Electroproduction of two light vector mesons in next-to-leading order BFKL: recent results

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Outline

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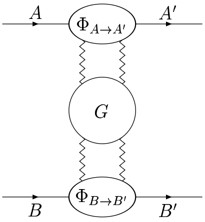
Introduction and Motivations

Scattering $A + B \longrightarrow A' + B'$ in the Regge kinematical region $s \to \infty$, t fixed

- BFKL approach: convolution of the Green's function of two interacting Reggeized gluons and of the impact factors of the colliding particles.



- Valid both in
  LLA (resummation of all terms $(\alpha_s \ln(s))^n$)
  NLA (resummation of all terms $\alpha_s(\alpha_s \ln(s))^n$).
- The Green's function is determined through the BFKL equation.
  [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]
- The kernel of the BFKL equation is completely known in the NLA for the forward ($t = 0$) case ...
  [V.S. Fadin, L.N. Lipatov (1998)]
  [G. Camici, M. Ciafaloni (1998)]
- ... and for the non-forward ($t \neq 0$) case [V.S. Fadin, R. Fiore (2005)]

- Impact factors have been calculated in the NLA for
  colliding partons [V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)]
  [M. Ciafaloni and G. Rodrigo (2000)]
  forward jet production [J. Bartels, D. Colferai, G.P. Vacca (2003)]

- Colorless NLA impact factors
  - $\gamma^* \to \gamma^*$, close to completion
    [J. Bartels, D. Colferai, S. Gieseke, A. Kyrieleis (2002)]
    [V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2003)]
    [J. Bartels, A. Kyrieleis (2004)]
  - $\gamma^* \to V$, with $V = \rho^0, \omega, \phi$, forward case
    [D.Yu. Ivanov, M.I. Kotsky, A. P. (2004)]

The (forward) $\gamma^*\gamma^* \longrightarrow V\ V$ amplitude is the first amplitude of a physical process completely calculable within perturbative QCD in the NLA.

(For the Born non-forward case, see [B. Pire, L. Szymanowsky, S. Wallon (2004)]; for the LLA case and for an estimated NLA result, see [R. Enberg, B. Pire, L. Szymanowsky, S. Wallon (2005)])

- Theoretical importance:
  - possibility to understand the role and the optimal choice of energy scales in the BFKL approach
  - comparison between different approaches (BFKL *vs.* DGLAP, etc.)

- Phenomenological interest:
  - first step toward the application of the BFKL approach to the description of
  - $\gamma^* p \to Vp$, at HERA
  - $\gamma^*\gamma^* \to V\ V$ or $\gamma^*\gamma \to VJ/\Psi$, at high-energy e^+e^- and $e\gamma$ colliders

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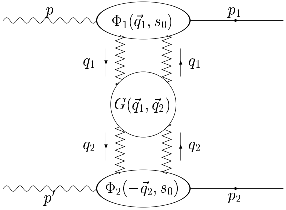
Kinematics and BFKL amplitude

$\gamma^*(p)\gamma^*(p') \to V(p_1)V(p_2)$



$p_1^2 = p_2^2 = 0\ , \qquad 2(p_1 p_2) = s$
(p_1 and p_2 Sudakov vectors)

virtual photon momenta:

$p \simeq p_1 - \frac{Q_1^2}{s} p_2\ , \qquad p' \simeq p_2 - \frac{Q_2^2}{s} p_1$

$s \gg Q_{1,2}^2 \gg \Lambda_{QCD}^2$

- Longitudinally polarized vector mesons are produced by longitudinally polarized photons; other helicity amplitudes power suppressed by $\sim m_\rho / Q_{1,2}$;
  [D.Yu. Ivanov, M.I. Kotsky, A. P. (2004)]
- forward scattering, i.e. zero transverse momenta of the produced mesons

$$\mathcal{I}m_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^2} \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} \Phi_1(\vec{q}_1, s_0) \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} \Phi_2(-\vec{q}_2, s_0) \int\limits_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

