Measurement of the Top Quark Mass in the Lepton+Jets Channel using DØ Run II Data: The Low Bias Template Method

The DØ Collaboration

http://www-d0.fnal.gov

(Dated: March 1, 2005)

In this note, we present measurements of the top quark mass, using lepton+jets events collected in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV at the Fermilab Tevatron using the DØ detector. The dataset corresponds to an integrated luminosity of approximately 229 pb$^{-1}$ collected between April 2002 and March 2004. We reconstruct the top quark mass using a kinematic fitting technique and use a method which relies on templates of signal and background mass spectra to determine the top quark mass. Two event samples are used for this measurement. One sample is selected based on a discriminant which utilizes the unique topology of $tt$ events (topological analysis), the other sampling requires that at least one of the jets in the event is tagged as originating from the fragmentation of a $b$-quark in the final state ($b$-tagged analysis). We measure the top quark mass to be: $m_t = 169.9 \pm 5.8$ (stat) $^{+6.8}_{-7.1}$ (syst) GeV (topological analysis) and $170.6 \pm 4.2$ (stat) $\pm 6.0$ (syst) GeV ($b$-tagged analysis).

Preliminary Results for Winter 2005 Conferences
I. INTRODUCTION

One of the main goals of Run II of the Fermilab Tevatron is the detailed study of the top quark. The top quark was discovered by the DØ and CDF collaborations in 1995 during Run I of the Tevatron [1]. In Run I the two experiments each accumulated an integrated luminosity of about 100 pb$^{-1}$. From these data measurements of the top quark pair production cross section and the top quark mass were obtained.

The measurement of the top quark mass is interesting because it is an important parameter in many predictions of the Standard Model. In particular, precise knowledge of the top quark mass, together with the $W$ boson mass, constrains the mass of the Higgs boson in the framework of the Standard Model. The measurement of the top quark mass by the DØ collaboration from Run I data is $179.0 \pm 5.1$ GeV [2],[3].

The top quark decays to a $W$ boson and a $b$ quark. The $W$ boson can either decay to leptons ($e^{+}e^{-}$, $\mu^{+}\mu^{-}$, $\tau^{+}\tau^{-}$) or quarks ($u\bar{d}$, $c\bar{s}$). The quarks manifest themselves as jets in our detector. If the $W$ boson from the top quark decays to $e\nu$ or $\mu\nu$ and the $W$ boson from the antitop quark decays to quarks (or vice versa), the events contain one charged lepton and four jets (from the two $b$ quarks and the $W$). We call this the lepton+jets channel. It is characterized by a sizeable branching fraction ($\approx 30\%$) and small backgrounds. Events of this type kinematically constrain the top quark mass. We do not use events in which the $W$ decays to $\tau\nu$ because they are harder to identify and reconstruct.

Here we present the first measurement of the top quark mass by DØ using a $b$-tagged event sample. The integrated luminosity used for this measurement is about 229 pb$^{-1}$. We present measurements from the lepton+jets channel using the “Template” method, a technique very similar to that used for the first top quark mass measurement by DØ [3], which uses a kinematic fit of the event four-vectors. In this technique, templates are constructed from fitted top quark masses for $t\bar{t}$ Monte Carlo samples with various hypothesized input top quark masses and from the main background. The top quark mass is then extracted from the data using a binned maximum likelihood fit which estimates the most likely value of the top quark mass and the number of $t\bar{t}$ events in the sample.

We present results from the analysis of two samples of events selected using criteria designed to preferentially select $t\bar{t}$ events over background processes. Jets originating from a bottom quark ($b$-jets) are present in all top quark events while they are absent from the largest background sources. Hence, requiring evidence for one or more $b$-jets in an event improves the signal to background ratio in the sample. We call this analysis the “$b$-tagged analysis”. Another strategy is to exploit the unique topology of $t\bar{t}$ events due to the large mass of the top quark in the event selection. This is the basis of the “topological analysis”. In this note, we present a measurement utilizing both methods to select samples that are highly enriched in $t\bar{t}$ events.

II. THE DØ DETECTOR

The DØ detector is a typical multipurpose collider detector, that consists of central tracking, calorimeter, and muon detection systems.

