Elastic proton-proton scattering from ISR to LHC energies, focusing on the *dip* region *

T. Csörgő[1], R. J. Glauber[2], and F. Nemes[1,3]

[1]Wigner Research Centre for Physics
H-1525 Budapest 114, P.O.Box 49, Hungary

[2] Lyman Laboratory of Physics, Harvard University
17 Oxford St, Cambridge, MA02138, USA

[3] CERN, CH-1211 Geneva 23, Switzerland

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Abstract

A detailed analysis of pp elastic scattering data is performed, based on a quark-diquark model that generalizes earlier models of Bialas and Bzdak, and a model of Glauber and Velasco. The differential cross-section of elastic proton-proton collisions is reported from the energy range of $\sqrt{s}$ = 23.5 GeV to 7 TeV. These studies suggests that the increase of the total pp cross-section is mainly due to an increase of the separation of the quark and the diquark with increasing energies. Within the investigated class of models, two simple and model-independent phenomenological relation were discovered that connect the total p+p scattering cross-section to the effective quark, diquark size and their average separation, and predict the position of the dip of elastic pp and pA scattering for future colliding energies.

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1 Diffraction - a historical perspective

Diffractive scattering of electrons on various nuclei resulted in important insights to the charge density distributions of spherical nuclei. The detailed analysis resulted in simple observations by Hofstadter and colleagues, that were summarized in the Nobel lecture of Hofstadter as follows:

- The volume of spherical nuclei is proportional to the mass number A
- The surface thickness is constant, independent of A

These observations revealed structures in atomic nuclei on the femtometer scale. They imply also that the central charge density of large nuclei is approximately constant. For more details, we recommend the Nobel Lecture (1961) by R. Hofstadter [1]. The results summarized there were one of the first observations of images on the femtometer scale, corresponding to nuclear charge density distributions. The more recent historical overview of ref. [2] discusses applications of multiple diffraction theory to high energy particle and nuclear physics. Recently, with the 7 and 8 TeV colliding energy of protons at CERN LHC, the resolution of diffraction based images of elastic proton-proton scattering reached the sub-femtometer scales, as we demonstrate below.

Our talk at the Low-X 2013 conference discussed two model classes: the Bialas-Bzdak model and the Glauber-Velasco model. Our conference contribution follows the same lines of presentation, except for the details of the results from the Bialas-Bzdak model, for which we direct the interested readers to suitable references.

2 Diffraction in quark-diquark models

In a series of papers, Bialas and Bzdak discussed a quark-diquark model of elastic proton proton scattering [3, 4, 5, 6]. Recently,this Bialas-Bzdak or BB model was tested in details on elastic proton-proton scattering data both at the ISR and LHC energies [7] and developed further to have a more realistic description in the dip region of elastic pp scattering in ref. [8].

In the BB model, the differential cross-section of elastic proton-proton scattering is given by the following formula

$$\frac{d\sigma}{dt} = \frac{1}{4\pi} \left| T\left(\Delta\right) \right|^2 , \tag{1}$$

where the amplitude of the elastic scattering in momentum space is given by

$$T(\vec{\Delta}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} t_{el}(\vec{b}) e^{i\vec{\Delta}\cdot\vec{b}} \mathrm{d}^2 b = 2\pi \int\limits_{0}^{+\infty} t_{el}\left(b\right) J_0\left(\Delta b\right) b \mathrm{d}b, \tag{2}$$

using the notation $b = |\vec{b}|$. From unitarity conditions one obtains

$$t_{el}(\vec{b}) = 1 - \sqrt{1 - \sigma(\vec{b})}. \tag{3}$$

The inelastic proton-proton cross-section in the impact parameter space for a fixed impact parameter $\vec{b}$ is given by the following integral

$$\sigma(\vec{b}) = \int\limits_{-\infty}^{+\infty} ... \int\limits_{-\infty}^{+\infty} \mathrm{d}^2 s_q \mathrm{d}^2 s_q' \mathrm{d}^2 s_d \mathrm{d}^2 s_d' D(\vec{s_q}, \vec{s_d}) D(\vec{s_q}', \vec{s_d}') \sigma(\vec{s_q}, \vec{s_d}; \vec{s_q}', \vec{s_d}'; \vec{b}), \quad (4)$$

where the integral is taken over the two-dimensional transverse position vectors of the quarks $\vec{s_q}$, $\vec{s_q}'$ and diquarks $\vec{s_d}$, $\vec{s_d}'$.

