Momentum space analysis of the Triple Pomeron Vertex

Krzysztof Kutak
Hamburg University
Institute of Nuclear Physics Kraków

in collaboration with
Jochen Bartels

1. **Motivations**
2. **Two gluons $\longrightarrow$ four gluons**
   effective transition vertex
3. **The Vertex in the forward direction**
4. **The Triple Pomeron Vertex**
   - **Collinear case**
   - **Anticollinear case**
5. **Angular averaged TPV in Balitzki-Kovchegov equation written for unintegrated gluon density.**
6. **Conclusions**

1.Motivations

$\downarrow$

- **One of basic elements of effective field theory based on BFKL pomeron**
- **Understanding of fan diagram equation for gluon density**

- **Some aspects of the of the pomeron loop**

2.Two gluons→four gluons effective transition vertex.

(Batels,Wusthoff) Let us consider:

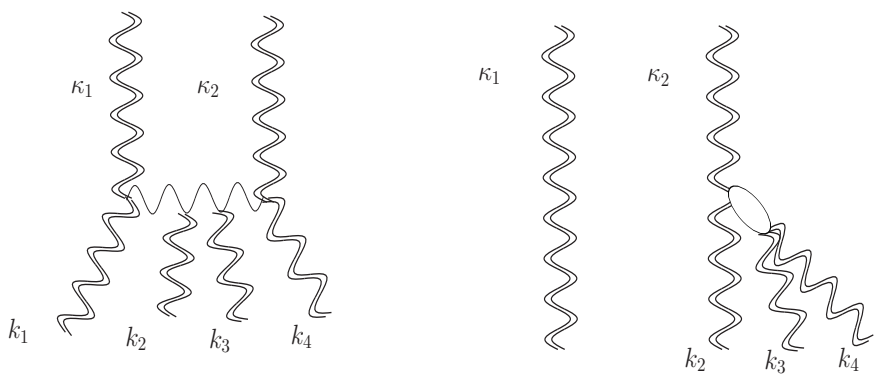

Figure 1: Example of diagrams that correspond to the Vertex

$$\mathcal{V}_{a_1' a_2'; a_1 a_2 a_3 a_4}(\kappa_1, \kappa_2; k_1, k_2, k_3, k_4) = \tag{1}$$

$$\frac{\sqrt{2}\pi \delta_{a_1' a_2'}}{\sqrt{N_c^2 - 1}} \Big[\delta_{a_1 a_2} \delta_{a_3 a_4} V(1,2,3,4) + \delta_{a_1 a_3} \delta_{a_2 a_4} V(1,3,2,4) + \delta_{a_1 a_4} \delta_{a_2 a_3} V(1,4,2,3) \Big]$$

where

$$V(1,2,3,4) \equiv V(\kappa_1, \kappa_2; k_1, k_2, k_3, k_4), \tag{2}$$

$$V(1,2,3,4) = \frac{1}{2} g^4 [G(1, 2+3, 4) + ... \tag{3}$$

$$G(a,b,c) = G_{real}(a,b,c) + G_{virtual}(a,b,c)$$

and $G_{real} = K_{2\to 3}$

In forward case and when $c = -a$

$$G(a, -, -a) \to K_{BFKL}(a, -a)$$

Vertex satisfies Ward identities i.e. $\mathcal{V}(k_1, k_2, k_3, k_4) \to 0$ **if any** $k_i = 0$

3.Vertex in the forward direction

Let us consider configuration:

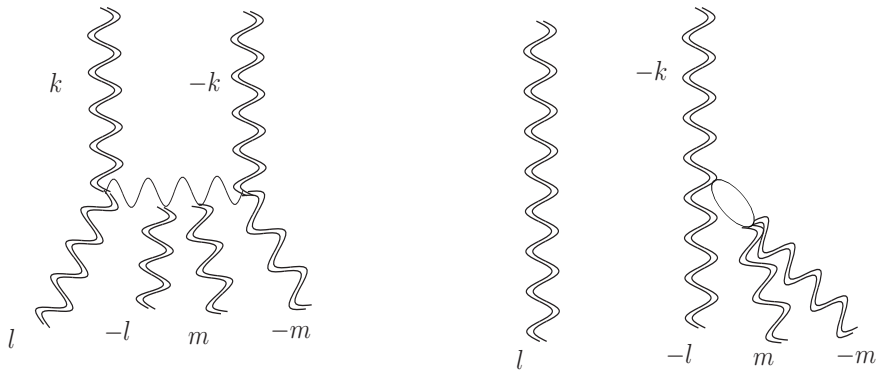

Figure 2:

Vertex simplifies:

$$\mathcal{V}_{a_1'a_2';a_1a_2a_3a_4}(k,-k;l,-l,m,-m) = \frac{\sqrt{2}\pi\delta_{a_1'a_2'}}{\sqrt{N_c^2-1}}$$
$$\times\Bigg[\delta_{a_1a_2}\delta_{a_3a_4}\Big(G(l,-m)+G(-l,-m)+G(l,m)+G(-l,m)\Big)$$
$$+\delta_{a_1a_3}\delta_{a_2a_4}\Big(G(l,-l)+G(m,-m)+G(-l,m)+G(l,m)-G(l,-l-m)$$
$$-G(-l,l+m)-G(m,-l-m)-G(-m,l+m)+G(l+m,-l-m)\Big)$$
$$+\delta_{a_1a_4}\delta_{a_2a_3}(G(l,-l)+...)\Bigg]$$

goal

$\downarrow$

twist structure of TPV.