The magnetic central-tracking system is comprised of a silicon microstrip tracker and a scintillating fiber tracker, both located within a 2 T superconducting solenoidal magnet [4]. Central and forward preshower detectors are located just outside of the coil and in front of the calorimeters. The liquid-argon/uranium calorimeter is divided into a central section covering $|\eta| \leq 1$ and two end calorimeters extending coverage to $|\eta| \leq 4$ [5]. In addition to the preshower detectors, scintillators between the calorimeter cryostats provide sampling of developing showers at $1.1 < |\eta| < 1.4$.

The muon system is located outside the calorimeter and consists of a layer of tracking detectors and scintillation trigger counters before 1.8 T toroids, followed by two similar layers outside the toroids. Tracking at $|\eta| < 1$ relies on 10 cm wide drift tubes [5], while 1 cm mini-drift tubes are used at $1 < |\eta| < 2$.

The trigger and data acquisition systems are designed to accommodate the high luminosities of Run II. Based on information from tracking, calorimeter, and muon systems, the output of the first level of the trigger is used to limit the rate for accepted events to $\approx 1.5$ kHz. At the next trigger stage, with more refined information, the rate is reduced further to $\approx 800$ Hz. These first two levels of triggering rely mainly on hardware and firmware. The third and final level of the trigger, with access to all the event information, uses software algorithms and a computing farm, and reduces the output rate to $\approx 50$ Hz, which is written to tape.

III. SIMULATION OF SIGNAL $t\bar{t}$ AND BACKGROUND $W$+JET EVENTS

Simulation of the $t\bar{t}$ events was performed by the ALPGEN [6] Monte Carlo (MC) program. ALPGEN was interfaced with PYTHIA [7] to simulate the parton shower, hadronization and hadron decays. The scale for calculation of the $t\bar{t}$ processes is $Q = m_t$. The $t\bar{t}$ samples were generated at nine top quark masses ranging between 150 and 200 GeV. We rely on Monte Carlo to predict the flavor composition of the jets produced in association with a $W$ boson. The
W+jets events with different jet multiplicity and flavor are generated using ALPGEN interfaced with PYTHIA. For the W+jets background simulation, the default dynamical scale of the interaction $Q^2$ is set to $M_W^2 + \sum p_T^2$.

Events are then processed through the full DØ detector simulation based on GEANT [8]. The same object reconstruction algorithms are applied to collider data and Monte Carlo events.

IV. DATA SET AND SELECTION

The data sample used is the same as for the topological and the lifetime tagging cross section analyses in the lepton+jets channel[9, 10]. The first step is to identify events which have a high $p_T$ electron or a muon accompanied by substantial $E_T$, indicative of W boson production in the final state. We require isolated electron (muon) candidates to have $p_T > 20$ GeV, be within the pseudorapidity range $|\eta| < 1.1$ ($|\eta| < 2.0$) and to satisfy tight quality requirements. A minimum $E_T$ of 20 GeV is also imposed on the events. We call this sample the base sample.

For the topological analysis, we start from the base sample and select events with at least 4 jets with transverse momentum $p_T > 20$ GeV. The jets must all be in the pseudorapidity range $|\eta| < 2.5$. There are 87 $e+$jets and 80 $\mu+$jets events after these requirements. Next, we impose a requirement on the event topology. We construct a discriminant $D$ using the kinematics of the event (see section V). We select events with $D$ greater than 0.4. A final selection on the kinematic fit of the event to the $t\bar{t}$ hypothesis is applied (see section VI). We require that at least one jet permutation which fits the top quark decay hypothesis has $\chi^2 < 10$.

To obtain the sample for the b-tagged analysis, we also start from the base sample and select events with at least 4 jets with transverse momentum $p_T > 15$ GeV and $|\eta| < 2.5$. We further require the identification of one or more jets as b-jets. The jets are tagged as b-jets based on identifying secondary vertices reconstructed from the charged particle tracks associated with calorimeter jets. At this stage of the selection, we have 47 (29) e+jets ($\mu+$jets) events. These are then fit to the $t\bar{t}$ hypothesis (see section VI). We find that for 42 (27) e+jets ($\mu+$jets) events the kinematic fit converges in a configuration where the lowest $\chi^2$ solution is consistent with the b-tagged jet permutation hypothesis.

