

Cross sections and Amplitudes for Soft Scattering at the LHC *

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Abstract

We describe elements of the GLM model that successfully describes soft hadronic interactions at energies from ISR to LHC. This model is based on a single Pomeron with a large intercept $\Delta_{\mathbb{P}} = 0.23$ and slope $\alpha'_{\mathbb{P}} = 0.028$, and so provides a natural matching with perturbative QCD. We analyze the elastic, single diffractive and double diffractive amplitudes, and compare the behaviour of the GLM amplitudes to those of other parameterizations. We summarize the main features and results of competing models for soft interactions at LHC energies.

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1 Introduction

The recent measurements of the proton-proton cross sections at the LHC at an energy of $W = 7$ TeV, allows one to appraise the numerous models that have been proposed to describe soft interactions. The classical Regge pole model *à la* Donnachie and Landshoff [1], which provided a reasonable description of soft hadron-hadron scattering upto the Tevatron energy, fails when extended to LHC energies [2]. In addition it has the intrinsic problem of violating the Froissart-Martin bound [3].

At present there are a number of models based on Reggeon Field Theory that provide an acceptable description of proton-proton scattering data over the energy range from ISR to LHC. I will describe the essential features of the GLM model [4] as an example of a model of this type, before comparing its results with other competing models on the market.

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1.1 Basic features of the GLM model

We utilize the simple two channel Good-Walker (GW) [5] model, to account for elastic scattering and for diffractive dissociation into states with masses that are much smaller than the initial energy. and impose the unitarity constraint by requiring that

$$2 \operatorname{Im} A_{i,k}(s, b) = |A_{i,k}(s, b)|^2 + G_{i,k}^{in}(s, b)$$

where, $A_{i,k}$ denotes the diagonalized interaction amplitude and $G_{i,k}^{in}$, the contribution of all non GW inelastic processes.

A general solution for the amplitude satisfying the above unitarity equation is:

$$A_{i,k}(s, b) = i \left(1 - \exp \left(-\frac{\Omega_{i,k}(s, b)}{2} \right) \right) \quad (1)$$

the opacities $\Omega_{i,k}$ are arbitrary. In the eikonal approximation $\Omega_{i,k}$ are assumed to be real, and taken to be the contribution of a single Pomeron exchange.

GLM parameterize the opacity :

$$\Omega_{i,k}(s, b) = g_i(b) g_k(b) P(s)$$

where $P(s) = s^\Delta$, and $g_i(b)$ and $g_k(b)$ are the Pomeron-hadron vertices given by:

$$g_l(b) = g_l S_l(b) = \frac{g_l}{4\pi} m_l^3 b K_1(m_l b).$$

$S_l(b)$ is the Fourier transform of $\frac{1}{(1+q^2/m_l^2)^2}$,

where q is the transverse momentum carried by the Pomeron, $l = i, k$. The form of $P(s)$ used by GLM, corresponds to a Pomeron trajectory slope $\alpha'_{\mathcal{P}} = 0$. This is compatible with the exceedingly small fitted value of $\alpha'_{\mathcal{P}}$, (0.028 GeV⁻²) and in accord with $N=4$ SYM.

For the case of $\Delta_{\mathcal{P}} \rightarrow 0$, the Pomeron interaction leads to a new source of diffraction production with large mass ($M \propto s$), which cannot be described by the Good-Walker mechanism. Taking $\alpha'_{\mathcal{P}} = 0$, allows one to sum all diagrams having Pomeron interactions [6, 7]. This is the advantage of such an approach. The GLM model only takes into account triple Pomeron interaction vertices ($G_{3\mathcal{P}}$), this provides a natural matching to the hard Pomeron, since at short distances $G_{3\mathcal{P}} \propto \alpha_s^2$, while other vertices are much smaller. A full description of the procedure for summing all diagrams (enhanced + semi-enhanced) is contained in [6, 7, 8]. We would like to emphasize that in the GLM model, the GW sector contributes to both low and high diffracted mass, while the non-GW sector contributes only to high mass diffraction ($\log(M^2/s_0) \approx 1/\Delta_{\mathcal{P}}$).

