From High Energy QCD to Statistical Physics
... and Back

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Based on:

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E.I., D. Triantafyllopoulos (hep-ph/0411405 & 0501193)

Outline

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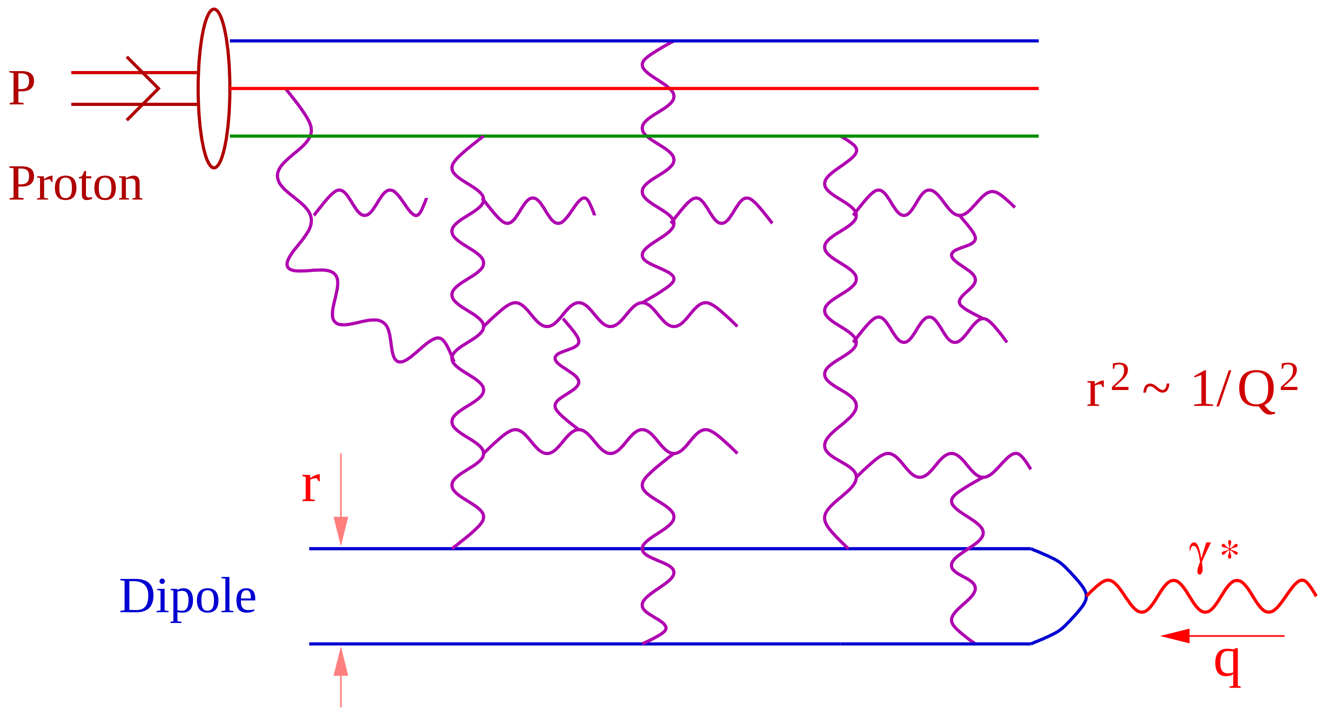
High–Energy Evolution in QCD
QCD vs. Statistical Physics
Mean Field Approximation
Fluctuating pulled fronts
Conclusions

- High–energy QCD evolution: a classical stochastic process
  - BFKL evolution $\Longrightarrow$ The rise of the gluon density
  - Gluon recombination $\Longrightarrow$ Saturation, CGC
  - Gluon splitting (bremsstrahlung) $\Longrightarrow$ Correlations
- The Mean Field Approximation (BK equation)

  & Its breakdown at high energy.
- The asymptotic behaviour $(s \to \infty, \quad \alpha_s \to 0)$ is universal !

  The same universality class as the reaction–diffusion problem in statistical physics.
- From recent (1997–2004) results in statistical physics to exact (asymptotic) results in QCD !
- How to go beyond ‘asymptotia’ ? See the next coming talks!

Deep Inelastic Scattering at High Energy



- DIS probes the gluon distribution with a small 'color dipole' :

$$Q^2 \sim 1/r^2 \gg \Lambda^2_{\rm QCD}\,, \qquad r^2 = (\boldsymbol{x}-\boldsymbol{y})^2 \;:\; \text{dipole size}$$

- At high energy, the dipole 'sees' a very complicated gluon configuration

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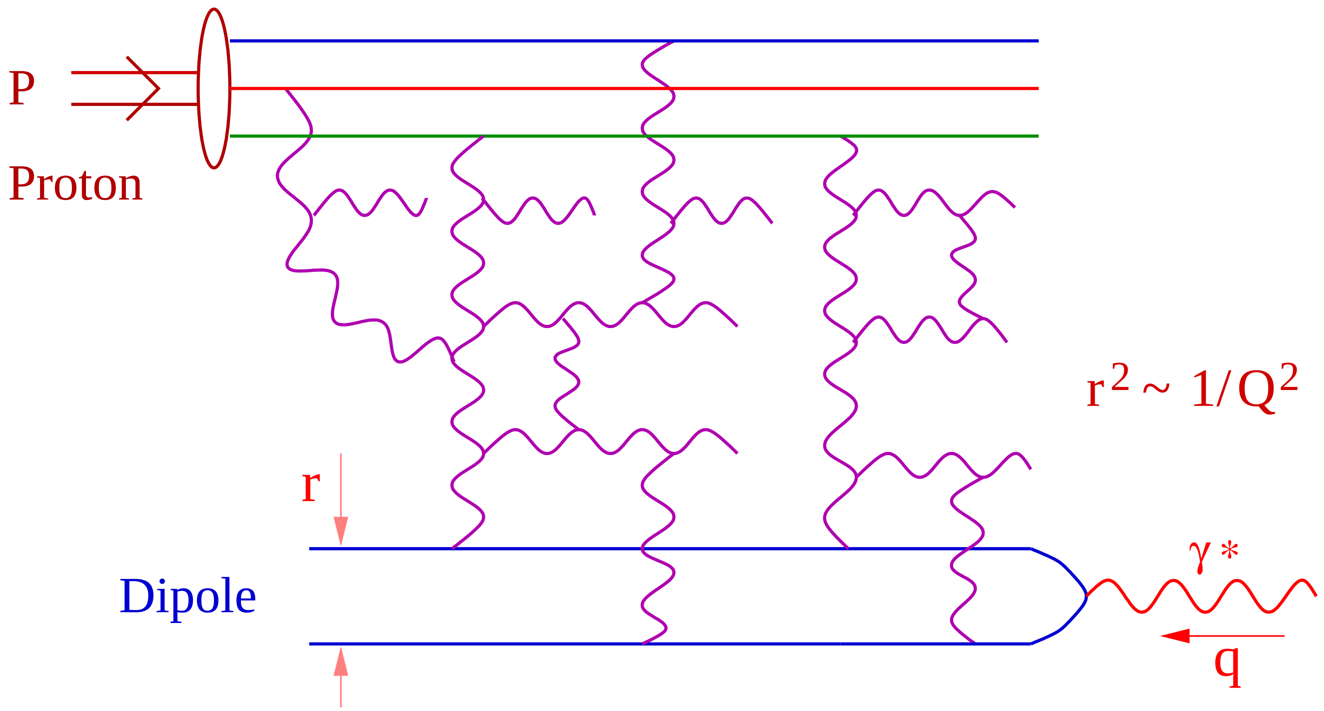