The dominant source of background is production of a W boson, which decays to $e\nu$ or $\mu\nu$, and several light-quark and gluon jets (topological analysis) or heavy flavor jets (b-tagged analysis). The jets in these events tend to be softer than the jets from top quark decays. The $p_T$ cuts in the selection are designed to reduce this background. Another source of background is from multijet events where one of the jets is misidentified as a lepton and there is significant $p_T$ imbalance due to detector resolution. In the $\mu+$jets channel, muons from heavy flavor quark decays can fake isolation if the hadronic activity in the unreconstructed jet fluctuates low. Jets can be mis-identified as electrons when there is a leading $\pi^0$ that overlaps with the track of a charged particle and muons can originate from the decay of $\pi$ and $K$ mesons. These mis-identification background sources are minimized by requiring that the missing $p_T$ is acollinear with the lepton direction.

The numbers of events that pass the final event selection are shown in Table I, together with the estimated composition of the sample. The sample composition for the topological analysis is determined via a fit to a similar topological likelihood as described in the $t\bar{t}$ production cross section analysis note [9]. The background contribution for the b-tagged analysis is obtained by weighting the W+jets Monte Carlo events with different flavor compositions with the probability to find a b-tagged jet for that given combination of jets. The multijet background is determined also by weighting an un-tagged sample with the probability to b-tag a jet in that sample. The details of the background determination technique can be found elsewhere [10].

<table>
<thead>
<tr>
<th>Channel</th>
<th>Topological Analysis</th>
<th>b-tagged Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>e+jets</td>
<td>$p+$jets</td>
</tr>
<tr>
<td>Number of Events</td>
<td>49</td>
<td>45</td>
</tr>
<tr>
<td>Sample Composition</td>
<td>$t\bar{t}$</td>
<td>$W+$jets</td>
</tr>
<tr>
<td>$27.5 \pm 2.2$</td>
<td>$20.4 \pm 0.9$</td>
<td>$30.5 \pm 2.4$</td>
</tr>
<tr>
<td>$9.5 \pm 0.45$</td>
<td>$22.0 \pm 2.6$</td>
<td>$7.0 \pm 0.6$</td>
</tr>
<tr>
<td>$12 \pm 0.55$</td>
<td>$2.6 \pm 0.5$</td>
<td>$4.5 \pm 0.4$</td>
</tr>
</tbody>
</table>

TABLE I: Number of events passing the event selection and breakdown in signal and background contributions.

V. LOW BIAS DISCRIMINANT

In order to get discrimination between signal and background, we derive a discriminant (Low Bias Discriminant, LB) constructed from the topology of the events. Because of the large mass of the top quark, $t\bar{t}$ events have a unique
FIG. 1: $\mathcal{A}$ distributions (top left), $H_T'$ distributions (top right), $K_{T\min}'$ distributions (bottom left) and $E_T$ distributions (bottom right) for $t\bar{t}$ and background from Monte Carlo simulations.

topology. The discriminant is designed to be uncorrelated with the top quark mass. We developed this discriminant by closely following the work described in reference [3]. The four topological variables considered here are:

1. $E_T$.
2. $\mathcal{A}$
3. $H_T'$
4. $K_{T\min}'$

$\mathcal{A}$ is the aplanarity of the event and defined as $\frac{2}{3} \times$ smallest eigenvalue of $\mathcal{P}$. $\mathcal{P}$ is the normalized momentum tensor of the event derived from the momenta of the jets and the reconstructed $W$ boson. It is defined by:

$$\mathcal{P}_{ij} = \frac{\sum_a p_{a,i} p_{a,j}}{\sum_a |p_a|^2},$$

where $i$ and $j$ label the spatial components of the momentum, and $a$ runs over all jets and the reconstructed $W$ boson. $\mathcal{A}$ has a range between 0 to 0.5. It can be shown [11] that the decay products from a massive particle have large values of aplanarity.

$H_T'$ is defined as the ratio of the scalar sum of the $|p_T|$ of the jets excluding the leading jet and the scalar sum of $|p_z|$ of the jets, isolated lepton, and the neutrino. $H_T'$ gives rather good discrimination and has only a small correlation with the fit mass. Although variables such as $H_T$, the scalar sum of $p_T$ of all jets in the events, have notably better discrimination power, Run I experience showed that the correlation with the fit mass was unacceptably large [3].

$$K_{T\min}' \equiv \min(\Delta R_{ij},) \min(E_T', E_T'),$$

and is a measure of the jet separation folded together with “transverse energy” of the reconstructed leptonic $W$ boson. $\Delta R_{ij}$ is the distance between jet $i$ and jet $j$ in $\eta - \phi$ space. Of the six possible $\Delta R_{ij}$ between the four leading jets, the smallest is chosen. $E_T'$ is defined as the scalar sum of the lepton $p_T$ and $E_T$.