3 Bialas - Bzdak model of elastic pp scattering

The BB model approximates the quark-diquark distribution inside the proton with a Gaussian shape [3, 4, 5, 6]

$$D\left(\vec{s_q}, \vec{s_d}\right) = \frac{1+\lambda^2}{\pi R_{qd}^2} e^{-(s_q^2+s_d^2)/R_{qd}^2} \delta^2(\vec{s_d} + \lambda \vec{s_q}), \ \lambda = m_q/m_d. \quad (5)$$

and, in order to define a model that can be analytically integrated and compared to data in a straight-forward way, the model is formulated in simple and if possible Gaussian terms. The BB model also supposes that protons are scattered elastically if and only if all of their constituents are scattered elastically

$$\sigma(\vec{s_q}, \vec{s_d}; \vec{s_q}', \vec{s_d}'; \vec{b}) = 1 - \prod_{a,b \in \{q,d\}} \left[1 - \sigma_{ab}(\vec{b} + \vec{s_a}' - \vec{s_b}')\right], \quad (6)$$

where the inelastic differential cross-sections of the constituents are parametrized with Gaussian distributions as well

$$\sigma_{ab}\left(\vec{s}\right) = A_{ab} e^{-s^2/R_{ab}^2}, \ R_{ab}^2 = R_a^2 + R_b^2. \quad (7)$$

Here A_{ab} are suitably chosen normalization factors, a, b stand for q, d, denoting quarks and/or diquarks, while and R_q and R_d stand for the Gaussian radii, that characterize in the BB model the quark and diquark inelastic scattering cross-sections, respectively.

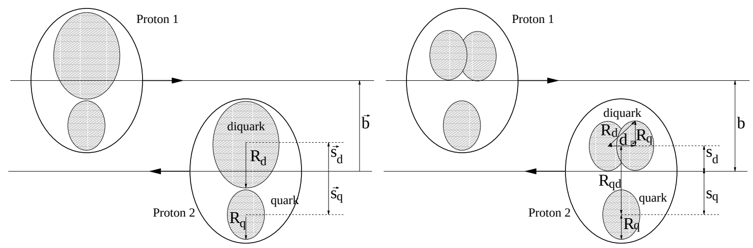

Figure 1: Scheme of the scattering of two protons, when the proton is assumed to have a quark-diquark structure. The diquark is assumed to be scattered as a single entity (left) or as composition of two quarks (right). This figure is a snapshot and all the model parameters follow a Gaussian distribution. Note, that a center of mass energy dependent Lorentz-contraction determines the longitudinal scale parameters.

This BB model comes in two different realizations, corresponding to two different pictures of the proton: in one of the cases, the diquark is assumed to have a structureless Gaussian distribution, while in the other case, the diquark is assumed to scatter as a loosely bound state of two correlated quarks. These scenarios are indicated by the $p = (q, d)$ and the $p = (q, (q, q))$ labels. As noted by Bialas and Bzdak, models with three uncorrelated quarks in the proton, labelled as $p = (q, q, q)$ were tested before at ISR energies and they are known to fail, with other words, we know that the quarks are correlated inside the protons [9]. In its original form, the BB model has been integrated analytically, both for the $p = (q, d)$ and the $p = (q, (q, q))$ scenarios, *assuming* that the real part of forward scattering is negligible.

