled by the ratio of lifetimes. By this indirect method we obtain  $P_8 \approx 0.041 \pm 0.010(\text{stat}) \pm 0.041(\text{syst})$ . (ii) Using Monte Carlo we obtain  $P_8 \approx 0.047 \pm 0.034(\text{stat})$ .

## VI. SYSTEMATIC ERRORS OF $A$

We add in quadrature the following errors. A summary is presented in Table VI.

Detector effects. Before averaging over magnetic field polarities, the maximum single muon charge asymmetry  $|A^+|$  (for toroid polarity  $\beta = 1$ ) or  $|A^-|$  (for toroid polarity  $\beta = -1$ ) for any cut using dimuons is 0.002. The corresponding dimuon charge asymmetry is  $A < 0.006$ . After averaging over magnetic fields (with appropriate weights), the error in the asymmetry is  $|A_{fb} \cdot A_{det}| < 0.0049 \cdot 0.030 = 0.00015$ , where we have used the largest  $|A_{fb}|$  and  $|A_{det}|$  of any dimuon cut, and any of the detectors (central, forward or all).

Inaccuracy of  $e \equiv \epsilon^+/\epsilon^-$ . We have obtained the ratio of integrated luminosities with toroid polarity  $\beta = 1$  over  $\beta = -1$  by counting dimuons. This procedure introduces no bias since we count all dimuons regardless of the charges of each muon. We take  $\Delta e = 0.03$  from the largest difference between any cuts. Multiplying by the asymmetry before averaging,  $< 0.006$ , we obtain  $\Delta A < 0.00018$ .

Prompt  $\mu + K^\pm$ -decay. The single muon charge asymmetry of kaon decay is  $a = 0.026 \pm 0.005$  as explained in Section V. We take  $P_8 = 0.098 \pm 0.014$  from Table V. The corresponding correction to  $A$ , explained in Section III, is  $\delta A = -0.5aP_8/A_{den} = -0.5 \times (0.026 \pm 0.005) \times (0.098 \pm 0.014)/0.45 = -0.0028 \pm 0.0007$  ( $A_{den} = 0.45$  is the denominator of Equation (8)). This error is by far the dominating systematic error of the entire measurement.

Dimuon cosmic rays. We take  $P_7 < 0.007$  from Table V. From cuts that select cosmic rays, we obtain the apparent dimuon charge asymmetry  $A = -0.0095 \pm 0.0117$ . Then the corresponding error of  $A$  is  $< 0.007 \times 0.28 \times 0.021/A_{den} = 0.0001$  (see Table IV;  $0.021 = |-0.0095| + 0.0117$ ).

Prompt  $\mu +$  single cosmic ray. We take  $P_9 < 0.0002$ , see Table V. From cuts that select cosmic rays, we obtain a single muon charge asymmetry  $a = 0.026 \pm 0.002$ . Then the error in  $A$  is  $< 0.0002 \times 0.5 \times 0.028/A_{den} = 6 \cdot 10^{-6}$  (see Table IV;  $0.028 = 0.026 + 0.002$ ).

Wrong local muon sign.  $P_{13} < 0.001$ , see Table V. Even if the asymmetry of wrong tracks is 0.1 (overestimate), the corresponding error of  $A$  is small:  $< 0.001 \times 0.69 \times 0.1/A_{den} = 0.00015$  (see Table IV).

Punch-through. Take  $P_{10} < 0.002$ , see Table V. The measured charge asymmetry of tracks with  $p_T > 3.0$  GeV/c is  $0.0049 \pm 0.0005$  due to showers on matter instead of antimatter. Then the error in  $A$  is  $< 0.002 \times 0.5 \times 0.0054/A_{den} =$

Source of error	$\Delta f$
$P_2$	0.087
$P_7$	0.002
$P'_8$	0.049
$\chi_d$	0.010
$f_d$	0.017
$\chi_s$	0.0002
$f_s$	0.014
Total	0.103

TABLE VII: Systematic errors of  $f$ .

