

Conformal Pomeron and Odderon in Strong Coupling*

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Abstract

We discuss how exact conformal invariance in the strong coupling leads naturally through AdS/CFT correspondence to a systematic expansion for the Pomeron and Odderon intercepts in power of $\lambda^{-1/2}$, with $\lambda = g^2 N_c$ the 't Hooft coupling. We also point out the importance of confinement for a realistic treatment of DIS in the HERA energy range.

1 Introduction

In the past decade overwhelming evidence has emerged for a conjectured duality between a wide class of gauge theories in d -dimensions and string theories on asymptotically AdS_{d+1} spaces. It has been shown, in a holographic or AdS/CFT dual description for QCD at high energies, the Pomeron can be identified with a reggeized Graviton in AdS_5 [1,2] and, similarly, an Odderon with a reggeized anti-symmetric Kalb-Ramond B -field [3]. This approach has been successfully applied to the study of HERA data [4], both for DIS at small- x [5] and for

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deeply virtual Compton scattering (DVCS) [6]. More recently, this treatment has also been applied to the study of diffractive production of Higgs at LHC [7] as well as other near forward scattering processes.

In this talk, we first briefly describe “Pomeron-Graviton” duality and its application for deep inelastic scattering (DIS) at small- x and deep virtual Compton scattering (DVCS) to HERA data. We next turn to a discussion on Pomeron and Odderon intercepts in the conformal limit and their relation to the anomalous dimensions.

2 Pomeron-Graviton Duality and Applications:

It can be shown for a wide range of scattering processes that the amplitude in the Regge limit, $s \gg t$, is dominated by Pomeron exchange, together with the associated s -channel screening correction, e.g., via eikonalization. We will use here a formulation based on gauge/gravity duality, or the *AdS/CFT* correspondence, of which one particular example is the duality between $\mathcal{N} = 4$ SYM and Type-IIB string theory on $AdS_5 \times S^5$. This approach has the advantages of allowing us to study the strong coupling region, providing a unified soft and hard diffractive mechanism, and as we will see it also fits well the experimental data.

Traditionally the Pomeron has been modeled at weak coupling using perturbative QCD; in lowest order, a bare Pomeron was first identified by Low and Nussinov as a two gluon exchange corresponding to a Regge cut in the J -plane at $j_0 = 1$. Going beyond the leading order, Balitsky, Fadin, Kuraev and Lipatov (BFKL) summed generalized two gluon exchange diagrams to first order in $\lambda = g^2 N_c$ and *all* orders in $(\lambda \log s)^n$, giving rise to the so-called BFKL Pomeron¹, which corresponds to a J -plane cut at $j_0 = 1 + \log(2)\lambda/\pi^2$.

In a holographic approach, the weak coupling Pomeron is replaced by the “Regge graviton” in AdS space, as formulated by Brower, Polchinski, Strassler and Tan (BPST) [1] which has both hard components due to near conformality in the UV and soft Regge behavior in the IR. Strong coupling corrections lower the intercept from $j = 2$ to

$$j_0 = 2 - 2/\sqrt{\lambda}. \tag{1}$$

In Fig. 1a, we compare the BPST Pomeron intercept with the weak coupling BFKL intercept for $\mathcal{N} = 4$ YM as a function of 't Hooft coupling λ . A typical

¹See [8] and references cited therein.

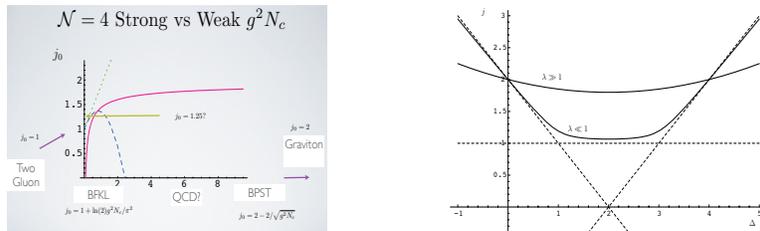


Figure 1: On the left, (a), intercept as a function of λ for the BPST Pomeron (solid red) and for BFKL (dotted and dashed to first and second order in λ respectively). On the right, (b), the conformal invariant $\Delta - j$ curve which controls both anomalous dimensions and the Pomeron intercept.

phenomenological estimate for this parameter for QCD is about $j_0 \simeq 1.25$, which suggests that the physics of diffractive scattering is in the cross over region between strong and weak coupling. A corresponding treatment for Odderons has also been carried out [3].