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1. The $\gamma^*\gamma^* \to V\ V$ forward amplitude
   - Introduction and Motivations
   - Kinematics and BFKL amplitude
   - The $\gamma^* \to V$ impact factor
   - Representations of the BFKL amplitude
2. "Pure" BFKL regime
   - Evidence for large NLA corrections
   - PMS method
   - FAC method
   - BLM method
   - Comparison with an approach based on collinear improvement
3. "Collinear factorization" regime
4. Conclusions

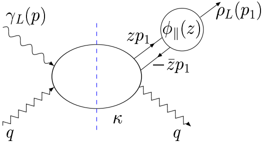
The $\gamma^* \to V$ impact factor

$$\Phi_{1,2}(\vec{q}) = \alpha_s D_{1,2} \left[C^{(0)}_{1,2}(\vec{q}^{\,2}) + \bar{\alpha}_s C^{(1)}_{1,2}(\vec{q}^{\,2}) \right]$$

$$D_{1,2} = -\frac{4\pi e_q f_V}{N_c Q_{1,2}} \sqrt{N_c^2 - 1}$$

$$e_q \longrightarrow \frac{e}{\sqrt{2}}\,, \qquad \frac{e}{3\sqrt{2}}\,, \qquad -\frac{e}{3}$$

$$\rho^0\,, \qquad \omega\,, \qquad \phi$$



- Leading order (photon virtuality Q^2): $C^{(0)}(\vec{q}^{\,2}) = \int_0^1 dz\, \frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + z\bar{z}Q^2} \phi_\parallel(z, \mu_F)$
- Next-to-leading order: $C^{(1)}(\vec{q}^{\,2}) = \frac{1}{4N_c} \int_0^1 dz\, \frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + z\bar{z}Q^2} [\tau(z) + \tau(1-z)] \phi_\parallel(z, \mu_F)$

$\tau(z)$ – 2-page-long expression, ... skipped

$\phi_\parallel(z, \mu_F)$ is the twist-2 meson distribution amplitude $\longrightarrow \phi_\parallel^{as}(z) = 6z(1-z)$

Outline

Representations of the BFKL amplitude

$$\frac{\mathcal{I}m_s\left(\mathcal{A}\right)}{D_1 D_2} = \frac{s}{(2\pi)^2}\int_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar\alpha_s(\mu_R)\chi(\nu)} \alpha_s^2(\mu_R) c_1(\nu) c_2(\nu)\left[1 + \bar\alpha_s(\mu_R)\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)} + \frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right)\right.$$

$$\left. + \bar\alpha_s^2(\mu_R) \ln\left(\frac{s}{s_0}\right)\left(\bar\chi(\nu) + \frac{\beta_0}{8N_c}\chi(\nu)\left[-\chi(\nu) + \frac{10}{3} + i\frac{d\ln\left(\frac{c_1(\nu)}{c_2(\nu)}\right)}{d\nu} + 2\ln(\mu_R^2)\right]\right)\right]$$

$$c_1(\nu) = \int d^2\vec q\ \frac{C_1^{(0)}(\vec q^{\,2})}{(\vec q^{\,2})} \frac{\left(\vec q^{\,2}\right)^{i\nu - \frac{1}{2}}}{\pi\sqrt{2}} \qquad c_2(\nu) = \int d^2\vec q\ \frac{C_2^{(0)}(\vec q^{\,2})}{(\vec q^{\,2})} \frac{\left(\vec q^{\,2}\right)^{-i\nu - \frac{1}{2}}}{\pi\sqrt{2}}$$

(analogously for $c_1^{(1)}(\nu)$ and $c_2^{(1)}(\nu)$)

$$\bar\alpha_s(\mu_R) = \frac{\alpha_s(\mu_R) N_c}{\pi} \qquad \chi(\nu) = 2\psi(1) - \psi\left(\frac{1}{2} + i\nu\right) - \psi\left(\frac{1}{2} - i\nu\right)$$

If only the "allowed" terms in the NLA are kept, $(\alpha_s \ln(s))^n)$ and $\alpha_s(\alpha_s \ln(s))^n$, the dependence on s_0 and μ_R disappears.

