In the Shadow of the Color Glass

Edmond Iancu
SPhT Saclay & CNRS

In the Shadow of the Color Glass

A fully illustrated introduction to Gluon Saturation, Balitsky–JIMWLK, Pomeron Loops, and all that

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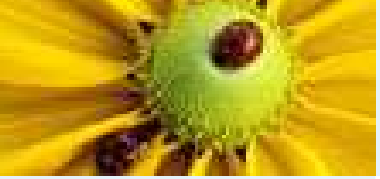

Outline

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Motivation
Gluon evolution
Pomeron loops
Backup

- Motivation : Gluons at HERA
- Non–linear gluon evolution: Color Glass Condensate
- Gluon–number fluctuations: Pomeron loops

Gluons at HERA

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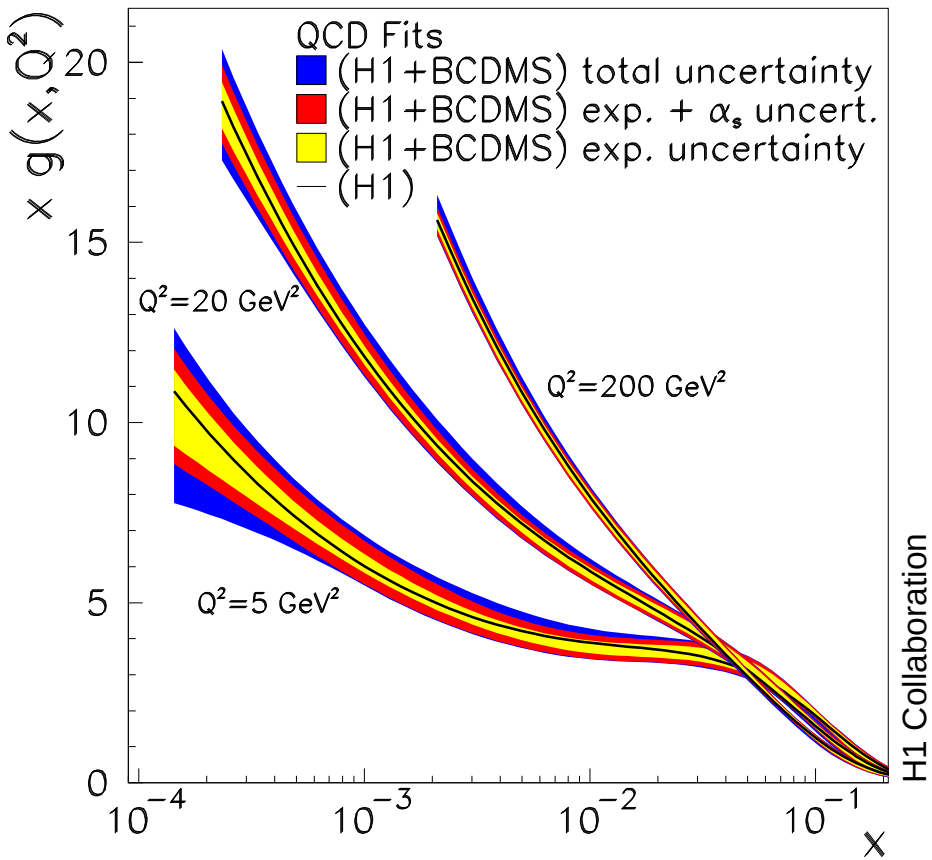
▷ The gluon distribution rises very fast at small x ! $(\sim 1/x^{\lambda})$



$xG(x,Q^2) \approx$ # of gluons with transverse size $\Delta x_\perp \sim 1/Q$ and $k_z = xP$

High energy = High density = Weak Coupling

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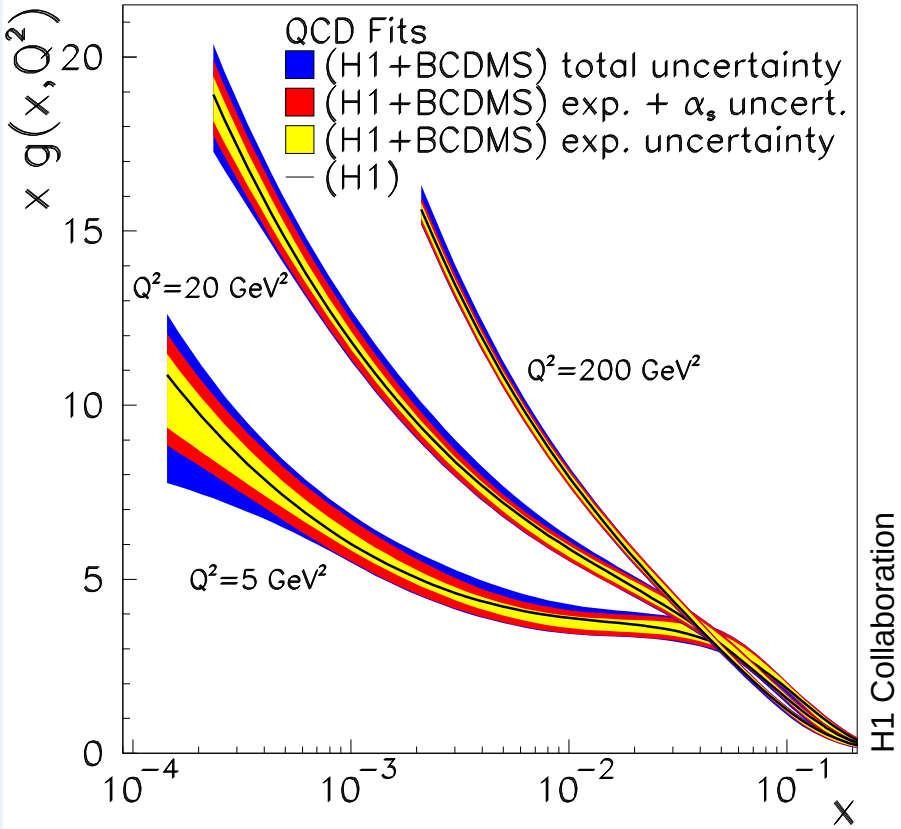

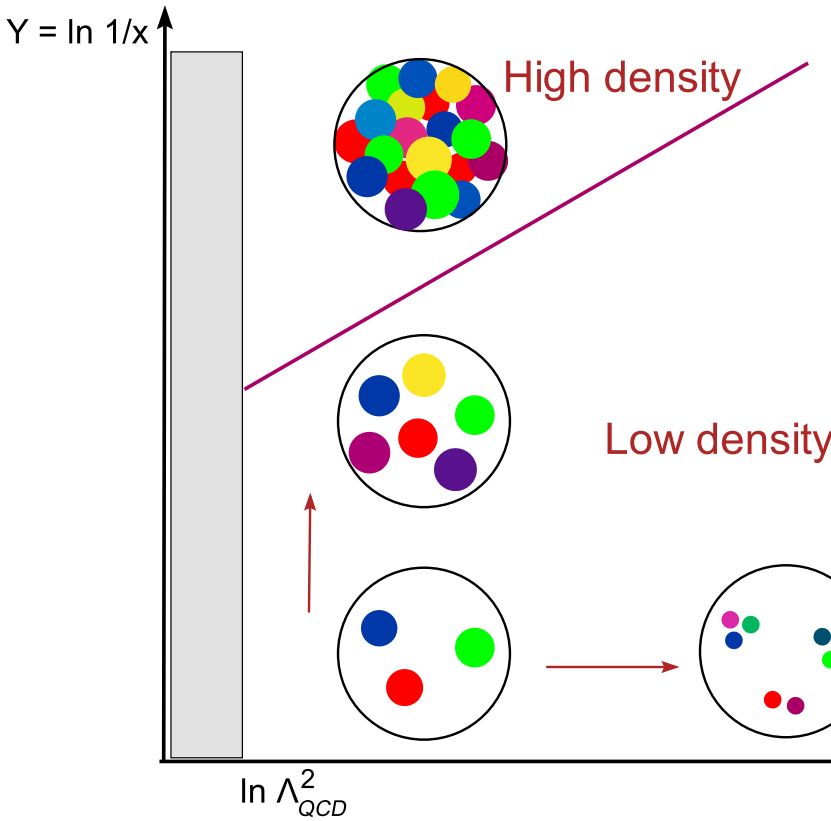

▷ High–energy evolution : An evolution towards increasing density.

▷ High density partonic matter is weakly coupled !

High density = Non–linear

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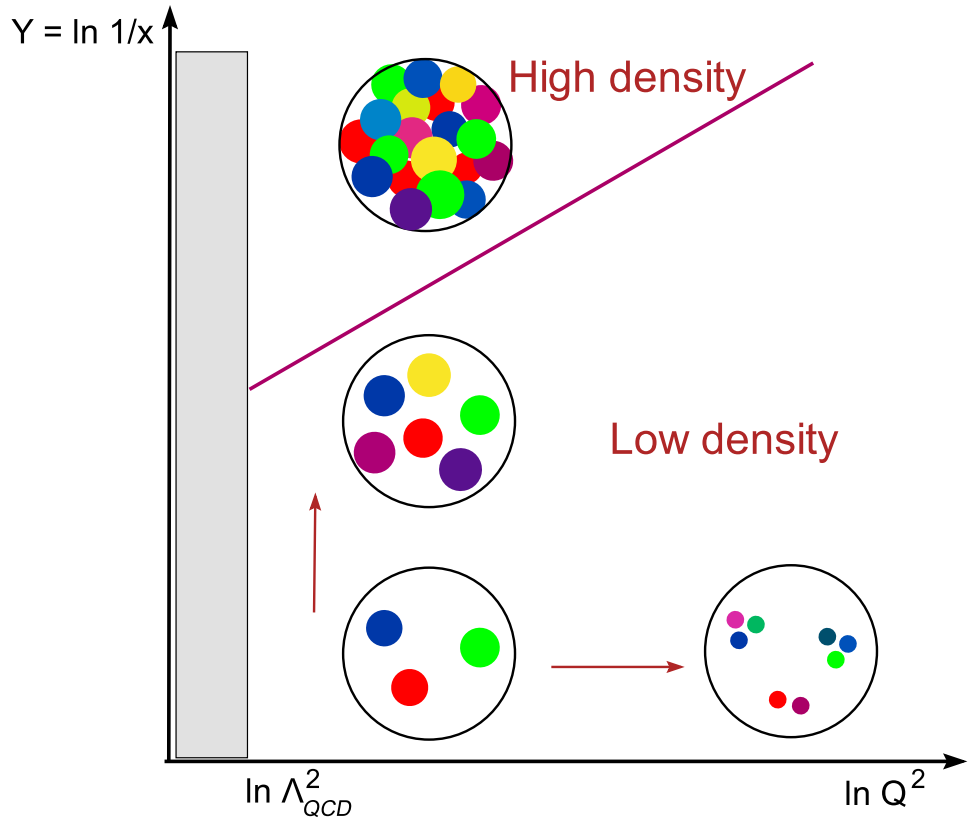

▷ A challenging problem though !

High density $\Longrightarrow$ weak coupling but strong non–linear effects

High density = Non–linear

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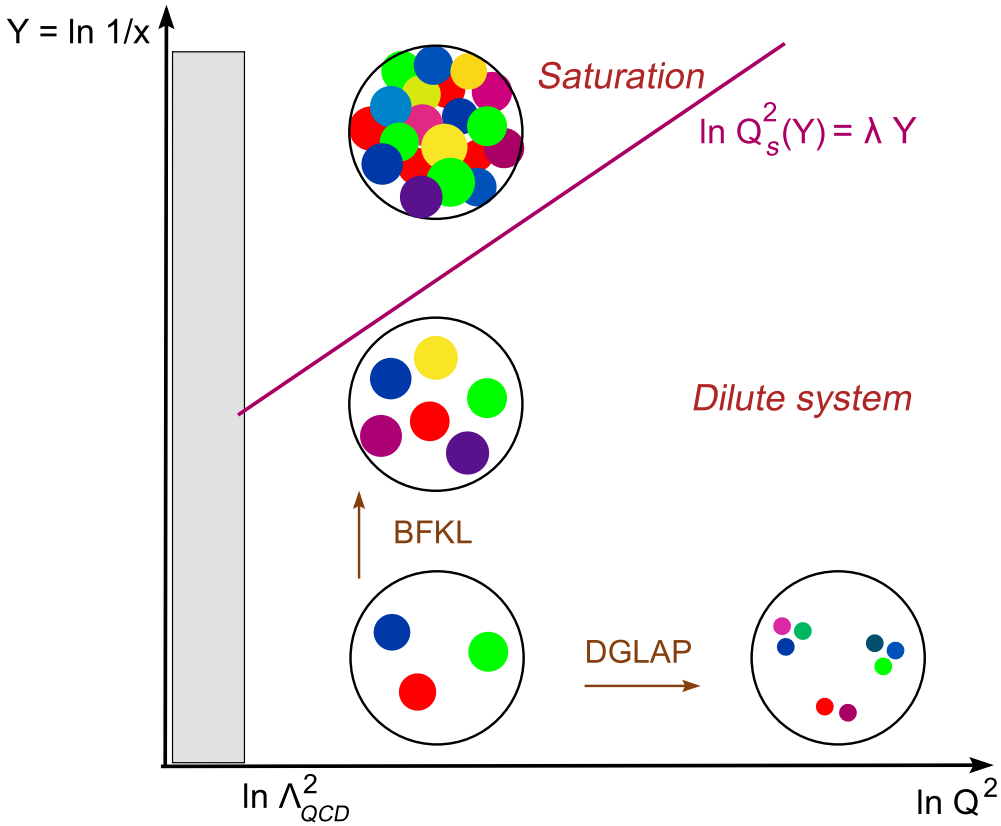

▷ DGLAP equation : naturally linear !

▷ BFKL equation : a linear approximation (intermediate energies)

Dipole factorization for DIS

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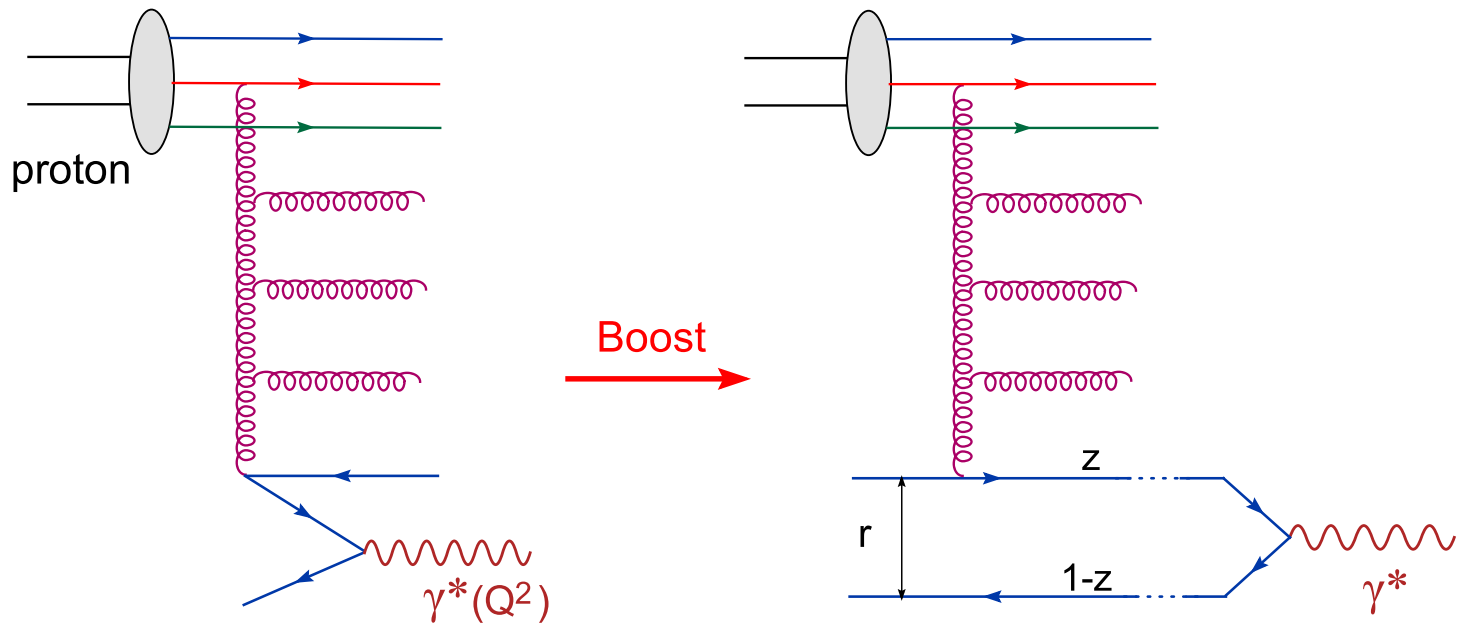
- Lorentz boost to the 'dipole frame'

  γ^* fluctuates into a $q\bar{q}$ pair which then scatters off the proton.



$$\sigma_{\rm dipole}(x,r) \;=\; 2\int {\rm d}^2 b \;\; T(x,r,b)$$

- $T \equiv 1 - S$: The dipole–proton scattering amplitude
- Unitarity bound: $T \leq 1$ ($T = 1$: 'black disk limit')

Unitarity problem: Single scattering

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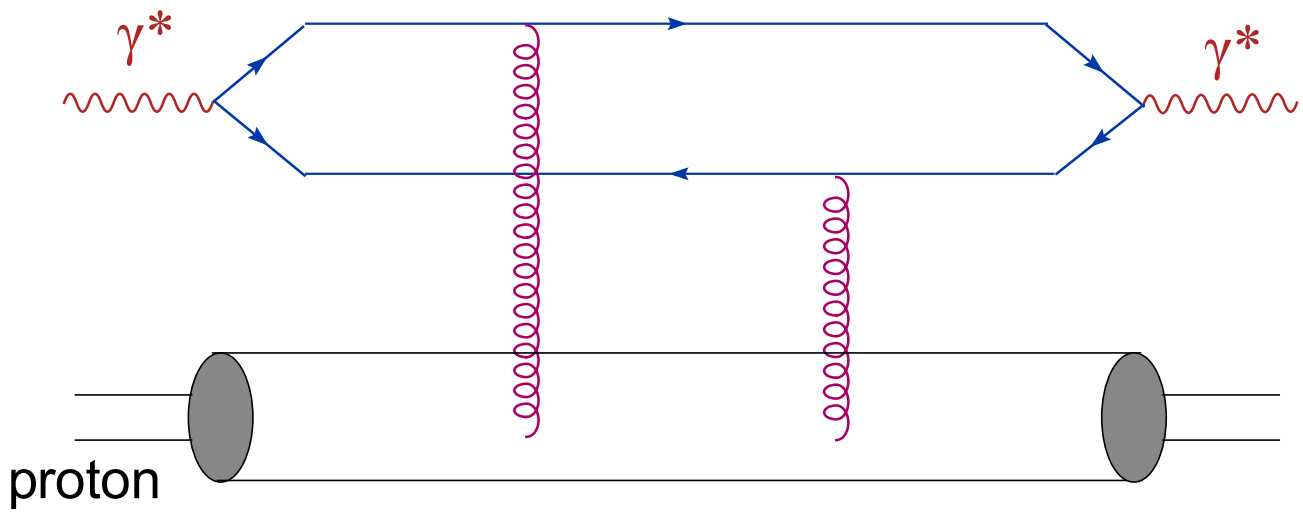
$$V(\boldsymbol{r}) \simeq g t^a \boldsymbol{r} \cdot \boldsymbol{E}_a \implies T(x,r,b) \propto g^2 r^2 \langle \boldsymbol{E}_a \cdot \boldsymbol{E}_a \rangle_x$$



$$T(x,r,b) \simeq \alpha_s\, r^2\, \frac{xG(x,1/r^2)}{\pi R^2} \equiv \alpha_s\, n(x, Q^2 \sim 1/r^2)$$

Strong scattering ($T \sim 1$) $\iff$ High gluon occupation ($n \sim 1/\alpha_s$)

- What happens when $T \sim 1$ (i.e., $n \sim 1/\alpha_s$) ?