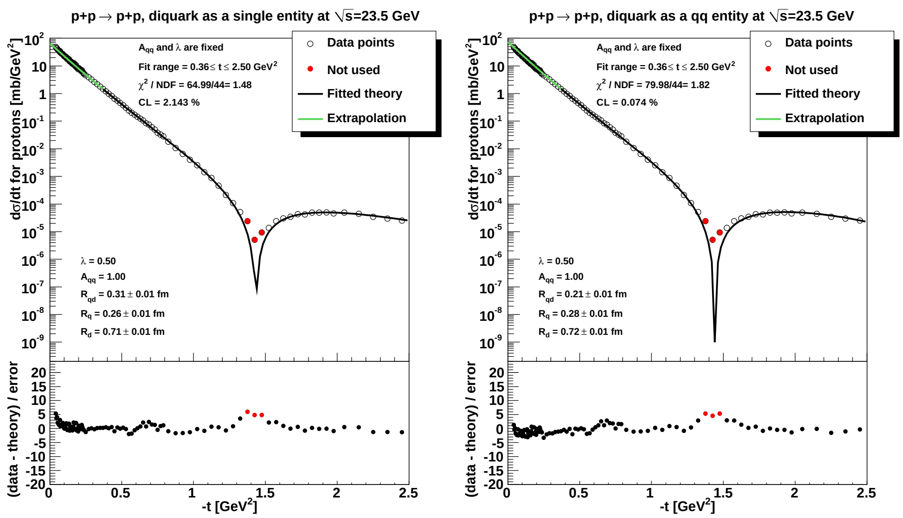

Figure 2: (Color online.) Results of Minuit fits of both versions of the Bialas-Bzak model at ISR energies. Left panel indicates the scenario $p = (q, d)$, where the diquark is assumed to scatter as a single entity while the right panel indicates the scenario $p = (q, (q, q))$, where the diquark inside the proton is considered to be a scattering object consisting of two quarks.

Figures 2 indicate CERN Minuit fit results of the BB model to differential cross-section data on elastic proton-proton scattering at $\sqrt{s} = 23.5$ GeV data, at lower ISR energies. Left plots correspond to the scenario $p = (q, d)$ while the right panel stands for the scenario $p = (q, (q, q))$. The top panels show the data points and the result of the best fit, while the lower panel shows the relative deviation of the model from data in units of measured error bars. As the original BB model is singular around the dip, 3 data points, located closest to the diffractive minimum, and indicated with filled (red) circles in this Figure, were left out from the optimalization. The fit range was restricted to the 0.36 - 2.5 GeV $|t|$, so that a fair comparison could subsequently be made with the first TOTEM results on proton-proton elastic scattering at LHC energy of 7 TeV, [10]. The best fits indicated with a solid (black) line in this range and its extrapolation to low values of t are also shown. The confidence levels, after fixing the values of λ and A_{qq}, are very close to 0.1%, which indicate tha the fit quality is similar, statistically acceptable in both scenarios. Similar quality fits were reported at each of the ISR energies of 30.7 GeV, 52.8 GeV and 62.5 GeV, see ref. [7] for details.

In Figure 3, we show the comparison of the BB model to TOTEM data on elastic pp scattering at 7 TeV LHC energies, indicating a qualitative change, as compared to the fit results at ISR energies: the quality of this fit is statistically not acceptable, CL is below 0.1%, and the fit deviates from the data in particular in the dip region. The bottom panel indicates, that the shape of the differential cross-section around the first diffractive minimum is not reproduced correctly by the original BB model at 7 TeV LHC energies, and, also be seen on this Figure 3 this shortcoming cannot be fixed by leaving out a few data points around the diffractive minimum from the optimalization procedure. The details of this fit are described in ref. [7].

