BFKL Pomeron in string models

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Gluon reggeization at high energies

Elastic scattering

$$A + B \to A' + B'$$

Regge kinematics

$$s = 4E^2 \gg -t = |q|^2 \approx E^2\,\theta^2$$

QCD Born amplitude

$$M_{AB}^{A'B'}(s,t)|_{Born} = g\,T^c_{A'A}\,\delta_{\lambda_{A'}\lambda_A}\,\frac{2s}{t}\,g\,T^c_{B'B}\,\delta_{\lambda_{B'}\lambda_B}$$

Leading logarithmic approximation

$$M_{AB}^{A'B'}(s,t) = M_{AB}^{A'B'}(s,t)|_{Born}\,s^{\omega(t)},$$

$$\alpha_s \ln s \sim 1\,,\;\; \alpha_s = \frac{g^2}{4\pi} \ll 1$$

Gluon Regge trajectory in LLA

$$\omega(-|q|^2) = -\frac{\alpha_s N_c}{4\pi^2}\int d^2k\,\frac{|q|^2}{|k|^2|q-k|^2} \approx -\frac{\alpha_s N_c}{2\pi}\,\ln\frac{|q^2|}{\lambda^2}$$

Non-perturbative string-type corrections

$$\omega(t) = \omega(t) + \alpha' t + ...$$

String model

Duality in the string model

$$A(s,t,u) = A(s,t) + A(u,t) + A(s,u)\,,$$

$$A(s,t) = \sum_i \frac{c_i(s)}{t-t_i} = \sum_i \frac{c_i(t)}{s-s_i}$$

Linear Regge tajectories

$$j = j_0 + \alpha' t$$

Critical intercepts for Graviton and gluon

$$j_0^G = 2\ ,\ \ j_0^g = 1$$

Vertex for the RNS super-string

$$V(z,\vartheta;k,\xi) = \xi\, D\, X\, e^{-ikX}\ ,\ \ D(z,\vartheta) = \partial_z + \vartheta\,\partial_\vartheta$$

Superfield correlator

$$\left\langle X^M(z,\vartheta) X^N(z',\vartheta') \right\rangle = 2\alpha' \eta^{MN} \ln(z - z' - \vartheta\vartheta')$$

Chan-Paton factors

$$A_n(\{k_j,\xi_j\}) = \sum_{(r)} T_{(r)} A_n^{(r)}(\{k_j,\xi_j\})\,,\ \ T_{(r)} = tr \prod_{r=1}^{n} \lambda_{j_r}$$

Elastic scattering amplitude

Multi-gluon correlator

$$A_n^{(r)}(\{k_j,\xi_j\}) = g^{n-2}(z_{j_1}-z_{j_2})(z_{j_1}-z_{j_n})(z_{j_2}-z_{j_n})$$

$$\times \int \theta(z_{j_2}-z_{j_3}) \prod_{s=3}^{n-1} \theta(z_{j_s}-z_{j_{s+1}}) dz_{j_s} \prod_{r=1}^{n} d\vartheta_{j_r}$$

$$\times \left\langle V(z_{j_1},\vartheta_{j_1};k_{j_1},\xi_{j_1}) \ldots V(z_{j_n},\vartheta_{j_n};k_{j_n},\xi_{j_n}) \right\rangle$$

Asymptotics of the elastic amplitude

$$A_{(ab)}^{(a'b')} = -2g^2\alpha' s\Gamma(-\alpha' t)(\alpha' s)^{\alpha' t}(\xi_a\xi_{a'})(\xi_b\xi_{b'})$$

$$\times \left\{ \sum_s f_{j_a j_{a'} s} f_{s j_b j_{b'}} (e^{-\pi i\alpha' t}+1) \right.$$

$$\left. +(e^{-\pi i\alpha' t}-1)\left[\frac{\delta_{j_a j_{a'}}\delta_{j_b j_{b'}}}{n} + \sum_s d_{j_a j_{a'} s} d_{s j_b j_{b'}} \right] \right\}$$