The GLM model has 14 parameters describing the Pomeron and Reggeon sectors. The values of these parameters are determined by fitting to data for σ_{tot} , σ_{el} , σ_{sd} , σ_{dd} and B_{el} in the ISR-LHC range [8]. We find the best fit value for $\alpha_{\mathcal{P}} = 0.21$, however to be in accord with the LHC data we have tuned $\alpha_{\mathcal{P}}$ to 0.23. The fitted values for $\alpha'_{\mathcal{P}}$ is 0.028 GeV⁻², while the triple Pomeron vertex $G_{3\mathcal{P}} = 0.03$ GeV⁻¹.

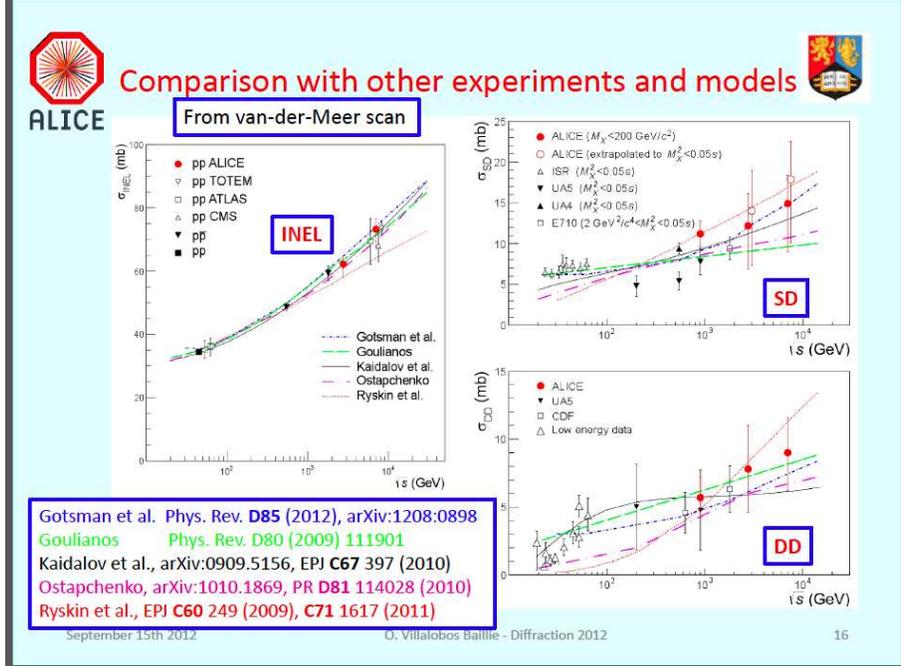


Figure 1: Comparison of Models with LHC data from Villalobos Baillie's talk at Diffraction 2012. [9]

2 Experimental Data and GLM results

Our results for σ_{inel} , σ_{sd} and σ_{dd} , are given in Fig. 1 which is taken from the talk given by Orlando Villalobos Baillie (for the Alice collaboration) (see reference [9]), where the experimental data, our results and the results of other models are displayed.

The comparison of our results with experimental data for σ_{tot} , σ_{el} and for B_{el} is shown in Fig. 2.

To summarize our results at high energy, we obtain an excellent reproduction of TOTEM's values for σ_{tot} and σ_{el} . The quality of our good fit to B_{el} is maintained. As regards σ_{inel} , our results are in accord with the higher values obtained by ALICE [10] and TOTEM [11]; ATLAS [12] and CMS [13] quote lower values with large extrapolation errors, see [14]. We refer the reader to [14] who suggests that the lower values found by ATLAS and CMS maybe due to the simplified Monte Carlo that they used to estimate their diffractive background.

There are also recent results at $W = 57$ TeV by the Auger Collaboration [15] for σ_{tot} and σ_{inel} . In Table 1 we compare the experimental results at $W = 7$ and 57 TeV and the GLM model.