The Saturation Momentum

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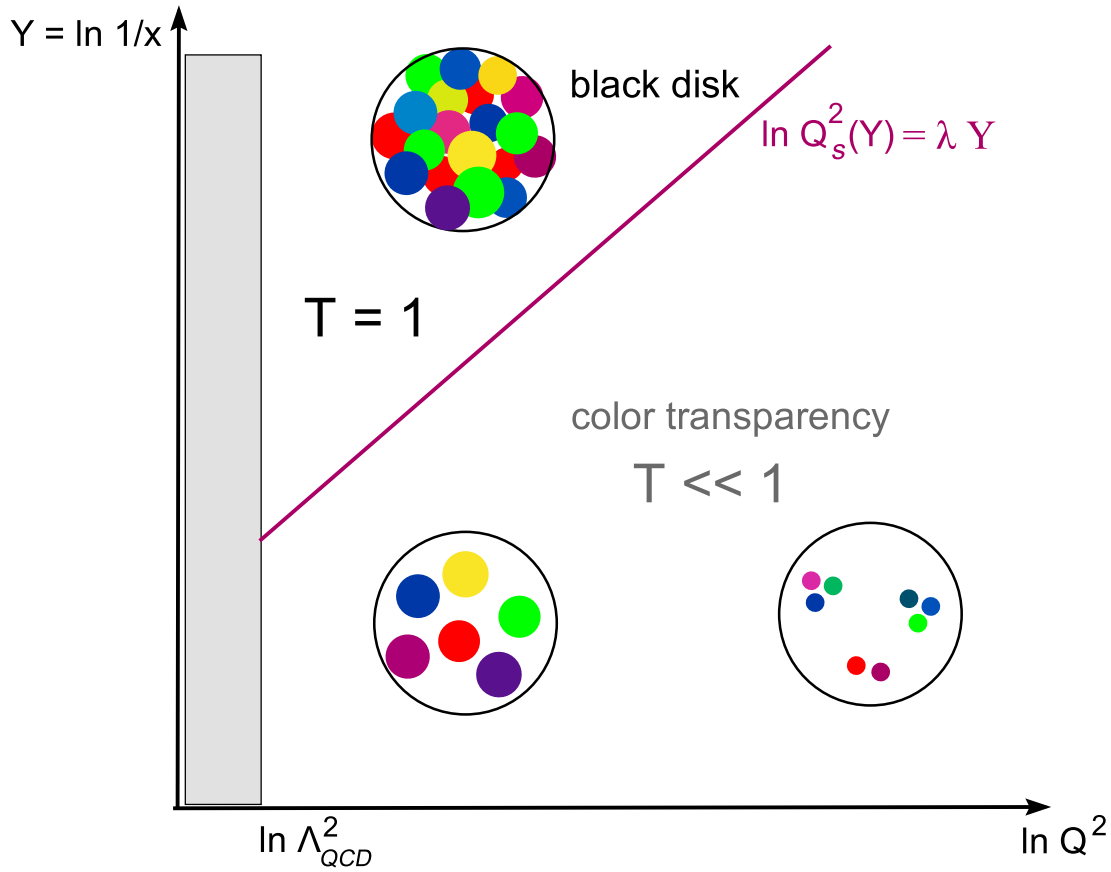
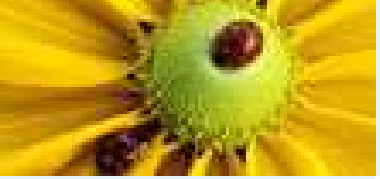
▷ Onset of unitarity corrections: $T(x,r) \sim 1 \iff n(x,Q^2) \sim 1/\alpha_s$

$$Q_s^2(x) \simeq \alpha_s \frac{xG(x,Q_s^2)}{\pi R^2} \sim x^{-\lambda}$$





BFKL evolution

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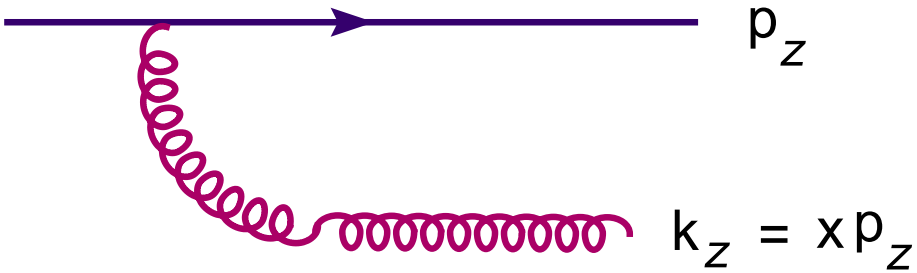
- The ‘infrared sensitivity’ of bremsstrahlung favors the emission of ‘soft’ (= small–x) gluons



$$\mathrm{d}\mathcal{P} \;\propto\; \alpha_s \frac{\mathrm{d}k_z}{k_z} \;=\; \alpha_s \frac{\mathrm{d}x}{x} \;\equiv\; \alpha_s\, \mathrm{d}Y$$

$$Y \;\equiv\; \ln \frac{1}{x} \;\sim\; \ln s \;\Longrightarrow\; \mathrm{d}Y \;=\; \frac{\mathrm{d}x}{x} : \quad \text{“rapidity”}$$

- A probability of $\mathcal{O}(\alpha_s)$ to emit one gluon per unit rapidity.

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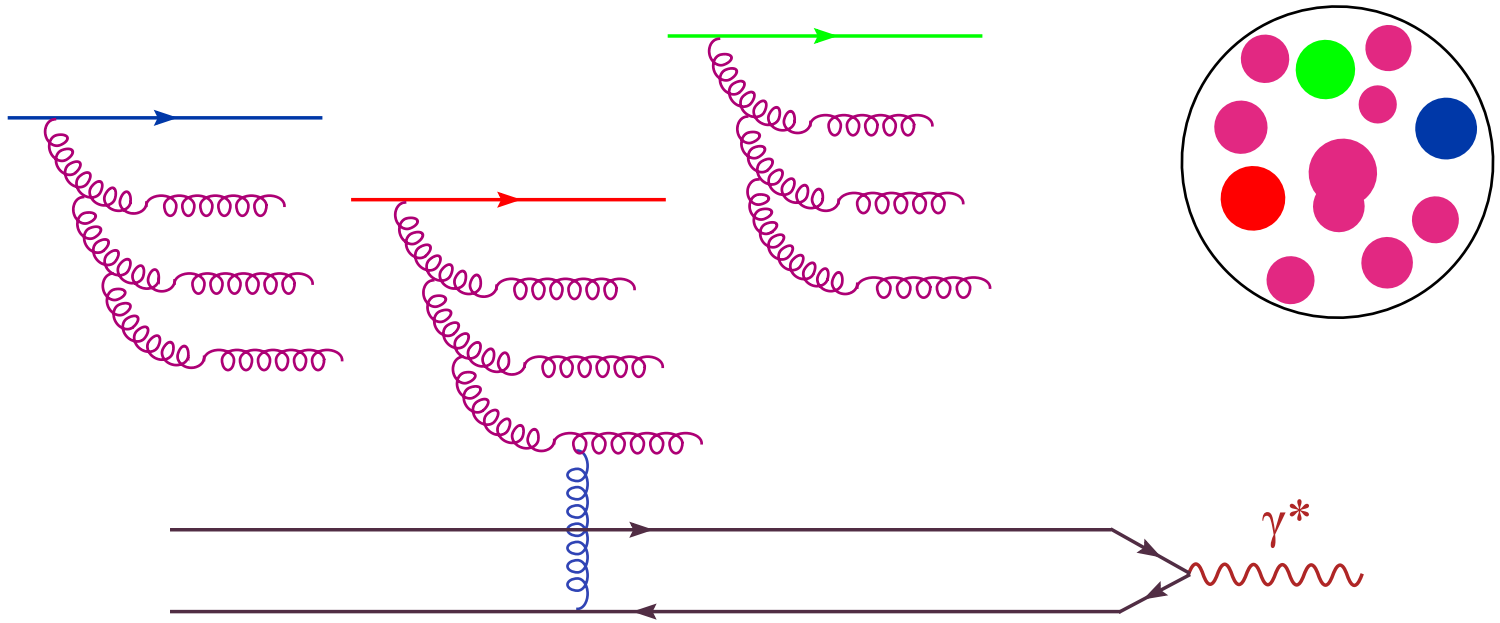
BFKL evolution

- The ‘last’ gluon at small x can be emitted off any of the ‘fast’ gluons with $x' > x$ radiated in the previous steps

$$\frac{\partial n}{\partial Y} \simeq \alpha_s n \qquad \Longrightarrow \qquad n(Y) \propto \mathrm{e}^{\omega \alpha_s Y}$$



- Cartoon version of BFKL eq. *(Balitsky, Fadin, Kuraev, Lipatov)*
- Linear $\Longrightarrow$ unstable growth of the gluon distribution !

Non–linear evolution: Saturation

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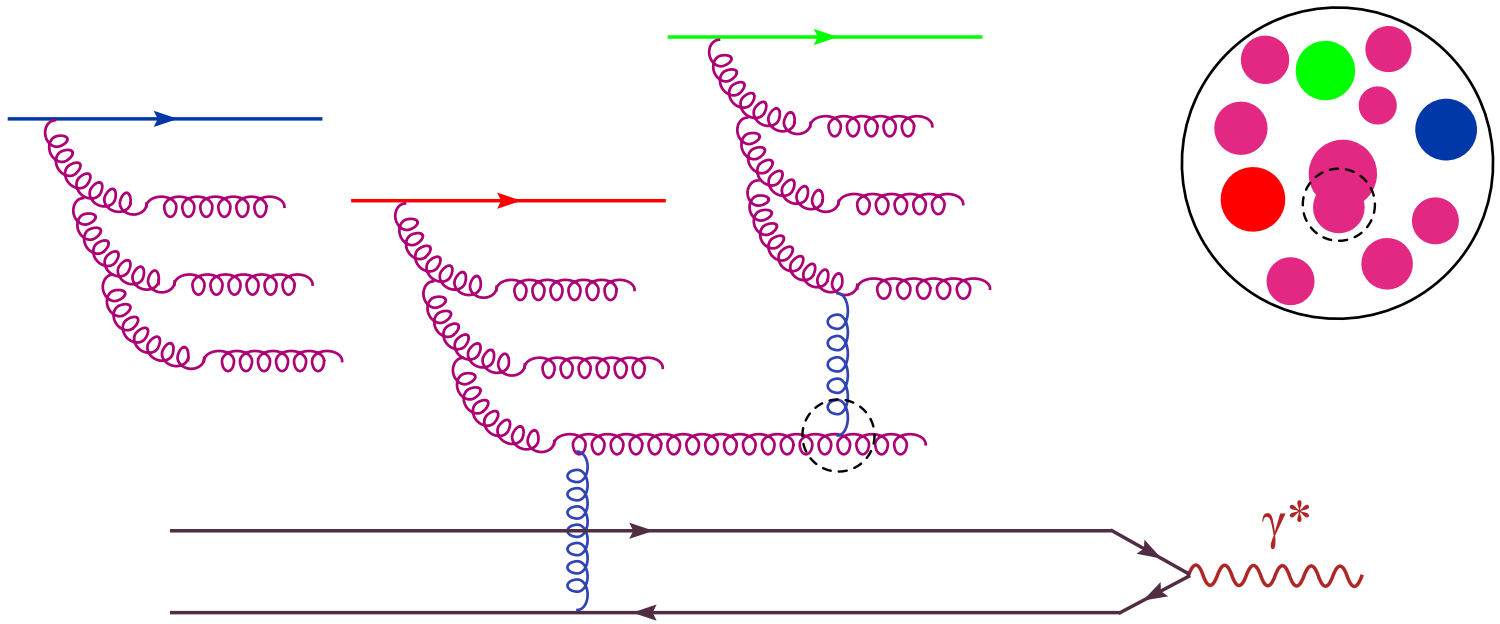
- High density: recombination processes leading to saturation



$$\frac{\partial n}{\partial Y} \simeq \alpha_s n - \alpha_s^2 n^2 = 0 \quad \text{when} \quad n \sim \frac{1}{\alpha_s} \gg 1$$

- Cartoon version of the GLR – MQ equation

  L. Gribov, Levin, Ryskin (83), Mueller, Qiu (86)

- Non–linear $\Longrightarrow$ stable fixed point at high energy !

Non–linear evolution: Unitarization

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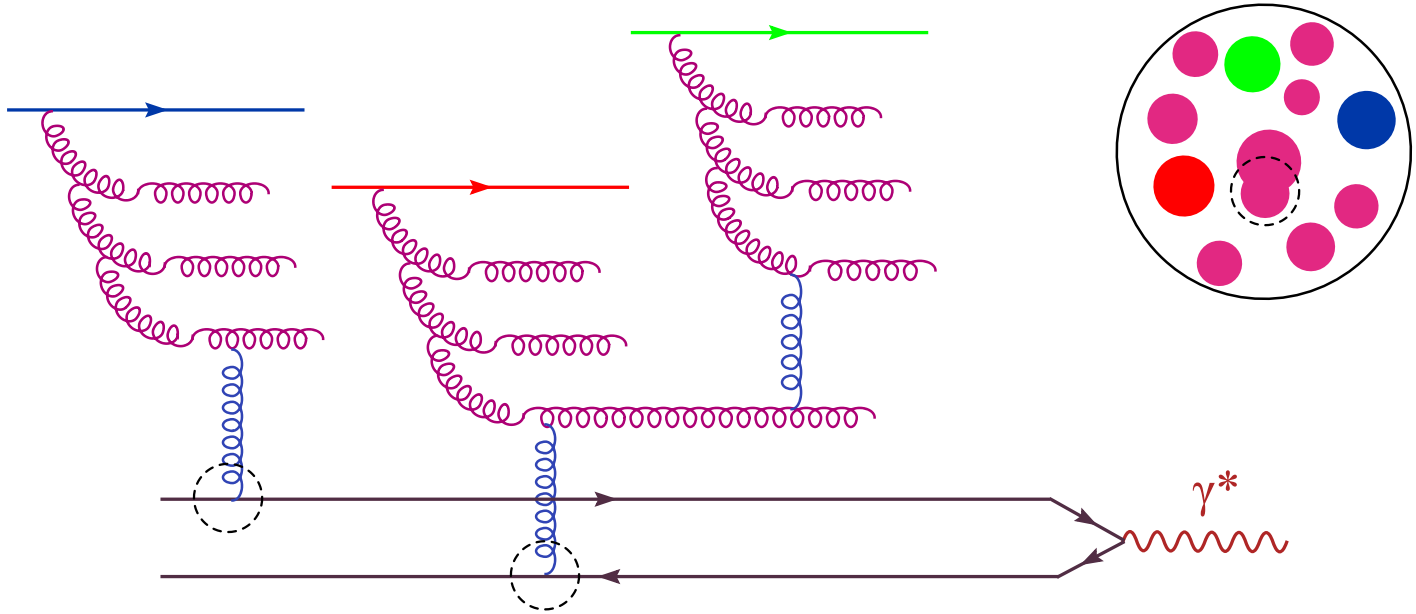
- Multiple scattering is important too : $\alpha_s n \sim (\alpha_s n)^2 \sim 1 \ldots$



- The behaviour of T (hence, of DIS) near the unitarity limit :

  The combined effect of :

  BFKL evolution + saturation + multiple scattering !