The distributions for the individual variables are shown in Fig. 1. The likelihood is constructed by the procedure described in [3]. The discriminant is shown for signal and background in Fig. 2.

In Fig. 3 we show the low bias discriminant versus fitted mass for the topological analysis and for the $b$-tagged sample. It is obvious that the $b$-tagged events (which are enriched in top quark signal) cluster at larger values of the discriminant. This demonstrates the power of the discriminant to distinguish signal and background.
FIG. 2: Discriminant for $t\bar{t}$ (blue) and background from Monte Carlo simulation (red) for $e$-jets and $\mu$-jets events combined.

FIG. 3: Low Bias Discriminant versus fitted mass for the events in the collider data sample. The left plot is for the topological analysis sample and the right plot shows events that are in the $b$-tagged analysis.

VI. CONSTRAINED KINEMATIC FIT

We use a constrained kinematic fit to extract mass information from the events. The fit technique is the same as used in the Run I template analysis [3]. The object resolutions used in the fit were updated to reflect those of the Run II D0 detector.

In order to reconstruct the mass of the top quark from its decay products, we need to measure the four-vectors of the final state particles: the four jets with the highest $p_T$, charged lepton, and neutrino. While the jets and the charged lepton can be directly observed, and their properties measured by the D0 detector, the neutrino cannot. The event missing transverse energy is taken to be the magnitude of the transverse momentum of the neutrino. However, this procedure cannot be extended to the longitudinal or $z$ component of the neutrino momentum since there are particles with very small transverse momentum that escape down the beam pipe. The $p_Z$ of the neutrino is the only kinematic unknown in the event.

In addition to these 17 measured numbers, three constraints can be imposed. The invariant mass of the decay products of the two $W$ bosons must equal the $W$ boson mass and the invariant mass of the two $Wb$ pairs must be equal. Since we need 18 numbers to completely define the six-particle final state we can perform a 2-C fit to the top-antitop quark decay hypothesis. In order to start the fitting procedure, one must have an initial value for all of the kinematic variables. The $p_Z$ of the neutrino is found by forcing the invariant mass of the charged lepton and the neutrino to be that of the $W$ boson. This leads to a quadratic equation and two possible solutions. We use the smallest absolute value of the neutrino $p_Z$ solution. This yields the correct solution in approximately 60% of the
cases [3]. There are 12 possible ways to assign the four jets to the $b$ and $\bar{b}$ quarks and the two quarks from the decay of the $W$ boson. The fit routine considers all 12 permutations and for each permutation returns a best fit top quark mass and a fit $\chi^2$.

VII. MASS SPECIFIC JET ENERGY SCALE CORRECTIONS

Due to fragmentation and detector effects, the measured energy in a jet cone is not equal to the energy of the original parton. Thus the jet energies have to be calibrated before we can measure the top quark mass.

The first step in calibrating the jet energy scale is to correct the measured energy of the jets to be equal on average to the sum of energies of the particles in the jet cone. This is done in the same way for data and events from Monte Carlo simulation so that both are on equal footing. The measured jet energy ($E_{\text{meas}}$) is corrected using the following expression:

$$E_{\text{corr}} = \frac{E_{\text{meas}} - O}{R \times S},$$

where $R$ is the calorimeter response (determined requiring $p_T$ balancing in $\gamma + \text{jets}$ events). $O$ is the energy offset due to the underlying event, energy pile-up, multiple interactions, electronic noise and uranium noise from the uranium absorber. $O$ is determined from energy densities in minimum bias events. $S$ is the fraction of shower energy that remains inside the jet cone ($\Delta R = 0.5$) in the calorimeter and is determined from the measured energy profiles of jets. These corrections are applied to all jets.

In addition, jets originating from $b$ quarks ($b$ jets) may contain a lepton from the semileptonic decay of the original $B$ hadron and in this case the jet does not include the energy of the escaping neutrino. In the case of an electron all the energy of the lepton will be contained in the calorimeter jet, while for a muon only a small amount of energy is deposited (typically on the order of 2 GeV). Thus for semileptonic decays to muons the energy of the $b$ jet is corrected for the muon energy and the energy of the neutrino. The correction factor is derived using Monte Carlo samples of $b$-quark semileptonic decays.