2.1 BPST Pomeron Intercept in Conformal Limit:

Let us begin by first examining briefly the concept of a BPST Pomeron in the general context of conformal field theories (CFT). A CFT 4-point correlation function $\mathcal{A} = \langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle$ can be analyzed in an operator product expansion (OPE) by summing over allowed primary operators $\mathcal{O}_{k,j}$, with integral spin j and dimensions $\Delta_k(j)$, and their descendants. It can be shown that the leading behavior in the Regge limit for the crossing even CFT correlation functions, appropriate for Pomeron exchange, are controlled by a set of dominant single-trace primary operators $\mathcal{O}_{G,j}$, one for each j and j even, with conformal dimensions $\Delta_G(j)$. (We will return to this treatment in Sec. 3.) It is useful to express these scaling dimensions as

$$\Delta_G(j) = \tau_G + j + \gamma_G(j), \quad (2)$$

where τ_G is the twist ² and $\gamma_G(j)$ are the anomalous dimensions. The lowest j in this set has $j = 2$, which is the energy-momentum tensor. Due to energy-momentum conservation, $\gamma_G(2)$ vanishes and $\Delta_G(4) = 4$. For simplicity, we will drop the subscript G in what follows.

Using AdS/CFT in strong coupling, it was shown in [1] that $\gamma(j)$ is analytic in j , so that one can expand $\Delta(j)$ about $j = 2$ as $\Delta(j) = 4 + \alpha_1(\lambda)(j - 2) +$

² $\tau_G = 2$ is the minimal twist which dominates the amplitude.

$O((j-2)^2)$, with the coefficient $\alpha_1(\lambda) = \sqrt{\lambda}/4 + O(1)$ in the strong coupling limit. Equivalently, one has an expansion

$$(\Delta(j) - 2)^2 = 4 + 4\alpha_1(\lambda)S + O(S^2) \quad (3)$$

where we have simplified the expression by introducing S for $j-2$. This notation is also useful for the discussion in Sec. 3 where we generalize the treatment to higher order in $1/\sqrt{\lambda}$ and also to the case of Odderons.

It was stressed in [1] that the $\Delta - j$ curve must be symmetric about $\Delta = 2$ due to conformal invariance, and, by inverting $\Delta(j)$, one has

$$j(\Delta) = j(2) + \alpha_1(\lambda)^{-1}(\Delta - 2)^2 + O((\Delta - 2)^4) \quad (4)$$

At large λ , the curve $j(\Delta)$ takes on a minimum at $\Delta = 2$, as exhibited in Fig. 1b. The Pomeron intercept is simply the minimum of $j(\Delta)$ curve at $\Delta = 2$, that is, $j_0 = j(2)$. In particular, it admits an expansion in $1/\sqrt{\lambda}$, $\alpha_P = j_0 = 2 - \frac{2}{\lambda^{1/2}} + \frac{a_2}{\lambda} + \frac{a_3}{\lambda^{3/2}} + \frac{a_4}{\lambda^2} + \dots$. The leading term corresponds to a graviton exchange and the first order correction comes from the classical contribution from string modes. We will return to higher order terms in Sec. 3.