Non–linear evolution: JIMWLK equation

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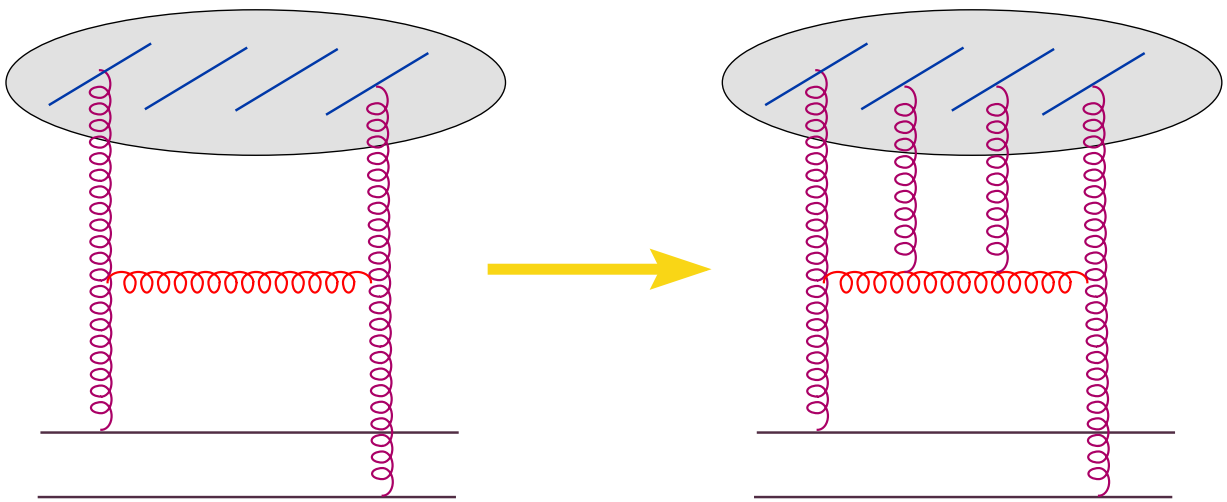
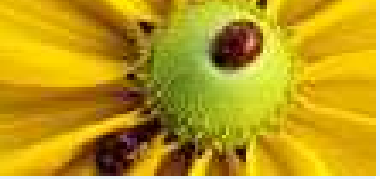
- No genuine recombination, but saturation of the gluon emission rate due to high–density effects

$$\frac{\partial n}{\partial Y} = \chi(n) \approx \begin{cases} \alpha_s n & \text{if } n \ll 1/\alpha_s \implies n \sim \mathrm{e}^{\alpha_s Y} \\ 1 & \text{if } n \gtrsim 1/\alpha_s \implies n \sim Y \end{cases}$$





Non–linear evolution: JIMWLK equation

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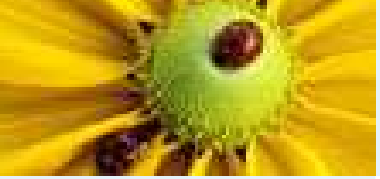
- No closed equation for $n = \langle E^i E^i \rangle$, but an infinite hierarchy of equations for the n–point functions of the color field A_a^+

$$\langle A(1)\, A(2)\, \ldots \rangle_Y = \int \mathrm{D}[A]\, W_Y[A]\, A(1)\, A(2)\, \ldots$$

- $W_Y[A]$: Probability distribution for the target color fields (a ‘generalized pdf’ describing all the n–point correlations)



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- Functional equation for the probability distribution $W_Y[A]$

$$\frac{\partial W_Y[A]}{\partial Y} = \frac{1}{2}\frac{\delta}{\delta A_a}\chi^{ab}[A]\frac{\delta}{\delta A_b}W_Y[A]$$

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, and Kovner (97–00)

$\chi[A] \sim \alpha_s A A$ if $A \ll 1/g$, but $\chi[A] \sim 1$ if $A \gtrsim 1/g$

The Color Glass Condensate

(McLerran, Venugopalan, 1994; E.I., Leonidov, McLerran, 2000)

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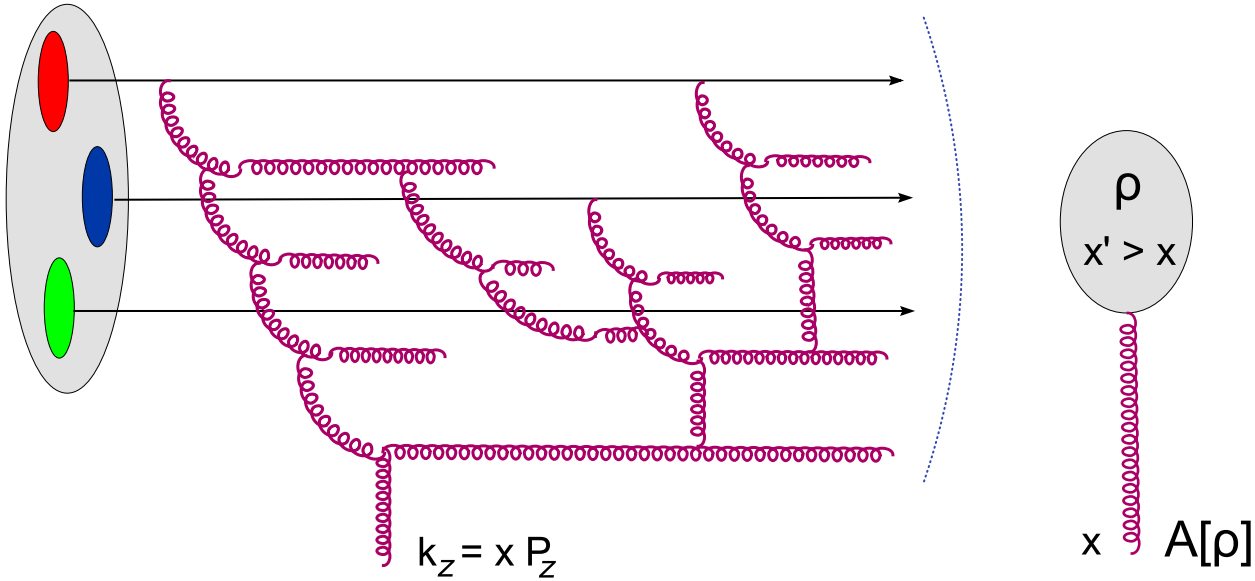
- Saturation : $n \sim 1/\alpha_s \Longleftrightarrow A \sim 1/g$: strong color fields !
- A classical effective theory for the small–x gluons :
  Classical color fields radiated by fast color sources
  (gluons with $x' \gg x$) 'frozen' in some random configuration



- Renormalization group in QCD at small–x : JIMWLK eq.

Deep Inelastic Scattering off the CGC

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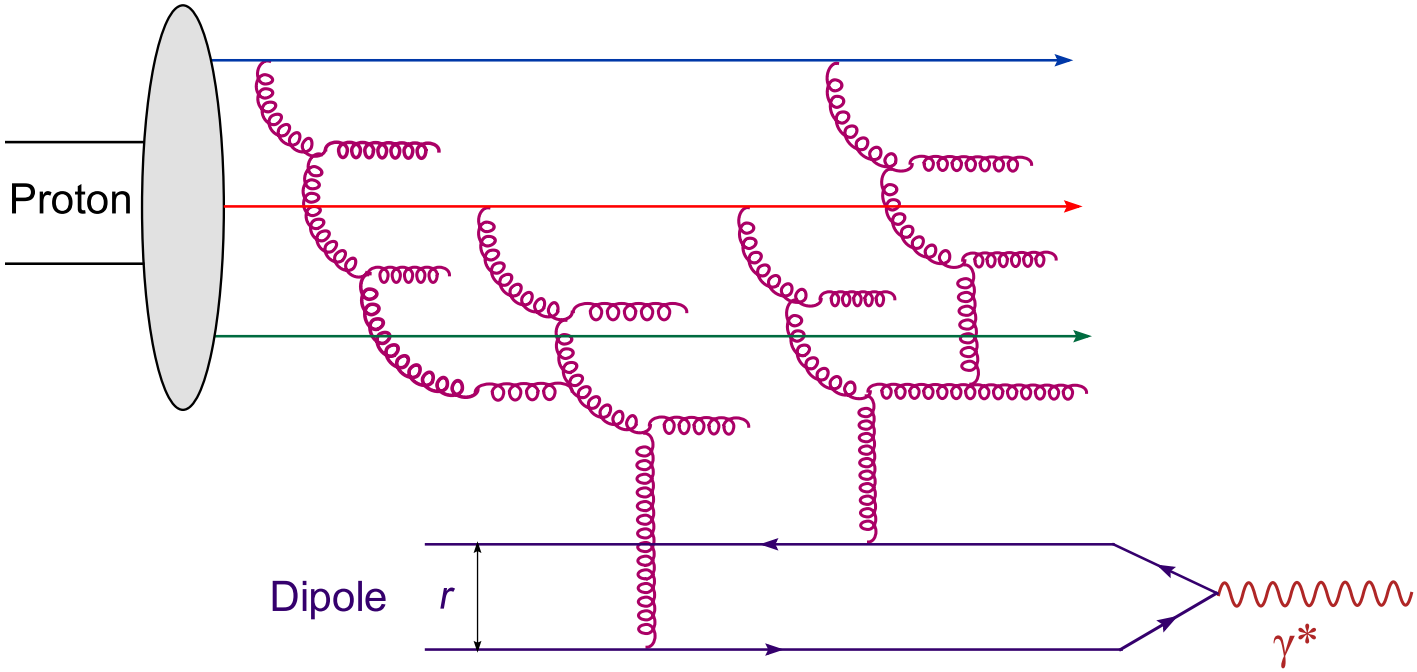

- Multiple scattering off a given configuration of (strong) color fields $A \implies T(r)[A]$ (eikonal approximation)
- Average over A with weight function $W_Y[A]$ ('glass')

$$\langle T(r,Y)\rangle = \int \mathrm{D}[A]\ W_Y[A]\ T(r)[A]$$

Deep Inelastic Scattering off the CGC

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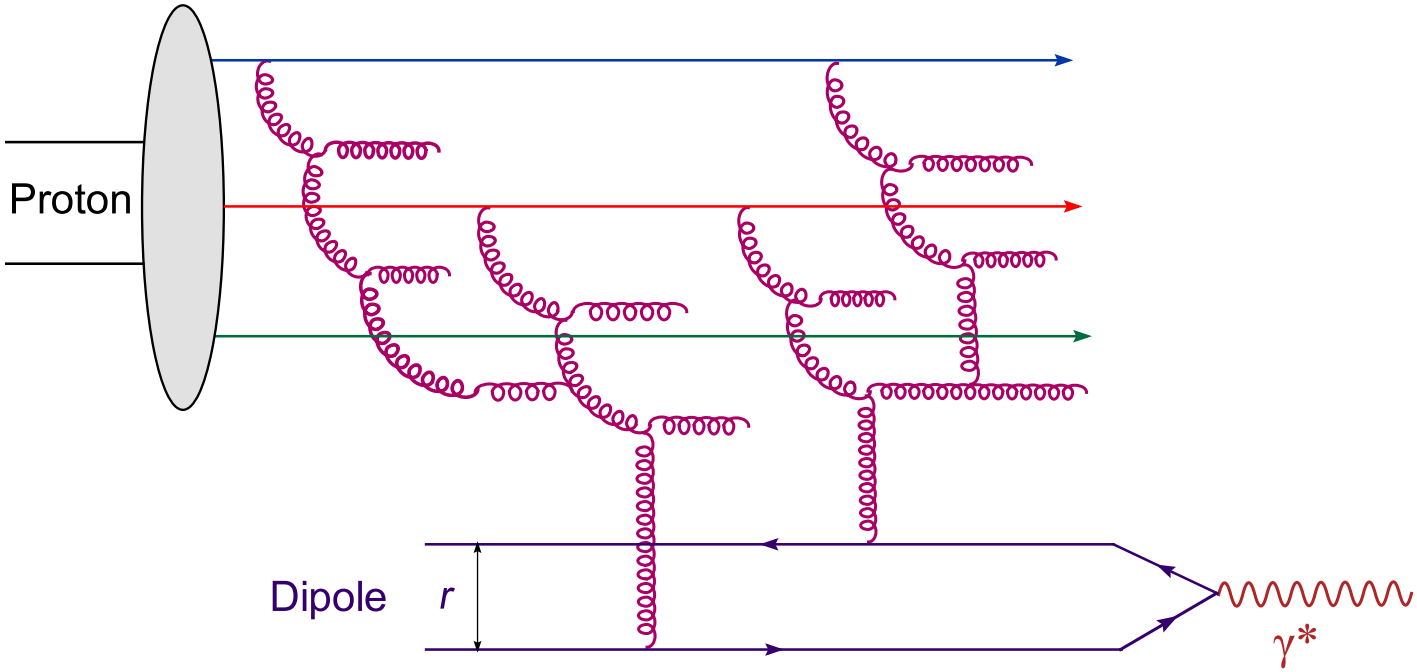

$$\partial_Y \langle T(r,Y)\rangle_Y = \int \mathrm{D}[A]\, \partial_Y W_Y[A]\; T(r)[A]$$

- JIMWLK eq. for $W_Y \implies$ Balitsky–Kovchegov eq. for $\langle T\rangle_Y$

$$\partial_Y \langle T\rangle = \langle T\rangle - \langle T^2\rangle \implies T \to 1 \quad \text{as} \quad Y \to \infty$$

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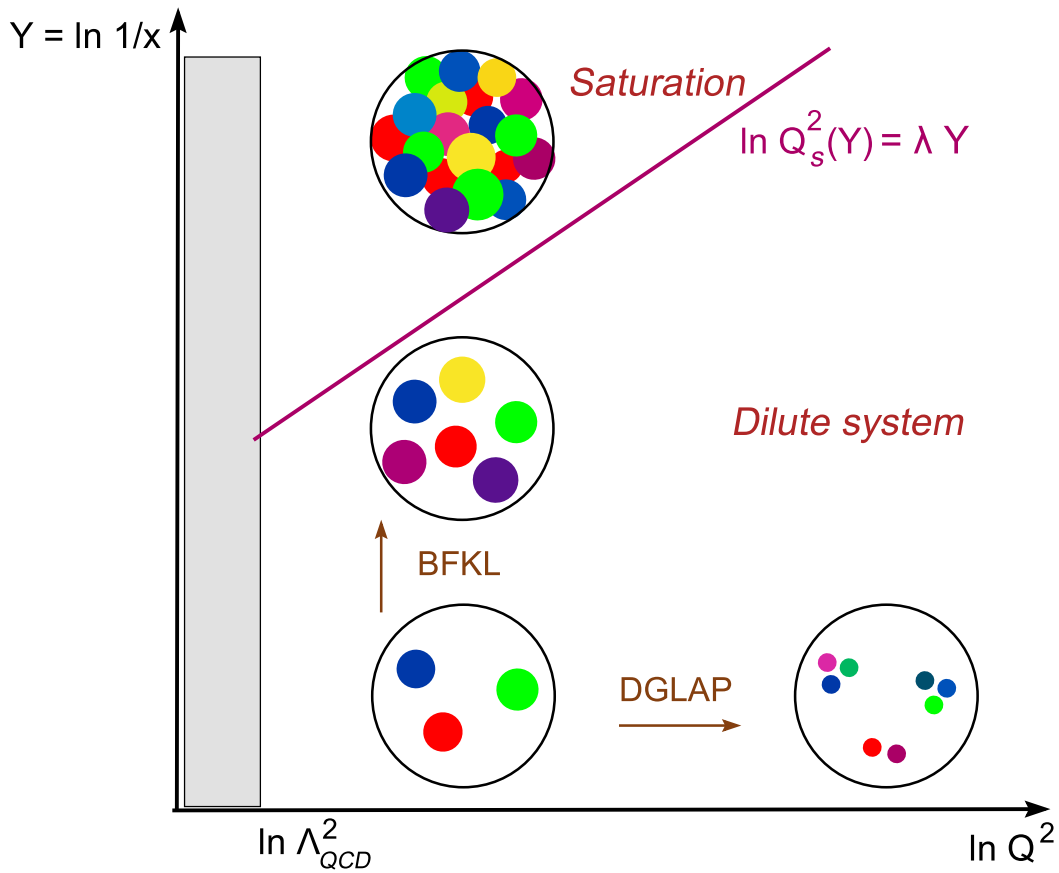
- Saturation condition : $T(r,Y) \sim 1 \quad \text{for} \quad 1/r^2 = Q_s^2(Y)$



- NLO BFKL + saturation $\Longrightarrow$

$\ln Q_s^2(Y) \simeq \lambda Y$ with $\lambda \approx 0.3$ *(Triantafyllopoulos, 02)*