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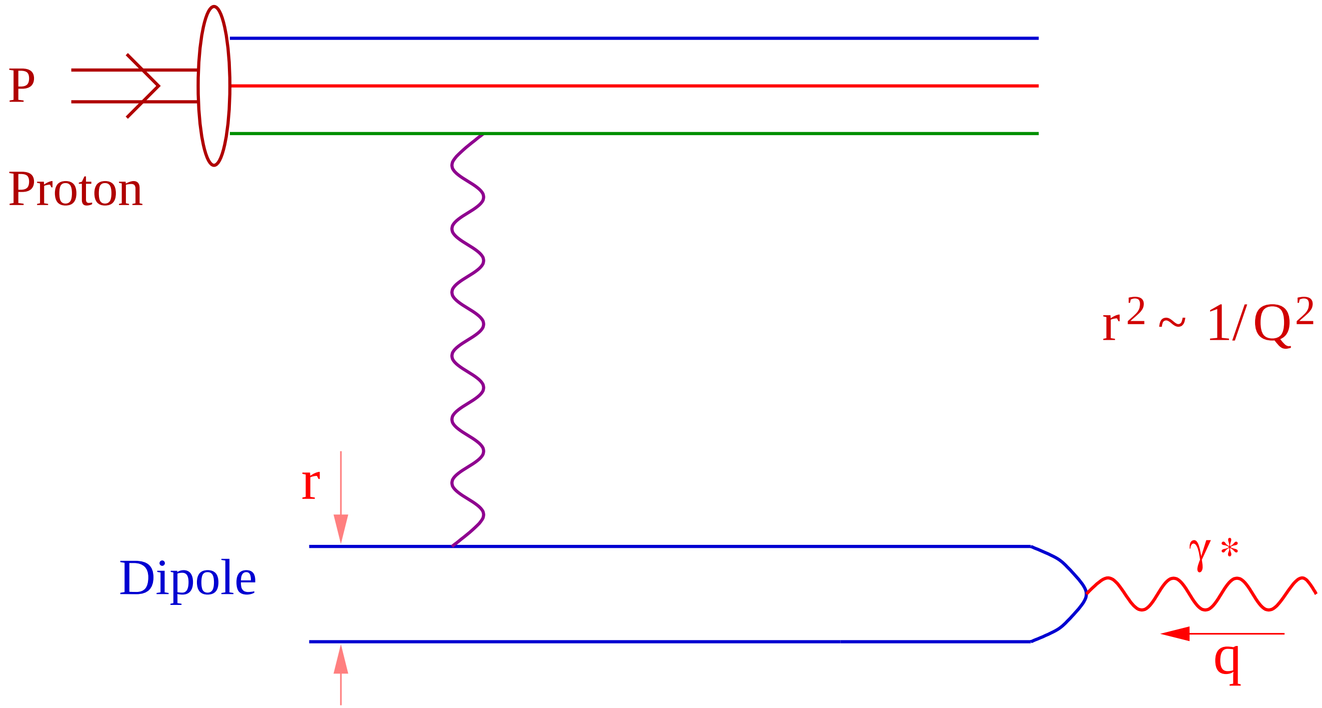
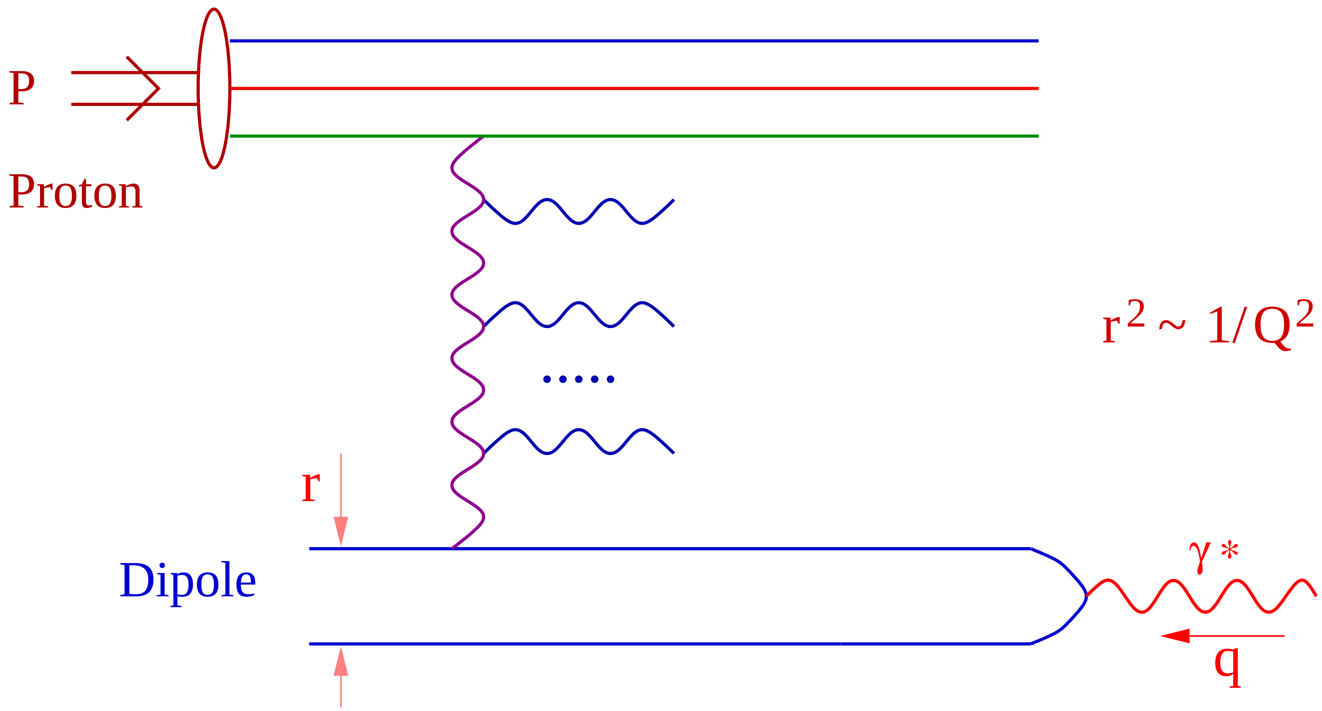
Lowest order