2.2 Holographic Treatment of DIS and DVCS:

In the holographic approach, the impact parameter space (b_\perp, z) is 3-dimensional, where $z \geq 0$ is the warped radial 5th dimension. Conformal dilatations, ($z \rightarrow cz$ with c a constant), take one from the UV boundary at $z = 0$ deep into the IR $z = \text{large}$. The near forward elastic amplitude $A(s, t)$, where $t = -q_\perp^2$, in a transverse AdS_3 representation, $A(s, t) = \int d^2b e^{i\vec{q}\cdot\vec{b}} \int dz dz' \tilde{A}(s, b, z, z')$, can be written in an eikonal form

$$\tilde{A}(s, b, z, z') = 2isP_{13}(z)P_{24}(z')\{1 - e^{i\chi(s, b, z, z')}\}. \quad (5)$$

When expanded to first order in the eikonal function, it leads to the contribution from exchanging a single Pomeron, with $\chi(s, b, z, z') = \frac{g_0^2}{2s}(\frac{R^2}{zz'})^2\mathcal{K}(s, b, z, z')$, where $\mathcal{K}(s, b, z, z')$, is the BPST Pomeron kernel in a transverse AdS_3 representation [1, 2]. In the conformal limit, a simple expression for $\mathcal{K}(s, b, z, z')$ can be found [2]. Confinement can next be introduced, e.g., via a hardwall model $z < z_{cut-off}$. The effect of saturation can next be included via the full transverse AdS_3 eikonal representation (5).

An important unifying feature for our treatment is factorization in the AdS space. For hadron-hadron scattering, $P_{ij}(z) = \sqrt{-g(z)}(z/R)^2\phi_i(z)\phi_j(z)$ involves a product of two external normalizable wave functions for the projectile

and the target respectively. For scattering involving external currents, we can simply replace P_{13} by product of the appropriate unnormalized wave-functions.

We next make use of the fact that the DIS cross section can be related to the imaginary part of the forward amplitude via the optical theorem, $\sigma = s^{-1}\text{Im}A(s, t = 0)$. Therefore, the AdS/CFT amplitude gives expressions for all structure functions F_i . We shall focus here on $F_2(x, Q^2) = \frac{Q^2}{\pi^2\alpha_{em}}(\sigma_T(\gamma^*p) + \sigma_L(\gamma^*p))$. In the conformal limit, P_{13} was calculated in [9] in terms of Bessel functions, so that, to obtain F_2 , we simply replace in (5), $P_{13}(z) \rightarrow P_{13}(z, Q^2) = \frac{1}{z}(Qz)^2(K_0^2(Qz) + K_1^2(Qz))$. For DVCS, states 1 and 3 are replaced by currents for an off-shell and an on-shell photon respectively, with P_{13} given by product of unnormalized wave-functions for appropriate R-currents. We can calculate these by evaluating the R-current - graviton Witten diagram in AdS, and we get $P_{13}(z) = -C \frac{\pi^2}{6} z^3 K_1(Qz)$. Here C is a normalization constant that can be calculated in the strict conformal limit. The DVCS cross section and differential cross section can then be calculated from $A(s, t)$ via $\frac{d\sigma}{dt}(x, Q^2, t) = \frac{|A|^2}{16\pi s^2}$ and $\sigma(x, Q^2) = \frac{1}{16\pi s^2} \int dt |A|^2$.

2.3 Pomeron Kernel:

The leading order BFKL Pomeron has remarkable properties. It enters into the first term in the large N_c expansion with zero beta function. Thus it is in effect the weak coupling cylinder graph for the Pomeron for a large N_c conformal theory, the same approximations used in the AdS/CFT approach albeit at strong coupling. Remarkable BFKL integrability properties allows one to treat the BFKL kernel as the solution to an $SL(2, \mathcal{C})$ conformal spin chain. Going to strong coupling, the two gluon exchange evolves into a closed string of infinitely many tightly bound gluons, but the same underlying symmetry persists—referred to as Möbius invariance in string theory or the isometries of the transverse AdS_3 impact parameter geometry. The position of the j -plane cut moves from $j_0 = 1 + \log(2)\lambda/\pi^2$ up to $j_0 = 2 - 2/\sqrt{\lambda}$.