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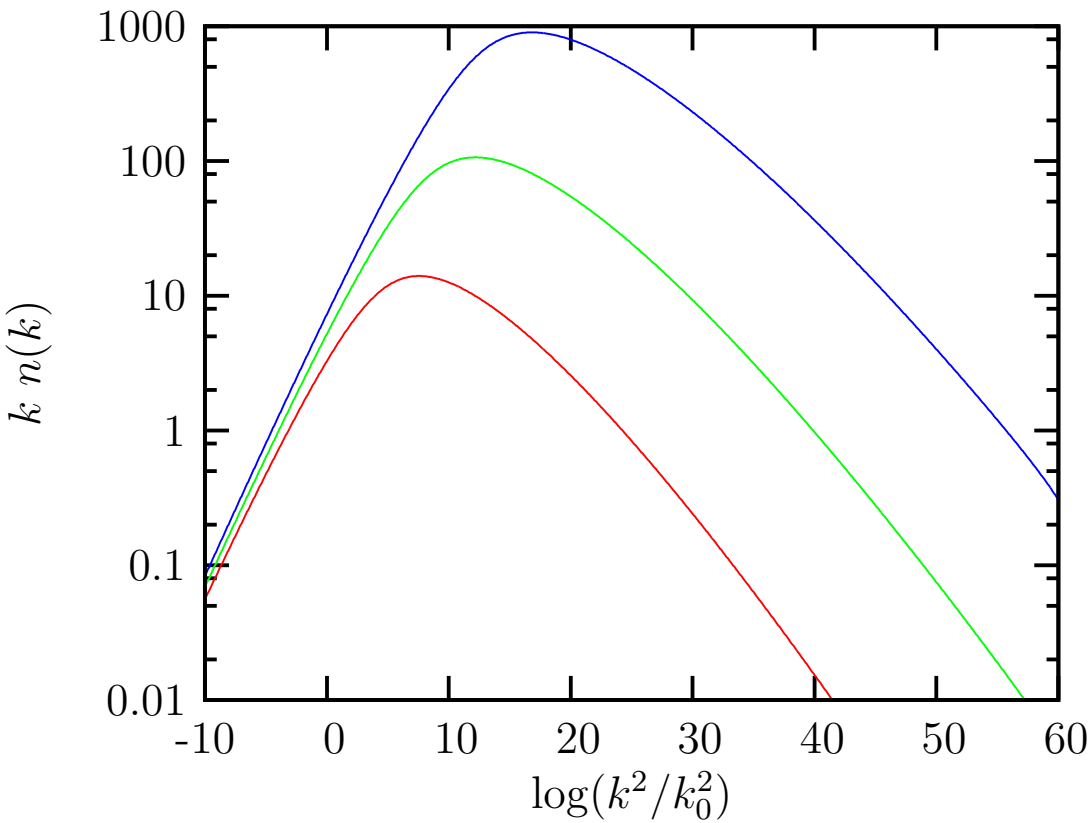
- The typical transverse momentum of the gluon distribution



$$xG(x,Q^2) = \int^Q \mathrm{d}k\ k\,n(k)$$

Geometric scaling

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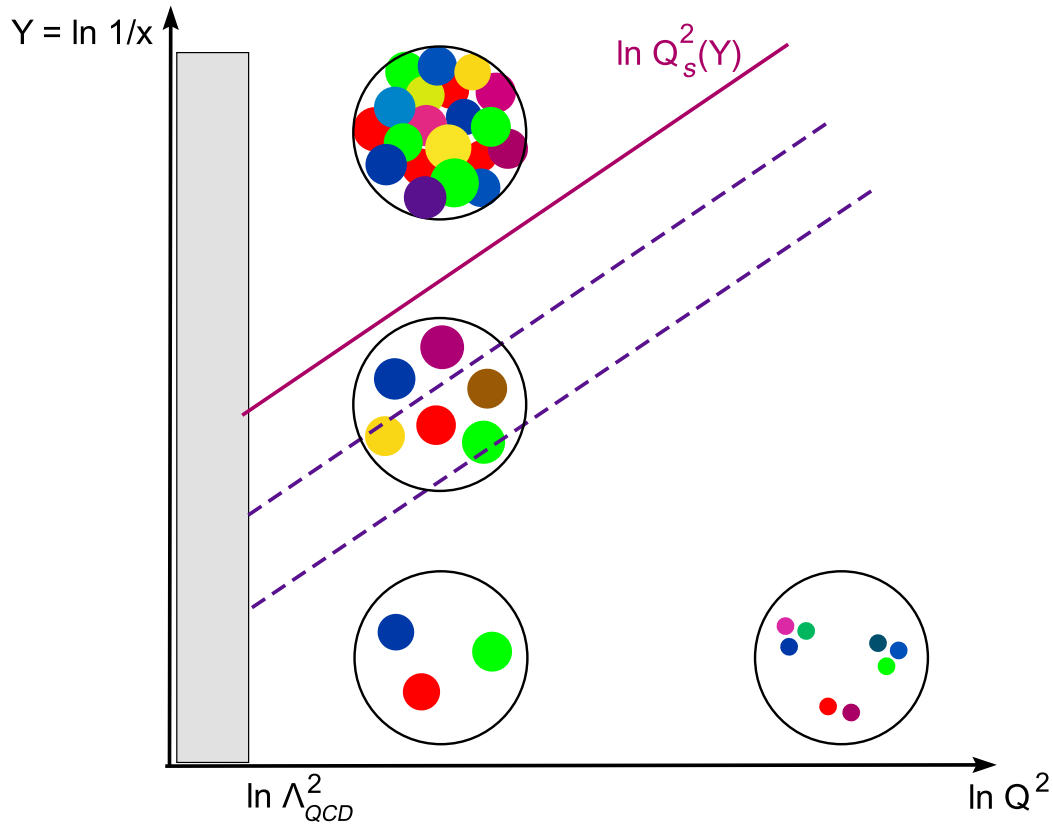

Physics is invariant along any line parallel to the saturation line.

- The observables depend only upon the difference

$$\ln Q^2 - \ln Q_s^2(Y) \;=\; \ln[Q^2/Q_s^2(Y)]$$

that is, they 'scale' as functions of the ratio $Q^2/Q_s^2(Y)$.

Geometric scaling

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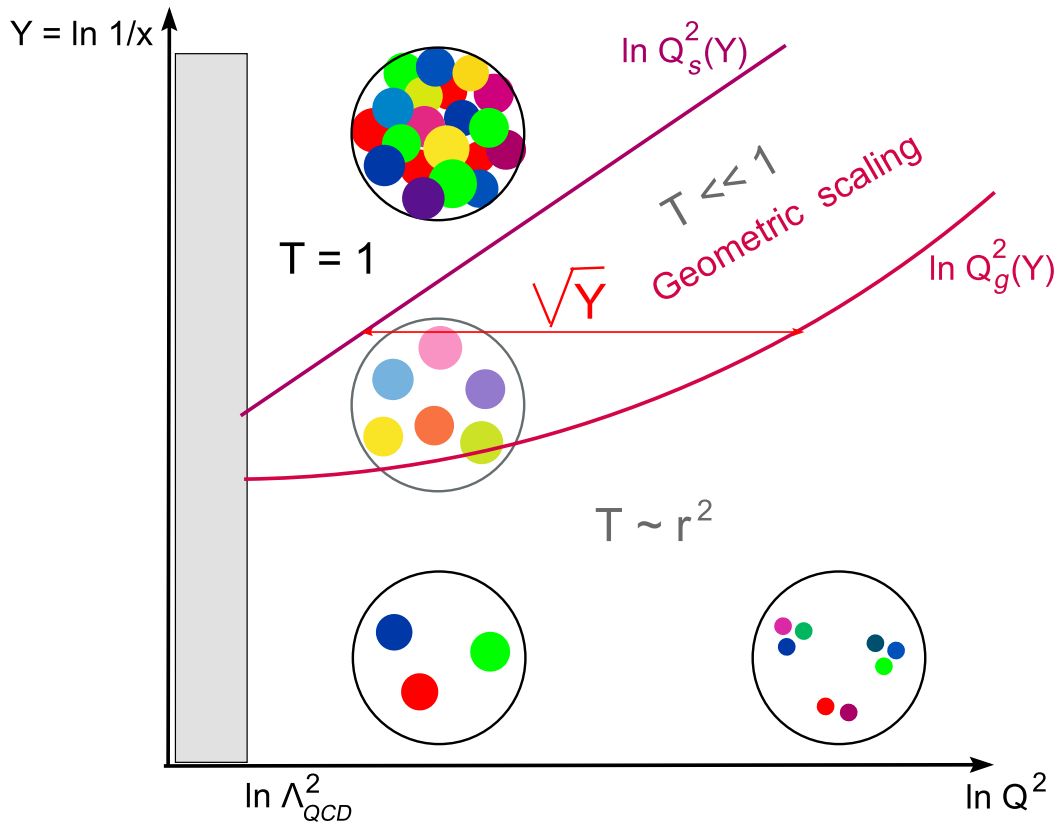
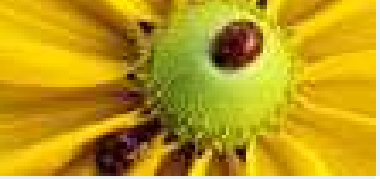


- A consequence of saturation which extends well above the saturation momentum ($Q^2 \gg Q_s^2$), in the dilute regime !

  E.I., Itakura, McLerran, 02; Mueller, Triantafyllopoulos, 02

- Relation with statistical physics: 'FKPP' *(Munier, Peschanski, 03)*



Geometric Scaling at HERA (1)

(Staśto, Golec-Biernat and Kwieciński, 2000)

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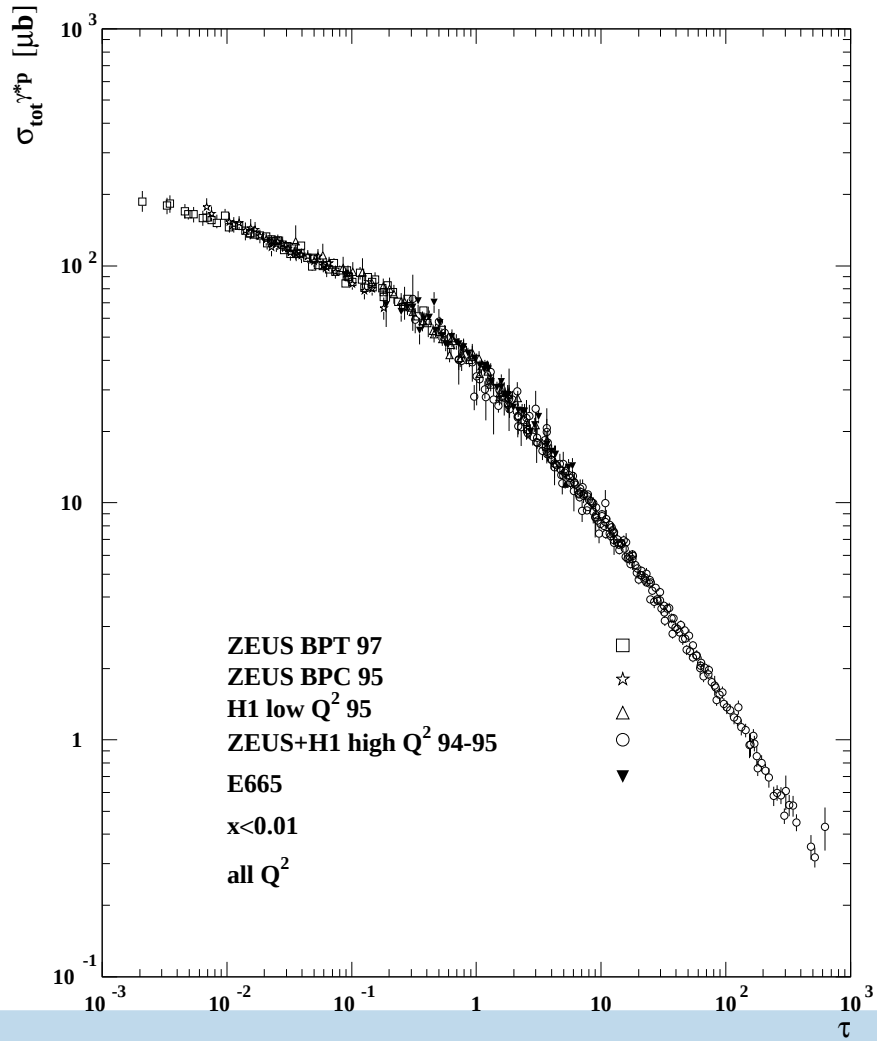
$$\sigma(Y, Q^2) \approx \sigma(\tau) \quad \text{with} \quad \tau \equiv Q^2/Q_s^2(Y), \quad Q_s^2(Y) = (x_0/x)^\lambda \, \mathrm{GeV}^2 \,, \quad \lambda \simeq$$

$x \leq 0.01$

$Q^2 \leq 450$ GeV^2

$Q_s^2 \sim 1$ GeV^2

for $x \sim 10^{-4}$



Geometric Scaling at HERA (2)

(Marquet and Schoeffel 2006)

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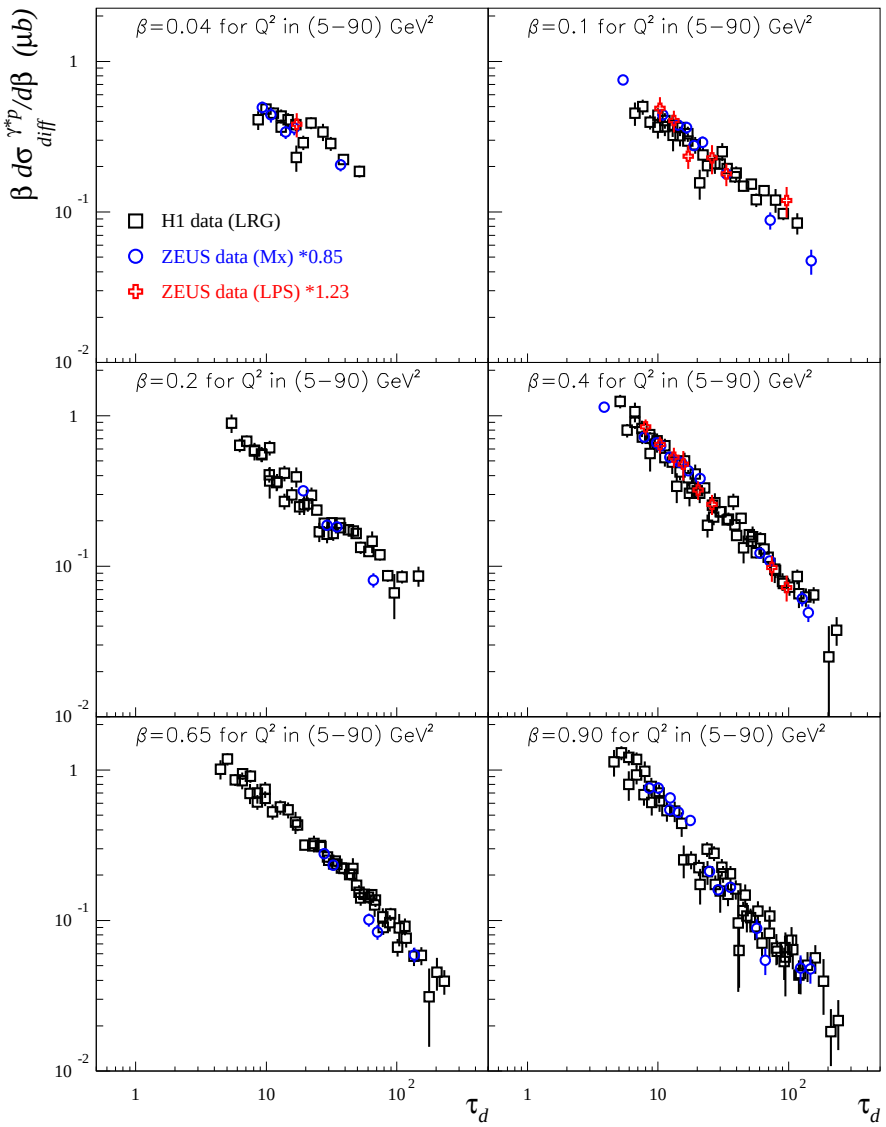

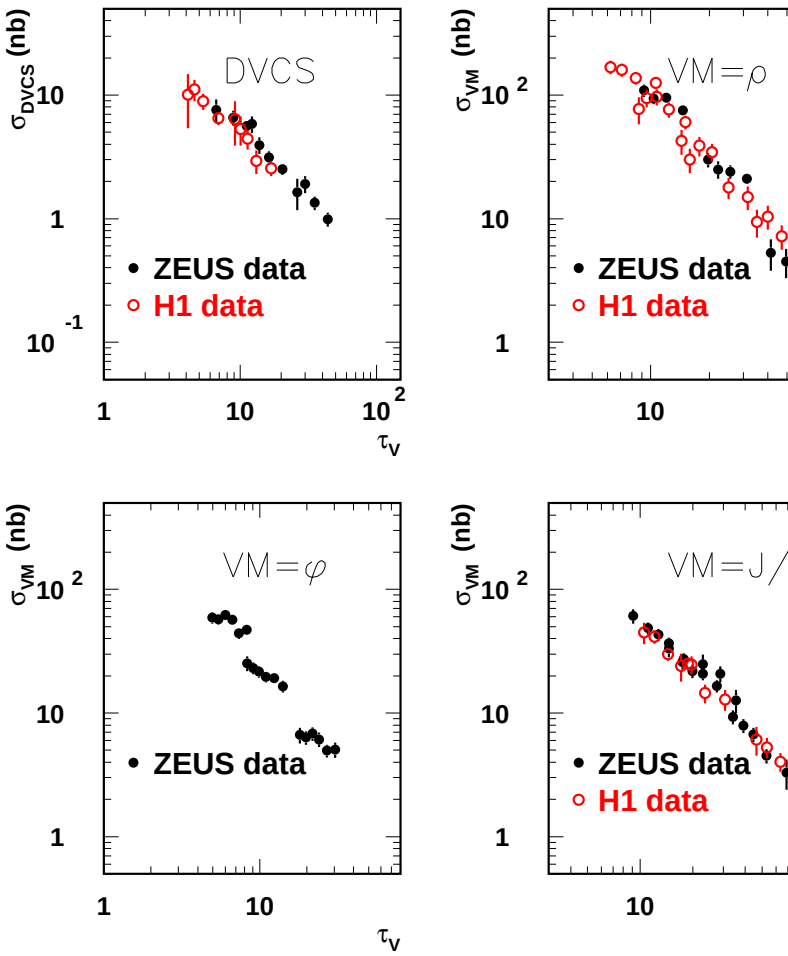

From high–energy QCD to statistical physics

(Mueller and Shoshi, 04; E.I., Mueller, and Munier, 04)

Gluon–number fluctuations modify the high–energy behaviour !