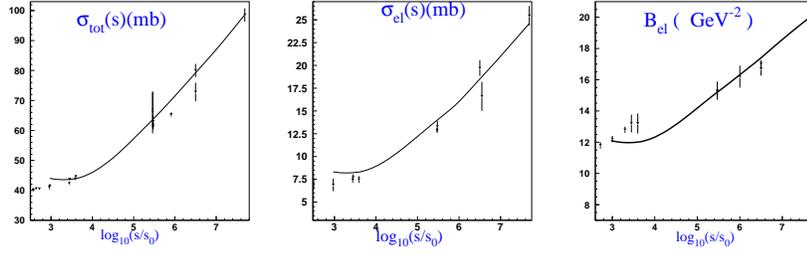


Figure 2: The GLM results compared to data for σ_{tot} , σ_{elas} and B_{elas}

W	$\sigma_{tot}^{model}(\text{mb})$	$\sigma_{tot}^{exp}(\text{mb})$	$\sigma_{el}^{model}(\text{mb})$	$\sigma_{el}^{exp}(\text{mb})$
7 TeV	98.6	TOTEM: 98.6 ± 2.2	24.6	TOTEM: 25.4 ± 1.1
W	$\sigma_{in}^{model}(\text{mb})$	$\sigma_{in}^{exp}(\text{mb})$	$B_{el}^{model}(\text{GeV}^{-2})$	$B_{el}^{exp}(\text{GeV}^{-2})$
7 TeV	74.0	CMS: $68.0 \pm 2^{syst} \pm 2.4^{lumi} \pm 4^{extrap}$ ATLAS: $69.4 \pm 2.4^{exp} \pm 6.9^{extrap}$ ALICE: $73.2 (+2./ - 4.6)^{model} \pm 2.6^{lumi}$ TOTEM: $73.5 \pm 0.6^{stat} \pm 1.8^{syst}$	20.2	TOTEM: 19.9 ± 0.3
W	$\sigma_{sd}^{model}(\text{mb})$	$\sigma_{sd}^{exp}(\text{mb})$	$\sigma_{dd}^{model}(\text{mb})$	$\sigma_{dd}^{exp}(\text{mb})$
7 TeV	$10.7^{GW} + 4.18^{nGW}$	ALICE : $14.9(+3.4/-5.9)$	$6.21^{GW} + 1.24^{nGW}$	ALICE: 9.0 ± 2.6
W	$\sigma_{tot}^{model}(\text{mb})$	$\sigma_{tot}^{exp}(\text{mb})$		
57 TeV	130	AUGER: $133 \pm 13^{stat} \pm 17^{sys} \pm 16^{Glauber}$		
	$\sigma_{inel}^{model}(\text{mb})$	$\sigma_{inel}^{exp}(\text{mb})$		
	95.2	AUGER: $92 \pm 7^{stat} \pm 11^{syst} \pm 7^{Glauber}$		

Table 1:

Comparison of the values obtained from the GLM model with experimental results at $W = 7$ and 57 TeV.

3 Alternative Models

There are several models on the market today that manage to reproduce the LHC experimental results. The most promising of these are summarized here, and their results are compared with those of GLM [4] in Table 1.

The Durham group's approach for describing soft hadron-hadron scattering [16] is similar to the GLM [4] approach, they include both enhanced and semi-enhanced diagrams. The two groups utilize different techniques for summing the multi-Pomeron diagrams. The Durham group have a bare (prior to screening) QCD Pomeron, with intercept $\Delta_{bare} = 0.32$. This model [16] which was tuned to describe collider data, predicts values for σ_{tot} , σ_{el} and σ_{inel} , which are lower than the TOTEM [11] data. To be consistent with the TOTEM results, RMK [17] have proposed an alternative formulation, based on a 3-channel eikonal description, with 3 diffractive eigenstates of different sizes, but with