Source of error	$\Delta\xi$
$R$	0.0074
$P_2$	0.028
$P'_4$	0.015
$P_7$	0.0001
$P'_8$	0.014
$P_{13}$	0.0002
Total	0.035

TABLE VIII: Systematic errors of  $\xi = \chi + \bar{\chi} - 2\chi\bar{\chi}$ .  $P'_4 \equiv P_4 + P_5 + P_6$ .

$1 \cdot 10^{-5}$  (see Table IV;  $0.0054 = 0.0049 + 0.0005$ ).

## VII. OTHER CROSS-CHECKS

The sign of central muons was cross-checked using cosmic rays (which are charge asymmetric[1]). The sign of forward muons relative to the sign of central muons was cross-checked with  $J/\psi$ 's.

We find the dimuon asymmetry  $A$  stable (within statistical errors) for all triggers or a set of dimuon triggers, and across the different cuts, data subsets, opposite or equal toroid and solenoid polarities, central or forward muons, or flavor creation or flavor excitation events.

## VIII. RESULTS

We obtain  $A = 0.0028 \pm 0.0017(\text{stat})$  for opposite solenoid and toroid polarities, and  $A = 0.0001 \pm 0.0018(\text{stat})$  for equal solenoid and toroid polarities (see last line in Table III). Combining these measurements we obtain

$$A = 0.0015 \pm 0.0012(\text{stat}). \quad (10)$$

We add a correction  $\delta A = -0.0028 \pm 0.0007$  to  $A$  due to asymmetric  $K^\pm$ -decay (this effect is explained in Sections V and VI). The error of this correction dominates the systematic error of the CP-violation parameter. The final corrected value of the dimuon charge asymmetry is

$$A = -0.0013 \pm 0.0012(\text{stat}) \pm 0.0008(\text{syst}). \quad (11)$$

The breakdown of systematic errors of  $A$  are presented in Table VI.

From the dimuon charge asymmetry  $A$  we obtain

$$\frac{\Re(\epsilon_{B^0})}{1 + |\epsilon_{B^0}|^2} = \frac{A_{B^0}}{4} \equiv f \cdot A = -0.0011 \pm 0.0010(\text{stat}) \pm 0.0007(\text{syst}), \quad (12)$$

where

$$f = 0.861 \pm 0.103(\text{syst}). \quad (13)$$

The breakdown of systematic errors of  $f$ , calculated from information provided in preceding sections, is listed Table VII. In comparison, the Particle Data Group average of 2004[1] is  $\Re(\epsilon_{B^0})/(1 + |\epsilon_{B^0}|^2) = 0.0005 \pm 0.0031$ .

We measure the ratio  $R$  of like-sign to opposite-sign dimuons. For this measurement we require the invariant mass of the two muons to be in the windows 5.0 to 8.7 GeV/ $c^2$  or 11.5 to 30 GeV/ $c^2$ . These mass cuts are designed to reduce backgrounds from same-side direct-sequential decay and backgrounds from  $\psi$  and  $\Upsilon$ -decays to allow a measurement of  $B \leftrightarrow \bar{B}$  mixing. For this reason we set  $P_4 = P_6 = 0$  for the mixing analysis. We also require dimuon triggers from a list that excludes triggers requiring opposite sign muons. We obtain  $R = 0.474 \pm 0.001(\text{stat}) \pm 0.010(\text{syst})$ , and  $\xi = 0.235 \pm 0.001(\text{stat}) \pm 0.035(\text{syst})$ . The breakdown of systematic errors, calculated from information provided in preceding sections, is shown in Table VIII. The final result for the mixing probability, averaged over the mix of hadrons with a  $b$  quark, is

$$\langle \chi \rangle = 0.136 \pm 0.001(\text{stat}) \pm 0.024(\text{syst}). \quad (14)$$

The agreement with the world average[1] is a sensitive test of  $f$ , since the largest systematic errors of  $\xi$  and  $f$  are due to the same weight  $P_2$ .

Finally, we measure the tendency (if any) of  $\mu^+$  ( $\mu^-$ ) to go in the proton (antiproton) direction. We obtain the dimuon forward-backward asymmetry

$$A_{fb} = 0.0004 \pm 0.0004(\text{stat}) \pm 0.0001(\text{syst}). \quad (15)$$

$A_{fb}$  is defined in Section III.

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