The BPST Pomeron kernel in the J -plane, $G_j(t, z, z')$, obeys a Schrödinger equation on AdS_3 space, with j serving as eigenvalue for the Lorentz boost operators M_{+-} . In the conformal limit, $G_j(t, z, z') = \int_0^\infty \frac{dq^2}{2} \frac{J_{\tilde{\Delta}(j)}(zq)J_{\tilde{\Delta}(j)}(qz')}{q^2 - t}$, with $\tilde{\Delta}(j)^2 = 2\lambda(j - j_0)$. The full Pomeron kernel can then be obtained via an inverse Mellin transform. In the mixed-representation, one has

$$K(s, b, z, z') \sim - \int \frac{dj}{2\pi i} \tilde{s}^j \frac{e^{-i\pi j} + 1}{\sin \pi j} \frac{e^{(2-\Delta(j))\eta}}{\sinh \eta} \quad (6)$$

where $\cosh \eta$ is the chordal distance in AdS_3 . By integrating over \vec{b} , one obtains

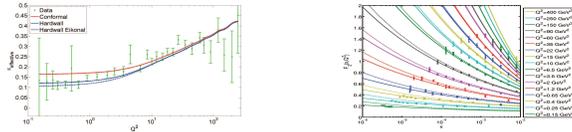


Figure 2: On the left, (a), Q^2 dependence for “effective intercept”. On the right, (b), fits by a single hardwall Pomeron vs hardwall eikonal.

for the imaginary part of the Pomeron kernel at $t = 0$

$$\text{Im } \mathcal{K}(s, t = 0, z, z') \sim \frac{s^{j_0}}{\sqrt{\pi \mathcal{D} \log s}} e^{-(\log z - \log z')^2 / \mathcal{D} \log s}, \quad (7)$$

which exhibits diffusion in the “size” parameter, $\log z$, for the exchanged closed string. This is analogous to the BFKL kernel at weak coupling where diffusion takes place in $\log(k_\perp)$, the virtuality of the off shell gluon dipole. The diffusion constant becomes $\mathcal{D} = 2/\sqrt{g^2 N_c}$ at strong coupling compared to $\mathcal{D} = 7\zeta(3)g^2 N_c/2\pi^2$ in weak coupling. The close analogy between the weak and strong coupling Pomeron suggests the development of a hybrid phenomenology leveraging plausible interpolations between the two extremes.

2.4 Fit to HERA Data

Both of these integrals, z and z' in (5), remain sharply peaked, the first around $z \sim 1/Q$ and the second around the inverse proton mass, $z' \equiv 1/Q' \sim 1/m_p$. We approximate both of them by delta functions. Under such an “ultra-local” approximation, all structure functions take on very simple form, e.g,

$$F_2(x, Q^2) = \frac{g_0^2}{8\pi^2 \lambda} \frac{Q}{Q'} \frac{e^{(j_0 - 1) \tau}}{\sqrt{\pi \mathcal{D} \tau}} e^{-(\log Q - \log Q')^2 / \mathcal{D} \tau} + \text{Confining Images},$$

with diffusion time given more precisely as $\tau = \log(s/QQ'\sqrt{\lambda}) = \log(1/x) - \log(\sqrt{\lambda}Q'/Q)$. Here the first term is conformal and, for the hardwall, the confining effect can be expressed in terms of image charges [5]. It is important to note that taking the $s \rightarrow \infty$ limit, the single-Pomeron amplitude grows asymptotically as $(1/x)^{j_0} \sim s^{j_0}$, thus violating the Froissart unitarity bound at very high energies. The eikonal approximation in AdS space [2, 10] plays the role of implementing “saturation” to restore unitarity via multi-Pomeron shadowing.

We have shown various comparisons of our results [11] to the small-x DIS data from the combined H1 and ZEUS experiments at HERA [4] in Fig. 2. Both

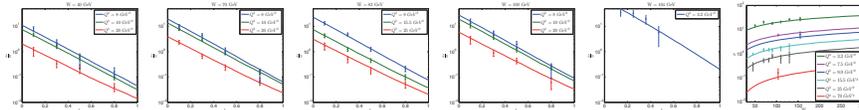


Figure 3: Fits by the hard wall Pomeron to HERA data. The first 5 correspond to the differential cross section, and the last one to the cross section.

