QCD and Strings

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LOW X MEETING, LISBON, PORTUGAL, 28/06 - 1/07, 2006

- A 35-years-old Puzzle
- AdS/CFT correspondence
- AdS/CFT and QCD Hydrodynamics
- Thermalization of a perfect fluid

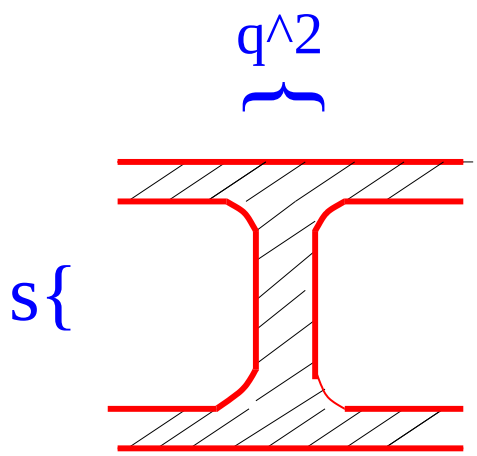
[a](with **Romuald Janik**, Cracow U.) Last papers: hep-th/0512162, hep-th/0606149

A 35-years-old Puzzle

A String Theory for **Strong** Interactions ?

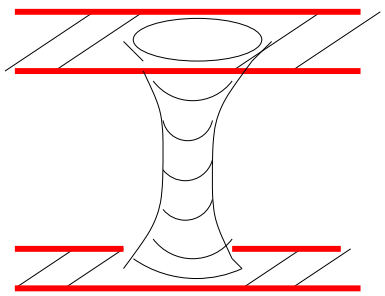
Strings $\Rightarrow$ Gauge Fields $\Rightarrow$ Strings

1968 $\Rightarrow$ 1974 $\Rightarrow$ 1998



Veneziano Amplitude

$A_R(s,q^2)$



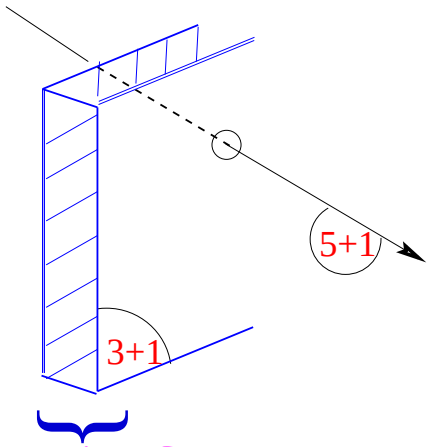
Shapiro-Virasoro Amplitude

$A_P(s,q^2)$

AdS/CFT Correspondence

J.Maldacena (1998)

AdS



Macroscopic

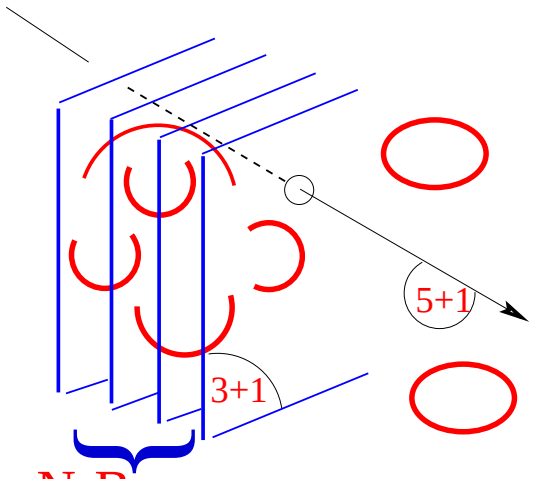
$(g_S N)^{-1} \to 0$

$AdS_5 \times S_5$ Superstring

Weak Gravity

Strong Gravity

CFT



Microscopic

$g_S N \to 0$

SU(N) Gauge Theory on the N-Branes

Strong Coupling

Weak Coupling

$\Longleftrightarrow$ Duality

More on AdS/CFT

- D_3-brane Solution of Super Gravity:

$$\boxed{ds^2 = f^{-1/2}(-dt^2 + \sum_1^3 dx_i^2) + f^{1/2}(dr^2 + r^2 d\Omega_5)}$$