Outline

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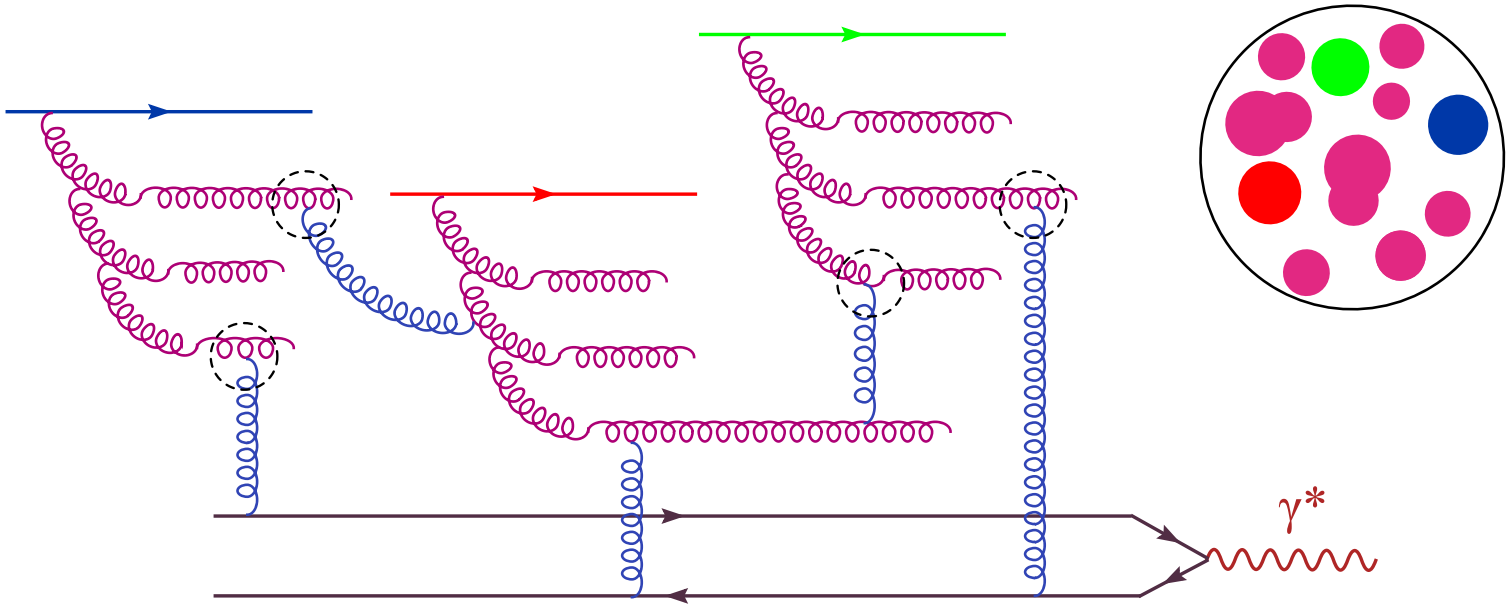
Gluon evolution

Pomeron loops
- Fluctuations
- Pomeron loops
- Black spots
- Diffusive scaling
- High–energy 'phase-diagram'

Backup



- Saturation & multiple scattering probe correlations among the gluons exchanged in the t–channel
- Such correlations are first generated via gluon–number fluctuations at low density

From high–energy QCD to statistical physics

(Mueller and Shoshi, 04; E.I., Mueller, and Munier, 04)

Gluon–number fluctuations modify the high–energy behaviour !

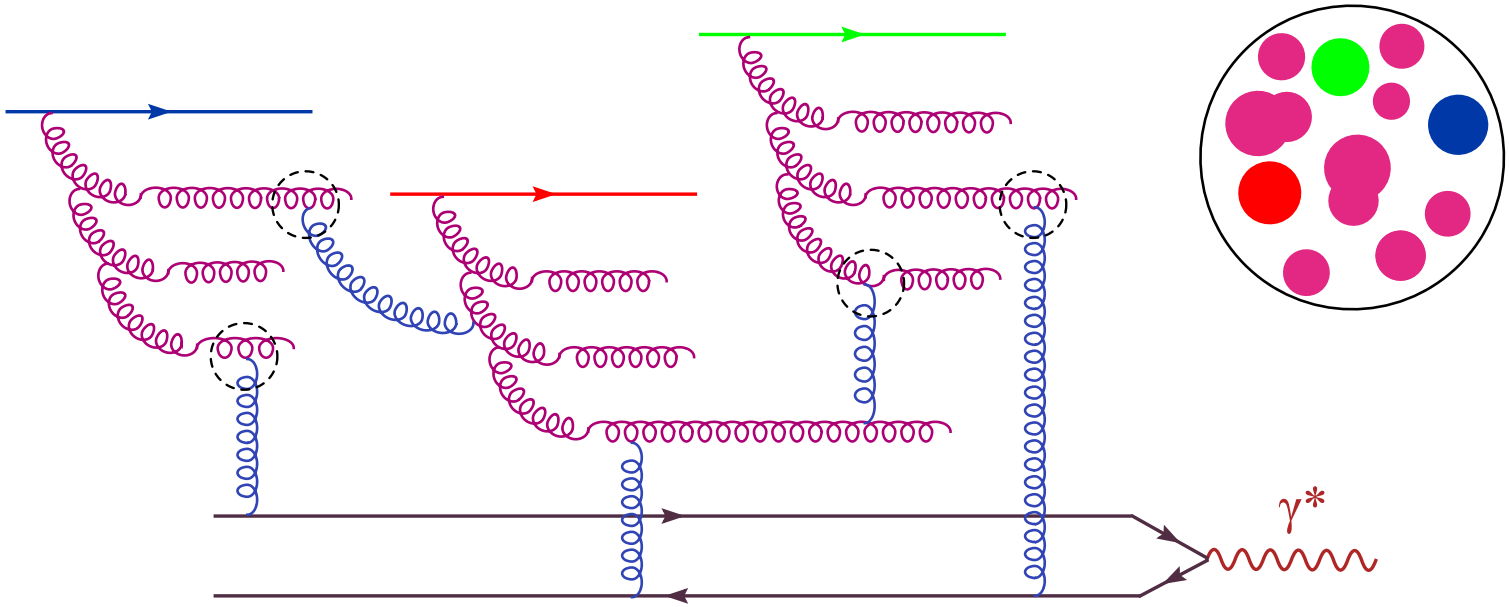
Outline

Motivation

Gluon evolution

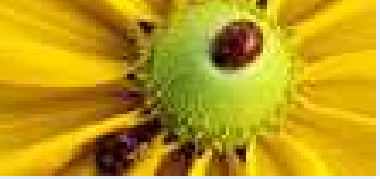
Pomeron loops
- Fluctuations
- Pomeron loops
- Black spots
- Diffusive scaling
- High–energy 'phase-diagram'

Backup



- High–energy QCD $\approx$ a 'reaction–diffusion' process $A \rightleftharpoons 2A$

$$\frac{\partial n}{\partial Y} \simeq \alpha_s n - \alpha_s^2 n^2 + \alpha_s \sqrt{\alpha_s n}\, \nu\,, \qquad \langle \nu(Y_1)\nu(Y_2)\rangle = \frac{1}{\alpha_s}\,\delta(Y_1 - Y$$



Pomeron loop equations

(E.I. and D. Triantafyllopoulos, 04)

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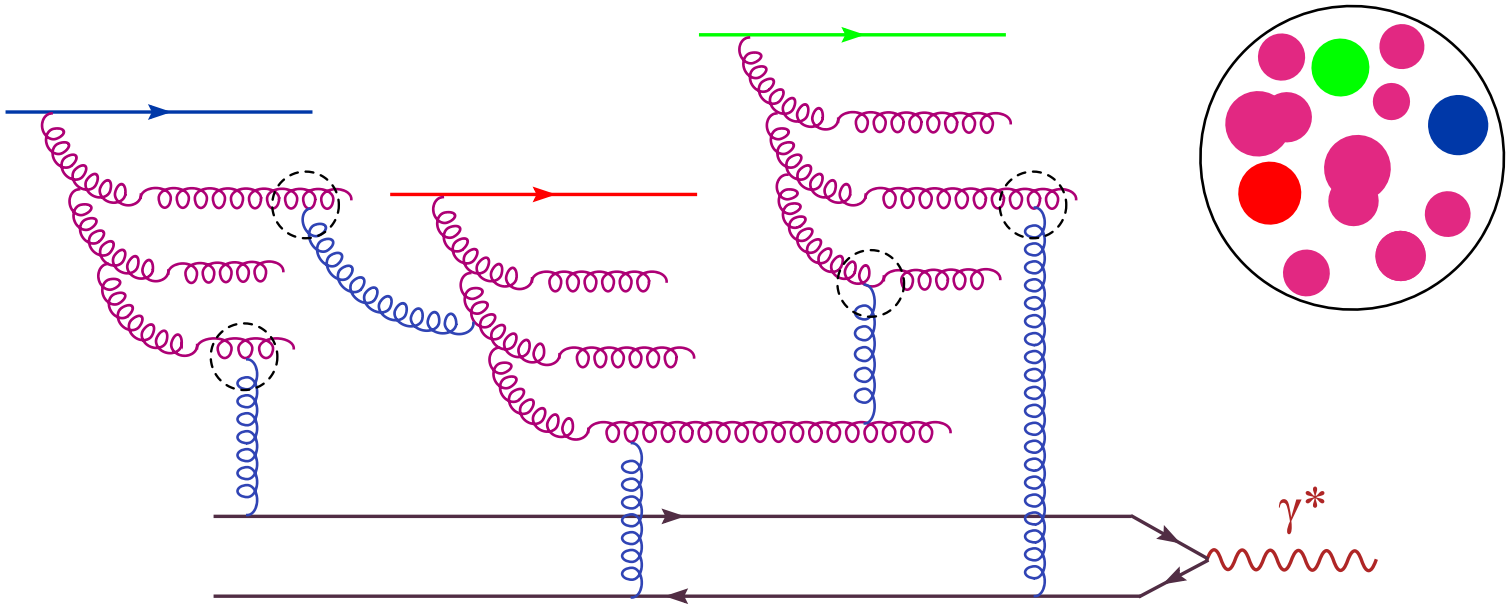
- The gluon–number fluctuations are not included in JIMWLK !



- New evolution equations in QCD at high energy and large N_c

  (JIMWLK at high density + Mueller's 'dipole picture' at low density)

- An intense activity over the last 18 months

  Mueller, Shoshi, and Wong; Levin and Lublinsky; Kovner and Lublinsky; Hatta, E.I., McLerran, Staśto, and D. Triantafyllopoulos; Balitsky; Braun

DIS with Pomeron loops

Outline
Motivation
Gluon evolution
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- Pomeron loops
- Black spots
- Diffusive scaling
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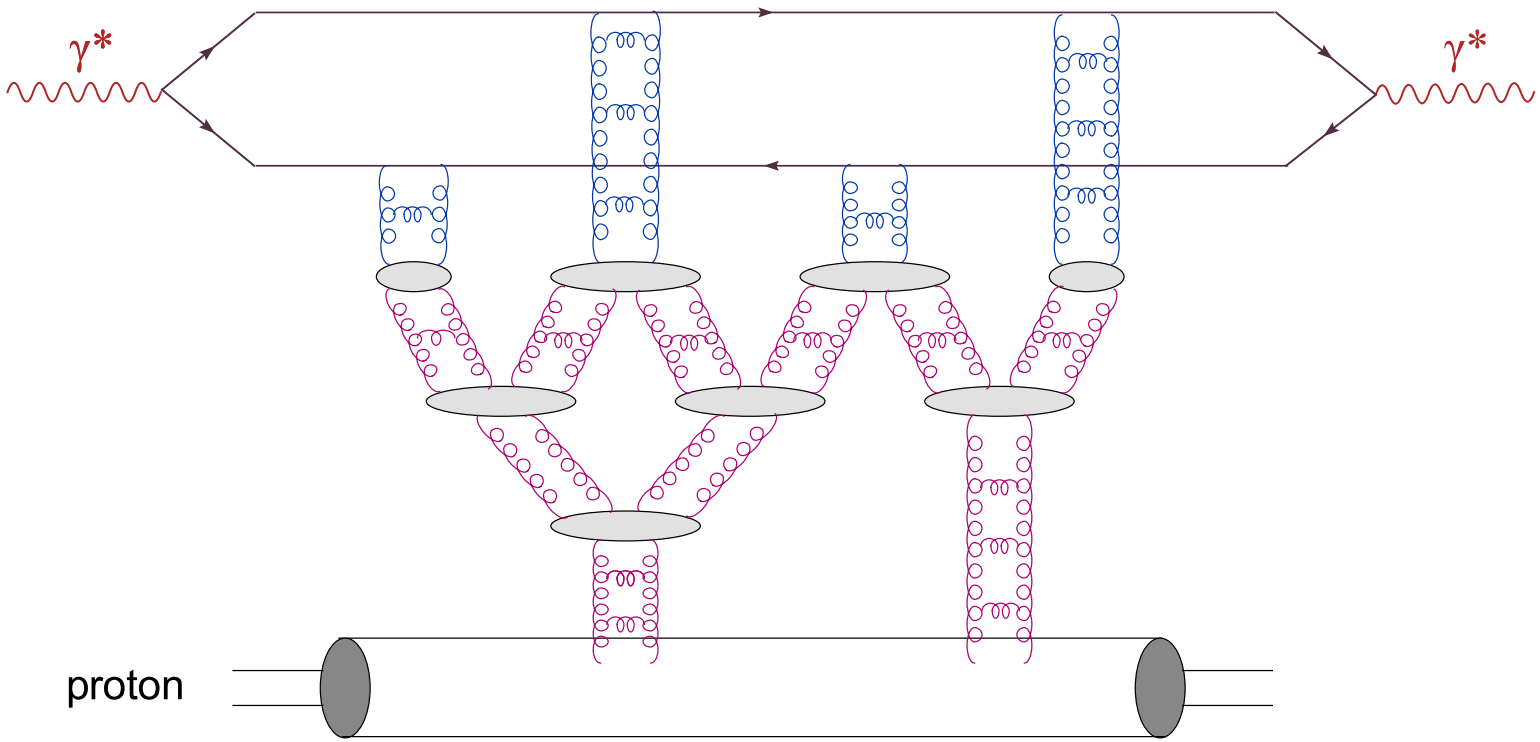
Backup

Effective theory for 'BFKL Pomerons': splitting, 'recombination', loops



▷ See also the talks by: *Levin, Kovner, Hatta, Soyez, Triantafyllopoulos, Munier, Soshi, Kozlov, Kutak, Motyka, Marquet*

▷ N.B. Saturation in QCD (JIMWLK) is not just 'recombination'

Black spots (1)

Outline
Motivation
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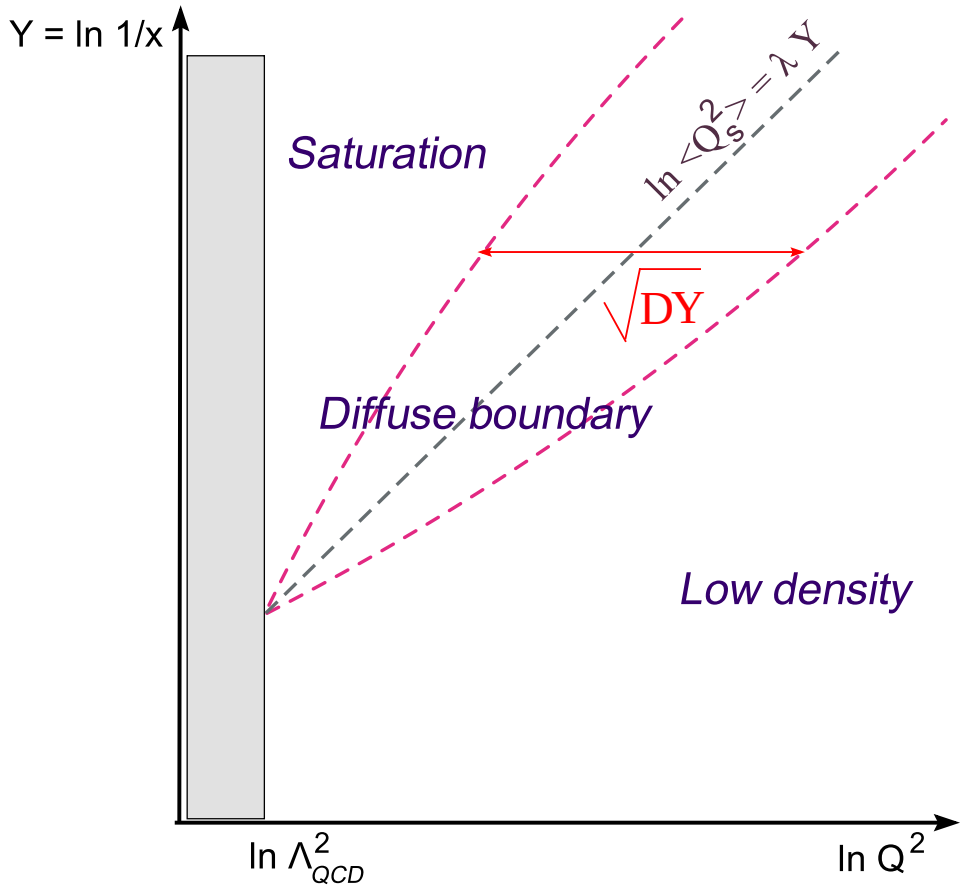
Backup