$W = 1.8 \text{ TeV}$	GLM	KMR2	Ostap(C)	BMR*	KP
$\sigma_{\text{tot}}(\text{mb})$	79.2	79.3	73.0	81.03	75.0
$\sigma_{\text{el}}(\text{mb})$	18.5	17.9	16.8	19.97	16.5
$\sigma_{SD}(\text{mb})$	11.27	5.9(LM)	9.2	10.22	10.1
$\sigma_{DD}(\text{mb})$	5.51	0.7(LM)	5.2	7.67	5.8
$B_{\text{el}}(\text{GeV}^{-2})$	17.4	18.0	17.8		
$W = 7 \text{ TeV}$	GLM	KMR2	Ostap(C)	BMR	KP
$\sigma_{\text{tot}}(\text{mb})$	98.6	97.4	93.3	98.3	96.4
$\sigma_{\text{el}}(\text{mb})$	24.6	23.8	23.6	27.2	24.8
$\sigma_{SD}(\text{mb})$	14.88	7.3(LM)	10.3	10.91	12.9
$\sigma_{DD}(\text{mb})$	7.45	0.9(LM)	6.5	8.82	6.1
$B_{\text{el}}(\text{GeV}^{-2})$	20.2	20.3	19.0		19.0
$W = 14 \text{ TeV}$	GLM	KMR2	Ostap(C)	BMR	KP
$\sigma_{\text{tot}}(\text{mb})$	109.0	107.5	105.	109.5	108.
$\sigma_{\text{el}}(\text{mb})$	27.9	27.2	28.2	32.1	29.5
$\sigma_{SD}(\text{mb})$	17.41	8.1(LM)	11.0	11.26	14.3
$\sigma_{DD}(\text{mb})$	8.38	1.1(LM)	7.1	9.47	6.4
$B_{\text{el}}(\text{GeV}^{-2})$	21.6	21.6	21.4		20.5

Table 2: Comparison of results of the different models for $W = 1.8, 7$ and 14 TeV.

only one Pomeron whose intercept and slope are: $\Delta_{\mathcal{P}} = 0.14$; $\alpha'_{\mathcal{P}} = 0.1 \text{ GeV}^{-2}$. Their results are shown in Table 2 in the column KMR2.

Ostapchenko [18] [pre LHC] has made a comprehensive calculation in the framework of Reggeon Field Theory, based on the resummation of both enhanced and semi-enhanced Pomeron diagrams. To fit the total and diffractive cross sections he assumes two Pomerons: (for his solution set C) "Soft Pomeron" $\alpha^{Soft} = 1.14 + 0.14t$ and a "Hard Pomeron" $\alpha^{Hard} = 1.31 + 0.085t$. His results are quoted in Table 2, in the column Ostap(C).

Kaidalov-Poghosyan [19] have a model which is based on Reggeon calculus, they attempt to describe data on soft diffraction taking into account all possible non-enhanced absorptive corrections to 3 Reggeon vertices and loop diagrams. It is a single Pomeron model and with secondary Regge poles, their Pomeron has the following intercept and slope: $\Delta_{\mathcal{P}} = 0.12$ and $\alpha'_{\mathcal{P}} = 0.22 \text{ GeV}^{-2}$. Their results are shown in Table 2, in the column KP.

Ciesielski and Goulianos have proposed an "event generator" [20] which is based on the BMR-enhanced PYTHIA8 simulation. In Table 2 their results are denoted by BMR.