- Series representation for the amplitude

$$\frac{Q_1 Q_2}{D_1 D_2} \frac{\mathcal{I}m_s \mathcal{A}}{s} = \frac{1}{(2\pi)^2} \alpha_s(\mu_R)^2 \left[b_0 + \sum_{n=1}^{\infty} \bar{\alpha}_s(\mu_R)^n \, b_n \left(\ln\left(\frac{s}{s_0}\right)^n + d_n(s_0, \mu_R) \ln\left(\frac{s}{s_0}\right)^{n-1} \right) \right]$$

- LLA

$$\frac{b_n}{Q_1 Q_2} = \int_{-\infty}^{+\infty} d\nu \, c_1(\nu) c_2(\nu) \frac{\chi^n(\nu)}{n!}$$

- NLA

d_n="complicated ν-integral, containing the NLA impact factors in the ν-representation, $c^{(1)}_{1,2}$, and the NLA BFKL kernel eigenvalues"

- Exponentiated form of the amplitude

$$\frac{\mathcal{I}m_s\left(\mathcal{A}\right)}{D_1 D_2} = \frac{s}{(2\pi)^2} \int\limits_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(\nu)+\bar{\alpha}_s^2(\mu_R)\left(\bar{\chi}(\nu)+\frac{\beta_0}{8N_c}\chi(\nu)[-\chi(\nu)+\frac{10}{3}]\right)} \alpha_s^2(\mu_R) c_1(\nu) c_2(\nu)$$

$$\times\left[1+\bar{\alpha}_s(\mu_R)\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)}+\frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right)+\bar{\alpha}_s^2(\mu_R)\ln\left(\frac{s}{s_0}\right)\frac{\beta_0}{8N_c}\chi(\nu)\left(i\frac{d\ln(\frac{c_1(\nu)}{c_2(\nu)})}{d\nu}+2\ln(\mu_R^2)\right)\right]$$

Outline

Evidence for large NLA corrections

$Q_1 = Q_2 \equiv Q$ "pure" BFKL regime

LLA: b_n coefficients (Q-independent)

$$b_0 = 17.0664 \quad b_1 = 34.5920 \quad b_2 = 40.7609 \quad b_3 = 33.0618 \quad b_4 = 20.7467$$
$$b_5 = 10.5698 \quad b_6 = 4.54792 \quad b_7 = 1.69128 \quad b_8 = 0.554475$$

NLA: $d_n(s_0, \mu_R)$ coefficients ($s_0 = Q^2 = \mu_R^2$, $n_f = 5$)

$$d_1 = -3.71087 \quad d_2 = -11.3057 \quad d_3 = -23.3879 \quad d_4 = -39.1123$$
$$d_5 = -59.207 \quad d_6 = -83.0365 \quad d_7 = -111.151 \quad d_8 = -143.06$$

NLA: $d_n^{\rm imp}(s_0, \mu_R)$ coefficients ($s_0 = Q^2 = \mu_R^2$, impact factor contribution)

$$d_1^{\rm imp} = -3.71087 \quad d_2^{\rm imp} = -8.4361 \quad d_3^{\rm imp} = -13.1984 \quad d_4^{\rm imp} = -18.0971$$
$$d_5^{\rm imp} = -23.0235 \quad d_6^{\rm imp} = -27.9877 \quad d_7^{\rm imp} = -32.9676 \quad d_8^{\rm imp} = -37.9618$$

- Large NLA corrections!
- Optimization of the perturbative expansion needed!

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PMS method

- Principle of minimal sensitivity (PMS) [P.M. Stevenson (1981)]: require the minimal sensitivity to the change of both s_0 and μ_R.

- Strategy: for each fixed s calculate the amplitude for varying s_0 at fixed μ_R and viceversa, up to finding the optimal values for which the amplitude is least sensitive to variations of them.

- In practice, there are wide regions in s_0 and μ_R where the amplitude is very weekly dependent on s_0 and μ_R.