The second step is to correct the energy of the jets to equal on average that of the original parton. Determining this parton-level correction requires knowledge of the parton momentum 4-vector and the reconstructed jet momentum. It can therefore only be determined from simulated Monte Carlo events. The parton-level corrections are derived as a function of energy for jets originating from the fragmentation of light quarks ($u, d, s, c$) and heavy quarks ($b$), and in three pseudorapidity bins. Using the information from the generated events, the primary partons from $W$-decay (before radiation) are matched to jets. Only uniquely matched jet-parton pairs are used to avoid biasing the corrections by the occasional hard gluon radiation that generates two distinct jets or overlap of jets from two or more partons. This is the same procedure as used in reference [3].

VIII. THE TEMPLATE MASS ANALYSIS

A. Method for mass extraction

The template analysis is based on comparing the fitted masses from the collider data with the results obtained from fitting simulated Monte Carlo data samples of known top quark masses. In this comparison we use the fitted top quark mass from the permutation with the smallest $\chi^2$ as the mass estimator. We apply the same event selection on the Monte Carlo events as on the collider data. For each hypothetical top quark mass, we create templates by constructing a histogram of fit masses with 10 GeV wide bins from 80 to 280 GeV. We also construct a background template from the most prominent background to our decay channel: $W + 4$ jet production.

In order to extract the top quark mass from this comparison, we use a binned likelihood fit. We write the probability distribution function for the mass estimator in terms of the number of signal events $n_s$ and the number of background events $n_b$ in our sample. We constrain the fraction of background events to the expected number using a Poisson probability term.

For each hypothesized top quark mass, the likelihood is maximized as a function of the number of signal and background events. The mass with the largest likelihood, or equivalently the smallest negative log likelihood ($-\ln(L)$) is identified and a parabola is fit to the values of $-\ln(L)$ for all hypothesized top quark masses within a small range around the mass with the largest likelihood. This range is set to $\pm 15$ GeV for the topological analysis and $\pm 10$ GeV for the $b$-tagged analysis. The minimum of the parabola is taken as the most likely top quark mass and the statistical uncertainty is extracted by finding the mass for which the fit to $-\ln(L)$ rises by $\frac{1}{2}$.
To extract the most likely number of signal events, we interpolate between the values of \(n_s\) at the two top quark masses which straddle the minimum of the fit to \(-\ln(L)\).

B. Performance on MC

In order to test our method of measuring the top quark mass, we performed a series of simulated Monte Carlo experiments. For these we fit ensembles of Monte Carlo events in the same way as the collider data. The size of each ensemble was fixed to the total number of events seen in data. The mean fraction of signal and background was adjusted to agree with the fit to the collider data sample. However, in each ensemble the number of \(t\bar{t}\) events and background events were allowed to vary according to binomial statistics. These ensemble tests were performed in order to evaluate the calibration of the method, the expected statistical uncertainty, and to verify that the uncertainties assigned by the fit are consistent with the statistical spread of the ensembles. The results shown in Figs. 4 and 5 demonstrate that the method is well calibrated for both the topological and \(b\)-tagged analyses. The uncertainties are consistent with the statistical spread seen in the ensembles. These results are also summarized in Table II.

C. Mass measurement from collider data

We find 94 data events in the topological analysis sample. The fit mass distribution of the events is shown in Fig. 6 (left plot). The fit for the top quark mass is shown in Fig. 6 (right plot). The topological cross-section analysis, with the slight differences in event selection taken into account, predicts that \(47.9 \pm 8.8\) of the 94 events are expected
TABLE II: Means and Pulls of ensemble tests for various input top quark masses.