- Lowest order process in perturbative QCD :

  One–gluon exchange (in the amplitude)

BFKL Evolution



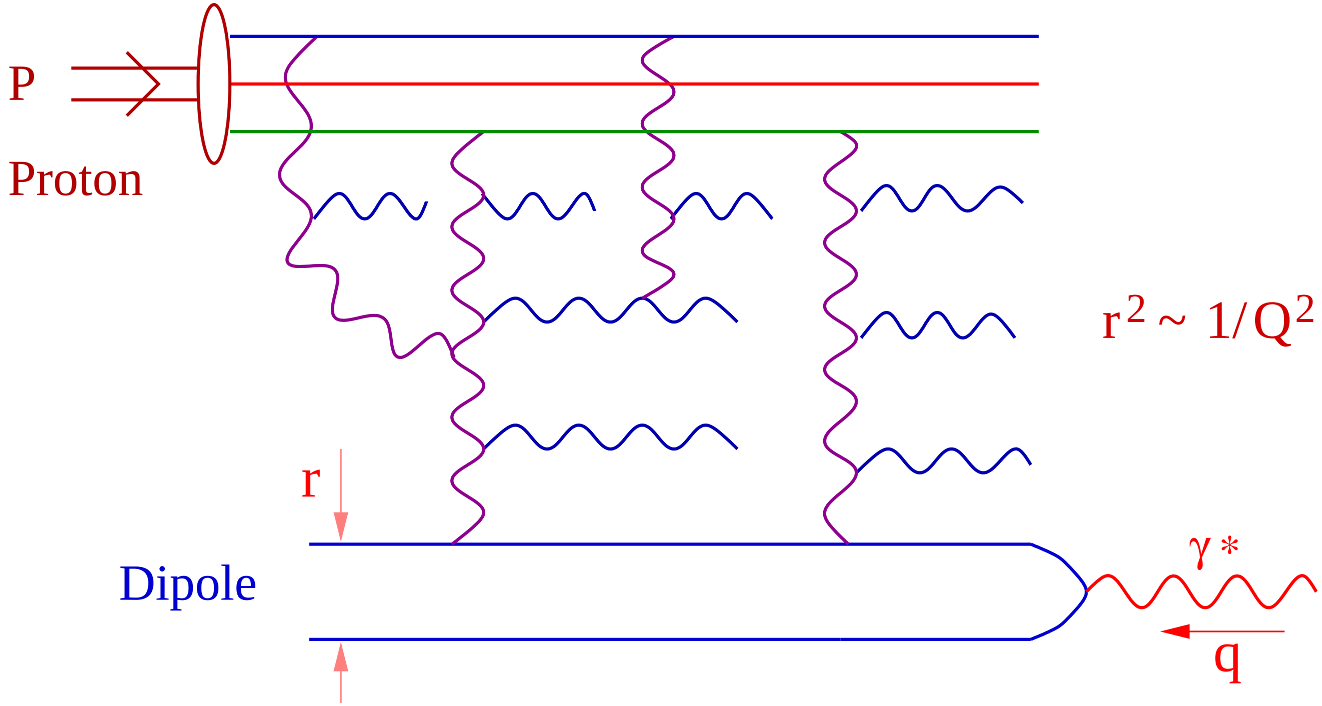
- n 'small–x' gluons in the s–channel $\Longrightarrow$ $T^{(n)} \sim (\alpha_s Y)^n/n!$

  $Y \equiv \ln 1/x \sim \ln s$: 'rapidity'

- BFKL equation : $T_{\rm BFKL}(r, Y) \sim \alpha_s^2\, r^{2\gamma}\, {\rm e}^{\omega Y}$

- Violation of the unitarity bound $(T \leq 1)$, Infrared diffusion ...

