The behaviour of single scale hard small x processes in QCD near the black disk limit

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(based on
B. Blok and L. Frankfurt
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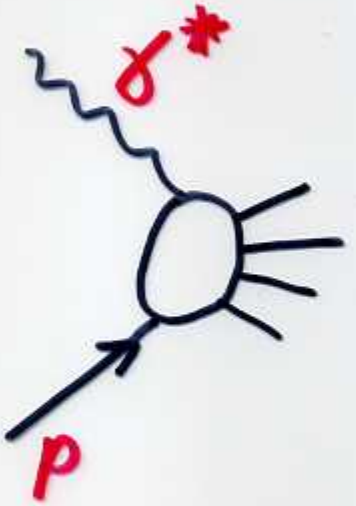
I Challenges of small x physics



$q^2 = -Q^2$

$\nu = 2pq$

$x = \frac{Q^2}{2pq}$

moderate x:

$\alpha_s \ll 1$, $\frac{\alpha_s}{\pi} \ln \frac{Q^2}{\Lambda^2_{QCD}} \sim 1$

$F_2(x, Q^2) = \sum f_i \left(x, \alpha_s \ln \frac{Q^2}{\Lambda^2}\right) \alpha_s^i$

for each f_i - QCD evolution equation.

For small x - new parameter

$\ln \frac{x_0}{x}$ - large $\Rightarrow$ DGLAP ?

One of the striking predictions of DGLAP (LO+NLO)

$$F_{2p}, \times G_p \sim x^{-n(Q^2)}$$

$n \sim 0.2$ for $Q^2 \sim 10\ GeV^2$
(consistent with HERA data)

Another approach - BFKL

$\alpha_s \ll 1$, $\alpha_s \ln \frac{x_0}{x} \sim 1$

$F_2, xG \sim x^{-M}$

$M = 4 \log 2 \frac{\alpha_s(10^2) N_c}{\pi} (1 - \ldots\ldots)$

(G. Salam)

However, LO+NLO BFKL
and LO+NLO DGLAP
give the same results
for $x < 10^{-7} \div 10^{-8}$
(G. Salam, M. Ciafaloni, D. Colferai
A. Staśto 1999, 2002)
Thus in the kinematics up to
small x pQCD calculations

However: problem with unitarity.

$$\sigma^{DGLAP}_{\gamma^*_T}(\text{inclusive}) \sim \frac{1}{x^{\sim 1(0^2)}} \frac{1}{Q^2}$$

violates Froissart bound

or, in impact parameter space representation

$$A(\text{dipole} + T \to \text{dipole} + T) =$$

$$= 2s \int d^2b\, e^{i q_t b} f(b, s, Q^2)$$

$|f(b^2, s, Q^2)| \leq 1$ must be,

But reaches

$f \sim 1$ for $Q^2 \sim 10\ \text{GeV}^2$

already at $x \sim 10^{-4} - 10^{-5}$

(LHC kinematics)

$$\sigma_{tot} = 2\pi b_{max}^2$$

$$b_{max} \sim \frac{1}{m} \ln s/s_0$$

(Froissart bound)

How to achieve such behaviour in perturbative QCD?

The observation —

- α_s series are only asymptotical and defined up to $O(\exp(-\frac{1}{\alpha_s}))$ terms

for sufficiently large energies the latter terms dominate, α_s expansion nonvalid - black disk limit (Froissart) behaviour

The challenge:
when the transition to BDL occurs and what is a state of the system?

that perturbative system is unstable is indicated by the increase of the current-current correlators

$$\langle p | j_n(y), j_0(0) | p \rangle \Big|_{\substack{y^2 \ll 1 \\ py \to \infty}} \sim (py)^{\nu}$$

$\nu > 0$

i.e. long-range correlations that usually indicate instability of the system

(Blok, Frankfurt PRD 2004, PL 2005)

II Effective field theory

Our aim - to describe scattering of two small dipoles (in this case the dominant configurations have large transverse momentum and large invariant mass - pQCD analogue of triple reggeon limit)

We consider one-scale processes like $\gamma_L^* + \gamma_L^* \to$ hadrons, $\gamma + \gamma$ etc