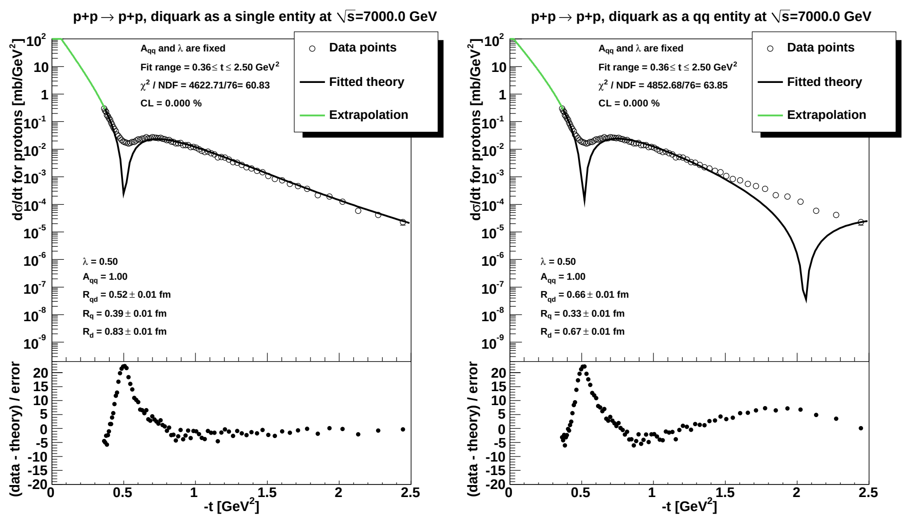

Figure 3: (Color online.) The result of the fit at LHC at 7 TeV when the diquark is assumed to scatter as a single entity (left) or as composition of two quarks (right).

Recently, we have generalized the Bialas-Bzdak model by adding a small real part to the forward scattering amplitude, to investigate, if this way the description of the dip region can be improved and can be made statistically acceptable. Our findings are described in detail in ref. [8]. The real part of the forward scattering amplitude was added by using an analogy of with the Glauber-Velasco model, and assuming that even if all the parton level scatterings are elastic, the proton-proton scattering can, with a small probability, become inelastic. This manner, a parton level ρ parameter was introduced. The results, detailed in ref. [8], indicate that a small real part indeed improves the agreement of the BB model with data in the dip region, and the fits become statistically acceptable in the whole t region, including all the data points from dip region, IF the energy of the collisions is limited to the ISR energy range of $\sqrt{s} = 23.5$ - 62.5 GeV. At the LHC energy of $\sqrt{s} = 7$ TeV, the generalized Bialas-Bzdak or the αBB model resulted in an improvement, that reduced the disagreement between the BB model and the data substantially and filled the dip region rather dramatically. However, the improvement did not result in a statistically acceptable fit quality to the differential cross-section of elastic proton-proton collisions at this LHC energy, although good quality fits were obtained in the dip region. Thus we kept on searching for a model that is able to describe elastic pp scattering data at LHC energies, and investigated the performance of the Glauber-Velasco model [11]. Before reporting the results, let us summarize what we have learned till now from the detailed fits using the original and the generalized versions of the Bialas-Bzdak model. This is discussed in the next section, based on the detailed results of ref. [7, 8].

4 What have we learnt so far ?

The original version of the Bialas-Bzdak model gave a statistically acceptable description of elastic pp scattering data at ISR energies, if the data points close to the diffractive minimum were left out from the fit. If these data points were included and also a small real part was added to the model, as detailed in ref. [8], the fits at the ISR energies from $\sqrt{s} = 23.5$ GeV to 62.5 GeV become statistically acceptable, good quality fits, in the fit range of $0.36 \leq -t \leq 2.5$ GeV^2. Two model parameters could be fixed at all energies ($A_q q = 1$ and $\lambda = 1$) while maintaining the statistically acceptable fit quality. The parameter α, that was introduced as a parton level ratio of the real to imaginary part of the forward scattering amplitude, remained indeed in the region of very small values, $\alpha = 0.01 \pm 0.01$ except at 52.8 GeV, where $\alpha = 0.02 \pm 0.01$ value was found. Although these α parameters are within errors consistent with zero, a small but non-vanishing value provided qualitatively better fits in the dip region, as detailed in ref. [8]. The best fit parameters, that described the quark structure of the protons geometrically, took also rather interesting values. For example, the quark radius R_q within 2 standard deviations was consistent with an energy independent value of $R_q = 0.27 \pm 0.01$ fm. The diquark size indicated a nearly constant value, varying between $R_d = 0.71 \pm 0.01$ to 0.77 ± 0.01 fm, sighlty increasing with increasing $\sqrt{s}$. Although the fit to the TOTEM data 7 TeV were not statistically acceptable, the best parameter values for the quark and diquark radii were in the same range, except a slight decrease of the diquark size in the $p = (q, (q, q))$ model at 7 TeV. We observed that the biggest variation, when the energy is increased to 7 TeV, is observable in the scale that measures the typical quark-diquark distance, R_{qd}. This value was in the range of $R_{qd} = 0.23 \pm 0.01$ fm at ISR energies in the $p = (q, (q, q))$ model, while it increased to the value of 0.73 ± 0.01 fm at 7 TeV. Similar trend of increasing quark-diquark separation is seen in the $p = (q, d)$ scenario. Graphically, the evolution of the proton elastic scattering structure is illustrated on Figure 4, where the best fit parameters are also indicated on the sub-plots, generated for the case of the $p = (q, (q, q))$ scenario. The same qualitative behaviour of increasing quark-diquark distance is observed also in the $p = (q, d)$ picture, see ref. [8] for further details.