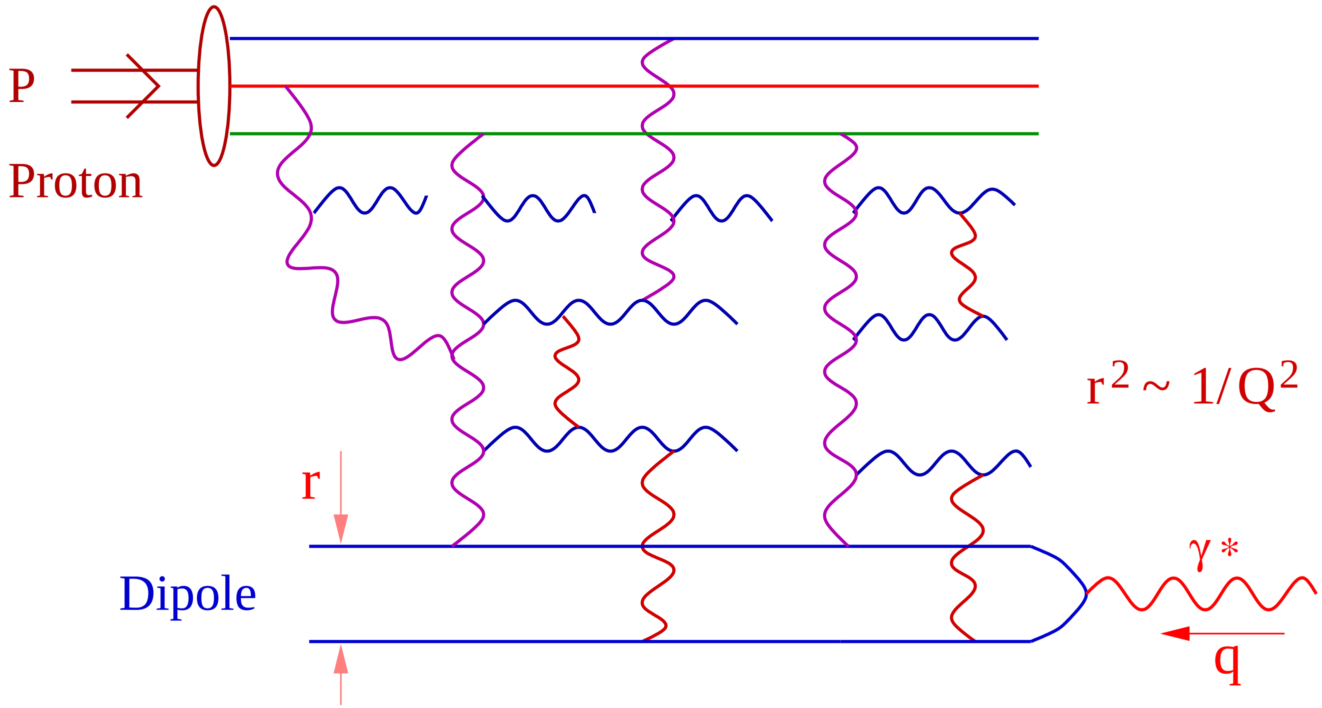
Gluon Saturation



- High density $\Longrightarrow$ Gluon recombination $\Longrightarrow$ Saturation
- Unitarity is restored: $T(r, Y) \simeq 1$ for $r \gtrsim 1/Q_s(Y)$
- Saturation momentum: $Q_s^2(Y) \propto \mathrm{e}^{\lambda Y}$

  λ = the saturation exponent

Bremsstrahlung



- How are the t–channel gluons generated in the first place ?
  Through bremsstrahlung in the high–energy evolution !
- The seed of the higher–point correlations
  ($\Longrightarrow$ fluctuations in the number of gluons)
- Bremsstrahlung + recombination $\Longrightarrow$ 'Pomeron loops'

From QCD to Statistical Physics

- All these phenomena are relevant for the dominant behaviour at high energy ! (saturation, unitarity)
- Currently, we know the QCD equations which encode these phenomena ... but they are complicated !
- The asymptotic ($s \to \infty$) behaviour is insensitive to the details of the dynamics beyond the BFKL equation
  - It follows from : the BFKL equation
  - + the existence of unitarity corrections
  - + the existence of gluon number fluctuations

  E.I., Mueller, Munier, 04
- The universality of the stochastic process !

  $\Longleftrightarrow$ The 'reaction–diffusion' problem in statistical physics

  chemical reactions, pattern formation, directed percolation, spreading of epidemics, solar activity, computer science ...

Reaction–diffusion process $A \rightleftharpoons 2A$

- Particles of type A distributed on an one-dimensional lattice
  - Particle splitting (rate α) : $A \xrightarrow{\alpha} A + A$
  - Particle merging (rate β) : $A + A \xrightarrow{\beta} A$
  - A particle can diffuse to a neighboring site
- At large t, $n(x,t)$ saturates at a value $N \equiv \alpha/\beta \gg 1$
- $N \to \infty$: $h(x,t) = n(x,t)/N$ obeys the F–KPP equation :

$$\partial_t h(x,t) = \underbrace{\partial_x^2 h(x,t)}_{\text{diffusion}} + \underbrace{h(x,t)}_{\text{growth}} \quad \underbrace{- \; h^2(x,t)}_{\text{recombination}}$$

- Two fixed points: $h = 0$ (unstable) and $h = 1$ (stable)
- “Traveling wave” : a front propagating into the unstable state

$$h(x,t) \simeq F(x - vt)\ , \quad F(z \to -\infty) \to 1\ , \quad F(z \gg 1) \sim \mathrm{e}^{-\gamma z}$$

- Finite N : Fluctuations in the particle number

Dipole scattering at high energy

- Evolution equation for the dipole amplitude *(Balitsky, JIMWLK)*

$$\frac{\partial}{\partial Y}\langle T(\boldsymbol{x},\boldsymbol{y})\rangle_Y = \frac{\alpha_s N_c}{\pi}\int_{\boldsymbol{z}} \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \Big\langle \underbrace{-T(\boldsymbol{x},\boldsymbol{y})+T(\boldsymbol{x},\boldsymbol{z})+T(\boldsymbol{z},\boldsymbol{y})}_{\text{BFKL (linear)}} + \underbrace{T(\boldsymbol{x},\boldsymbol{z})T(\boldsymbol{z},\boldsymbol{y})}_{\text{non–linear}}\Big\rangle_Y$$

- Non–linear effects $\langle T\,T\rangle \longleftrightarrow$ Gluon recombination
- Fluctuations $\langle T\,T\rangle - \langle T\rangle\langle T\rangle \longleftrightarrow$ Gluon splitting
- Mean field approximation : $\langle T\,T\rangle \approx \langle T\rangle\langle T\rangle$

  $\Longrightarrow$ Non–linear equation for $\langle T\rangle$: Balitsky–Kovchegov eq.
- BK eq. : the same universality class as the F–KPP equation

  (Munier & Peschanski, 03)