Soft Pomeron contribution

$$A_{(ab)}^{(a'b')} \sim \Gamma(-\alpha' t)\,(e^{-\pi i\alpha' t}-1)\,\delta_{\lambda_a\lambda_{a'}}\delta_{\lambda_b\lambda_{b'}}\,(\alpha' s)^{1+\alpha' t}$$

Gluon Regge trajectory

Perturbative expansion

$$\omega(t) = \alpha' t + \omega_1(t)$$

One loop correction in the $O(32)$-model

$$\omega_1(t) = -8g^2 n \int_{-1}^{1} \frac{d\lambda}{\lambda} \int_0^1 d\nu_2 \, \frac{(\sin \pi\nu_2)^2}{(1 + L_1} \left(\frac{L_2}{1 + L_1} \right)^{-\alpha' t}$$

$$L_1 = -2 \sum_{n=1}^{\infty} \frac{\lambda^n \, (1 - \lambda^n)^2}{(1 - 2\lambda^n \cos 2\pi\nu_2 + \lambda^{2n})^2}$$

$$L_2 = \prod_{n=1}^{\infty} \frac{(1 - \lambda^n)^4}{(1 - 2\lambda^n \cos 2\pi\nu_2 + \lambda^{2n})^2}$$

Renormalized correction in the $U(N_c)$-model

$$\omega_1(t) = -8g^2 N_c \int_0^1 \frac{d\lambda}{\lambda} \int_0^1 d\nu_2 \frac{(\sin \pi\nu_2)^2}{1 + L_1}$$

$$\times \left(\left(\frac{L_2}{1 + L_1} \right)^{-\alpha' t} - 1 \right)$$

Correction to the pomeron trajectory

$$\omega_1^P(t) \approx \omega_1(t)$$

Multi-particle production

Multi-Regge kinematics

$$s \gg s_r = (k_{r-1} + k_r)^2 \gg q_r^2 \,, \;\; \kappa_r^2 = \frac{s_r \, s_{r+1}}{(k_{r-1} + k_{r+1})^2}$$

Production amplitude

$$A = 2\, s\, g T^{c_1}_{A'A}\, \delta_{\lambda_{A'}\lambda_A} \frac{s_1^{\omega(t_1)}}{t_1} g\, T^{d_1}_{c_2 c_1} C(q_2, q_1) ... g T^{c}_{B'B}\, \delta_{\lambda_{B'}\lambda_B}$$

BFKL equation

$$\left(\omega - \omega_1(-l^2) - \omega_1(-(q-l)^2)\right) f_\omega(\omega; q; l, l_1)$$

$$= \widehat{I}(q; l, l_1) + \frac{g^2 N_c c_s}{(2\pi)^{D-1}} \int \widehat{I}(q; l, l') f_\omega^{(s)}(q; l', l_1) d^{D-2} l'$$

Massless contribution

$$(\kappa_0^2)^2 \widetilde{I}_0(q; l, l') = \alpha'[-q^2 + (l')^2 + (q-l')^2] J^{-1,0}_{l',l} J^{-1,0}_{q-l',q-l}$$

$$-\alpha'[l^2 J^{-1,-1}_{l',l} J^{0,0}_{q-l',q-l} + (q-l)^2 J^{0,0}_{l',l} J^{-1,-1}_{q-l',q-l}] - J^{0,0}_{l',l} J^{-1,0}_{q-l',q-l}$$

$$-J^{-1,0}_{l',l} J^{0,0}_{q-l',q-l} \,, \;\; J^{r',r}_{l',l} = \int_0^\infty dt e^{-t} t^a (z+t)^b \,,$$

$$a = -(-1)^{r'} \alpha' l' + r' \,, \;\; b = -(-1)^r \alpha' l + r \,, \;\; z = \alpha'(l - l')^2)$$

Massive particle production

Ultaviolet divergency in M_r^2

$$S_\omega(q;l,l') \sim \sum_{\alpha' M_r^2=1}^{\infty} \frac{(\alpha' M_r^2)^{-\omega-\alpha' q^2}}{M_r^2}$$