4.The Triple Pomeron Vertex

Collinear limit

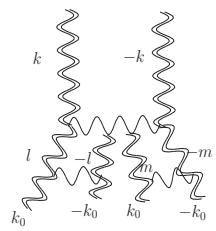

Figure 3:

Connected part

$$G_1(l,m) = \frac{l^2k^2}{(k-m)^2} + \frac{m^2k^2}{(k+m)^2} - \frac{(l+m)^2k^4}{(k-l)^2(k+m)^2} \tag{4}$$

$|k|\gg|l|, |m|$ **Expanding** $G(l,m)$**in** l/k, m/k **we identify:**
twist four

$$G_1(l,m) \simeq 4\pi\, k^2 \left[\frac{2(l\cdot m)^2 - l^2m^2}{k^4}\right] \tag{5}$$

Projection on color singlets in $\delta_{a_1a_2}\delta_{a_3a_4}$

$$\mathcal{V}^{LON_c}_{rl\{a'\}}(l,-l,m,-m) = \tilde{\delta}_{a_1'a_2'} 16\pi \frac{g^4}{2} k^2 \frac{(2(l\cdot m)^2 - l^2m^2)}{k^4} \tag{6}$$

$$K^l_{BFKL} = \frac{2k_0^2l^2}{|k_0^2 - l^2|} \simeq 2k_0^2 \tag{7}$$

$$K^m_{BFKL} = \frac{2k_0^2m^2}{|k_0^2 - m^2|} \simeq 2k_0^2 \tag{8}$$

and convolution with collinear approximated BFKL kernels gives:

$$(K_{2\to2}K_{2\to2}) \otimes \mathcal{V}^{LON_c}_{rl\{a'\}}$$

$$= \tilde{\delta}_{a_1'a_2'} 16\pi k^2 \frac{g^4}{2} \int_{k_0^2}^{k^2} \frac{d^2l}{(2\pi)^3} \int_{k_0^2}^{k^2} \frac{d^2m}{(2\pi)^3} \frac{2k_0^2}{l^4} \frac{2k_0^2}{m^4} \frac{(2(l\cdot m)^2 - l^2m^2)}{k^4} = 0 \tag{9}$$

In collinear approximation we do not get contribution from TPV. Leading twist vanishes→ colinear pole vanishes. Mismach with GLR.

Disconnected part

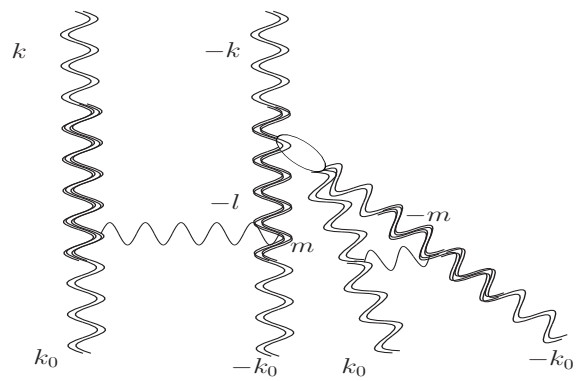

Figure 4:

G_2 **function reads:**

$$G_2(l,m) = -2k^4 ln\frac{|l|}{|l+m|}\delta(l^2-k^2) - 2k^4 ln\frac{|m|}{|l+m|}\delta(m^2-k^2) \tag{10}$$

Convoluting with impact factor and adding rungs we get:

$$\phi \otimes K_{BFKL} K_{BFKL} \otimes \mathcal{V} \simeq \frac{k_0^6}{Q_1^6} log \frac{k_0^6}{Q_1^6} \tag{11}$$

$\uparrow$

negligible in DIS limit

Anticollinear limit

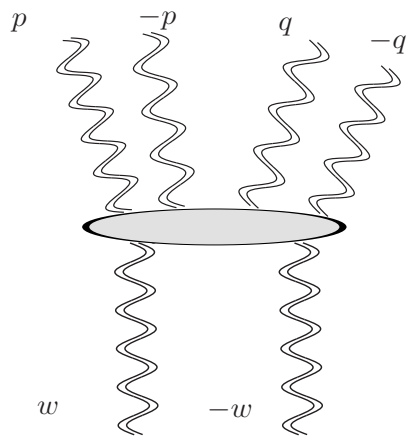

Figure 5:

$$G_1(a,b) = w^2 \left[\frac{1}{(1-\frac{w}{a})^2} + \frac{1}{(1+\frac{w}{b})^2} - \frac{(\frac{w}{a}+\frac{w}{b})^2}{(1-\frac{w}{a})^2(1+\frac{w}{b})^2} \right] \tag{12}$$