- Strong density fluctuations in the vicinity of the average saturation line $\langle Q_s^2(Y)\rangle$

$\Longrightarrow$ The saturation boundary gets diffuse, with a width $\sqrt{DY}$

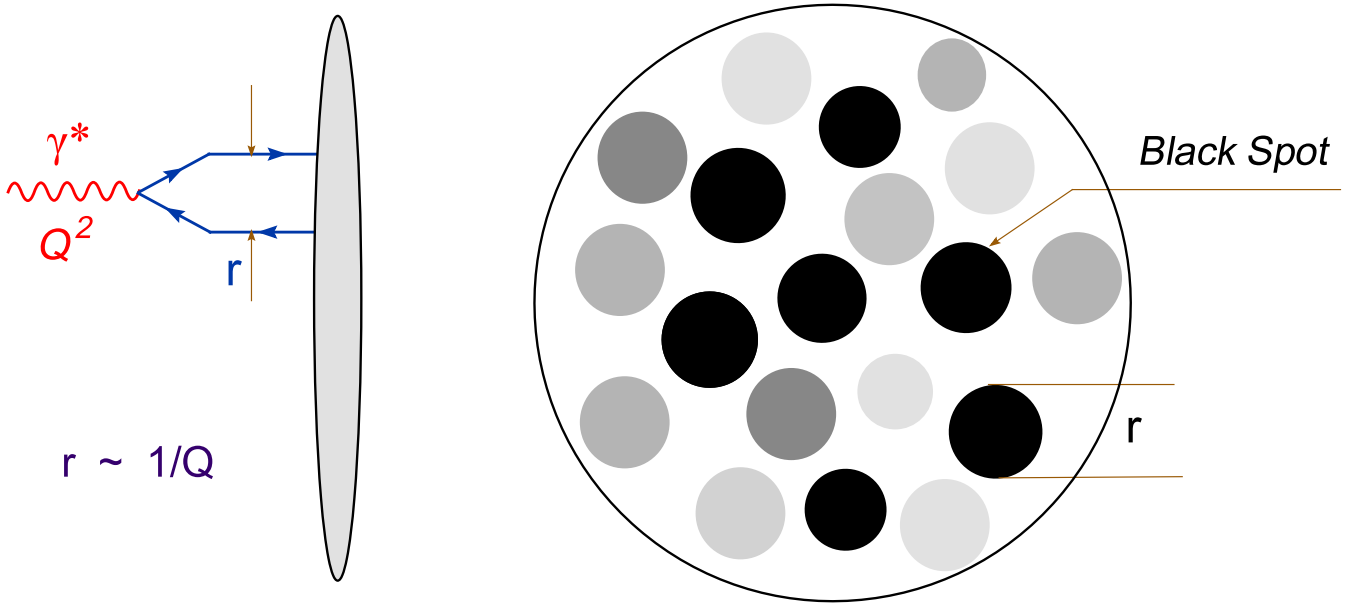


Black spots (2)

Outline
Motivation
Gluon evolution
Pomeron loops
- Fluctuations
- Pomeron loops
- Black spots
- Diffusive scaling
- High–energy 'phase-diagram'

Backup

- DIS at high energy and relatively large $Q^2 \gg \langle Q_s^2(Y)\rangle$



- The proton looks dilute on the average : $\langle T\rangle \ll 1$
- The cross–section is dominated by black spots: $Q_s^2 \gtrsim Q^2$
- 'Black spots' : Rare fluctuations with unusually large gluon density : $Q_s^2 \gg \langle Q_s^2(Y)\rangle$

The Black&White Hadron

(E.I. and D. Triantafyllopoulos, 04)

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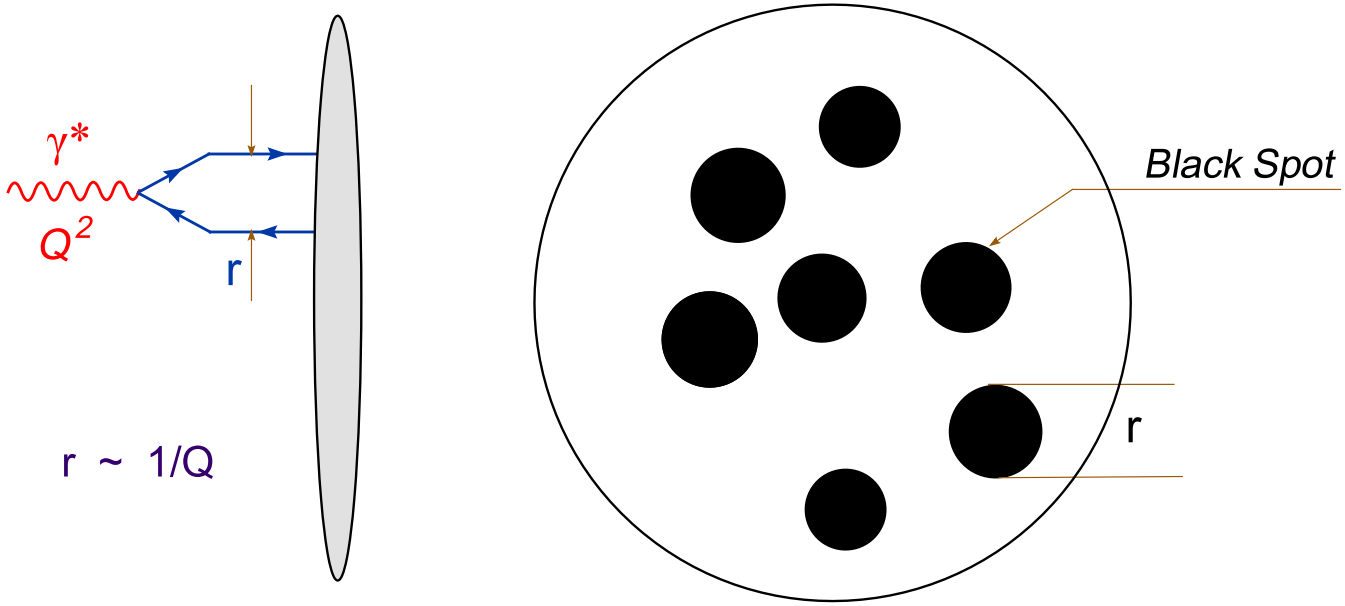
Backup

- The 'grey' spots are in fact irrelevant at very high energy



T = 0 or T = 1

- $\langle T(r) \rangle$ = the probability to find a 'black spot' with $Q_s^2 \gtrsim Q^2$.

  $P(Q_s^2) \approx$ A Gaussian in $\ln\left[Q_s^2/\langle Q_s^2(Y)\rangle\right]$ with width $\sqrt{DY}$.

Diffusive scaling

(E.I., Mueller, Munier 04; Hatta, E.I., Marquet, Soyez, Triantafyllopoulos, 06)

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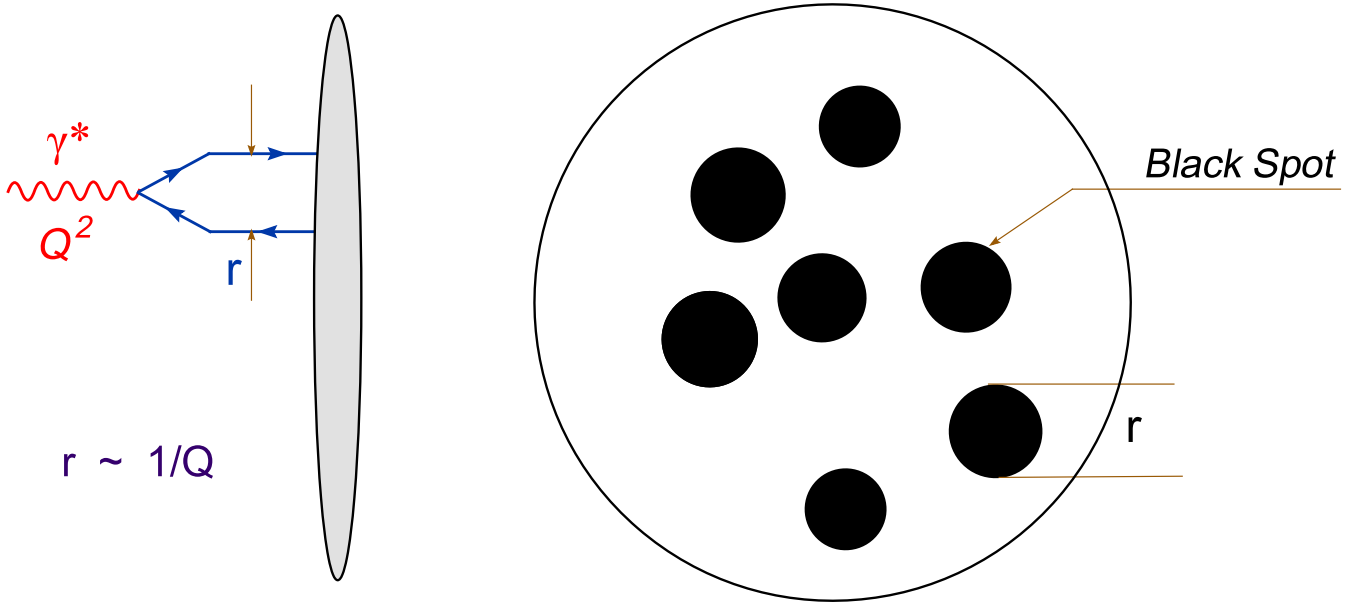
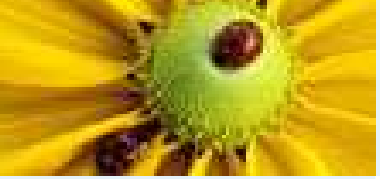


- With increasing energy, geometric scaling is progressively washed out ... and replaced by diffusive scaling :

$$\langle T(r,Y)\rangle \simeq \mathcal{T}\left(\frac{\ln\left[r^2\langle Q_s^2(Y)\rangle\right]}{\sqrt{DY}}\right) \quad \text{when} \quad DY \gg 1$$



The “phase–diagram” revisited

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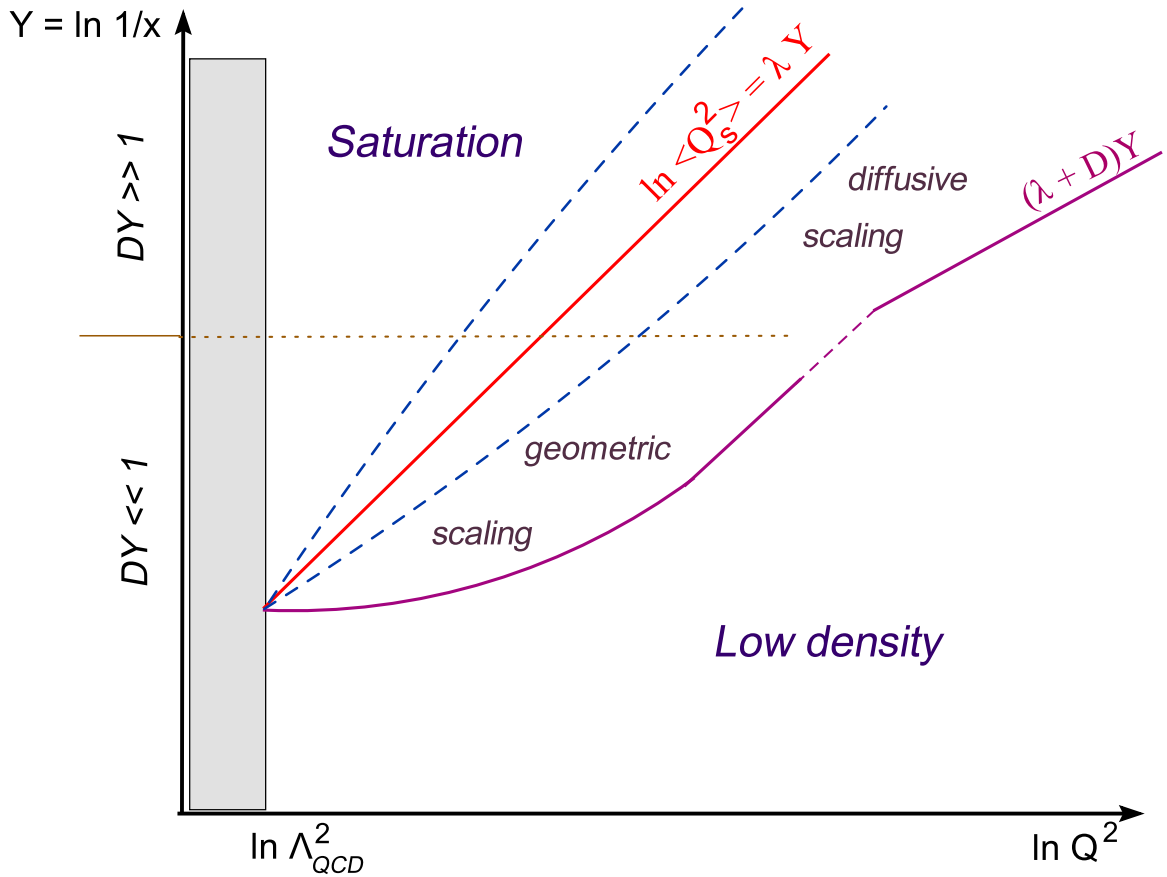
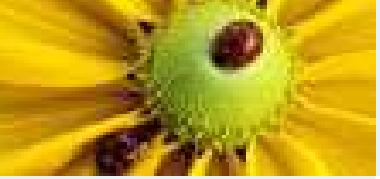


- HERA, RHIC $\approx$ intermediate energies (most likely)
- LHC ? ... We don’t really know ! *(large theoretical uncertainties)*
- Compare theoretical expectations with the data !



Eikonal Scattering in DIS

Outline

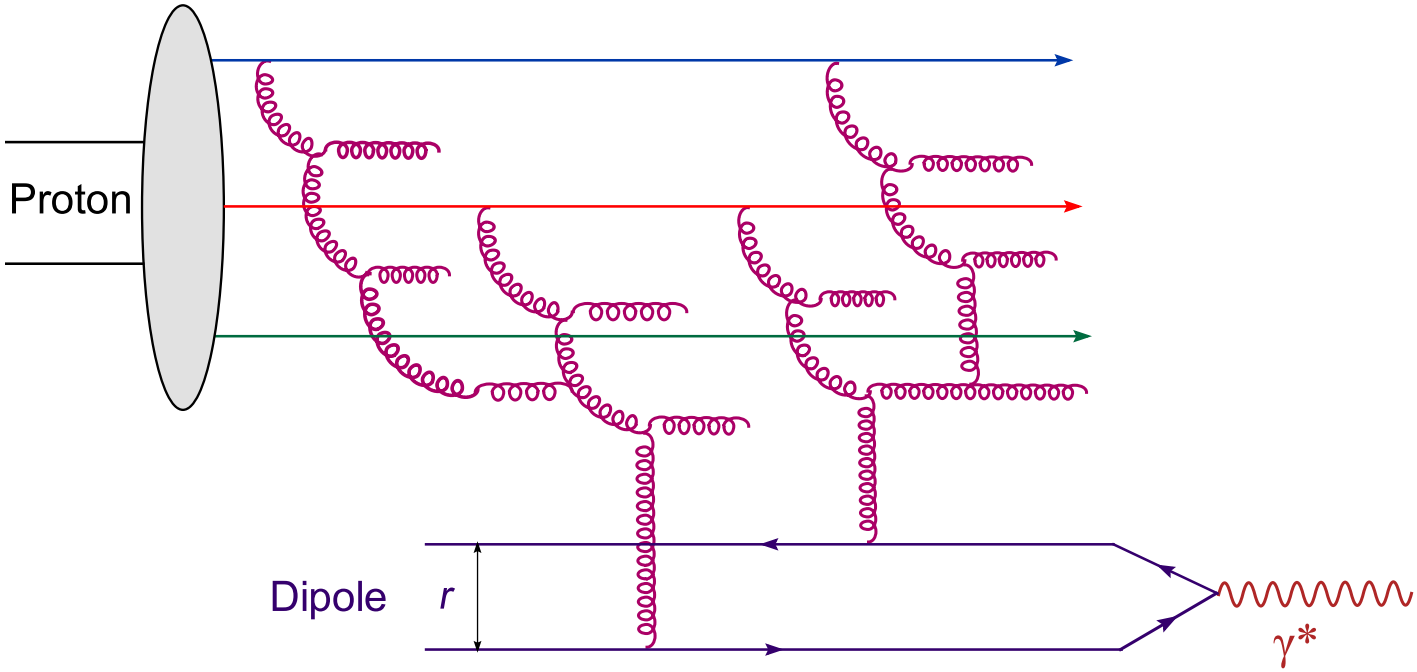
Motivation

Gluon evolution

Pomeron loops

Backup

- Eikonal
- DIS Diffraction
- Soft diffraction (?)
- (Semi)Hard diffraction
- DIS
- Gluons at HERA
- Black spots
- Cross–sections
- Reaction-Diffusion



- Multiple scattering off a given configuration of target fields A:

$$T(\boldsymbol{x},\boldsymbol{y})[A] \;=\; 1 \;-\; \frac{1}{N_c}\mathrm{tr}\big(V_{\boldsymbol{x}}^{\dagger}\,V_{\boldsymbol{y}}\big)$$

$$V(\boldsymbol{x}) \;\equiv\; \mathrm{P}\exp\Big(\mathrm{i}g\int dx^{-}A_a^{+}(x^{-},\boldsymbol{x})T^a\Big) \qquad \text{(Wilson line)}$$