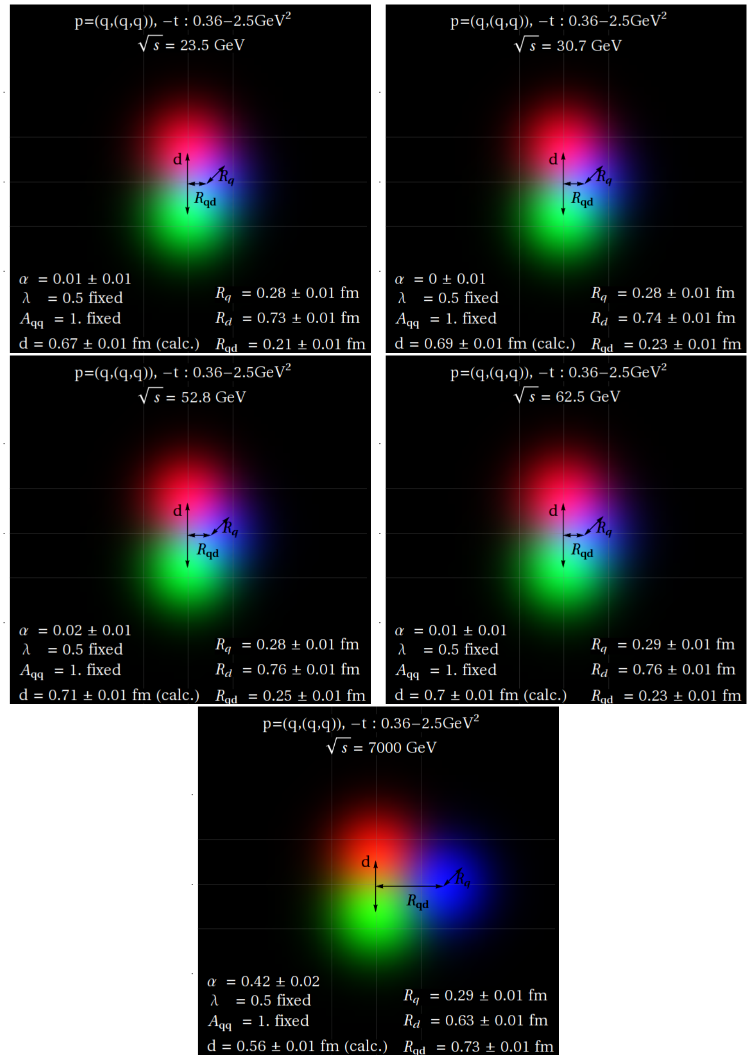

Figure 4: Visualization of the fit results of Bialas-Bzdak models, extended to a small real part, for the case of $p = (q,(q,q))$, when the diquark is assumed to be resolvable as a weekly bound state of two quarks. The main effect of increasing $\sqrt{s}$ is apparently the increasing value of R_{qd}, the typical quark-diquark distance.

As discussed both in refs. [7] and [8], the $p = (q,d)$ and the $p = (q,(q,q)$ models provide similar quality of data description both at ISR energies (where they are both statistically acceptable) and at 7 TeV LHC energy, (where both fail to describe TOTEM data in a statistically acceptable manner). Nevertheless we compare the best fit values of the different eneries, to try to get a qualitative insight assuming, that the missing element of the model will not modify drastically the best fit parameters at LHC energies.

Given that the $p = (q, d)$ and the $p = (q, (q, q))$ Bialas-Bzdak models correspond to two different assumptions about the internal structure of the protons, it was a kind of surprize for us, that the measured total pp cross-section σ_{tot} was phenomenologically related to the parameters of the BB model in a model-independent way, i.e. the following relation is approximately valid for both scenarios:

$$\sigma_{tot} \approx 2\pi R_{eff}^2 \ = \ 2\pi(R_q^2 + R_d^2 + R_{qd}^2). \tag{8}$$

This approximation was found to be valid within a relative error of about 9 % at ISR energies, while at the LHC energies it yields only an ball-park value, order of magnitude estimation ($\frac{\sigma_{tot}}{2\pi R_{eff}^2} = 1.42$).