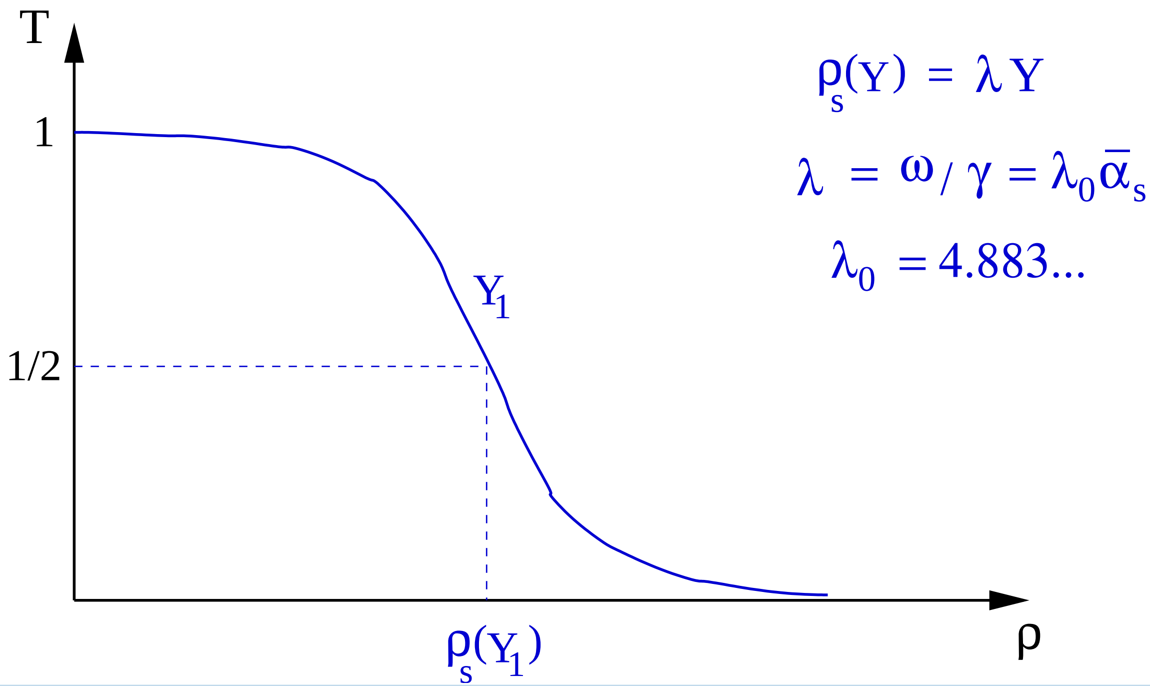
  $h(x,t) \longleftrightarrow T(\rho,Y)\,, \qquad \rho \equiv \ln 1/r^2 \sim \ln Q^2$

BK Equation: The Traveling Wave

- Two fixed points: $T = 0$ (unstable) and $T = 1$ (stable)
  - Low energy/small dipole $\Longrightarrow$ $\langle T \rangle \ll 1$ $\Longrightarrow$ BFKL growth

$$\langle T(r) \rangle_Y \sim r^{2\gamma}\, s^{\omega} \sim \mathrm{e}^{-\gamma\rho}\, \mathrm{e}^{\omega Y} \qquad \left(\rho \equiv \ln \frac{1}{r^2} \sim \ln k^2\right)$$

  - High energy/large dipole $\Longrightarrow$ $\langle T \rangle \to 1$ (unitarity limit)
- When increasing Y, the front propagates towards larger ρ



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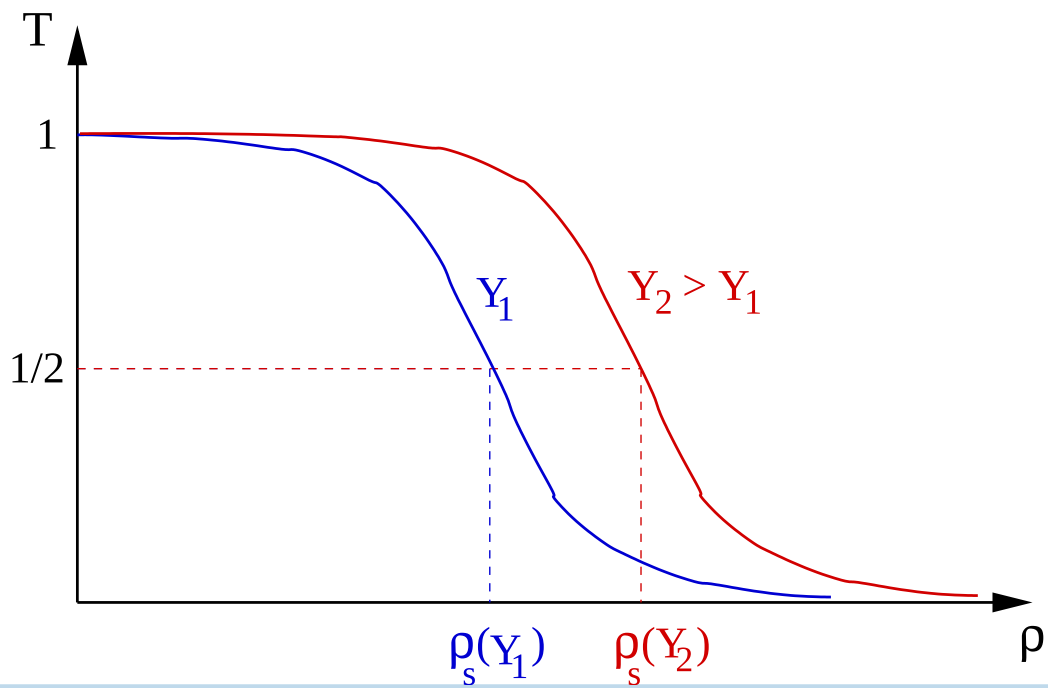
- Saturation scale : $\rho_s(Y) = \lambda Y$, with $\lambda = \omega/\gamma$ (from BFKL !)



Geometric Scaling

- The position of the front $\Longrightarrow$ Saturation momentum

$$\rho_s(Y) \equiv \ln Q_s^2(Y) = \lambda Y \Longrightarrow Q_s^2(Y) \propto \mathrm{e}^{\lambda Y}$$

- The shape of the front does not change in the course of the propagation $\Longrightarrow$ “Geometric scaling"

$$\langle T(\rho, Y)\rangle \simeq F(\rho - \rho_s(Y)) \equiv \mathcal{F}\big(r^2 Q_s^2(Y)\big)$$

(E.I., Itakura, McLerran, 02 ; Mueller, Triantafyllopoulos, 02)

- A natural explanation for a new scaling law identified in the HERA data for DIS at small–x