<table>
<thead>
<tr>
<th>Input Mass (GeV)</th>
<th>Topological Analysis</th>
<th>b-tagged Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average Topolog. Out</td>
<td>Width of Gaussian</td>
</tr>
<tr>
<td></td>
<td>Mass (GeV)</td>
<td>Fit to Pull</td>
</tr>
<tr>
<td>165</td>
<td>166.4 ± 0.60</td>
<td>0.87 ± 0.05</td>
</tr>
<tr>
<td>170</td>
<td>169.6 ± 0.55</td>
<td>0.89 ± 0.06</td>
</tr>
<tr>
<td>175</td>
<td>175.2 ± 0.55</td>
<td>0.93 ± 0.05</td>
</tr>
<tr>
<td>180</td>
<td>180.6 ± 0.60</td>
<td>0.87 ± 0.05</td>
</tr>
<tr>
<td>185</td>
<td>186.6 ± 0.65</td>
<td>0.87 ± 0.05</td>
</tr>
</tbody>
</table>

FIG. 6: The distribution of fit masses of the topologically selected events (left). $-\ln(L)$ distribution as a function of the fit top quark mass to these events (right). The red curve is the expectation from background only events (normalized to the fraction preferred by the fit) while the blue curve is the sum of the expectation from signal and background for the mass point closest to the fit result.

from $t\bar{t}$ decays. We fit a top quark mass of $m_t = 169.9 ± 5.8$ GeV with $44.2 ± 6.6$ $t\bar{t}$ events (statistical uncertainties only). As shown in Fig. 8 (left), the statistical uncertainty on the measurement seen in data is quite close to the expected statistical uncertainty from ensemble tests.

There are 69 events in the $b$-tagged event sample. The $b$-tagged cross-section analysis [10], corrected for the kinematic fit convergence, predicts the number of signal events to be $52.4 ± 4.2$. Fig. 7 shows the fit mass distribution for MC and data (left plot) and the fit to the $-\ln(L)$ versus the assumed top quark mass (right plot). The result of the likelihood fit leads to a measurement of the top quark mass of $170.6 ± 4.2$ GeV (statistical uncertainty only). The number of $t\bar{t}$ events is determined to be $49.2 ± 6.3$.

Figure 8 (right) shows that the statistical uncertainty on the measurement in data is quite close to the expected statistical uncertainty from ensemble tests.

D. Systematic uncertainties

We considered several sources of systematic uncertainty on the measurement: the jet energy scale, the jet energy resolution, gluon radiation, Monte Carlo $t\bar{t}$ signal model, background model, $b$-tagging, trigger bias, limited Monte Carlo statistics, and the calibration uncertainty.

The dominant systematic uncertainty originates, as expected, from the jet energy scale. This was evaluated by scaling the jet energy scale up and down by 1σ for both the signal and background Monte Carlo samples. The uncertainty on the jet energy scale has been conservatively taken to be 5% for jet $E_T > 30$ GeV. For jets with transverse energies below 30 GeV, the uncertainty rises linearly and is given by 30% $\times \frac{\text{Jet Energy}}{30 \text{ GeV}}$.

These events were then used in ensembles fit using templates with the nominal jet energy scale. The difference in the mean top quark mass obtained from these ensembles and the ensembles with the nominal energy scale was taken as the uncertainty associated with the jet energy scale calibration. This resulted in a variation of $+6.8/-6.5$ GeV for the topological analysis and $+4.7/-5.3$ GeV for the $b$-tagged analysis.

The jet energies in the Monte Carlo events were smeared to more closely model the detector performance. This
FIG. 7: The $-\ln(L)$ curve from the fit of the tagged event selection (right). The distribution of fit masses of the $b$-tagged events (left). The red curve is the expectation from background only events (normalized to the fraction preferred by the fit) while the blue curve is the sum of the expectation from signal and background for the mass point closest to the fit result.

FIG. 8: The distribution of statistical uncertainties from ensemble tests from a $t\bar{t}$ sample with input mass of 170 GeV for the topological analysis (left) and the $b$-tagged analysis (right). The results obtained for the collider data sample are marked by the large arrows.

smearing was varied by $\pm 1\sigma$ in both signal and background Monte Carlo events and ensemble tests repeated using templates with the nominal jet energy resolution. The resulting variation, $\pm 0.9$ GeV, was taken to be the systematic uncertainty for the jet energy resolution.