Here the contribution of QCD evolution is suppressed $\Rightarrow$ exists the factorisation of the information on the wave function of the projectile and target and the interaction, similar to the one that occurs in the Regge pole

now we can built EFT.

basic degree of freedom
- perturbative ladder
(has no relation direct to the
pomeron of sof QCD)

use of it as quasiparticle
- 1) due to factorisation
  2) $r(Q^2)$ dependence is weak
  (M. Ciafaloni et al, V. Fadin et al)

$$L = \frac{1}{2}(p\partial_y q - q\partial_y p) - r(Q^2)pq -$$

$$- \alpha' p \Delta q - \kappa p q (p+q) -$$

$$- C_{dipole} \int d^2b \exp\left(-\frac{bq}{2}\right) q(y, b-B)$$

$$\delta(y+Y)\,dy$$

$$- C_{dipole} \int \exp\left(-\frac{bQ}{2}\right) p(y, B-b) d^2b$$

$$\delta(y)$$

$$p = i\psi^+ \qquad \partial_y = \partial_{\log x_0/x}$$

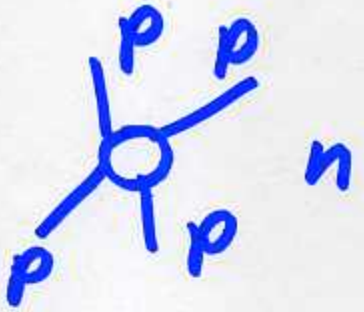
$$q = i\psi \qquad \text{- rapidity}$$

The parameteres can be estimated in QCD:

$$K \sim \frac{\alpha_c^2 N_c}{k} \qquad (L \sim Q)$$

$$\alpha_p' \sim \frac{N_c \alpha_s}{k^2} \qquad \kappa \sim \alpha_s N_c$$

$$G_{np} \sim \frac{\alpha_s^{2n-4} N_c^{n-2}}{k^{n-2}}$$ and is suppressed near extremums



so we can restrict by $n=3$ in L_{eff}.

note: kinetic term describes Gribov diffusion.

(BFKL type diffusion is strongly suppressed, M. Ciafaloni et al)

The result is consistent with

We assume $\alpha_s, \alpha_s N_c \ll 1$.
(pQCD regime).
Then L belongs to a group
of Lagrangians considered
by D. Amati, M. Ciafaloni,
Le Bellac, G. Marchesini
V. Alessandrini
in early 70-s (NPB)
so we can use results obtained
in those papers.

1. There are 3 extremums:

$p = q = 0$ — perturbative vacuum

$p = \frac{\pi}{k}, q = 0$, $p = 0, q = \frac{\pi}{k}$ — nonperturbative states

at each rapidity w.f. is
their linear combinations
(plus fluctuations).

2. In the limit $\alpha' = 0$ — quantum mechanics can be mapped to hermitean problem with tachyon & higgs-like potential (for $\alpha' \neq 0$ need numerical studies)

3. action of kinks $\sim \frac{M}{\varkappa} \sim \frac{1}{\alpha_s}(1-v)$ – look to be tunneling transitions

key role by critical kinks $S_{cr} = 0$

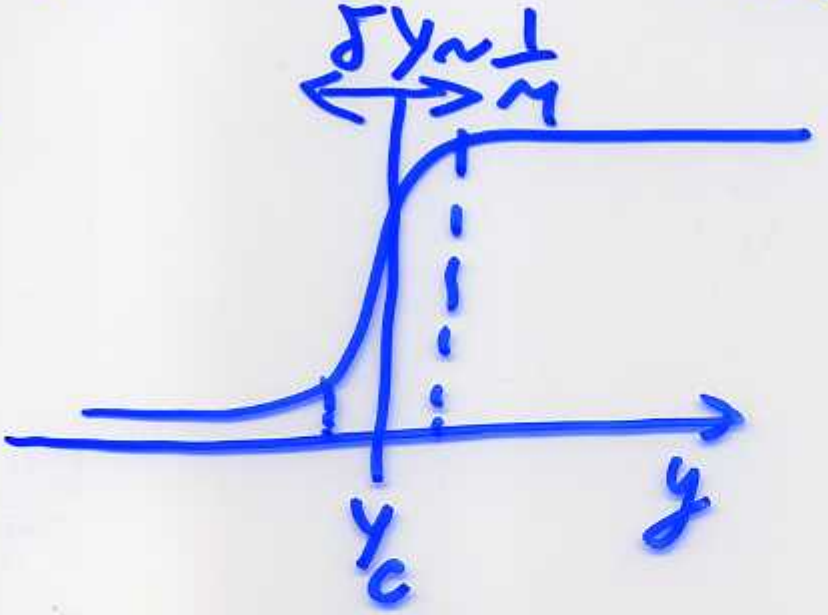
4. The key role is played by massless excitations around kinks – 2d „phonons".