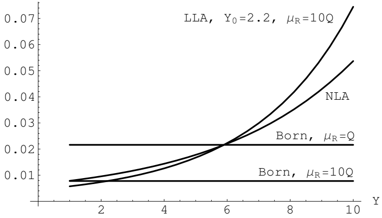
PMS method - Series representation

$$\frac{Q^2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s} = \frac{1}{(2\pi)^2}\alpha_s(\mu_R)^2\left[b_0+\sum_{n=1}^{\infty}\bar{\alpha}_s(\mu_R)^n\, b_n\left((Y-Y_0)^n+d_n(s_0,\mu_R)(Y-Y_0)^{n-1}\right)\right]$$

$$Y \equiv \ln\left(\frac{s}{Q^2}\right)\ ,\quad Y_0 \equiv \ln\left(\frac{s_0}{Q^2}\right)$$

$\frac{Q^2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s}$ vs Y

Q^2=24 GeV^2, $n_f = 5$



Lessons

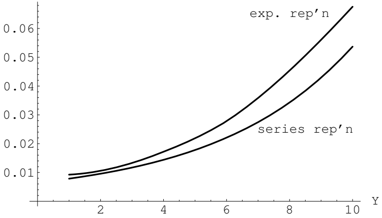
- The Born approximation does not give necessarily the estimate from below
- The optimal values for μ_R are "unnaturally" larger than Q (new scale or nature of the BFKL series?).

[D.Yu. Ivanov, A. P. (2005)]

PMS method - Exponentiated representation

$\frac{Q^2}{D_1 D_2} \frac{\mathcal{I}m_s \mathcal{A}}{s}$ vs Y

Q^2=24 GeV^2, $n_f = 5$



- Rough agreement with the PMS method applied to the series representation.
- The optimal value for μ_R is slightly smaller than in the previous case.

Outline

FAC method

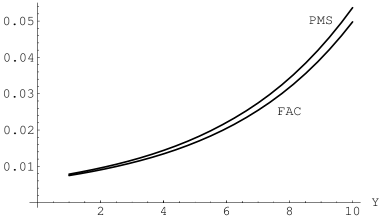
- Fast apparent convergence (FAC) [G. Grunberg (1980)]: require that the NLO corrections identically vanish.
- Strategy: for each fixed Y calculate determine the line of Y_0 and μ_R values for which the NLO corrections cancel; then, the optimal values of Y_0 and μ_R along this line are chosen according to "minimum sensitivity".

FAC method - Series representation

$$\frac{Q^2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s} = \frac{1}{(2\pi)^2}\alpha_s(\mu_R)^2\left[b_0+\sum_{n=1}^{\infty}\bar{\alpha}_s(\mu_R)^n\, b_n\left((Y-Y_0)^n+d_n(s_0,\mu_R)(Y-Y_0)^{n-1}\right)\right]$$

$\frac{Q^2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s}$ vs Y

Q^2=24 GeV^2, $n_f = 5$



- Despite the very different strategy, FAC and PMS give quite consistent results

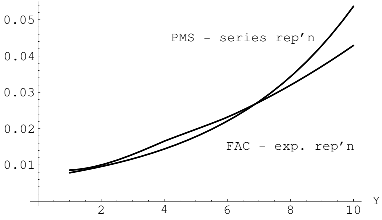
FAC method - Exponentiated representation

$$\frac{\mathcal{I}m_s\left(\mathcal{A}_{NLA}\right)}{D_1D_2} = \frac{s}{(2\pi)^2}\int\limits_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar\alpha_s(\mu_R)\chi(\nu)+\bar\alpha_s^2(\mu_R)\left(\bar\chi(\nu)+\frac{\beta_0}{8N_c}\chi(\nu)\left[-\chi(\nu)+\frac{10}{3}\right]\right)} \alpha_s^2(\mu_R)c_1(\nu)c_2(\nu)$$

$$\times\left[1+\bar\alpha_s(\mu_R)\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)}+\frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right)+\bar\alpha_s^2(\mu_R)\ln\left(\frac{s}{s_0}\right)\frac{\beta_0}{8N_c}\chi(\nu)\left(i\frac{d\ln\left(\frac{c_1(\nu)}{c_2(\nu)}\right)}{d\nu}+2\ln(\mu_R^2)\right)\right]$$

$$\frac{\mathcal{I}m_s\left(\mathcal{A}_{LLA}\right)}{D_1D_2} = \frac{s}{(2\pi)^2}\int\limits_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar\alpha_s(\mu_R)\chi(\nu)} \alpha_s^2(\mu_R)c_1(\nu)c_2(\nu)$$

$$\mathcal{I}m_s\left(\mathcal{A}_{NLA}\right) = \mathcal{I}m_s\left(\mathcal{A}_{LLA}\right) + \left[\mathcal{I}m_s\left(\mathcal{A}_{NLA}\right) - \mathcal{I}m_s\left(\mathcal{A}_{LLA}\right)\right]$$



$$\frac{Q^2}{D_1D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s} \ \text{vs}\ Y$$