Massive particle contribution

$$S_\omega(q;l,l') \approx \frac{\alpha'^{\,3}[q^2-l^2-(q-l)^2][q^2-l'^{\,2}-(q-l')^2]}{\omega+\alpha' q^2}$$

$$\times\Big[\tilde{F}(\alpha'(l')^2,\alpha'(q-l')^2;\alpha' q^2-\alpha'(l')^2-\alpha'(q-l')^2)$$

$$+\tilde{F}(1-\alpha' q^2,\alpha'(q-l')^2;\alpha' q^2-\alpha'(l')^2-\alpha'(q-l')^2)$$

$$+\tilde{F}(1-\alpha' q^2,\alpha'(l')^2;\alpha' q^2-\alpha'(l')^2-\alpha'(q-l')^2)\Big],$$

$$\tilde{F}(a,b,c)=\int_0^1 df\int_0^1 dy f^{a+b-1}(1+f+fy)^{c-1}[y^{b-1}+y^{a-1}$$

Small momenta asymptotics and soft Pomeron

$$S_\omega^{sing}(q;l,l')=\frac{[q^2-l^2-(q-l)^2][q^2-l'^{\,2}-(q-l')^2]}{\alpha'^{\,-1}(\omega+\alpha' q^2)(l')^2(q-l')^2}$$

Four-dimension case

Equation with linear gluon trajectories

$$\omega\phi = \hat{K}\,\phi\,,\;\; \hat{K} = \hat{K}_{BFKL} - \alpha'|p|_1^2 - \alpha'|p|_2^2$$

Small-t asymptotics

$$\alpha'\vec{q}^{\,2} \ll g^2\,N_c$$

Solution at small ρ

$$E_{m,\widetilde{m}}(\vec{q},\vec{\rho}) \sim \rho^m\,(\rho^*)^{\widetilde{m}} + e^{i\,\delta_{m,\widetilde{m}}(\vec{q})}\,\rho^{1-m}\,(\rho^*)^{1-\widetilde{m}}\,,$$

$$e^{i\,\delta_{m,m}(\vec{q})}_{|_{n=0}} = \left(\frac{|q|}{4}\right)^{-4i\nu}\,\frac{\Gamma^2(1+i\nu)}{\Gamma^2(1-i\nu)}$$

Diffusion approximation

$$E\phi = \left(-4\ln 2 - 14\,\zeta(3)\,(\partial/\partial z)^2 + \lambda e^z\right)\phi\,,$$

$$z = \ln(\alpha' k^2)\,,\;\; \lambda = \frac{4\pi^2}{g^2 N_c}$$

Quantization of Pomeron trajectories

$$2\sqrt{\frac{E_r + 4\ln 2}{14\,\zeta(3)}}\,\ln\left(\frac{7\zeta(3)\,g^2\,N_c}{2\pi^2\alpha'\vec{q}^{\,2}}\right) = 2\pi(r+1/4)$$

BFKL equation in $N = 4$ SUSY (A.K., L.L.)

Eigenvalue of the kernel

$$\omega = 4\,\hat{a}\;\chi(n,\gamma) + 4\;\hat{a}^2\,\Delta(n,\gamma)\,,\;\; \hat{a} = g^2 N_c/(16\pi^2)$$

Non-analyticity in QCD

$$\Delta_{QCD}(n,\gamma) = c_0\delta_{n,0} + c_2\delta_{n,0} + ...$$

Hermitial separability in $N = 4$ SUSY

$$\Delta(n,\gamma) = \phi(M) + \phi(M^*) - \frac{\rho(M) + \rho(M^*)}{2\hat{a}/\omega}\,,\; M = \gamma + \frac{|n|}{2}\,,$$

$$\rho(M) = \beta'(M) + \frac{1}{2}\zeta(2)\,,\; \beta'(z) = \frac{1}{4}\left[\Psi'\left(\frac{z+1}{2}\right) - \Psi'\left(\frac{z}{2}\right)\right]$$

Property of maximal complexity

$$\phi(M) = 3\zeta(3) + \Psi''(M) - 2\Phi(M) + 2\beta'(M)\Big(\Psi(1) - \Psi(M)\Big)$$

$$\Phi(M) = \sum_{k=0}^{\infty} \frac{\beta'(k+1)}{k+M} +$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k+M}\left(\Psi'(k+1) \;-\; \frac{\Psi(k+1) - \Psi(1)}{k+M}\right)$$