"On-Branes$\times$Out-Branes"

$$f = 1 + \frac{R^4}{r^4} \ ; \ R = 4\pi g_{YM}^2 \alpha'^2 N$$

- "Maldacena limit":

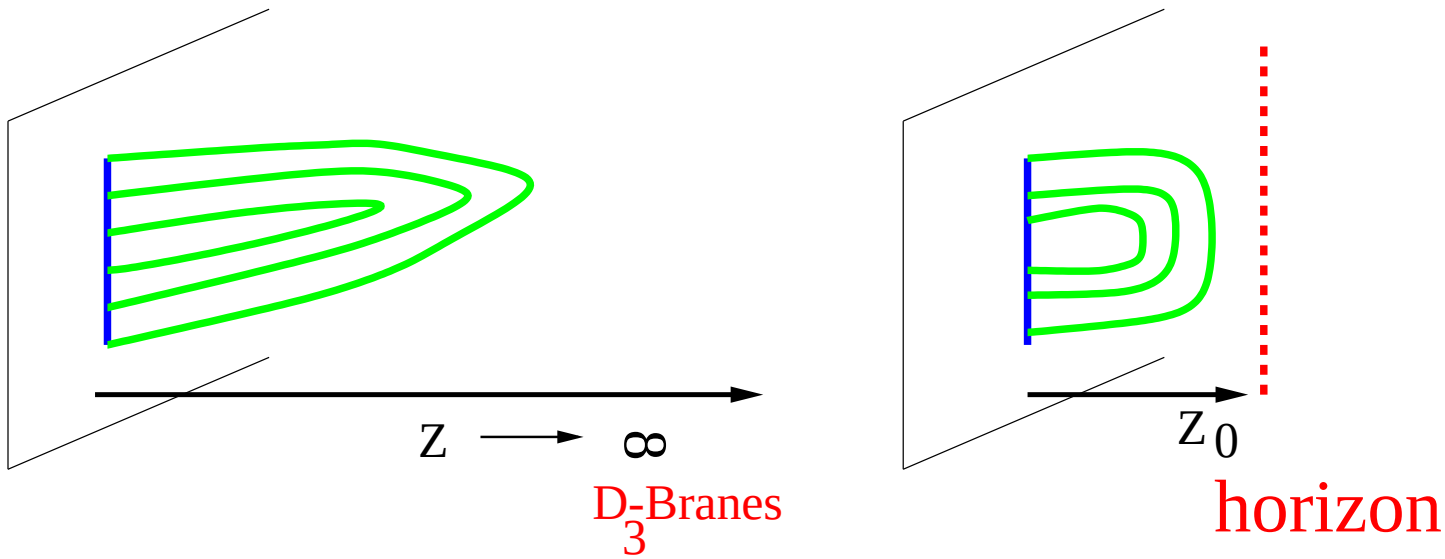
$$\frac{\alpha'(\to 0)}{r(\to 0)} \to z \ , \ R \ fixed \ \Rightarrow g_{YM}^2 N \to \infty$$

Strong coupling limit

$$\boxed{ds^2 = \frac{1}{z^2}(-dt^2 + \sum_{1-3} dx_i^2 + dz^2) + R^2 d\Omega_5}$$