the conformal, the hard-wall model as well as the eikonalized hard-wall model can fit the data reasonably well. This can best be seen in Fig. 2a to the left which exhibits the Q^2 dependence of an effective Pomeron intercept. This can be understood as a consequence of diffusion. However, it is important to observe that the hard-wall model provides a much better fit than the conformal result for Q^2 less than $2 \sim 3 \text{ GeV}^2$. The best fit to data is obtained using the hard-wall eikonal model, with a $\chi^2 = 1.04$. This is clearly shown by Fig. 2b to the right, where we present a comparison of the relative importance of confinement versus eikonal at the current energies. We observe that the transition scale $Q_c^2(x)$ from conformal to confinement increases with $1/x$, and it comes before saturation effect becomes important. For more details, see Ref. [11].

We now compare our model to the measurements at HERA. Related papers using AdS/CFT correspondence applied to DVCS can be found in [6]. We use conformal form for the photon wave functions and a delta function for the proton. Note that Eq. (7) is for the conformal model, and the hard wall expression would include another term with the contribution due to the presence of the hard wall. We obtain a good agreement with experiment, with χ^2 varying from $0.51 - 1.33$ depending on the particular data and model we are considering. We find that confinement starts to play a role at small $|t|$, and the hardwall fits the data better in this region. In Fig. 3 we present the plots for our fits. (See [6] for details.)

3 Regge Limit in CFT, Anomalous Dimensions, Conformal Pomeron and Odderon

Let us return to take a closer look at the Regge limit for a CFT. Consider a connected 4-point correlation function for a scalar field $\phi(x)$ of dimension Δ_0 , $\mathcal{A} = \langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle$, which can be expressed as $\mathcal{A} = x_{12}^{-\Delta_0} x_{34}^{-\Delta_0} F(u, v)$. Here $x_{ij}^2 = (x_i - x_j)^2$, and (u, v) are two conformal invariant cross ratios,

$u = x_{12}^2 x_{34}^2 / x_{14}^2 x_{23}^2$, and $v = x_{13}^2 x_{24}^2 / x_{14}^2 x_{34}^2$. It can formally be expanded in an operator product expansion, which, in turn, can be expressed in a conformal partial-wave expansion, $F(u, v) = \sum_k \sum_{j, \Delta_{k,j}} C_{k,j} G_j(u, v; \Delta_{k,j})$. Here $G_j(u, v; \Delta_{k,j})$ are the ‘‘conformal blocks’’, associated with a primary, $\mathcal{O}_{k,j}$, which includes contributions from its descendants. For each j , there can be many such primaries, labelled by the index k . Of these, we will be interested in the family of primaries, $\mathcal{O}_{G,j}$, for even j , which interpolate with the energy-momentum tensor at $j = 2$. Since we are dealing with a CFT with AdS dual, this family of primaries are that dual to string modes associated with the graviton, with spin $j = 2, 4, \dots$, i.e., those given by Eq. (2). The challenge is to find the associated anomalous dimensions in the strong coupling limit.

In a coordinate treatment, it can be shown that the Regge limit corresponds to the simultaneous limits of $u \rightarrow 0$ and $v \rightarrow 1$, with $\xi = (1 - v) / \sqrt{u}$ fixed. In the Euclidean region, $G_j(u, v; \Delta_{k,j}) \sim u^{\Delta_{k,j}/2} \sim 0$. However, the Regge limit of interest is defined in the Minkowski region, where

$$G_j(u, v; \Delta_{k,j}) \sim u^{(1-j)/2} H(\xi; j, \Delta_{k,j})$$

diverges as $u \rightarrow 0$. The j -sum can be performed via a Sommerfeld-Watson resummation. After taking into account the analytic structure, e.g., Eq. (4), one finds $F(u, v) \sim u^{(1-j_0)/2} H(\xi; j_0, \Delta_{k,j_0})$. With $\sqrt{u} \sim 1/s$ and ξ identified with the chordal distance, this precisely corresponds to that obtained earlier for the conformal Pomeron kernel given in a mixed representation, e.g., Eqs. (6) and (7) [1, 3, 12, 13].