$$E_k = i2\sqrt{\alpha' M}\,k = 2\sqrt{\alpha' M}\,p_{cl}$$

5. Black disk limit occurs at

$$(R_0^2 + 4\alpha' y)\, n y \sim b_{max}^2$$

6.



color inflation

at Y_c condensate of ladders is formed

$$T_I \sim \frac{\exp(\frac{1}{\alpha_s})}{Q} \sim \frac{10^2}{Q} \ll T_c \sim \frac{1}{Q x^{1-\alpha}}$$

T_I: time of transition to ladder condensate regime.

x: Ioffe coherence length

Technically, we use

$$S(B, Y, g, f) = \int dp \int dq \; \exp(-L(B, Y, g, f))$$

one finds classical kinus by solving equations of motion

$$-\frac{dp}{dy} = \alpha' \Delta p - \mu p + \alpha(2pq + qp^2) + C_{dipole} \int d^2B \exp(-BQ/2) q(B - \frac{B}{2}y) \delta(y - Y)$$

$$\frac{dq}{dy} = \alpha' \Delta q - \mu q + K(2pq + q^2) + C_{dipole} \int d^2B \exp(-\underline{BQ}) p(y, B - \frac{B}{2}) \delta(y + Y)$$

approximately, kink in 2+1 D GRFT (3+1 physical theory)

$$p(q) \sim \theta(v\sqrt{\alpha' \mu}(y - y_0) - |\vec{B} - \vec{B}_0|)$$

after kinks are found in $(y, \vec{B})$ space the theory is quantised semiclassicaly around classical solution

Then $S(B,Y) = \exp\left[-\frac{r}{\varkappa}\,\theta\left(2\sqrt{\alpha' r}\,Y - B\right)\right]$

in the same way one can calculate 2-Reggeon green function:

$$D(B,Y) = \langle \phi_0 | p(0,\vec{0}) \exp(i\vec{p}\vec{B}) \exp(-HY)\, q(0,\vec{0}) | \phi_0 \rangle$$

$$D(B,Y) = \left(\frac{r}{\varkappa}\right)^2 \Theta\left(-B + 2\sqrt{\alpha' r}\,Y\right)$$

or, after proper renormalisation

$$G(B,Y) = \frac{2}{(J-1)^2 + v_0^2 k^2}$$

$v_0 = 2\sqrt{\alpha' r}$ – critical kink speed.

– black disk regime.

III Color network

during Bose-Einstein condensation of ladders

$$N \sim \frac{\cancel{\mu}^2}{k^2} k^2(y)$$ ladders are created.

The transverse distance of the ladders (between them)

$$l_t \sim \frac{k}{\mu} \sim \frac{\alpha_s}{\lambda}$$

The characteristic scale

$$d_t^2 \sim \frac{N_c \alpha_s}{\lambda^2} \quad \left(\frac{1}{\alpha'_p}\right)$$

$$\sqrt{\frac{\alpha_s}{N_c}} \sim \frac{l_t}{d_t} \ll 1$$

i.e. ladders overlap and this indicates that color network is formed. The color breely moves along a macroscopic volume.

Thus QCD predicts not screening by exchange of ladders, but color network.