Relation between BFKL and DGLAP equations

Operator expansion of the Green function

$$< \phi(\overrightarrow{\rho_1})\, \phi(\overrightarrow{\rho_2})\, \phi(\overrightarrow{\rho_{1'}})\, \phi(\overrightarrow{\rho_{2'}}) >$$

$$\sim < \phi(\overrightarrow{\rho_1})\, \phi(\overrightarrow{\rho_2}) \sum_n |\rho_{1'2'}|^{2\Gamma_\omega(n)} \left(\frac{\rho_{1'2'}}{\rho^*_{1'2'}}\right)^{\frac{|n|}{2}} O_{|n|,\nu_\omega}(\overrightarrow{\rho_{1'}}) >$$

Perturbative expansion

$$\Gamma_\omega = 1 + \frac{|n|}{2} - \gamma_n(\omega)\,, \;\; \gamma_n(\omega)|_{\omega\to 0} = \frac{g^2 N_c}{4\pi^2\omega} + O(g^4)$$

Hypothesis about $\gamma(j)$ (A.K., L.L. (2000))

$$\lim_{j\to -r} \gamma(j) = \lim_{|n|\to -r-1} \gamma_n(j+r)$$

Predictions (A.K., L.L. (2003))

$$\gamma(j)|_{j\to -r} \;=\; \frac{g^2 N_c}{4\pi^2}\frac{1}{j+r} + \left(\frac{g^2 N_c}{4\pi^2}\right)^2 \frac{1+(-1)^r}{2(j+r)^3} + \ldots$$

Independent calculation (L.L. (1997))

$$\gamma(j) \;=\; \frac{g^2 N_c}{16\pi^2}\gamma^{LLA}(j), \;\; \gamma^{LLA}(j) \;=\; 4\Big(\Psi(1) - \Psi(j-1)\Big)$$

Integrable Heisenberg spin model (L.L. (1997))

One loop anomalous dimensions

Wilson twist-2 operators

$$\mathcal{O}^g_{\mu_1,\ldots,\mu_j} = \hat{S} G^a_{\rho\mu_1} D_{\mu_2} D_{\mu_3} \ldots D_{\mu_{j-1}} G^a_{\rho\mu_j} \,,$$

$$\tilde{\mathcal{O}}^g_{\mu_1,\ldots,\mu_j} = \hat{S} G^a_{\rho\mu_1} D_{\mu_2} D_{\mu_3} \ldots D_{\mu_{j-1}} \tilde{G}^a_{\rho\mu_j} \,,$$

$$\mathcal{O}^q_{\mu_1,\ldots,\mu_j} = \hat{S} \bar{\Psi}^a \gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_j} \Psi^a \,,$$

$$\tilde{\mathcal{O}}^q_{\mu_1,\ldots,\mu_j} = \hat{S} \bar{\Psi}^a \gamma_5 \gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_j} \Psi^a \,,$$

$$\mathcal{O}^\varphi_{\mu_1,\ldots,\mu_j} = \hat{S} \bar{\Phi}^a D_{\mu_1} D_{\mu_2} \ldots D_{\mu_j} \Phi^a$$

Anomalous dimension matrix (L.L. (1999))

$$\gamma_{gg} = -\frac{8}{j-1} + \frac{8}{j} - \frac{8}{j+1} + \frac{8}{j+2} + 8S_1(j)\,, \quad \gamma_{q\varphi} = -\frac{16}{j}\,,$$

$$\gamma_{gq} = -\frac{8}{j-1} + \frac{8}{j} - \frac{4}{j+1}\,, \quad \gamma_{g\varphi} = -\frac{8}{j-1} + \frac{8}{j}\,,$$

$$\gamma_{qg} = -\frac{16}{j} + \frac{32}{j+1} - \frac{32}{j+2}\,, \quad \gamma_{qq} = -\frac{16}{j} + \frac{16}{j+1} + 8S_1(j),$$

$$\gamma_{\varphi g} = -\frac{24}{j+1} + \frac{24}{j+2}\,, \quad \gamma_{\varphi q} = -\frac{12}{j+1}\,, \quad \gamma_{\varphi\varphi} = 8S_1(j),$$

$$\widetilde{\gamma}_{gg} = -\frac{16}{j} + \frac{16}{j+1} + 8S_1(j)\,, \quad \widetilde{\gamma}_{gq} = -\frac{8}{j} + \frac{4}{j+1}\,,$$

$$\widetilde{\gamma}_{qg} = \frac{16}{j} - \frac{32}{j+1}\,, \quad \widetilde{\gamma}_{qq} = \frac{8}{j} - \frac{8}{j+1} + 8S_1(j)$$