Background Structure: $\mathbf{AdS}_5 \times S_5$

Holography

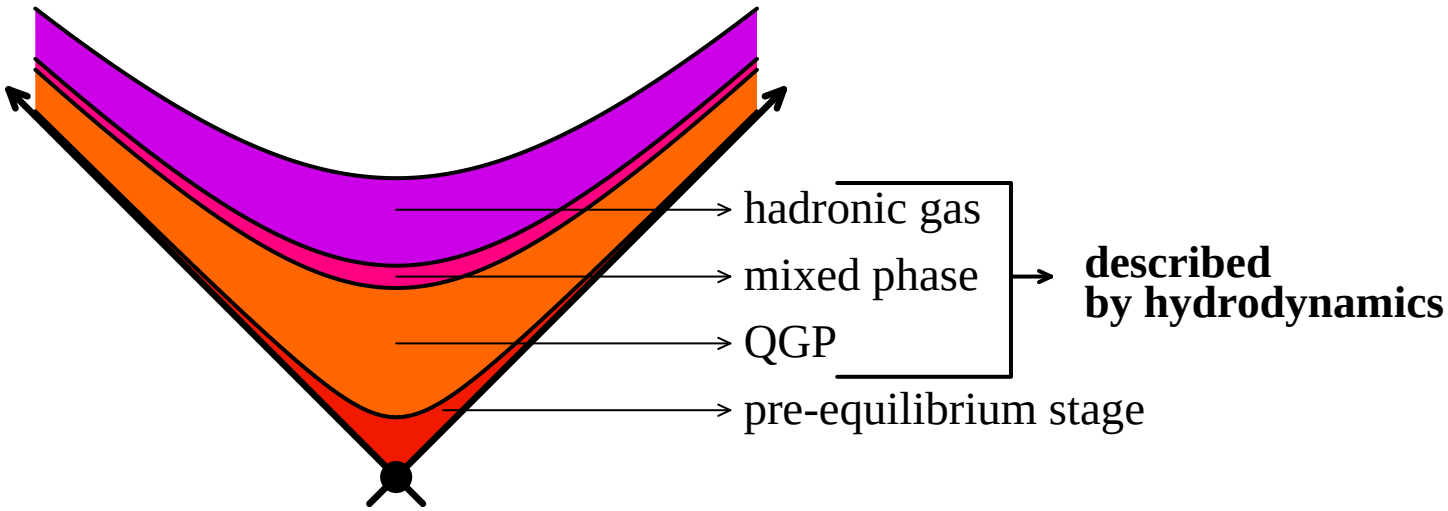


$$Area^{AdS}_{min} = \lim_{\chi\to\infty} L/(L/\chi) \qquad Area^{BH}_{min} = \lim L \times (L/\chi)$$

Confinement or Temperature: Minimal Surfaces with Horizon

AdS/CFT and QCD Hydrodynamics

QGP and Hydrodynamics: J.D.Bjorken (1982)



- **Boost Invariance**

$$\tau = \sqrt{x_0^2 + x_1^2} \; ; \; y = \frac{1}{2} \log \frac{x_0 + x_1}{x_0 - x_1} \; ; \; x_T \to x_1, x_2$$

- **QGP: Perfect fluid behaviour**

- **Pre-equilibrium stage: Fast**

The 4d Energy-Momentum Tensor

- **Constraint Equations**

$$\boxed{\begin{array}{c} \mathbf{T}^{\mu}{}_{\mu} \equiv -T_{\tau\tau} + \frac{1}{\tau^2} T_{yy} + 2T_{xx} = 0 \\ \mathbf{D}_{\nu} T^{\mu\nu} \equiv \tau \frac{d}{d\tau} T_{\tau\tau} + T_{\tau\tau} + \frac{1}{\tau^2} T_{yy} = 0 \end{array}}$$

- **Boost-invariant Tensor**

$$T_{\mu\nu} = \begin{pmatrix} f(\tau) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\tau^3 \frac{d}{d\tau} f(\tau) - \tau^2 f(\tau) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & f(\tau) + \frac{1}{2}\tau \frac{d}{d\tau} f(\tau) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{...} \end{pmatrix}$$

- **1-parameter family**

$f(\tau) \propto \tau^{-s}$: $(0 < s < 4 \; ; \; T_{\mu\nu} t^{\mu} t^{\nu} \geq 0)$

$f(\tau) \propto \tau^{-\frac{4}{3}}$: **Perfect Fluid**

$f(\tau) \propto \tau^{-1}$: **Free streaming**

$4 \rightarrow 5d$: Holographic Renormalization

K.Skenderis (2002)

- **Fefferman-Graham Coordinates:**

$$\boxed{ds^2 = \frac{g_{\mu\nu}dx^\mu dx^\nu + dz^2}{z^2}}$$

- $4d \Leftrightarrow 5d$ **metric:**

$$\boxed{g_{\mu\nu} = g^{(0)}_{\mu\nu}(= \eta_{\mu\nu}) + z^2 g^{(2)}_{\mu\nu}(= 0) + z^4 g^{(4)}_{\mu\nu}(\propto \langle T_{\mu\nu}\rangle) + \ldots}$$

$+\ldots$: **from Einstein Eqs.**

Technicalities...