We also have observed an interesting scaling property of the differential and the total proton-proton elastic scattering cross-section, namely the product of the total cross-section times the t of the dip is within errors a constant:

$$t_{dip}\sigma_{tot} \approx C \tag{9}$$

where $C = 54.8 \pm 0.7$ mb GeV^2 from a fit. We find that this relation is valid within 5 % relative error at each ISR and also at 7 TeV LHC energies. A similar relation holds for a light scattering from a black disc, however, with a significantly different constant value, $C_{blackdisc} \approx 35.9$ mb GeV^2.

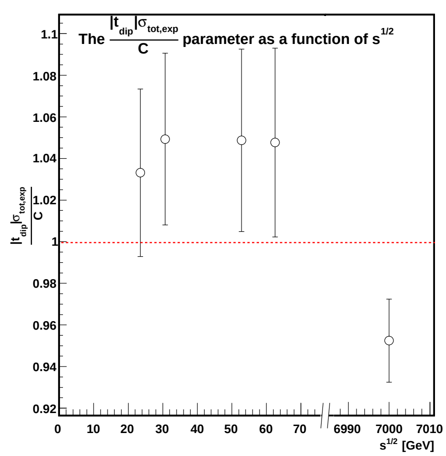

Figure 5: The $\frac{|t_{dip}|\cdot\sigma_{tot,exp}}{C}$ ratio, directly obtained from experimental data. The dashed line indicates 1, which value within errors is consistent with all the data from $\sqrt{s} = 23.5$ GeV to 7 TeV.

Given that there are theoretically well established formulas for the description of the rise of the total pp scattering cross-section with increasing energies, the above formula can be well used to predict the position of the (first) minimum or the dip in the differential cross-section of pp collisions and also can be extrapolated, or, predicted for pA and AB collisions [15].

Given that we could not find a statistically acceptable quality fit with the Bialas-Bzdak model to 7 TeV TOTEM data on elastic pp scattering at LHC, neither in the original form, nor when a small real part is added to the forward scattering amplitude of this model, we started to look for alternative interpretations and derivations of $d\sigma/dt$. One possibility is to allow for not only small values of the real part of the forward scattering amplitude, but still keep the basic structure of the Bialas-Bzdak model. The studies in this direction will be reported elsewhere. In the next section we report about the other natural direction, that we investigated in detail. In particular, when we added a small real part to the forward scattering amplitude to the Bialas-Bzdak model in ref. [8], we were introducing a parton level ρ parameter inspired by the Glauber-Velasco model of refs. [11, 12]. In the next section, we summarize this model and report about its first comparisions to TOTEM data.

5 GlauberVelasco model summary

In this section, we follow the lines of the presentation of the Glauber-Velasco model, as described in refs. [11, 12]. Glauber diffractive multiple scattering theory is utilized to describe elastic collisions of two nucleons, which are pictured as clusters of partons. The parton distributions are assumed to have form factors given by the experimentally measured electric charge form factors. Differential cross sections calculated in this way showed good agreement with the experimentally measured ones over a broad range of p-p and p - $\bar{p}$ energies when the parton-parton scattering amplitude is given a suitable parametrization [11, 12]. The range of the parton-parton interaction derived from these data is found to increase steadily with energy. The absorption processes that take place are localized in the overall nucleon-nucleon interaction by calculating the shadow profile function. The emerging picture corresponded to an opaque region of interaction that grows in radius with increasing energy. The surface region of the interaction seems however to maintain a remarkably fixed shape as the radius grows.