IV Conclusions

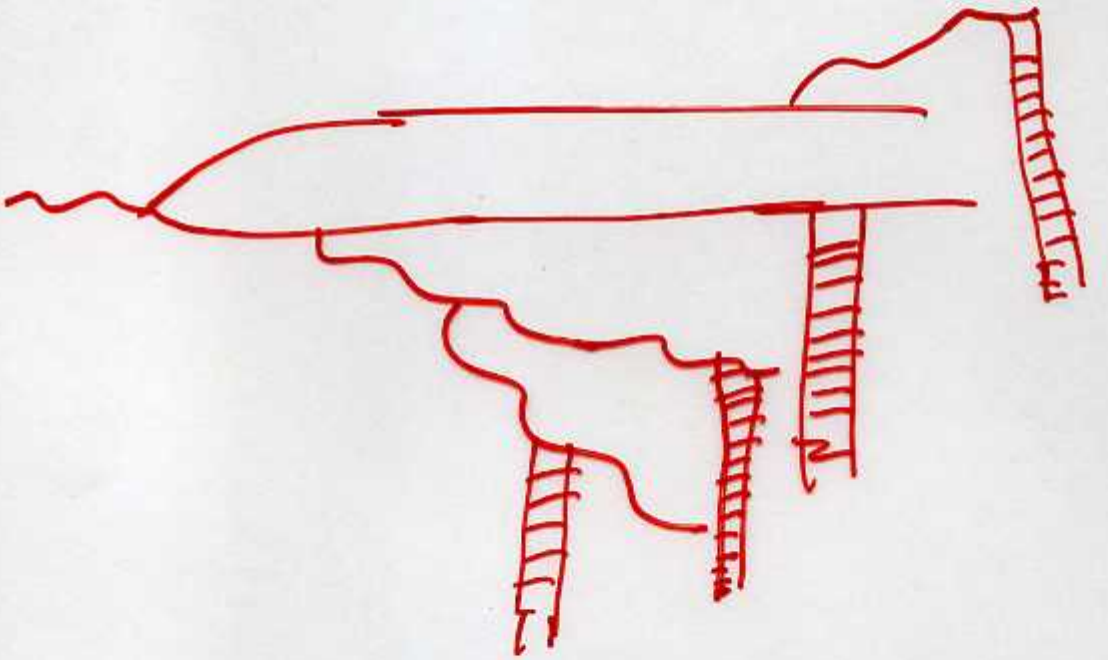
We expect near BDL

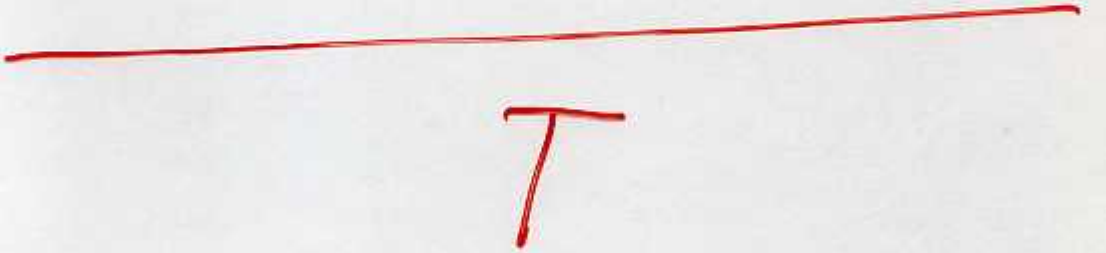
1) formation of a color network in space-time
2) Importance of tunneling (nonperturbative) effects
3) universality of structure functions & cross-sections (the transitions due to noncritical kinks $\sim \exp(-\frac{1}{\alpha_s})$)

unresolved problem:
hard processes at moderately small x, where multireggeon interactions are kinematically suppressed, but evolution is important. In other words: how to extend to L-scale processes?

eikonal coupling

energy conservation (evolution)





c) some experimental consequences

1) softening the parton distribution over the longitudinal momenta
threshold increase in multiplicity
no reason for AGK

2) cross-section of Mueller-Navelet jets

$p+p \to jet + X + jet$:

initial fast increase of the cross-section shall change to fast (pQCD) decrease at large y
due to color inflation
(disappearance of long range fluctuations in rapidity)