Q^2=24 GeV^2, $n_f = 5$

Outline

BLM method

- [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)] optimization method: perform a finite renormalization to a physical scheme and then choose the renormalization scale in order to remove the β_0-dependent part.

Strategy (applied to the amplitude in the series representation only):

- finite renormalization to the MOM-scheme (ξ=0)

$$\alpha_S \to \alpha_S \left[1 + T_{MOM}(\xi = 0)\frac{\alpha_S}{\pi}\right]$$

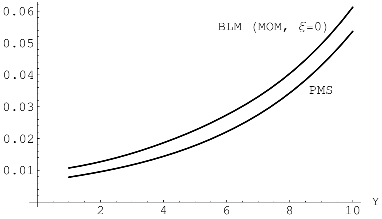
$$T_{MOM}(\xi = 0) = T_{MOM}^{conf} + T_{MOM}^{\beta}$$

$$T_{MOM}^{conf} = \frac{N_C}{8}\frac{17}{2}I \qquad T_{MOM}^{\beta} = -\frac{\beta_0}{2}\left[1 + \frac{2}{3}I\right] \qquad I \simeq 2.3439$$

- Y_0 and μ_R chosen in order to make the term proportional to β_0 in the resulting amplitude vanish (the β_0-dependence in the series representation of the amplitude is hidden into the d_n coefficients)
- optimal values for Y_0 and μ_R determined according to "minimum sensitivity"

$\frac{Q^2}{D_1 D_2} \frac{\mathcal{I}m_s \mathcal{A}}{s}$ *vs* Y

Q^2=24 GeV^2, $n_f = 5$



- Drawback: for each given Y, Y_0 "wants" to be as large as Y.

Outline

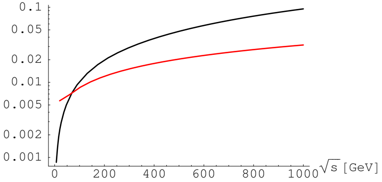
Comparison with an approach based on collinear improvement

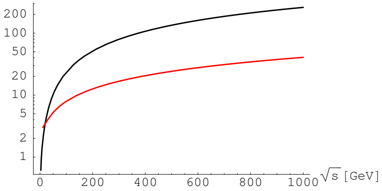
[R. Enberg, B. Pire, L. Szymanowsky, S. Wallon (2005)] made an estimate of NLA effects for the same amplitude, using the following ingredients:

- LO impact factors for the $\gamma^* \longrightarrow V$ transition;
- BLM scale fixing for the running of the coupling in the prefactor of the amplitude; the BLM scale is found using the amplitude with NLO $\gamma^* \longrightarrow V$ impact factors and two-gluon exchange;
- RG-resummed BFKL kernel; the resummation is performed on the LLA BFKL kernel at fixed coupling
  [V.A. Khoze, A.D. Martin, M.G. Ryskin, W.J. Stirling (2004)]

$\left.\frac{d\sigma}{dt}\right|_{t_{min}}$ in pb $\cdot$ GeV^{-2}

Q^2=16 GeV^2 $\longrightarrow$, $n_f = 4$





$\longleftarrow$ Q^2=4 GeV^2, $n_f = 3$

Courtesy R. Enberg for the curves in red.