$w \ll p, q$ **the leading term is:**

$$\mathcal{V}^{LON_c}_{rl\{b'\}}(p,-p,q,-q) \simeq \tilde{\delta}_{b'_1 b'_2} 2\pi \frac{g^4}{2} 2w^2 \tag{13}$$

$\uparrow$

corresponds to the anticollinear pole in Mellin space

Comments:

- **We do not get logarithmic contribution after convolution with BFKL kernel.**
- **One gets expected logarithms in when one allows non forward configuration.**
- **Disconnected pieces do not give expected fourth power of** log

3. Angular averaged TPV in BK written for unintegrated gluon density

BK$\leftarrow$ fan diagram equation(Bartels,Lipatov,Vacca)
Averaging over angles yields:

$$\downarrow$$

$$\mathcal{V}^{L0N_c}_{\{a\}}(a,-a,b,-b) = \tilde{\delta}_{a_1' a_2'} \frac{g^4}{2}(2\pi)^2 \Bigg[8k^2\theta(a^2-k^2)\theta(b^2-k^2)$$
$$+\frac{1}{2}k^4 ln\left(\frac{a^2}{b^2}\right)\delta(a^2-k^2)\theta(b^2-a^2) + \frac{1}{2}k^4 ln\left(\frac{b^2}{a^2}\right)\delta(b^2-k^2)\theta(a^2-b^2)\Bigg]$$

It's action on Green function $f^{(2)}(a,b) = f^{(1)}(a)f^{(1)}(b)$:

$$\downarrow$$

$$\mathcal{V}^{L0N_c}_{\{a\}}(a,-a,b,-b) \otimes f^{(2)}(a,b) = C\tilde{\delta}_{a_1' a_2'}\Bigg(k^2 \int_{k^2}^{\infty} \frac{da^2}{a^4} f(a^2) \int_{k^2}^{\infty} \frac{db^2}{b^4} f(b^2) +$$
$$+\frac{1}{2} f(k^2) \int_{k^2}^{\infty} \frac{da^2}{a^4} ln\left(\frac{a^2}{k^2}\right) f(a^2) + \frac{1}{2} f(k^2) \int_{k^2}^{\infty} \frac{db^2}{b^4} ln\left(\frac{b^2}{k^2}\right) f(b^2)\Bigg)$$

Comments:

- **main contribution from anticollinear pole**
- **the lower k the longer path of integration, and stronger nonlinear term**
- **presence of disconnected contribution**

The same structure in BK equation for unintegrated gluon density: (Kimber,Martin,Kwieciński),(Kwieciński,K.K)

$$\frac{\partial f(x,k^2)}{\partial \ln 1/x} = \frac{N_c \alpha_s}{\pi} K \otimes f(x,k^2) \tag{14}$$

$$-\frac{2\alpha_s^2}{R^2}\left[k^2\left(\int_{k^2}^{\infty}\frac{dk'^2}{k'^4}f(x,k'^2)\right)^2 + f(x,k^2)\int_{k^2}^{\infty}\frac{dk'^2}{k'^4}\ln\left(\frac{k'^2}{k^2}\right)f(x,k'^2)\right]$$

related through

$$f(x,k^2) = \frac{N_c}{4\alpha_s\pi^2}k^4\nabla_k^2\phi(x,k^2)$$

to:

$$\frac{\partial \phi(x,k^2)}{\partial \ln 1/x} = \overline{\alpha}\pi K \otimes \phi(x,k^2) - \overline{\alpha}\phi^2(x,k^2) \tag{15}$$

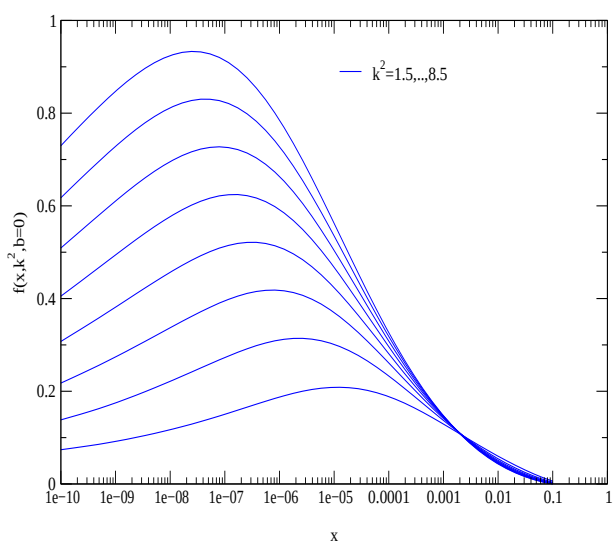

Figure 6:

Conclusions and additional comments

- $2 \longrightarrow 4$ **Vertex projected onto pomerons doesn't give contribution in colinear and large N_c limit**
- **main contribution comes from anticollinear pole**
- **collinear poles contribute at finite N_c**
- **disconnected diagrams give contribution at finite N_c**