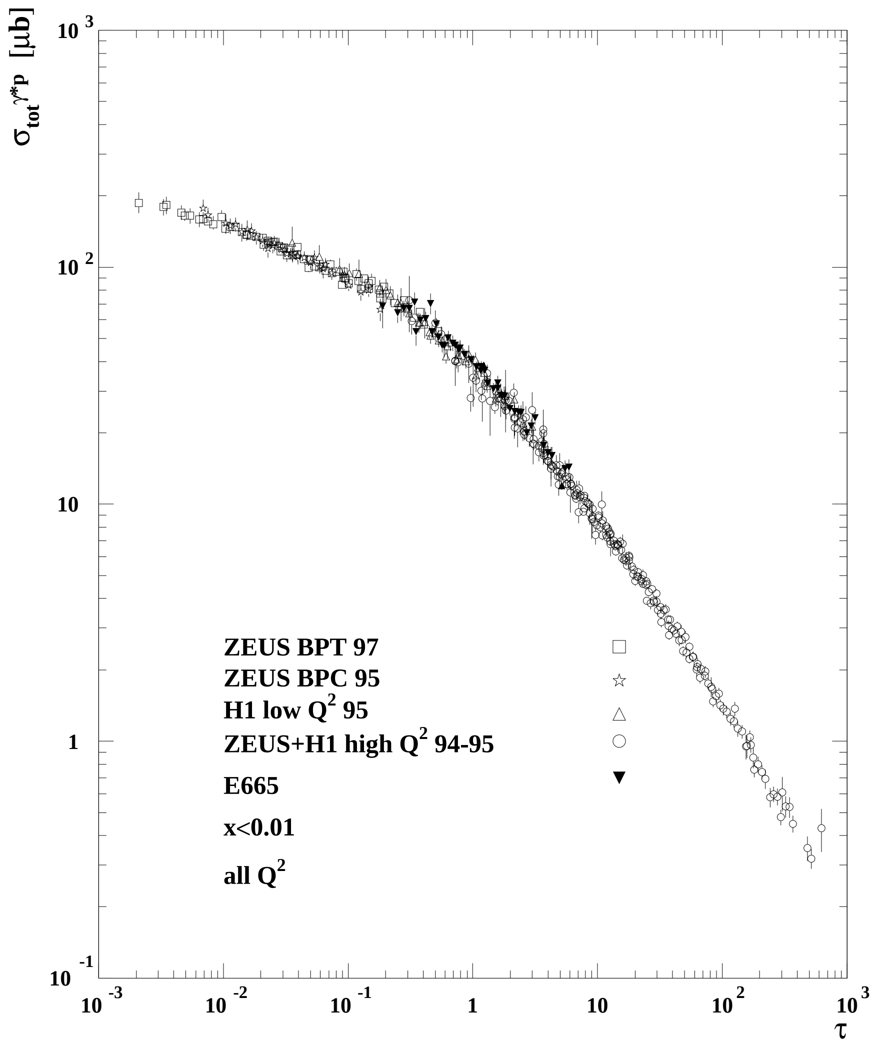
(Staśto, Golec-Biernat, and Kwieciński, 2000)

- Relevant for the high–p_T suppression observed in deuteron-gold collisions at RHIC

(Kharzeev, Levin, McLerran, 02 ; E.I., Itakura, Triantafyllopoulos, 04)

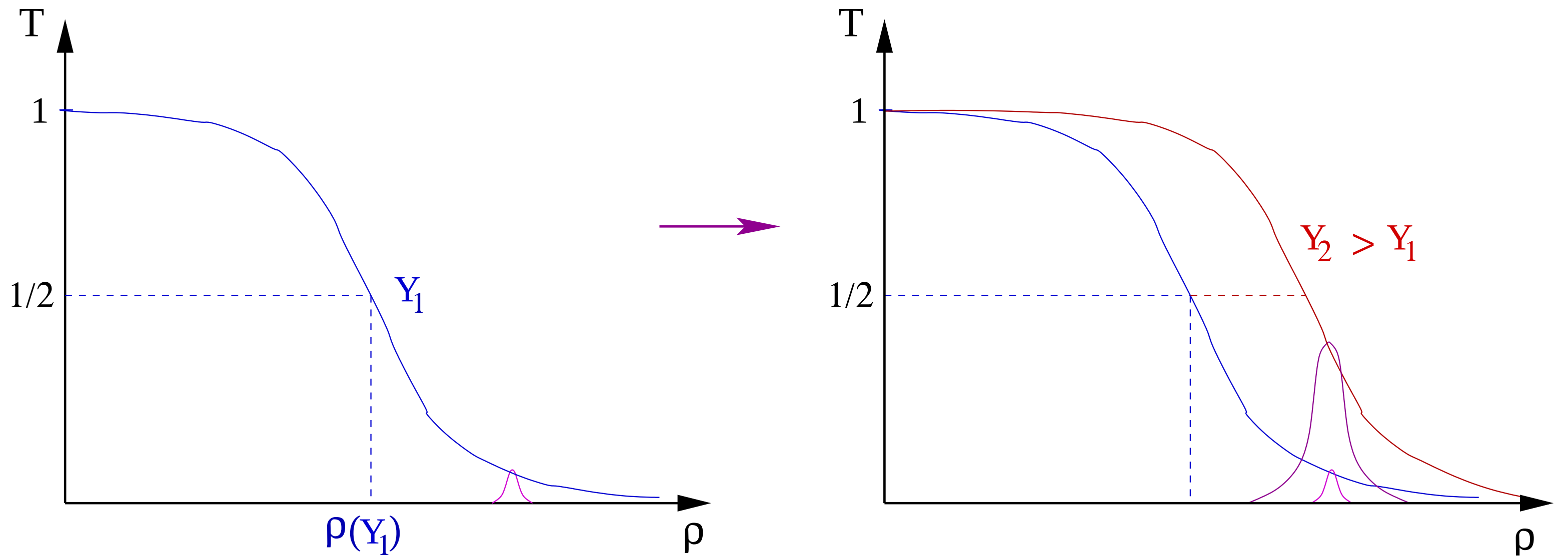
Geometric Scaling at HERA

$$\sigma_{\gamma^* p}(x, Q^2) \approx \sigma_{\gamma^* p}(\tau)\,, \quad \text{with} \quad \tau \equiv \frac{Q^2}{Q_s^2(x)} \quad \text{and} \quad Q_s^2(x) \sim 1/x^\lambda$$



Beyond the Mean Field Approximation

- Gluon number fluctuations $\Longrightarrow \langle T\,T\rangle - \langle T\rangle\langle T\rangle$
- Ignored by Balitsky–JIMWLK ! *(E.I. & Triantafyllopoulos, 04)*
- Important only in the dilute tail at $\rho \gg \rho_s(Y)$ (where $T \ll 1$)
- Front propagation is driven by the BFKL growth in the tail !



- The 'pulled front' dynamics is very sensitive to fluctuations !