4 Amplitudes

Until recently most of the comparison of models has been made on the level of cross-sections (which are areas), and only reveal the energy dependence, and

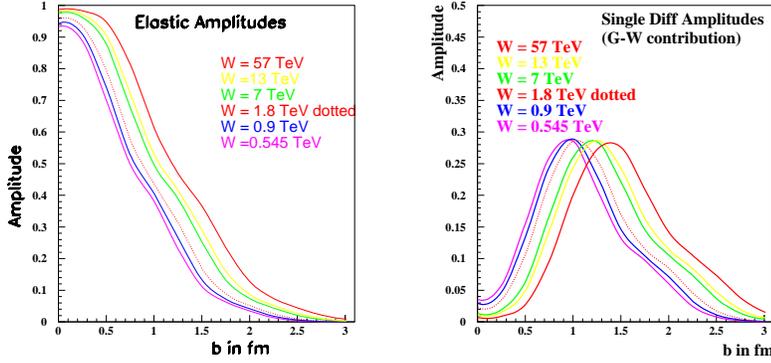


Figure 3: Left panel: GLM elastic amplitudes and Right panel: GLM-GW single diffraction amplitudes.

therefore are not very helpful to discriminate between the different models. Having the behaviour of the various amplitudes as functions of impact parameter (momentum transfer) would be more revealing. Unfortunately, there is a paucity of material available on amplitudes, and most refer only to the elastic amplitude.

In Fig.3 (left panel) we show elastic amplitudes emanating from the GLM model for various energies. We note the overall gaussian shape of the elastic amplitudes for all energies $0.545 \leq W \leq 57$ TeV, with the width and height of the gaussian growing with increasing energy. For small values of b the slope of the amplitudes decreases with increasing energy. The elastic amplitude (as $b \rightarrow 0$) becomes almost flat for $W = 57$ TeV, where it is still below the Unitarity limit $A_{el} = 1$.

In Fig. 3 (right panel) we show the energy behaviour of the GLM (G-W contribution) of the single diffractive amplitude as a function of impact parameter for different energies. In Fig. 4 (left panel) we display the behaviour of the double diffractive amplitude. A common feature of both diffractive amplitudes is that with increasing energy the peaks broaden and become more peripheral. In Fig. 4 (right panel) we show the elastic, single diffraction and double diffraction amplitudes as functions of b for $W = 7$ TeV. Note the completely different shapes of the three amplitudes, the elastic amplitude $A_{el}(b)$ is gaussian in shape, while the single diffractive amplitude $A_{sd}(b)$ and the double diffractive amplitude $A_{dd}(b)$ are very small at small b . A_{sd} has a peak at 1.25 fm, while A_{dd} 's maximum is at $b = 2.15$ fm.

The Durham group [17] have attempted to extract the form of the Elastic Opacity directly from the data. They assume that at high energies the real part of the scattering amplitude is very much smaller than the imaginary part, then to a good approximation

$$A(b) = i[1 - \exp(-\Omega(b)/2)]$$

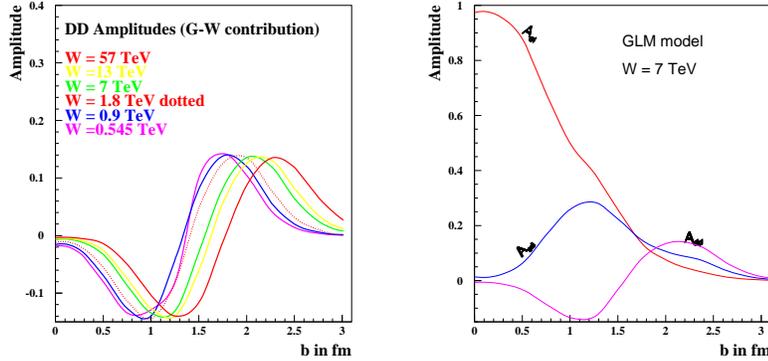


Figure 4: Left panel: GLM double diffractive amplitudes and Right panel: GLM elastic, single diffractive and double diffractive amplitudes for $W = 7$ TeV.

(see Eqn(1)). As $\Omega_{el} = -2\ln(1 - A_{el})$, they determine the Opacity directly from the data since

$$\text{Im}A(b) = \int \sqrt{\frac{d\sigma_{el}}{dt} \frac{16\pi}{1 + \rho^2}} J_0(q_t b) \frac{q_t dq_t}{4\pi},$$

where $q_t = \sqrt{|t|}$ and $\rho \equiv \text{Re}A/\text{Im}A$. Their results are shown in Fig. 5 (left panel) .