Two-loop universal anomalous dimension

Diagonalization in the Born Approximation

$$\begin{vmatrix} 8S_1(j-2) & 0 & 0 \\ 0 & 8S_1(j) & 0 \\ 0 & 0 & 8S_1(j+2) \end{vmatrix},$$

$$\begin{vmatrix} 8S_1(j-1) & 0 \\ 0 & 8S_1(j+1) \end{vmatrix}$$

Universal anomalous dimension

$$U\gamma U^+ = \gamma_{uni}(j)\,,\ \gamma^{(0)}_{uni}(j) = -4S_1(j-2)\,,\ S_r(j) = \sum_{i=1}^{j}\frac{1}{i^r}$$

Most transcendental functions (A.K.,L.L. (2000))

$$\gamma_{uni}(j) = \hat{a}\gamma^{(0)}_{uni}(j) + \hat{a}^2\gamma^{(1)}_{uni}(j) + \hat{a}^3\gamma^{(2)}_{uni}(j) + \ldots$$

Two-loop dimension (A.K.,L.L.,V.V (2003))

$$\gamma^{(1)}_{uni}(j+2)/8 = 2S_1(j)\left(S_2(j)+S_{-2}(j)\right)-2S_{-2,1}(j)+S_3(j)$$

$$+S_{-3}(j),\ S_{-r}(j) = \sum_{i=1}^{j}\frac{(-1)^i}{i^r}, S_{-2,1} = \sum_{m=1}^{j}\frac{(-1)^m}{m^2}S_1(m)$$

Three-loop anomalous dimension

QCD dimension matrix (S.M., J.V., A. V. (2004)

$$\gamma_{QCD}^{ik}(j) = ..., \;\; i,k = g,q$$

N=4 universal dimension (A.K.,L.L.,A.O.,V.V.)

$$\gamma_{uni}^{N=4}(j) = \hat{a}\gamma_{uni}^{(0)}(j) + \hat{a}^2\gamma_{uni}^{(1)}(j) + \hat{a}^3\gamma_{uni}^{(2)}(j) + \dots,$$

$$\gamma_{uni}^{(2)}(j+2)/32 = -12\,(S_{-3,1,1} + S_{-2,1,2} + S_{-2,2,1})$$
$$+6\,(S_{-4,1} + S_{-3,2} + S_{-2,3}) - 3\,S_{-5} - 2\,S_3\,S_{-2} - S_5$$
$$-2\,S_1^2\,(3\,S_{-3} + S_3 - 2\,S_{-2,1}) - S_2\,(S_{-3} + S_3 - 2\,S_{-2,1})$$
$$+24\,S_{-2,1,1,1} - S_1\left(8S_{-4} + S_{-2}^2 + 4S_2S_{-2} + 2S_2^2\right)$$
$$-S_1\,(3S_4 - 12S_{-3,1} - 10S_{-2,2} + 16S_{-2,1,1})$$

Harmonic sums

$$S_a(j) = \sum_{m=1}^{j} \frac{1}{m^a}\,, \;\; S_{a,b,c,\dots}(j) = \sum_{m=1}^{j} \frac{1}{m^a}\,S_{b,c,\dots}(m)\,,$$

$$S_{-a}(j) = \sum_{m=1}^{j} \frac{(-1)^m}{m^a}, \; S_{-a,b,\dots}(j) = \sum_{m=1}^{j} \frac{(-1)^m}{m^a}\,S_{b,\dots}(m),$$

$$\overline{S}_{-a,b,c\dots}(j) = (-1)^j\,S_{-a,b,\dots}(j) + S_{-a,b,\dots}(\infty)\left(1-(-1)^j\right)$$

Comparison with other approaches (A.K.,L.L.,A.O.,V.V.)