- **Boost-Invariant ansatz:**

$$\boxed{ds^2 = \frac{-e^{a(\tau,z)}d\tau^2 + \tau^2 e^{b(\tau,z)}dy^2 + e^{c(\tau,z)}dx_\perp^2}{z^2} + \frac{dz^2}{z^2}}$$

- **Einstein Equation(s):**

$$[a(\tau,z), b(\tau,z), c(\tau,z)] = [a(v), b(v), c(v)] + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right)$$

$$\boxed{v = \frac{z}{\tau^{s/4}}}$$

$$v(2a'(v)c'(v)+a'(v)b'(v)+2b'(v)c'(v))-6a'(v)-6b'(v)-12c'(v)+vc'(v)^2 = 0$$
$$3vc'(v)^2 + vb'(v)^2 + 2vb''(v) + 4vc''(v) - 6b'(v) - 12c'(v) + 2vb'(v)c'(v) = 0$$
$$2vsb''(v) + 2sb'(v) + 8a'(v) - vsa'(v)b'(v) - 8b'(v) + vsb'(v)^2 +$$
$$4vsc''(v) + 4sc'(v) - 2vsa'(v)c'(v) + 2vsc'(v)^2 = 0 \ .$$

- **Asymptotic Solution**

$$\begin{aligned} a(v) &= A(v) - 2m(v) \\ b(v) &= A(v) + (2s-2)m(v) \\ c(v) &= A(v) + (2-s)m(v) \end{aligned} \tag{1}$$

Dual of a Perfect Relativistic fluid

$$\boxed{v = \frac{z}{\tau^{1/3}}}$$

- **Asymptotic metric**

$$\boxed{ds^2 = \frac{1}{z^2}\left[-\frac{\left(1-\frac{e_0}{3}\ \frac{z^4}{\tau^{4/3}}\right)^2}{1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}}d\tau^2+\left(1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}\right)(\tau^2 dy^2+dx_\perp^2)\right]+\frac{dz^2}{z^2}}$$

- **Black Hole off in the 5th dimension**

$$Horizon: \ z_0 = \left(\frac{3}{e_0}\right)^{\frac{1}{4}} \cdot \tau^{\frac{1}{3}} \ .$$