  "Fluctuating pulled fronts" (see arXiv:cond–mat after 97)

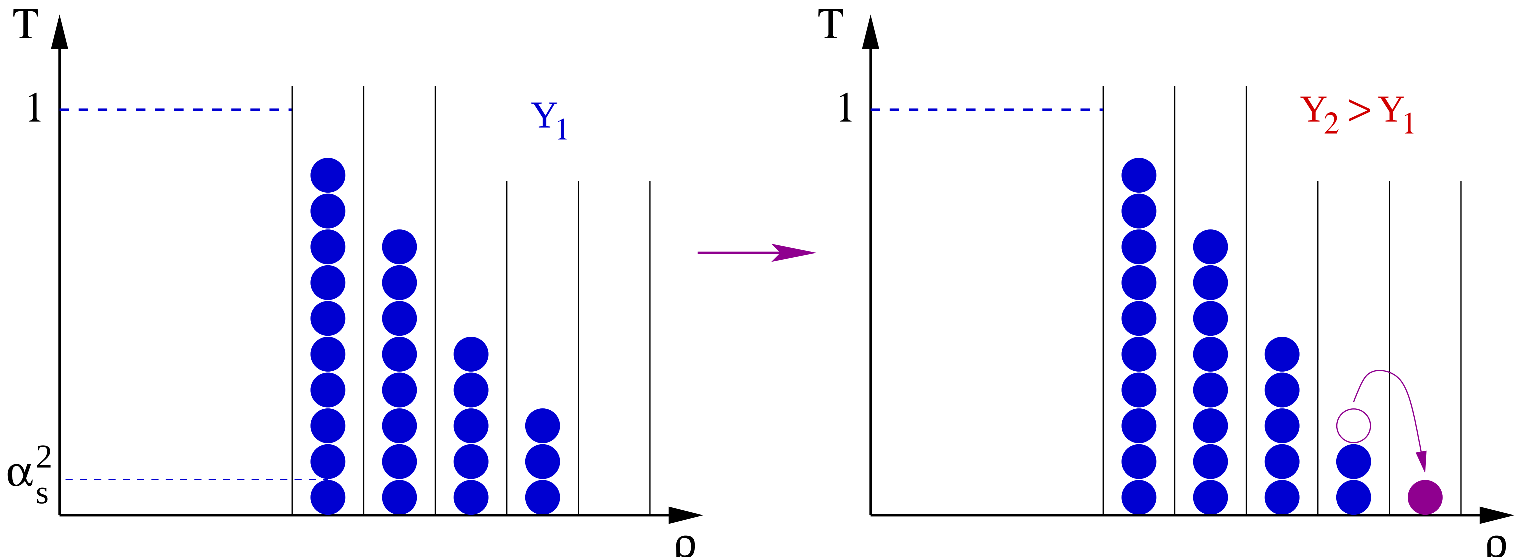
Particle number fluctuations

- At low density, the dipole counts the gluons in the target:

$$T(r,Y) \approx \alpha_s^2\, n(r,Y) \;\text{ with }\; n(r,Y) = 0, 1, 2, ...$$

$\Longrightarrow$ In an event–by–event description, T is discrete !

- Discreteness modifies the mechanism for front propagation
  - MFA : BFKL growth in the tail.
  - Discrete system : Diffusion of gluons in the foremost bin.



- As compared to the MFA, the front should slow down !

Saturation exponent with fluctuations

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High–Energy Evolution in QCD

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Mean Field Approximation

Fluctuating pulled fronts
- Pulled fronts
- Discreteness
- Saturation exponent
- Front diffusion
- Breakdown of BFKL

Conclusions

- Asymptotic behaviour is universal !
  - Large 'evolution time' Y, i.e. energy $\to \infty$
  - High occupancy at saturation ($1/\alpha_s^2 \gg 1$), i.e. $\alpha_s^2 \to 0$
- Brunet–Derrida, 97 (for the reaction–diffusion problem)

$$\lambda_s \equiv \frac{1}{\bar{\alpha}_s}\frac{d\rho_s(Y)}{dY} \approx \lambda_0 - \frac{\mathcal{C}}{\ln^2(1/\alpha_s^2)}\,, \qquad \lambda_0 \approx 4.88, \quad \mathcal{C} \approx 150\,(!)$$

Mueller, Shoshi (04); E.I., Mueller, Munier (04)

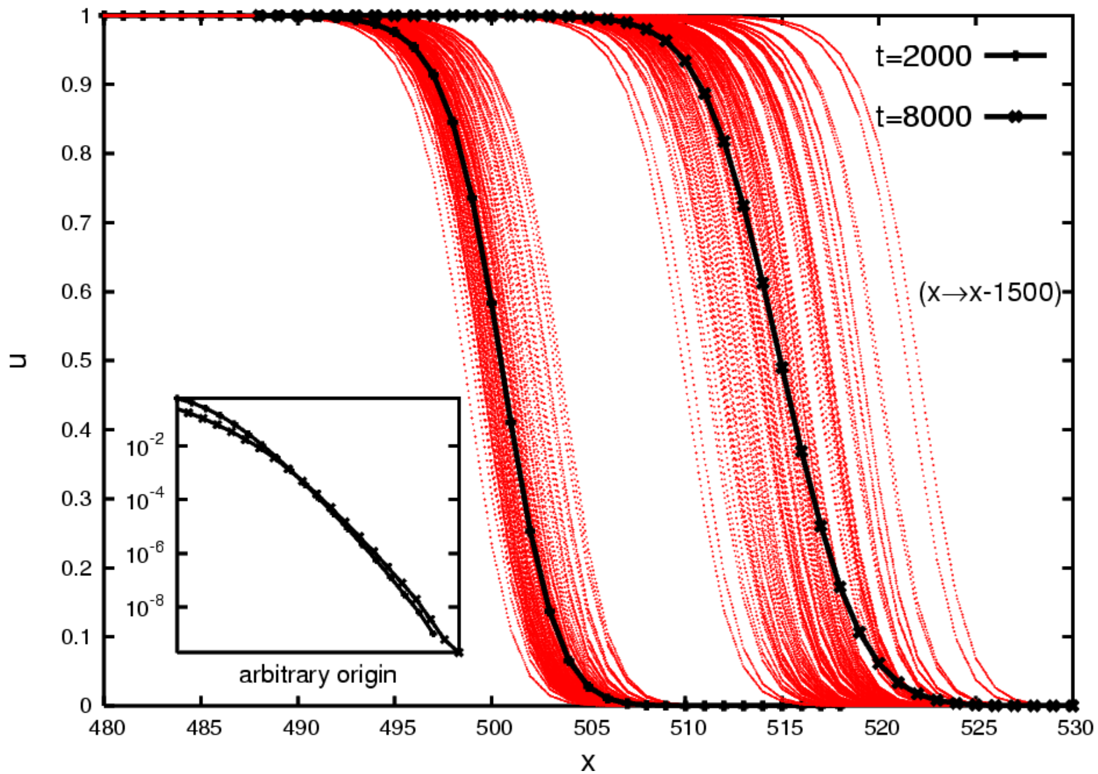
- An exact result in QCD in the limit $\alpha_s \to 0$