The Durham group [17] find that at $Spp\bar{S}$ and Tevatron energies the Opacity distributions have approximately a Gaussian form. The analogous GLM model results are shown in Fig. 5(Right), are in agreement with [17] regarding the shape of $\Omega_{el}(b)$, and in addition suggest that this is also true for the LHC energies. GLM find that with increasing energy, the intercept of the Opacity at $b = 0$ increases, while the slope at small b decreases.

Kopeliovich, et al [21] have calculated the proton-proton elastic amplitude within the framework of a two scale dipole model. We show their result in Fig. 6 (left panel). As well as that of Ferreira, Kodama and Kohara [22] who have recently made a detailed study of the proton-proton elastic amplitude for center of mass energy $W = 7$ TeV, based on the Stochastic Vacuum Model.

In Fig. 6 (left panel) we compare the GLM, KPPS and FKK elastic amplitudes at $W = 7$ TeV as a function of the impact parameter. Although the shapes are similar, the KPPS and FKK amplitudes have lower intercepts at $b = 0$. If we normalize the FKK amplitude to the GLM value at $b = 0$, we note that the amplitudes which are gaussian in shape, have very similar behaviour as a function of the impact parameter.

In Fig.6 (right panel) we show the single diffraction amplitude as given by the DIPSY Monte Carlo [23] (dashed line) at $W = 1.8$ and 14 TeV. This includes contributions both from the Good-Walker sector and enhanced and semi-enhanced sector. The full line is the GLM amplitude which only contains the

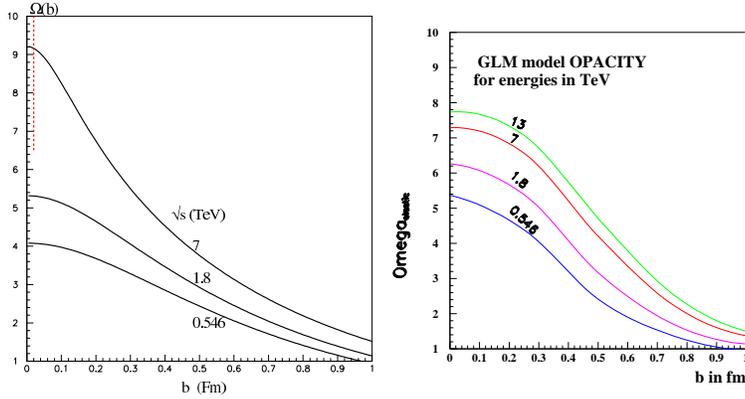


Figure 5: Left panel: The proton opacity $\Omega(b)$ determined directly from the pp $d\sigma_{el}/dt$ data at 546 GeV , 1.8 TeV and 7 TeV data. The uncertainty on the LHC value at $b = 0$ is indicated by a dashed red line. This figure is taken from [17] which should be consulted for details. Right panel: Opacities calculated using the GLM model.

Good-Walker contribution. Note, although the amplitudes for the same value of W , peak at the same value of b , the DIPSY amplitudes are broader and higher, due to the enhanced contributions.

5 Conclusions

We [4] have succeeded in building a model for soft interactions, which provides a very good description all high energy data, including the LHC measurements. The model is based on a Pomeron with a large intercept ($\Delta_P = 0.23$) and very small slope ($\alpha'_P = 0.028$). We find no need to introduce two Pomerons: i.e. a soft and a hard one. The Pomeron in our model provides a natural matching with the hard Pomeron in processes that occur at short distances. The qualitative features of our model are close to what one expects from $N=4$ SYM [6, 7], which is the only theory that is able to treat long distance physics on a solid theoretical basis.