"Collinear factorization" regime

$\gamma^*(Q_1)\gamma^*(Q_2) \to V\,V$ for strongly ordered photon virtualities, $Q_1^2 \gg Q_2^2$

$$\frac{Q_1 Q_2}{D_1 D_2}\frac{\mathcal{I}m_s \mathcal{A}}{s} = \frac{1}{(2\pi)^2}\alpha_s(\mu_R)^2 \times \left[b_0 + \sum_{n=1}^{\infty} \bar{\alpha}_s(\mu_R)^n\, b_n \left(\ln\left(\frac{s}{s_0}\right)^n + d_n(s_0,\mu_R) \ln\left(\frac{s}{s_0}\right)^{n-1} \right) \right] ,$$

$$\frac{b_n}{Q_1 Q_2} = \int_{-\infty}^{+\infty} d\nu\, c_1(\nu) c_2(\nu) \frac{\chi^n(\nu)}{n!}$$

$$c_1(\nu)c_2(\nu) = \frac{1}{Q_1 Q_2}\left(\frac{Q_1^2}{Q_2^2}\right)^{i\nu} \frac{9\,\pi^3(1+4\nu^2)\sinh(\pi\nu)}{32\,\nu\,(1+\nu^2)\cosh^3(\pi\nu)}$$

$$\chi(\nu) = 2\psi(1) - \psi\left(\frac{1}{2}+i\nu\right) - \psi\left(\frac{1}{2}-i\nu\right)$$

$d_n(s_0,\mu_R)$ is a (much) more complicate ν-integral, involving the NLA impact factors and the NLA BFKL eigenvalues in the ν-representation

- Leading-twist approximation: close the ν-integration contour in the upper plane and take only the residue at the pole $\nu = i/2$.

- LLA: near the pole, $\nu = \frac{i}{2} + \Delta$

$$c_1(\Delta)c_2(\Delta) = \left(\frac{Q_1^2}{Q_2^2}\right)^{i\Delta} \frac{1}{Q_1^2} \left(\frac{-3}{\Delta^2} + \frac{i}{\Delta} + \frac{34}{3} + O(\Delta)\right)$$

$$\chi(\Delta) = -\frac{i}{\Delta} + O(\Delta^2)$$

$$\text{Residue}_{\Delta=0} \left(\frac{(Q_1^2/Q_2^2)^{i\Delta}}{\Delta^n}\right) = i^{n-1} \frac{\log^{n-1}(Q_1^2/Q_2^2)}{(n-1)!}$$

$$b_n = \frac{2\pi}{3} \frac{Q_2}{Q_1} L^{n-1} \frac{9L^2 - 3L(n+1) + 34n(n+1)}{n!(n+1)!} + O(L^{n-2})$$

$$L \equiv \log(Q_1^2/Q_2^2)$$

In the NLA the problem arises from the very complicated expression for the impact factors ($C_{1,2}(\vec{q}^{\,2})$ below are the 2-page-long expressions):

$$c_1^{(1)}(\nu) = \int d^2\vec{q}\ C_1^{(1)}(\vec{q}^{\,2}) \frac{\left(\vec{q}^{\,2}\right)^{i\nu-\frac{3}{2}}}{\pi\sqrt{2}} = \int d^2\vec{q}\ C_1^{(1)}(\vec{q}^{\,2}) \frac{\left(\vec{q}^{\,2}\right)^{-2+i\Delta}}{\pi\sqrt{2}}$$

near the pole ($\nu = \frac{i}{2} + \Delta$); only the small-$\vec{q}^{\,2}$ limit of $C_1^{(1)}(\vec{q}^{\,2})$ needed

Analogously,

$$c_2^{(1)}(\nu) = \int d^2\vec{q}\ C_2^{(1)}(\vec{q}^{\,2}) \frac{\left(\vec{q}^{\,2}\right)^{-i\nu-\frac{3}{2}}}{\pi\sqrt{2}} = \int d^2\vec{q}\ C_2^{(1)}(\vec{q}^{\,2}) \frac{\left(\vec{q}^{\,2}\right)^{-1-i\Delta}}{\pi\sqrt{2}}$$

near the pole ($\nu = \frac{i}{2} + \Delta$); only the large-$\vec{q}^{\,2}$ limit for $C_2^{(1)}(\vec{q}^{\,2})$ needed