Although the nominal $t\bar{t}$ lepton + jets event has a final state of four jets, a charged lepton, and a neutrino, $\approx 40\%$ of these events are expected to have an extra jet from either initial or final state gluon radiation. There has been some theoretical work dedicated to the study of gluon radiation in top quark decays [12] but owing to the limited samples of top quark events produced there is little experimental work on the subject. Since we use only the four highest $p_T$ jets in the mass fit, the effect on the kinematic mass fit depends on the source of the gluon. If one of the initial state quarks radiates a hard gluon that hadronizes and becomes one of the four highest $p_T$ jets in the event it will be used in the mass fit. A priori, we know that regardless of what jet permutation is used, the fit mass will not be the correct mass for the event. This has a tendency to produce higher mass solutions and a broadening of the fit mass distribution. If the gluon radiates from one of the final state quarks at a large enough angle the energy of that quark is split into two jets and leads to lower mass solutions since not all the energy of the $t\bar{t}$ system is included in the fit. Since there is essentially no previous experimental guidance on gluon radiation in top quark decays, Monte Carlo must be used and trusted to provide the right number of gluon jets with the correct properties. To understand the effects this has on the top quark mass reconstruction, the events in Monte Carlo with only four partons hadronizing and forming four jets were compared to the events where one gluon jet was used in the kinematic fit. Taking a conservative stance, the systematic uncertainty was taken by selecting events with gluon radiation not present and events where one of the jets used in the kinematic fit was from a gluon. For the topological analysis the variation from nominal is 0.2 GeV for the events without gluon radiation and 2.6 GeV with the events with gluon radiation. For the $b$-tagged analysis, the variation from nominal is thus found to be 0.1 GeV for the events without gluon radiation and 2.4 GeV for the
events with gluon radiation. The two variations were then added in quadrature to yield a ±2.6 GeV uncertainty for the topological analysis and a ±2.4 GeV uncertainty for the b-tagged analysis.

The number of top quark events seen in data is very small. Therefore, the model of the kinematic properties of the events is taken purely from Monte Carlo simulation. Since the Monte Carlo simulation is used to produce the templates for the mass fit, deficiencies in the Monte Carlo model could lead to a bias in the top quark mass measurement. In order to conservatively estimate the size of this effect, in addition to the nominal sample for the tt signal, we generate another tt sample where an additional parton is produced in association with the tt pair. This sample is also generated using the ALPGEN MC interfaced to PYTHIA to perform the hadronization process. This process has a cross section of 2.5 pb, compared to 6.0 pb for the nominal tt signal sample. By analyzing the sample where along with tt production there is an extra hard quark in the events, we obtain an uncertainty of +2.3 GeV due the signal model for the topological analysis as well as the b-tagged analysis.

We investigated the effect on top quark mass on the dynamical scale at which the background W+jets MC were generated. The default W+jets samples were generated using $Q^2 = M^2_T + \sum p^2_T$. In order to test the dependence on this scale an alternate sample was generated with $Q^2 = \sum p^2_T$. A variation of +0.7 GeV was found forming ensembles using this sample for the background using the template with the default dynamical scale. In the b-tagged analysis, the background is expected to be composed essentially of heavy flavor quark production in conjunction with a W boson. The heavy quarks tend to have a slightly harder $p_T$ spectrum than the light quarks. The fragmentation process is different for heavy and light quarks [11] with heavy quarks tending to form hadrons that carry most of the energy of the jet. Finally, the efficiency for identifying the heavy quarks is strongly dependent upon the $p_T$ and $\eta$ of the jet. All of these lead to the fit mass distribution for W + heavy quark events having a somewhat larger average mass. The nominal background template for the b-tagged analysis was taken from the combination of Monte Carlo W+jet events in the fraction that is expected from the cross-section [10]. In order to ascertain the uncertainty due to this assumption, Monte Carlo ensembles were formed using only W + light flavor events with a background template composed of the nominal fractions. Then ensembles using only W + b-jets were created and fit using the same background template. This leads to a variation of 0.8 GeV and 0.4 GeV from the nominal. We combine this uncertainty with that due to the choice of dynamical scale to obtain a uncertainty of ±0.8 GeV associated with modeling the background in the b-tagged analysis.

To estimate the uncertainty due to the b-tagging algorithm, we vary the heavy quark tagging efficiency and light quark tagging rates determined from data and applied to the tt Monte Carlo samples by ±1 σ around the nominal. Using these samples, we find that the top quark mass varied by ±0.7 GeV for the b-tagged analysis only.

Although the calibration curve is consistent with zero offset and unit slope, there is of course point to point variation in the calibration curve. We take the parameters from the fit calibration along with the uncertainties and compute the uncertainty which is associated with the uncertainty in the calibration curve. This procedure produces an uncertainty of 0.5 GeV.