  ... but pretty useless for practical applications !
- Fluctuations are parametrically more important than the NLO BFKL corrections

Front diffusion & Geometric scaling violations

- The position $\rho_s(Y)$ of the front shows a diffusive wandering :

$$\langle\rho_s(Y)\rangle = \lambda_s\bar{\alpha}_s Y, \quad \langle\rho_s^2\rangle - \langle\rho_s\rangle^2 = D\bar{\alpha}_s Y, \quad D \sim \frac{1}{\ln^3(1/\alpha_s^2)}$$

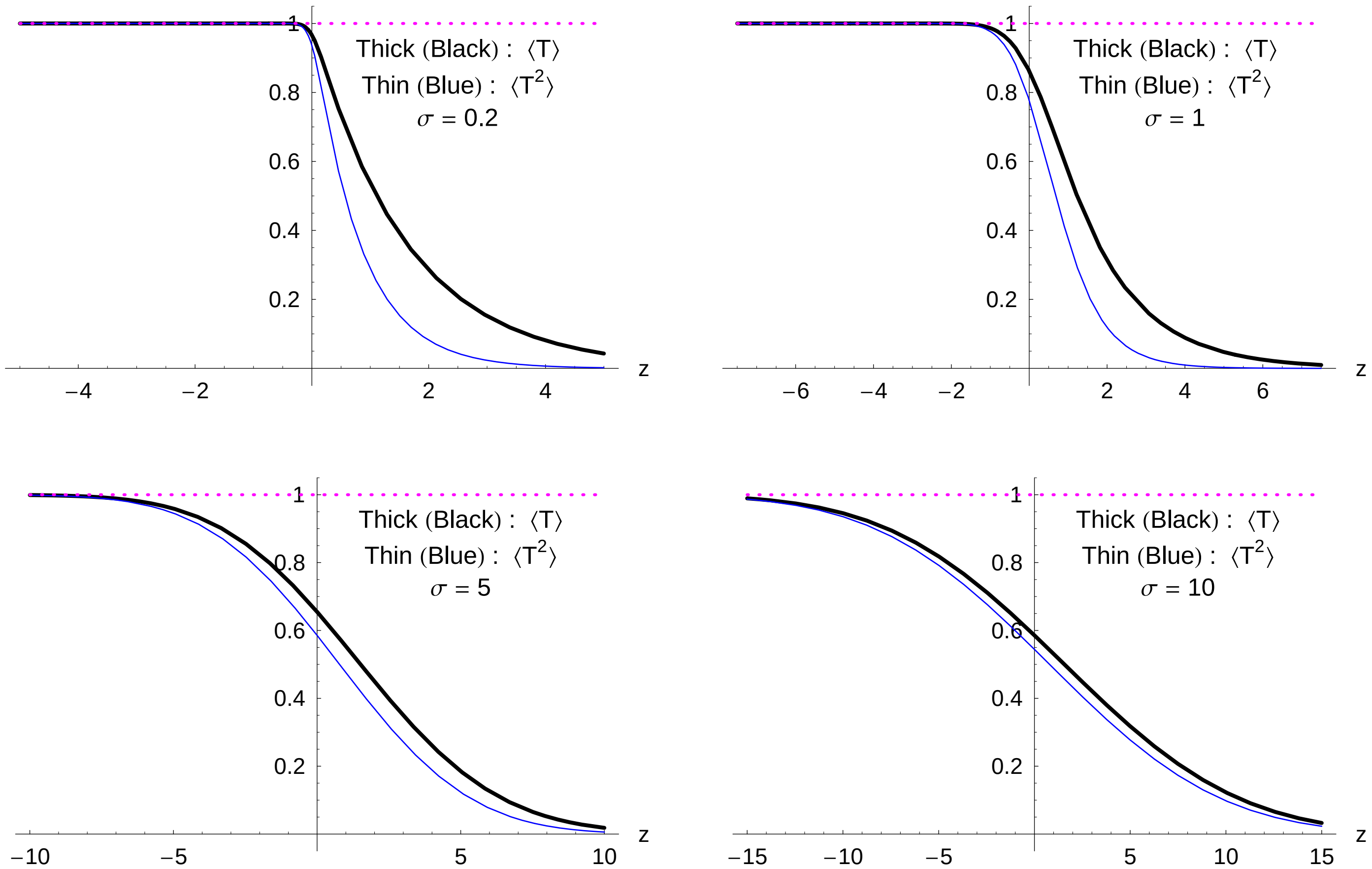


- For a given stochastic realization: $T(\rho, Y) \simeq F(\rho - \rho_s(Y))$

- For the average amplitude: $\langle T(\rho, Y)\rangle \simeq F\left(\frac{\rho - \langle\rho_s(Y)\rangle}{\sqrt{D\,\bar{\alpha}_s Y}}\right)$

The breakdown of the BFKL approximation

- In the MFA (BFKL) : $\langle T^2 \rangle \approx \langle T \rangle \langle T \rangle$ in the tail
- In the presence of fluctuations : $\langle T \rangle \approx \langle T^2 \rangle \cdots \approx \langle T^n \rangle$



- The dynamics of the tail is dominated by rare fluctuations

Conclusions & Open questions

- QCD evolution at high energy: Classical stochastic process
  - particle number fluctuations
  - non–linear effects $\Longrightarrow$ gluon saturation
  - unitarization of scattering amplitudes
  - geometric scaling and its violations
- Strong sensitivity to fluctuations in the low–density regime
- Universality of the asymptotic (high–energy and weak coupling) behaviour
- Relation to problems in statistical physics, chemistry, biology
- How to go beyond asymptotics ?
  - What are the relevant evolution equations ?

    (see next talks by Triantafyllopoulos, Lublinsky, Hatta)
  - How to solve these equations ?

    (see the talk by G. Soyez)