$$b_n d_n = 2\pi \frac{Q_2}{Q_1}\frac{1}{(n!)^2}\left\{ L^{n+1}\left[\frac{3\beta_0}{8N}\frac{n(n^2+n+2)}{n+1} - \frac{n}{n+1}\left(\frac{3}{4N^2}+\frac{17}{4}+\frac{11n}{4}\right) - \frac{n_f}{N^3}\frac{n(n-1)}{2(n+1)}\right.\right.$$

$$\left. + \frac{3n}{(n+1)}\log\frac{s_0}{Q_1^2}\right] + L^n\left[-n\log\frac{s_0}{Q_1^2} - \frac{11n(n-1)}{12}\frac{n_f}{N^3} + \frac{\beta_0}{4N}\left(-3n(n+1)\log\frac{Q_1^2}{\mu_R^2}\right.\right.$$

$$\left.\left.\left. -n\left(\frac{n(n-21)}{2}-10\right)\right) - n\left(\frac{19}{6}+\frac{11n}{3}+\frac{\pi^2}{2}-\frac{51}{24N^2}\right)\right]\right\}$$

$$L \equiv \log(Q_1^2/Q_2^2)$$

- Terms with $\alpha_s(\mu_R)^{n+2}L^{n+2}$ canceled out, as it should be
- The same procedure can be applied to the case of the $\gamma^*\gamma^*$ total cross section

Conclusions

- Closed analytical expression found for the $\gamma^* \gamma^* \longrightarrow V\ V$ forward amplitude in the Regge limit of QCD with next-to-leading order accuracy.
- For equal photons' virtualities, i.e. in the BFKL regime
  - the next-to-leading order corrections are large and of opposite sign with respect to the leading order contribution.
  - the PMS optimization method allows to get stable results for the amplitude; the optimal value of the renormalization scale μ_R turns out to be much larger than the kinematical scale of the problem; this could be a manifestation of the nature of the BFKL series.
  - the energy dependence of the amplitude is reasonably stable under change of representation (exponentiated vs series) or of optimization method (FAC, BLM).
  - there is discrepancy in the differential cross section at t_{min} with the determination by [R. Enberg et al. (2005)].
- For strongly ordered photons' virtualities, the structure of the amplitude is compatible with the leading-twist collinear factorization.

Diffraction 2006

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The Green's function

$$\mathcal{I}m_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^2}\int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\Phi_1(\vec{q}_1, s_0)\int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\Phi_2(-\vec{q}_2, s_0)\int\limits_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

BFKL equation: $\delta^2(\vec{q}_1-\vec{q}_2) = \omega\, G_\omega(\vec{q}_1, \vec{q}_2) - \int d^2\vec{q}\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_2)$

Transverse momentum notation: $\hat{\vec{q}}\;|\vec{q}_i\rangle = \vec{q}_i|\vec{q}_i\rangle$

$$\langle\vec{q}_1|\vec{q}_2\rangle = \delta^{(2)}(\vec{q}_1-\vec{q}_2) \qquad \langle A|B\rangle = \langle A|\vec{k}\rangle\langle\vec{k}|B\rangle = \int d^2k A(\vec{k})B(\vec{k})$$

$$\hat{1} = (\omega - \hat{K})\hat{G}_\omega \qquad \longrightarrow \qquad \hat{G}_\omega = (\omega-\hat{K})^{-1}$$

$$\hat{K} = \bar{\alpha}_s\hat{K}^0 + \bar{\alpha}_s^2\hat{K}^1\ , \qquad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

With NLA accuracy

$$\hat{G}_\omega = (\omega-\bar{\alpha}_s\hat{K}^0)^{-1} + (\omega-\bar{\alpha}_s\hat{K}^0)^{-1}\left(\bar{\alpha}_s^2\hat{K}^1\right)(\omega-\bar{\alpha}_s\hat{K}^0)^{-1} + \mathcal{O}\left[\left(\bar{\alpha}_s^2\hat{K}^1\right)^2\right]$$