6 Acknowledgments

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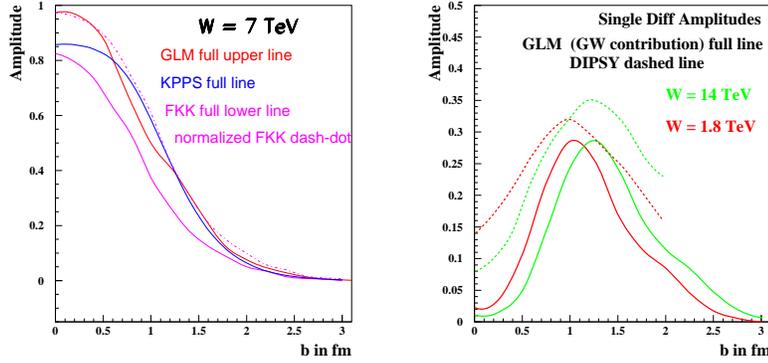


Figure 6: Left panel: Comparison of the elastic amplitude at 7 TeV determined by FKK [22], KPPS [21] and GLM and Right panel: Comparison of single diffraction amplitude, dashed line DIPSY [23] (which includes enhanced and semi-enhanced contributions) and full line GLM (only GW contribution)

References

- [1] A. Donnachie, and P. V. Landshoff, *Phys. Letters* **296**, 222 (1992).
- [2] A. Donnachie, and P. V. Landshoff, arXiv:1112.2485.
- [3] M. Froissart, *Phys. Rev.* **123**, 1053 (1961). A. Martin, *Nuovo Cimento* **42A**, 930 (1966) and arXiv:0812.0680v3.
- [4] E. Gotsman, E. Levin and U. Maor, *Phys. Letters* **B716**, 425 (2012), and references quoted there.
- [5] M. L. Good and W. D. Walker, *Phys. Rev.* **120**, 1857 (1960).
- [6] E. Gotsman, E. Levin and U. Maor, *Eur. Phys. J.* **C71**, 1553 (2011).
- [7] E. Gotsman, E. Levin, U. Maor and J. S. Miller, *Eur. Phys. J.* **C57**, 689 (2008).
- [8] E. Gotsman, E. Levin and U. Maor, *Phys. Rev. D* **85**, 094007 (2012).
- [9] O. Villalobos Baillie, talk at Diffraction 2012, Lanzarote, September 2012.
- [10] M. G. Poghosyan *et al* [ALICE Collaboration], *J. Phys. G* **38**, 124044 (2011).
- [11] F. Ferro [TOTEM Collaboration], *Europhys. Lett.* **96**, 21002 (2011).
- [12] G. Aad *et al.* [ATLAS Collaboration], *Nature Commun.* **2**, 463 (2011).
- [13] S. Chatrchyan *et al.* [CMS Collaboration], arXiv:1210.6718 [hep-ex].

- [14] J. P. Revol, *Proton-proton cross sections with multiplicities with ALICE at the LHC*, talk at the Low-x Meetings, Phafos, Cyprus, June 27- July 1, 2012.
- [15] P. Abreu, *et al.* [Auger Collaboration], *Phys. Rev. Letters* **109**, 062002 (2012).
- [16] M. G. Ryskin, A. D. Martin and V. A. Khoze, *Eur. Phys. J.* **C71**, 1617 (2011).
- [17] M. G. Ryskin, A. D. Martin and V. A. Khoze, *Eur. Phys. J.* **C72**, 1937 (2012).
- [18] S. Ostapchenko, *Phys. Rev.* **D81**, 114028 (2010).
- [19] A. B. Kaidalov and M. G. Poghosyan, arXiv:09095156 [hep-ph].
- [20] R. Ciesielski and K. Goulios, arXiv:1205.1446.
- [21] B.Z. Kopeliovich, I.K. Potoshnikova, B. Povh and Ivan Schmidt, *Phys.Rev.* **D76**, 044020 (2007).
- [22] E. Ferreira, T. Kodama and A.K. Kohara, *Eur. Phys. J.* **C73**, 2326 (2013).
- [23] C. Flensburg and G. Gustafson, *JHEP* **1010**, 014 (2010).