We simulate the effect of trigger efficiencies by removing events from the ensembles by generating a random number and comparing to the probability for that event to satisfy the trigger. This is done for all the Monte Carlo templates. In order to ascertain the uncertainty associated with a possible trigger bias, we prepare Monte Carlo ensembles in which the trigger efficiency is not taken into account. This prescription leads to an uncertainty of ±0.5 GeV.

Since there is a finite number of Monte Carlo events, statistical fluctuations in the signal and background templates can lead to an uncertainty in the extracted top quark mass. In order to quantify this effect we divided the Monte Carlo sample into four subsamples to produce four different sets of background and signal templates. Then ensembles were produced from the full Monte Carlo set and fit with the four sets of signal and background templates. In this case the ensembles were fixed and the template sets were varied. The uncertainty was computed as the rms of the results from the four sets of templates divided by $\sqrt{N-1}$, where $N = 4$ is the number of different templates to account for the fact that we have four times the number of events in our templates. The result is an uncertainty of 0.5 GeV.

The systematic uncertainties are summarized in Table III. After adding the uncertainties in quadrature we measure the top quark mass to be $m_t = 169.9 \pm 5.8$ (stat) $^{+7.8}_{-7.1}$ (syst) GeV (topological analysis) and $m_t = 170.6 \pm 4.2$ (stat)$\pm 6.0$ (syst) GeV (b-tagged analysis).

In order to cross check the results, a slightly different fitting method was used. In this case, the constraint on the signal fraction to that expected from the cross-section measurement was removed from the likelihood function. In the b-tagged analysis we find the top quark mass to be 171.8 ± 4.8 GeV with 39.9 ± 11.6 tt events in the sample. For the topological analysis, we determine the top quark mass to be 170.7 ± 6.5 GeV with 40.2 ± 13.3 tt events. Both are within a GeV of the constrained results indicating that the result is quite stable with respect to the presumed signal fraction.
TABLE III: Systematic Uncertainties. This Table shows the variation of the top quark mass that is seen when various quantities which enter the top quark mass estimation are varied within ±1σ of their known values.

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Topological Analysis</td>
</tr>
<tr>
<td>Jet Energy Scale</td>
<td>+6.8 / -6.5</td>
</tr>
<tr>
<td>Jet Resolution</td>
<td>±0.9</td>
</tr>
<tr>
<td>Gphon Radiation</td>
<td>±2.6</td>
</tr>
<tr>
<td>Signal Model</td>
<td>+2.3</td>
</tr>
<tr>
<td>Background Model</td>
<td>+0.7</td>
</tr>
<tr>
<td>b-tagging</td>
<td>-</td>
</tr>
<tr>
<td>Calibration</td>
<td>±0.5</td>
</tr>
<tr>
<td>Trigger Bias</td>
<td>±0.5</td>
</tr>
<tr>
<td>Limited Monte Carlo Statistics</td>
<td>±0.5</td>
</tr>
<tr>
<td>Total</td>
<td>+7.8 / -7.1</td>
</tr>
</tbody>
</table>

FIG. 9: The results from this paper compared to previous measurements of the top quark mass.

IX. CONCLUSION

In conclusion, we present a preliminary measurement of the mass of the top quark in the lepton+jets channel based on an integrated luminosity of about 229 pb⁻¹ of data from Run II of the Tevatron. Two methods have been used to measure the mass of the top quark. The first, based on a topological event selection, yields a result of $m_t = 169.9 ± 5.8^{+7.8}_{-7.1}$ (syst) GeV with 44.2 ± 6.6 $t\bar{t}$ events. The second method, based on the identification of hadronic jets from b quarks, yields a result of $m_{t,\text{op}} = 170.6 ± 4.2$(stat) ± 6.0(syst) GeV with 49.2 ± 6.3 $t\bar{t}$ events. In Fig. 9, these results are compared to the previous measurements of the top quark mass. Along with being one of the first measurements during the new run, this is the first measurement to utilize a b-tagged sample to measure the top quark mass at the DØ experiment. The results obtained from the b-tagged sample is presently the most precise measurement of the top quark mass from Run II.
[10] The DØ Collaboration, “Measurement of the $t\bar{t}$ Production Cross-Section in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV Using b-tagged lepton + jet events”, DØ Conference Note 4565, August 2004.