Basis of eigenfunctions of the LLA kernel: $\{|\nu\rangle\}$

$$\boxed{\hat{K}^0|\nu\rangle = \chi(\nu)|\nu\rangle} \qquad \chi(\nu) = 2\psi(1) - \psi\left(\tfrac{1}{2}+i\nu\right) - \psi\left(\tfrac{1}{2}-i\nu\right)$$

$$\langle\vec{q}|\nu\rangle = \frac{1}{\pi\sqrt{2}}\left(\vec{q}^{\,2}\right)^{i\nu-\frac{1}{2}} \qquad \langle\nu'|\nu\rangle = \int \frac{d^2\vec{q}}{2\pi^2}\left(\vec{q}^{\,2}\right)^{i\nu-i\nu'-1} = \delta(\nu-\nu')$$

Action of the full NLA kernel on the LLA eigenfunctions:

$$\begin{aligned}\hat{K}|\nu\rangle &= \bar{\alpha}_s(\mu_R)\chi(\nu)|\nu\rangle + \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(\nu) + \frac{\beta_0}{4N_c}\chi(\nu)\ln(\mu_R^2)\right)|\nu\rangle \\ &+ \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(\nu)\left(i\frac{\partial}{\partial\nu}\right)|\nu\rangle\end{aligned}$$

$$\chi^{(1)}(\nu) = -\frac{\beta_0}{8\,N_c}\left(\chi^2(\nu) - \frac{10}{3}\chi(\nu) - i\chi'(\nu)\right) + \bar{\chi}(\nu)$$

$$\bar{\chi}(\nu) = -\frac{1}{4}\left[\frac{\pi^2-4}{3}\chi(\nu) - 6\zeta(3) - \chi''(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} + \frac{\pi^2\sinh(\pi\nu)}{2\nu\cosh^2(\pi\nu)}\left(3 + \left(1+\frac{n_f}{N_c^3}\right)\frac{11+12\nu^2}{16(1+\nu^2)}\right) + 4\phi(\nu)\right]$$

$$\phi(\nu) = 2\int_0^1 dx\,\frac{\cos(\nu\ln(x))}{(1+x)\sqrt{x}}\left[\frac{\pi^2}{6} - \mathrm{Li}_2(x)\right]$$

Series representation

$$\frac{Q_1Q_2}{D_1D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s} = \frac{1}{(2\pi)^2}\alpha_s(\mu_R)^2\Bigg[b_0+\sum_{n=1}^{\infty}\bar{\alpha}_s(\mu_R)^n\, b_n\left(\ln\left(\frac{s}{s_0}\right)^n\right.$$
$$\left.\left. + \quad d_n(s_0,\mu_R)\ln\left(\frac{s}{s_0}\right)^{n-1}\right)\right]$$

LLA

$$\frac{b_n}{Q_1Q_2} = \int_{-\infty}^{+\infty} d\nu\, c_1(\nu)c_2(\nu)\frac{\chi^n(\nu)}{n!}$$

$$b_0 = \frac{9\pi}{4}(7\zeta(3)-6)$$

Born (2-gluon exchange limit)

[B. Pire, L. Szymanowsky, S. Wallon (2004)]

NLA

$$d_n = n\ln\left(\frac{s_0}{Q_1Q_2}\right) + \frac{\beta_0}{4N_c}\left((n+1)\frac{b_{n-1}}{b_n}\ln\left(\frac{\mu_R^2}{Q_1Q_2}\right) - \frac{n(n-1)}{2}\right.$$
$$\left. + \frac{Q_1Q_2}{b_n}\int_{-\infty}^{+\infty} d\nu\,(n+1)f(\nu)c_1(\nu)c_2(\nu)\frac{\chi^{n-1}(\nu)}{(n-1)!}\right)$$
$$+\frac{Q_1Q_2}{b_n}\left(\int_{-\infty}^{+\infty} d\nu\, c_1(\nu)c_2(\nu)\frac{\chi^{n-1}(\nu)}{(n-1)!}\left[\frac{\bar{c}_1^{(1)}(\nu)}{c_1(\nu)} + \frac{\bar{c}_2^{(1)}(\nu)}{c_2(\nu)} + (n-1)\frac{\bar{\chi}(\nu)}{\chi(\nu)}\right]\right)$$
$$f(\nu) = \frac{5}{3} + \psi(3+2i\nu) + \psi(3-2i\nu) - \psi\left(\frac{3}{2}+i\nu\right) - \psi\left(\frac{3}{2}-i\nu\right)$$