Let us now return to the determination of the Pomeron intercept j_0 ; as discussed earlier, j_0 is fixed by knowing $\Delta_{G,j}$, analytically continued to small j . In a remarkable paper [14], Basso, taking advantage of many recent ingenious calculations of anomalous dimensions for $\mathcal{N} = 4$ YM based on integrability, has been able to generalize the work of [1] so that the coefficient $\alpha_1(\lambda)$ in Eq. (3), is known exactly, for all λ . Furthermore, coefficient for next few orders in the $(j-2)$ -expansion are also known through these studies. Taking advantage of the supersymmetry, it is sufficient to work with the scalars and investigate anomalous dimensions of the corresponding Konishi multiplets. These can be symbolically represented by $\text{tr} D^S Z^J + \text{mixing}$, where D is a covariant derivative and Z is a complex scalar. (In [14], J is the twist. We shall use τ instead.) For these operators, conformal dimensions admit a small S expansion, $\Delta = \tau + \alpha_1(\lambda, \tau) S + O(S^2)$.

In making connection with our problem at hand, we need to identify Δ above with $\Delta - 2$, $\tau \rightarrow 2$, and $S \rightarrow j - 2$, leading to Eq. (3). Maintaining the symmetry

in $\Delta \leftrightarrow 4 - \Delta$, and keeping enough terms to evaluate up to $O(\lambda^{5/2})$, we have

$$(\Delta - 2)^2 = \tau^2 + \left(2\sqrt{\lambda} - 1 + \frac{\tau^2 - \frac{1}{4}}{\sqrt{\lambda}} + \dots\right)S + \left(\frac{3}{2} - \frac{b_0}{\sqrt{\lambda}} + \frac{b_1}{\lambda} + \dots\right)S^2 \\ + \left(-\frac{3}{8\sqrt{\lambda}} + \dots\right)S^3 + O(S^4)$$

where the b_i are unknown. After inverting, and evaluating the minimum of $j(\Delta)$ at $\Delta = 2$, one obtains [13, 15] (note: the b_0 term is completely determined by this procedure)

$$\alpha_P = j_0 = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\lambda^{3/2}} + \frac{6\zeta(3) + 2}{\lambda^2} + \frac{-2b_1 + 9\zeta(3) + \frac{245}{64}}{\lambda^{5/2}} + \dots$$

A similar analysis can also be applied for Odderons. Recall that, from AdS/CFT, it has been shown that Odderons can be identified with modes associated with the anti-symmetric Kalb-Ramond fields in AdS. We note, in particular, the spin approaches $j = 1$ in the super-gravity limit. It follows that a similar expansion in $\lambda^{-1/2}$ can also be carried out, leading to an expansion where $\alpha_O = j_0^- = 1 + \frac{a_1^-}{\lambda^{1/2}} + \frac{a_2^-}{\lambda} + \frac{a_3^-}{\lambda^{3/2}} + \frac{a_4^-}{\lambda^2} + \dots$. In [3], two Odderon solutions were found. Solution-A has $a_{1,A}^- = -8$. The second solution, solution-B, surprisingly remains at 1 in the large λ limit, i.e., $a_{1,B}^- = 0$. For solution-B, one immediate question is whether this pattern will survive at higher order.

We have recently extended the analysis of [3] to higher order, leading to a similar expansion as Eq. (3). For solution-A, we find

$$\alpha_{O,A} = j_{0,A}^- = 1 - \frac{8}{\lambda^{1/2}} - \frac{4}{\lambda} + \frac{13}{\lambda^{3/2}} + \frac{96\zeta(3) + 41}{\lambda^2} + \frac{-81(45 + 32b_1 - 144\zeta(3))}{256\lambda^{5/2}} + \dots$$

Let us turn next to solution B. In order to match the vanishing first order correction, we find that $a_{i,B}^- = 0$ for all $i = 1, 2, \dots$ recursively. This leads to a surprising conclusion that

$$\alpha_O = 1$$

to all orders in $1/\sqrt{\lambda}$ in a strong coupling treatment. It is also interesting that this finding is consistent with that reported from a weak-coupling analysis³. More detailed analysis will be presented in a forthcoming report⁴.

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