DIS Diffraction

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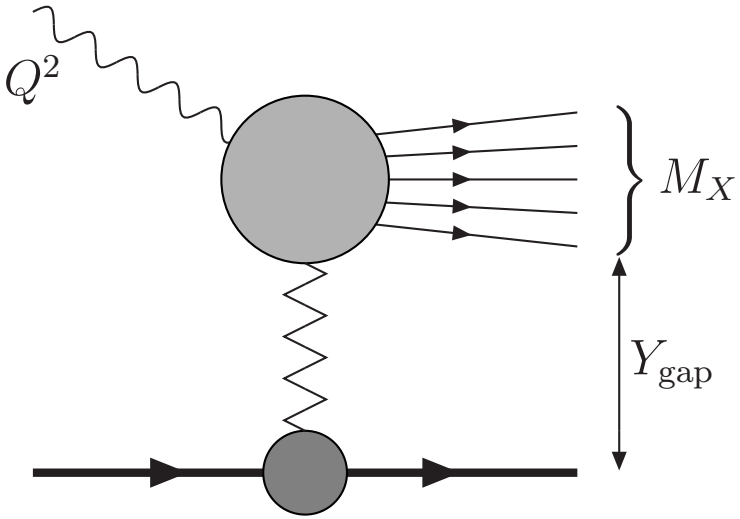

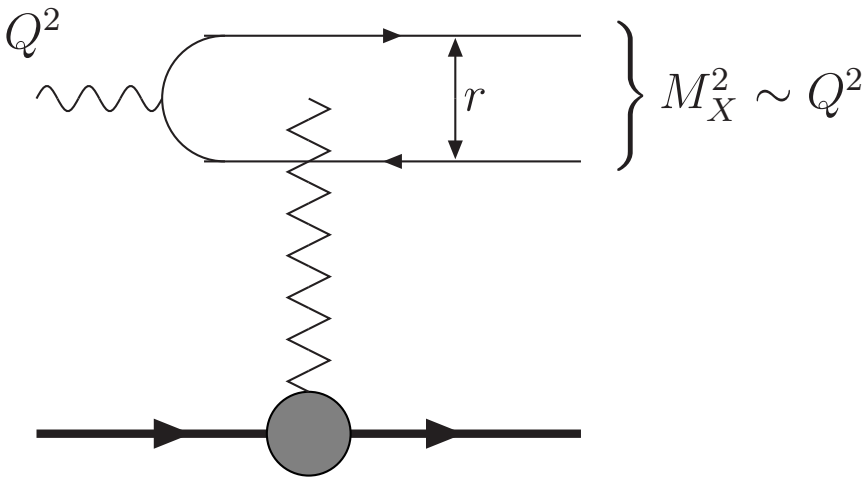

- An ideal laboratory to study saturation/unitarity effects
  - sensitive to relatively large dipole sizes
  - sensitive to theoretical models (or prejudices)

DIS Diffraction

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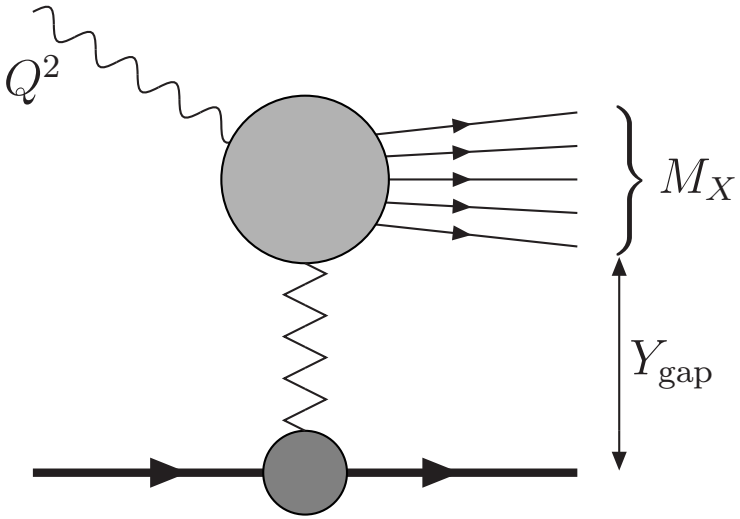

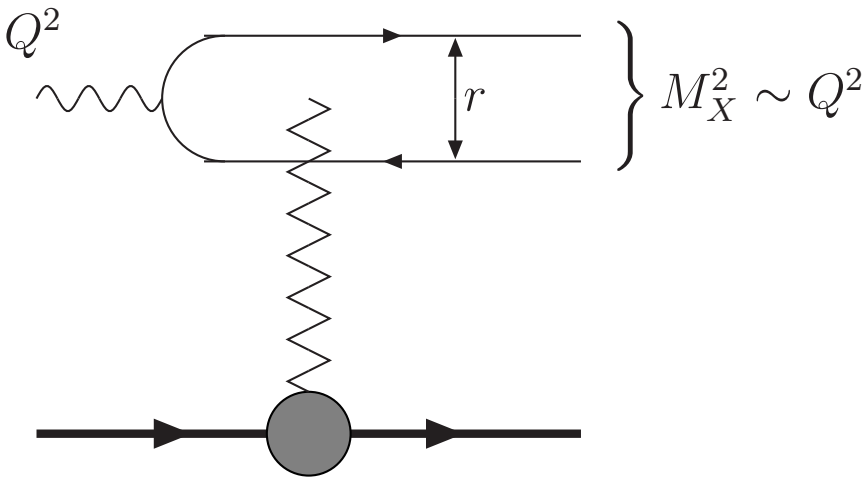

- An ideal laboratory to study saturation/unitarity effects
  - sensitive to relatively large dipole sizes
  - sensitive to theoretical models (or prejudices)

- Original prejudice: "Even for large Q^2, diffraction is soft"

$\sigma_{\rm diff} \propto x^{-2(\alpha_{\mathbb{P}}-1)}$ and hence $\dfrac{\sigma_{\rm diff}}{\sigma_{\rm tot}} \sim x^{-(\alpha_{\mathbb{P}}-1)}$ at small x ✓

Diffractive over inclusive ratio at HERA

Golec-Biernat, Wüsthoff (99) ; Bartels, Golec-Biernat & Kowalski (02)

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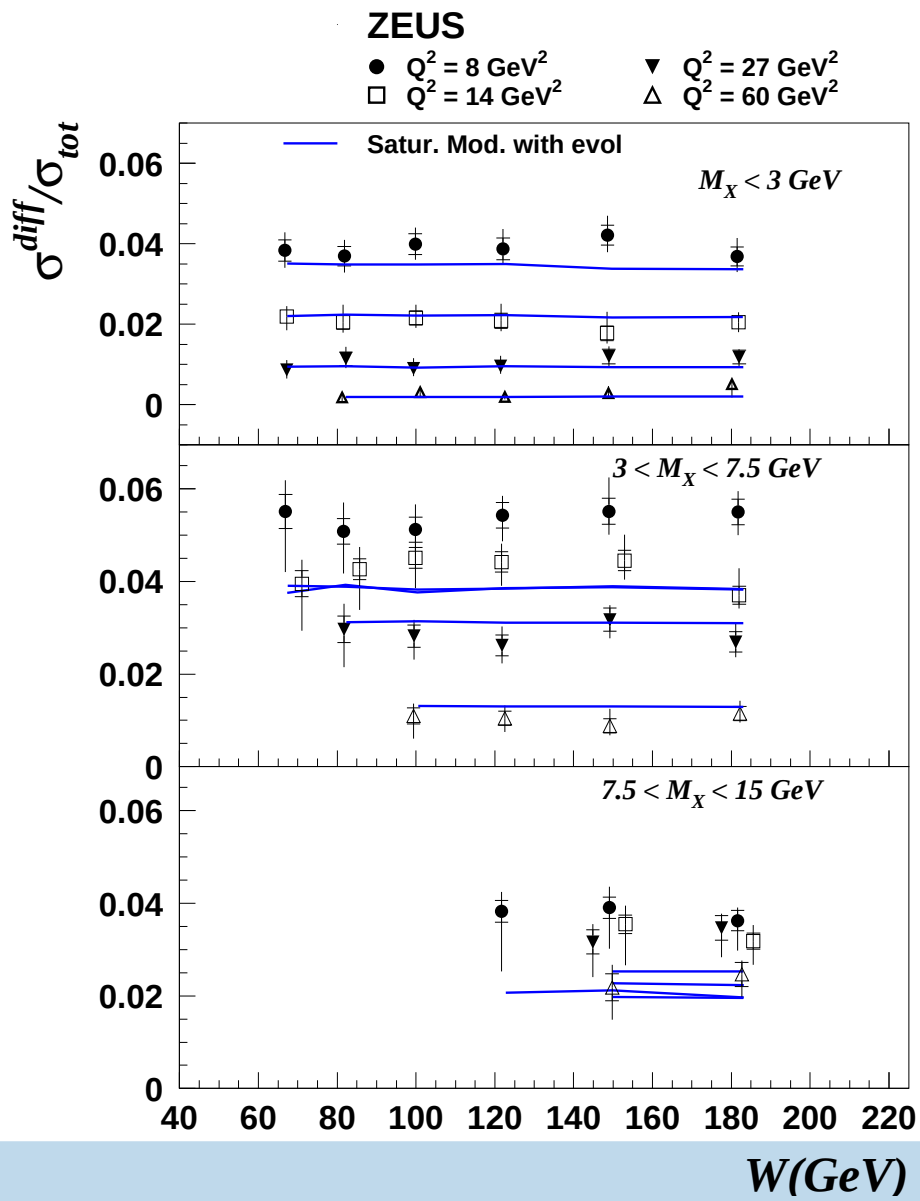

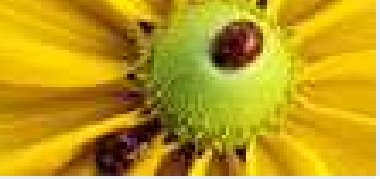

Diffractive dissociation of the virtual photon

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$$\frac{\mathrm{d}\sigma_{\mathrm{diff}}}{\mathrm{d}^2 b} = \int \mathrm{d}z\, \mathrm{d}^2\boldsymbol{r}\, |\Psi_\gamma(z,r;Q)|^2\ \langle T(r,Y)\rangle^2$$

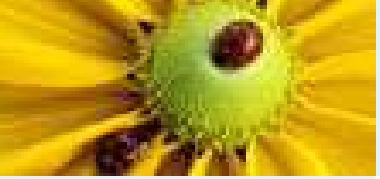
- The photon wavefunction favors small dipoles ($r \sim 1/Q$)

$$\frac{\mathrm{d}\sigma_{\mathrm{diff}}}{\mathrm{d}^2 b} \sim \frac{1}{Q^2} \int_{1/Q^2}^{\infty} \frac{\mathrm{d}r^2}{r^4} \langle T(r,Y)\rangle^2$$

- The dipole amplitude favors relatively large dipoles :

$$T(r) \ \propto\ r^2 \qquad \text{(single scattering)}$$

- "The integral is dominated by large, non–perturbative, dipoles with size $r \sim 1/\Lambda_{\mathrm{QCD}}$, hence the soft pomeron ! "



Hardening the diffraction (1)

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- At sufficiently high energy, gluon saturation cuts off the large dipoles already on the 'semi–hard' scale $1/Q_s$!

$$\frac{\mathrm{d}\sigma_{\mathrm{diff}}}{\mathrm{d}^2 b} \sim \frac{1}{Q^2} \int_{1/Q^2}^{1/Q_s^2} \frac{\mathrm{d}r^2}{r^4} \left(r^2 Q_s^2(x)\right)^2 \sim \frac{Q_s^2(x)}{Q^2} \propto x^{-\lambda}$$

  - σ_{diff} is dominated by dipole sizes $r \sim 1/Q_s(x)$!
  - $\sigma_{\mathrm{diff}} \propto x^{-\lambda}$: single, hard pomeron increase with $1/x$ (instead of double soft !)
  - $\sigma_{\mathrm{diff}}/\sigma_{\mathrm{tot}} \approx$ constant ! ✓

- 'Semi–hard diffraction' ... at intermediate energies !

Hardening the diffraction (2)

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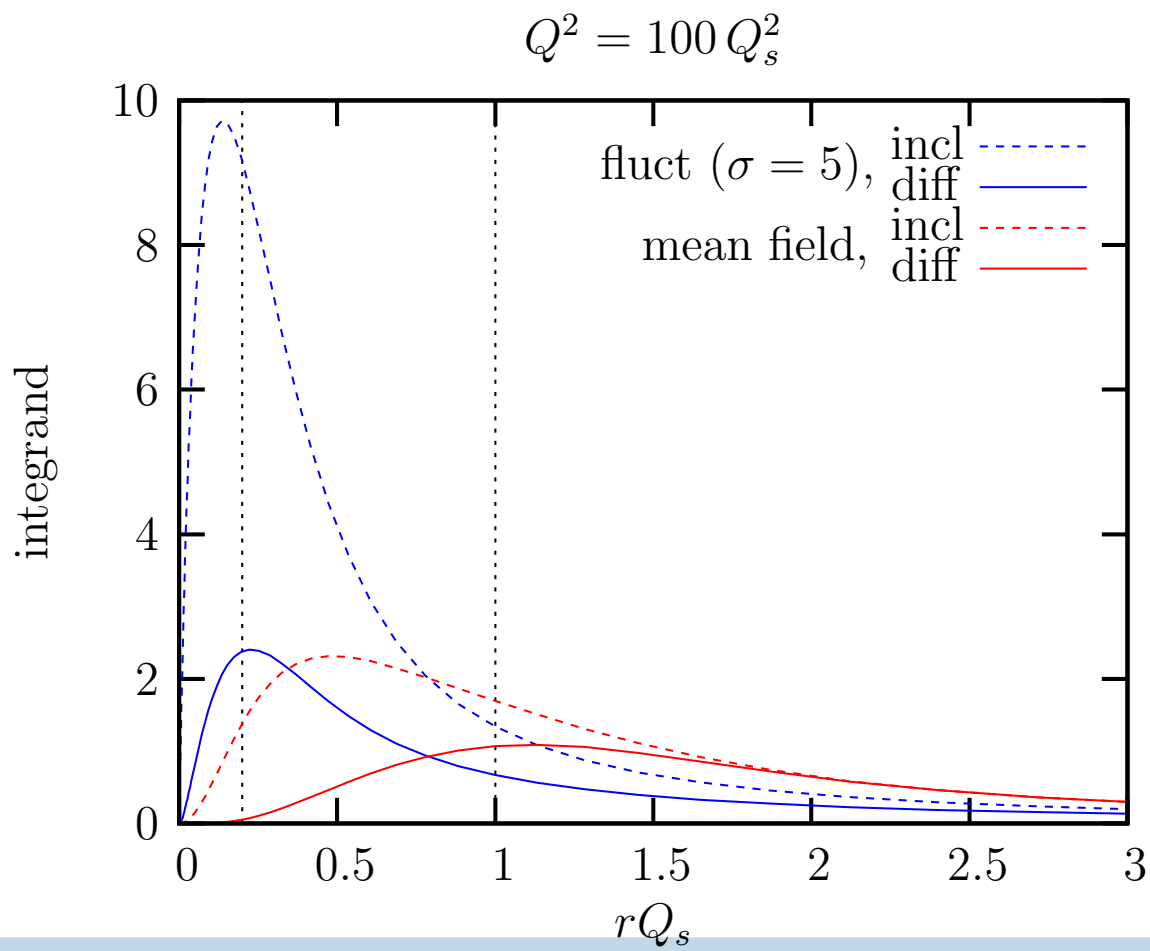
- Very high energies: 'black spots' with $Q_s^2 \sim Q^2$
  - $\sigma_{\rm diff}$ is dominated by the hard scale $r \sim 1/Q$!
  - no 'pomeron' (power–like) increase, diffusive scaling, ...