Singularities at $j = 1 + \omega \to 0$

$$\gamma_{uni}^{N=4}(j) = \hat{a}\frac{4}{\omega} - 32\zeta_3\,\hat{a}^2 + 32\zeta_3\,\hat{a}^3\frac{1}{\omega} + \ldots$$

DL resummation at $j + 2r = \omega \to 0$

$$\gamma_{uni} = 4\,\frac{\hat{a}}{\omega} + \frac{\gamma_{uni}^2}{\omega}$$

Anomalous dimensions at large j

$$\gamma_{uni} = a(z)\,\ln j\,,\;\; z = \frac{\alpha N_c}{\pi} = 4\hat{a}$$

Perturbative results

$$a = -z + \frac{\pi^2}{12}\,z^2 - \frac{11}{720}\pi^4 z^3 + \ldots$$

Polyakov AdS/CFT prediction

$$\lim_{z\to\infty} a = -z^{1/2} + \frac{3\ln 2}{4\pi} + \ldots$$

Resummation

$$\widetilde{a} = -z + \frac{\pi^2}{12}\,\widetilde{a}^2 = -z + \frac{\pi^2}{12}\,z^2 - \frac{1}{72}\pi^4 z^3 + \ldots$$

Pomeron and graviton at $N = 4$ SUSY (OKLV)

BFKL Pomeron in a diffusion approximation

$$j = 2 - \Delta - D\,\nu^2$$

Anomalous dimension of twist-2 operators

$$\gamma = 1 + \frac{j-2}{2} + i\nu$$

Constraint from the conservation of $T_{\mu\nu}$

$$\gamma = (j-2)\left(\frac{1}{2} - \frac{1/\Delta}{1+\sqrt{1+(j-2)/\Delta}}\right)$$

AdS/CFT for the graviton Regge trajectry

$$j = 2 + \frac{\alpha'}{2}\,t\,,\ t = E^2/R^2\,,\ \alpha' = \frac{R^2}{2}\,\Delta$$

Large coupling asymptotics

$$\gamma_{|z\to\infty} = -\sqrt{j-2}\,\Delta^{-1/2}_{|j\to\infty} = \sqrt{\pi j}\,z^{1/4}$$

Pomeron intercept at large α and its perturbative estimate

$$j = 2 - \Delta\,,\ \Delta = \frac{1}{\pi}\,z^{-1/2} \approx \frac{\sqrt{3}}{2\pi}z^{-1/2}$$

Resummation of the DGLAP and BFKL equations

Slope of the anomalous dimension

$$\gamma'(2) = \frac{1}{2} - \frac{1}{2\Delta} = -\frac{\pi^2}{6} z + \frac{\pi^4}{72} z^2 - \frac{\pi^6}{540} z^3 + \ldots$$

Resummation equation

$$\frac{\pi^2}{6} z = -\widetilde{b} + \frac{1}{2}\widetilde{b}^2 \,, \;\; b = \gamma'(2)$$

Weak and strong coupling asymptotics

$$\widetilde{b} = -\frac{\pi^2}{6} z + \frac{\pi^4}{72} z^2 - \frac{\pi^6}{432} z^3 + \ldots \,, \;\; \widetilde{\Delta} = \frac{\sqrt{3}}{2\pi} z^{-1/2}$$

Two loop BFKL results

$$\omega_0 = 1 - \Delta = 4 \ln 2 \, z - a_1 \, z^2 + \ldots$$

$$D = 14 \, \zeta(3) \, z - b_1 z^2 + \ldots \,, \;\; \zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3}$$

Resummation for large couplings

$$4 \ln 2 \, z = \frac{(1-\Delta) - \left(2 - a_1 \, (4 \, \ln 2)^{-2}\right) (1-\Delta)^2}{\Delta^2}$$

$$14 \, \zeta(3) \, z = \frac{1 - \sqrt{1 - 2 \, D^3 + c D^4}}{D^2}$$

Discussion

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2. Soft pomeron in string theories
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