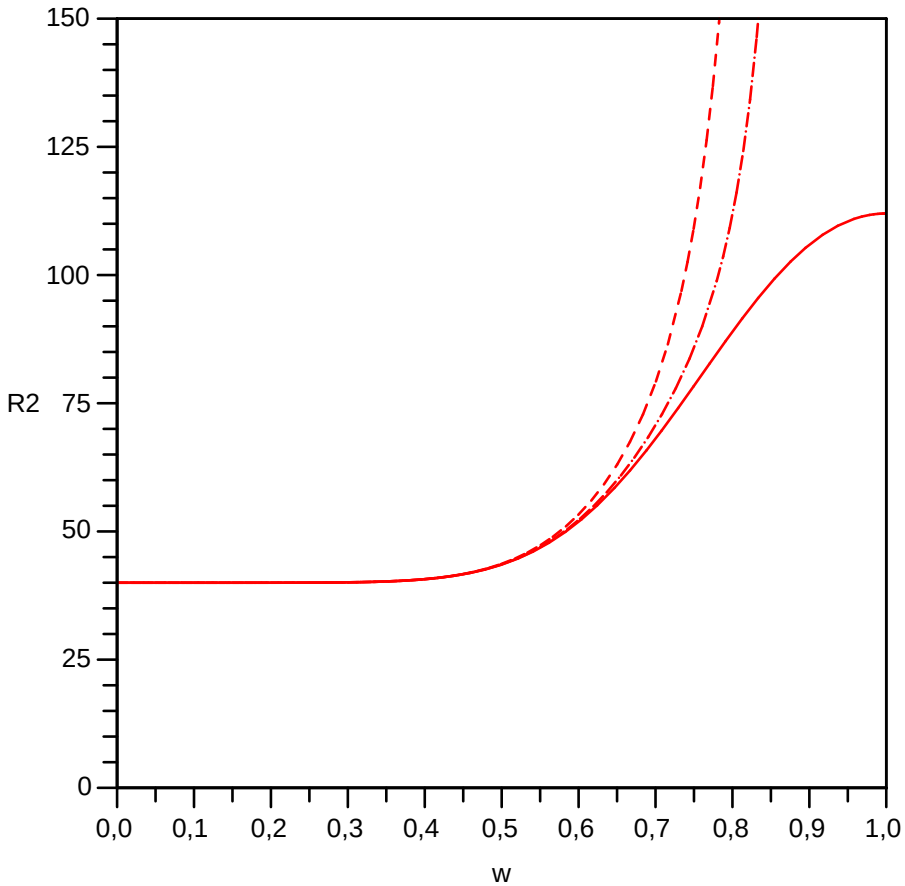
$$Temperature: \ T(\tau) \sim \frac{1}{z_0} \sim \tau^{-\frac{1}{3}}$$

$$Entropy: \ S(\tau) \sim Area \sim \tau \cdot \frac{1}{z_0^3} \sim const$$

- **Other cases: True Singularities $\neq$ BH**

AdS/CFT: Selection of the Perfect Fluid

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \quad s = \frac{4}{3} \pm .1$$



Thermalization of a perfect fluid

- **Thermal stability from scalar excitations:**

$$\boxed{\Delta\phi \equiv \frac{1}{\sqrt{-g}}\partial_i\left(\sqrt{-g}g^{ij}\partial_j\phi\right) = 0}$$

$$\left[\partial_z \to \tau^{-\frac{1}{3}}\partial_v \ ; \ \partial_\tau \to \partial_\tau - \frac{1}{3}\tau^{-\frac{4}{3}}\partial_v\right]$$

$$-\frac{1}{v^3}\frac{(1+v^4)^2}{1-v^4}\partial_\tau^2\phi(\tau,v) + \tau^{-\frac{2}{3}}\ \partial_v\left(\frac{1}{v^3}(1-v^8)\partial_v\phi(\tau,v)\right) = 0$$

- **Scaling solution**

$$\partial_\tau^2 f(\tau) = -\omega^2\tau^{-\frac{2}{3}}f(\tau) \ \Rightarrow \ f(\tau) = \sqrt{\tau}J_{\pm\frac{3}{4}}\left(\frac{3}{2}\omega\tau^{\frac{2}{3}}\right) \ \sim \ \tau^{\frac{1}{6}}\ e^{\frac{3}{2}i\omega\tau^{\frac{2}{3}}}$$

$$\boxed{\partial_v\left(\frac{1}{v^3}(1-v^8)\partial_v\phi(v)\right) + \omega^2\frac{1}{v^3}\frac{(1+v^4)^2}{1-v^4}\phi(v) = 0}$$

- **Dominant Decay Frequency**

$$\frac{\omega_c}{\pi T} \sim 3.1194 - 2.74667\ i$$

Conclusions

- **AdS/CFT and Strings**
  Construction of "Gravity Duals"
- AdS/CFT and QGP Hydrodynamics
  Construction of the "Dual" of relativistic fluids
- Selection of the perfect fluid
  Non-singular 5d Horizon $\Rightarrow$ Asymptotic
  4d perfect fluid
- Thermalization:
  The perfect fluid is "very stable"

Unexpected AdS/CFT consequences for QCD at strong coupling?