In multiple diffraction theory, the elastic scattering amplitude for diffractive collisions can be written as an impact parameter integral

$$F(t) = i \int_0^\infty J_0\left(b\sqrt{-t}\right)\left\{1 - \exp\left[-\Omega\left(b\right)\right]\right\} bdb \tag{10}$$

Any particular model is characterized by the opacity function $\Omega(b)$, which in general may be a complex valued function. If we picture the two colliding nucleons as clusters of partons that scatter one another with the averaged scattering amplitude $f(t)$, then the opacity function can be written in the form of an integral over momentum transfers q,

$$\Omega\left(b\right)=\frac{\kappa}{4\pi}\left(1-i\alpha\right)\int_{0}^{\infty}J_{0}\left(q\,b\right)G_{p,E}^{2}\left(-t\right)\frac{f(t)}{f(0)}qdq \tag{11}$$

The constants κ and α in this expression are real-valued and must be determined empirically. The function $G_{p,E}(t)$ is the form factor for the parton density in the proton, and we shall assume it to be the same as the observed electric form factor for the proton.

One choice of parametrization we have investigated is

$$\frac{f(t)}{f(0)}=\frac{e^{i\left(b_{1}\left|t\right|+b_{2}\,t^{2}\right)}}{\sqrt{1+a\left|t\right|}} \tag{12}$$

The BSWW form factor, corresponding to the distribution of electric charge in the proton, is described with a four-pole parametrization [14]

$$G_{p,E}\left(q^{2}\right)=\sum_{i=1}^{n}\frac{a_{i}^{E}\left(m_{i}^{E}\right)^{2}}{\left(m_{i}^{E}\right)^{2}+q^{2}}\,,\,\sum_{i=1}^{n}a_{i}^{E}=1\,,\,G_{p.E}(0)=1 \tag{13}$$

The differential cross-section for elastic pp collisions is evaluated as

$$\frac{d\sigma_{el}}{d\left|t\right|}=\pi\left|F(t)\right|^{2}. \tag{14}$$

The parameters of the BSWW form factor are given by the following table:

a_i^E	$(m_i^E)^2$ (fm^{-2})
0.219	3.53
1.371	15.02
-0.634	44.08
0.044	154.20

Table 1: Best fit parameters [14] of the four-pole fit Eq. (13)
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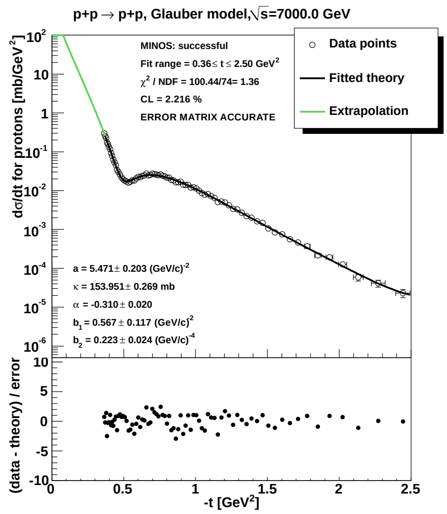

Figure 5 indicates, that the Glauber-Velasco model is able to describe successfully the differential scattering cross-section of elastic pp collisions at the 7 TeV LHC energies: the fit quality is statistically acceptable, with CL > 0.1 %. We have tested the model at the ISR energy range of 23.5 GeV - 62.5 GeV too, where the similarly good quality fits were found. The detailed results will be reported in a manuscript that is currently under preparation.

6 Summary

In summary, we have analized elastic proton-proton scattering data from the 23.5 GeV ISR energies to 7 TeV LHC energies, using various forms of the Bialas-Bzdak model. We found that the scenario when the proton is considered to be a quark-diquark state provides a fit quality that is similar to the case when the diquark is resolved as a correlated quark-quark system within the framework of the same model. Adding a small real part to the forward scattering amplitude of the original Bialas-Bzdak model provides a statistically acceptable description of elastic pp scattering data at the ISR energies, however, even this generalized Bialas-Bzdak model fails to describe TOTEM data on elastic pp scattering at 7 TeV. Given that the generalization of the Bialas-Bzdak model followed the lines of the Glauber-Velasco model, we tested also the performance of the Glauber-Velasco model in its original form, and found that it was describing elastic proton-proton scattering both at ISR and at LHC energies when the fit range was restricted to $0.36 < -t < 2.5$ GeV^2. The details of this investigation will be reported elsewhere, but the first details of the Glauber-Velasco model results at LHC are released in this conference contribution.

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