Deep Inelastic Scattering at small–x

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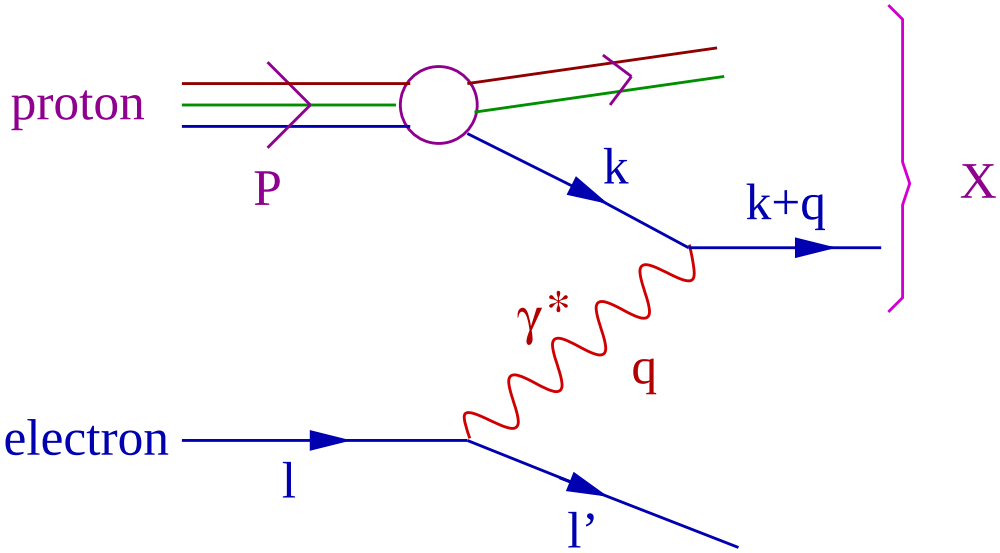
electron (l) + proton (P) $\longrightarrow$ electron (l') + X (P_X)



- Two independent kinematical invariants :
  - $Q^2 \equiv -q^\mu q_\mu \geq 0$
  - $x \simeq Q^2/s$ with $s \equiv (P+q)^2 \gg Q^2$
- Virtual photon absorbed by a quark excitation of the proton
  - with transverse size $\Delta x_\perp \sim 1/Q$
  - and longitudinal momentum $k_z = xP$

Motivation: Gluons at HERA

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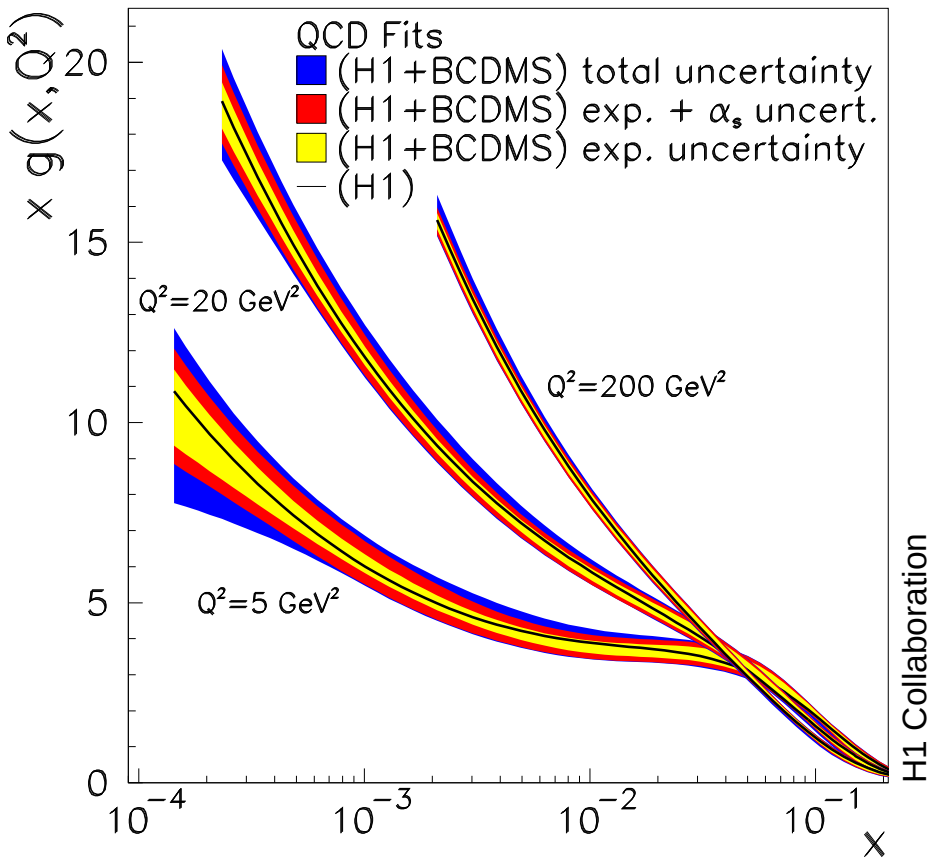
▷ The gluon distribution rises very fast at small x ! $(\sim 1/x^{\lambda})$



$xG(x,Q^2) \approx$ # of gluons with transverse size $\Delta x_\perp \sim 1/Q$ and $k_z = xP$

Motivation: Gluons at HERA

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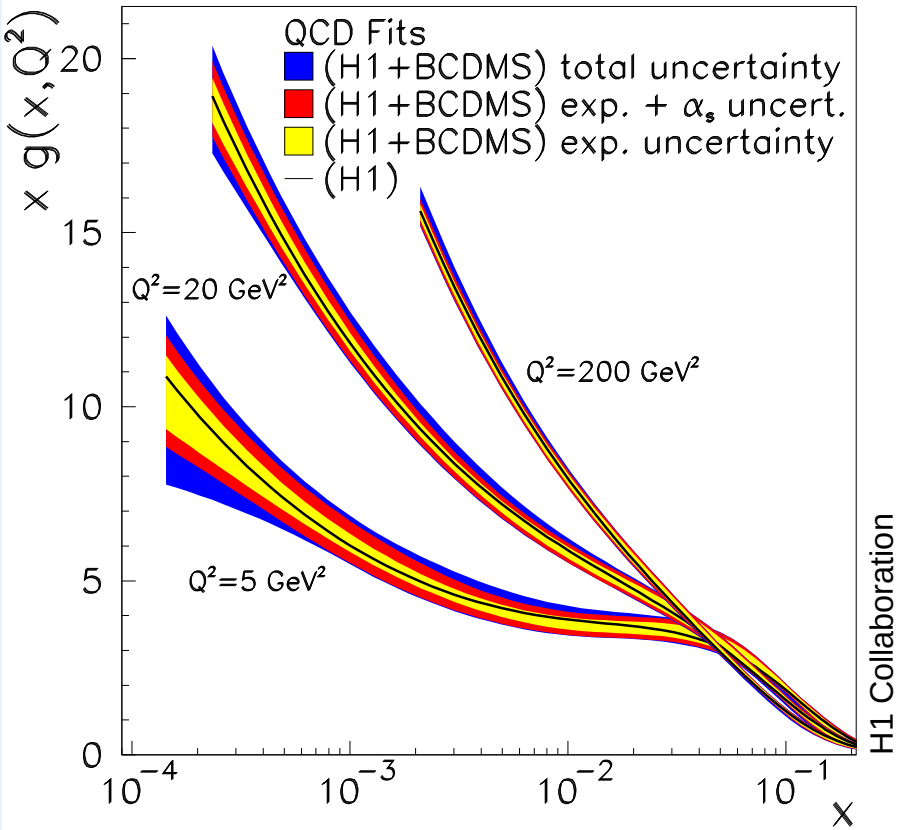

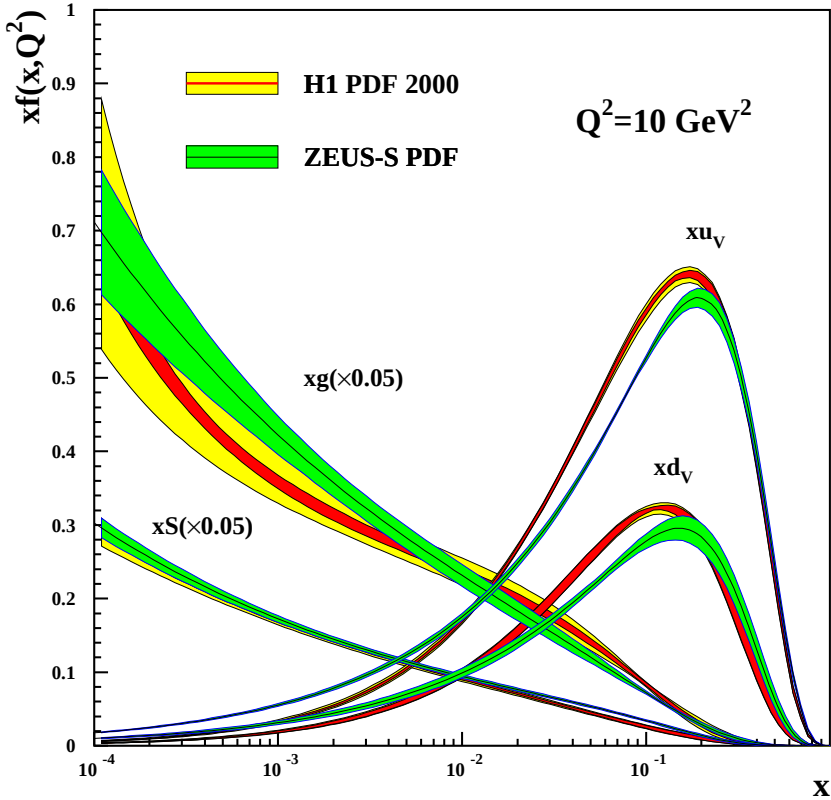

▷ The gluons dominate the proton wavefunction at small–x

Black spots

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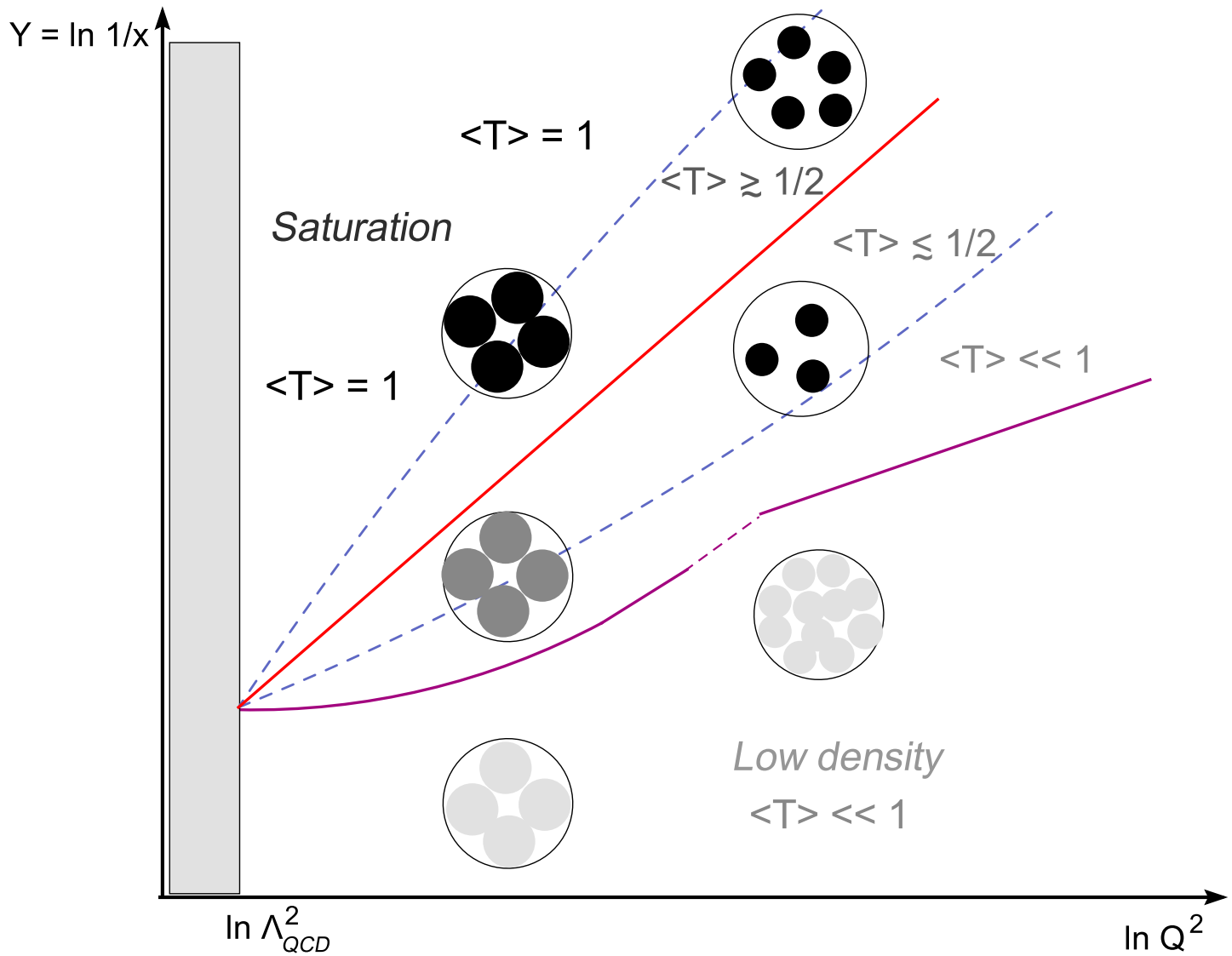

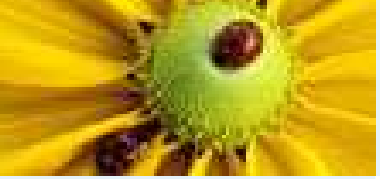

DIS in the high–energy limit

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- The DIS cross–sections in the 'diffusive scaling window' :

$$\frac{\mathrm{d}\sigma_{\mathrm{tot}}}{\mathrm{d}^2 b} \simeq \frac{\sqrt{\pi} F}{6}\, \sigma\, \frac{\exp\left(-Z^2\right)}{Z^2}$$

$$\frac{\mathrm{d}\sigma_{\mathrm{diff}}}{\mathrm{d}^2 b} \simeq \frac{F}{24}\, \sigma\, \frac{\exp\left(-2Z^2\right)}{Z^3}$$

$$F \equiv \frac{N_c \alpha_{\mathrm{em}}}{2\pi^2} \sum_f e_f^2, \qquad Z \equiv \frac{\ln(Q^2/\langle Q_s^2\rangle)}{\sigma}$$

$$\ln \frac{\langle Q_s^2(Y)\rangle}{Q_0^2} = \lambda Y, \qquad \sigma^2(Y) = DY$$

- Just one free parameter: the initial condition Q_0^2.

From high–energy QCD to statistical physics

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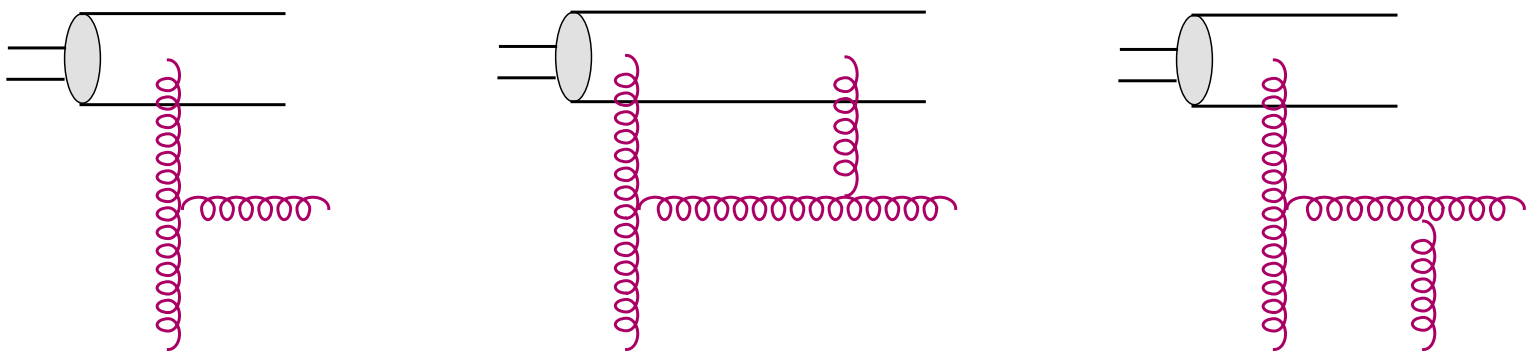

BFKL recombination splitting

- Infinite hierarchy of equations for the N–body densities :

$$\frac{\partial \langle n \rangle}{\partial Y} \simeq \alpha_s \langle n \rangle - \alpha_s^2 \langle nn \rangle$$

$$\frac{\partial \langle nn \rangle}{\partial Y} \simeq 2\alpha_s \langle nn \rangle - \alpha_s^2 \langle nnn \rangle + \alpha_s^2 \langle n \rangle \ \ldots$$

  - Early stages: Correlations are generated via fluctuations
  - Intermediate stages: ... then amplified by BFKL evolution
  - High density: ... and eventually lead to saturation !

From high–energy QCD to statistical physics

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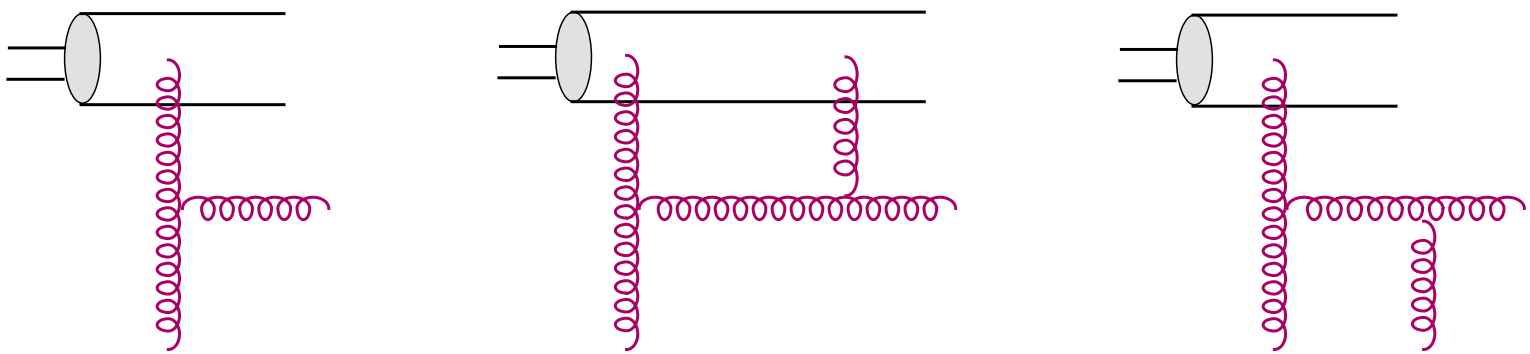

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$$\frac{\partial\langle nn\rangle}{\partial Y} \simeq 2\alpha_s\langle nn\rangle - \alpha_s^2\langle nnn\rangle + \alpha_s^2\langle n\rangle \ \ldots$$

- Cartoon version of the 'Pomeron loop' equations in QCD at large N_c *(E.I. and D. Triantafyllopoulos, 04)*
- A classical stochastic process: 'Reaction–diffusion' $A \rightleftharpoons 2A$