Pomeron Loops in Small-x Evolution

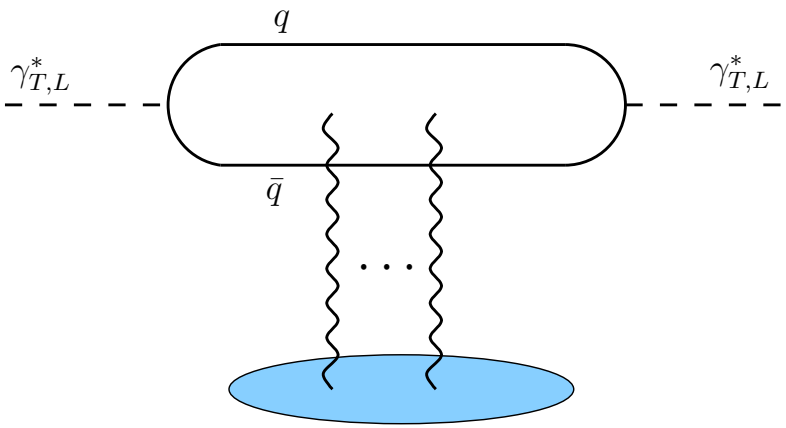
Arif Shoshi

Bielefeld University, Germany

Low-x Meeting, Lisbon, Portugal, June 27-July 1, 2006

Introduction

- A proton (nucleus) probed with a dipole:



- Total cross section:

$$\sigma^{\gamma^*_{T,L}\ p}(Q, x_{Bj}) = \int d^2\boldsymbol{r} \int dz\ |\psi_{T,L}(z,Q)|^2\ 2 \int d^2\boldsymbol{b}\ T(\boldsymbol{r}, x_{Bj}, \boldsymbol{b})$$

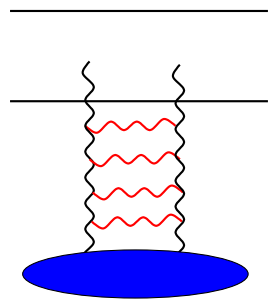
- How does $T(\boldsymbol{r}, Y)$ change if $Y \to Y + dY$?

- Kinematics:

$Q^2 = -q^2$	–	photon virtuality
$x_{Bj} \approx \frac{Q^2}{s} \ll 1$	–	Bjorken variable (small-x_{Bj})
$Y = \ln(1/x_{Bj}) \sim \ln(s)$	–	total rapidity

"Mean Field Equations"

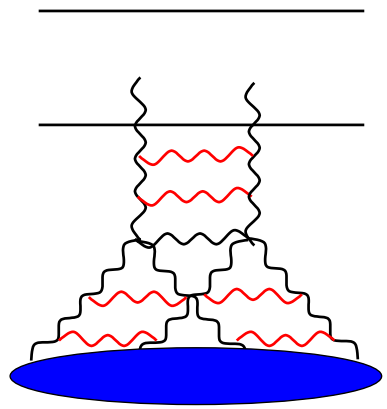
- Linear BFKL Evolution:



$$\frac{\partial}{\partial Y}\langle T\rangle_Y \propto \bar{\alpha}_s \ \langle T\rangle_Y$$

$*$ $\langle T\rangle_Y \sim \exp[c\bar{\alpha}_s Y] \rightarrow$ unitarity violation!

- Kovchegov equation:



$$\frac{\partial}{\partial Y}\langle T\rangle_Y \propto \bar{\alpha}_s \ [\langle T\rangle_Y - \langle T\rangle_Y\langle T\rangle_Y]$$

$*$ $\langle T\rangle_Y\langle T\rangle_Y$; non-linear evolution, $\langle T\rangle_Y \leq 1$

$*$ $\langle T\rangle_Y \ll 1$; linear BFKL evolution

- Balitsky-JIMWLK equations:

$$\frac{\partial}{\partial Y}\langle T\rangle_Y \propto \bar{\alpha}_s \ [\langle T\rangle_Y - \langle T\ T\rangle_Y] \ ,$$

$$\frac{\partial}{\partial Y}\langle T\ T\rangle_Y \propto \bar{\alpha}_s \ [\langle T\ T\rangle_Y - \langle T\ T\ T\rangle_Y] \ , \quad \cdots$$

$*$ Fluctuations: $\langle TT\rangle_Y \neq \langle T\rangle_Y\langle T\rangle_Y$?

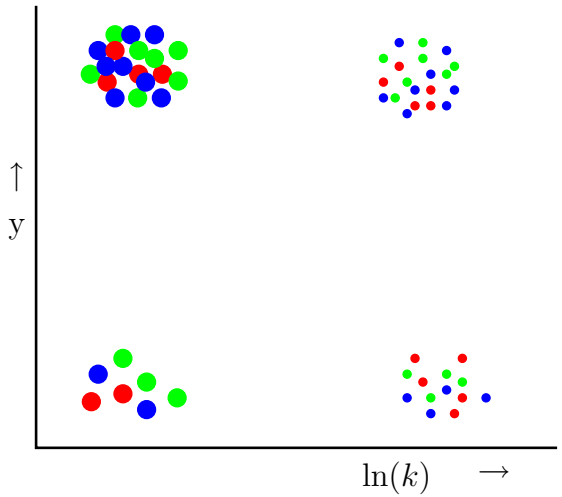
$*$ Mean field approximation: $\langle TT\rangle_Y \approx \langle T\rangle_Y\langle T\rangle_Y \quad \rightarrow \quad$ Kovchegov Eq.

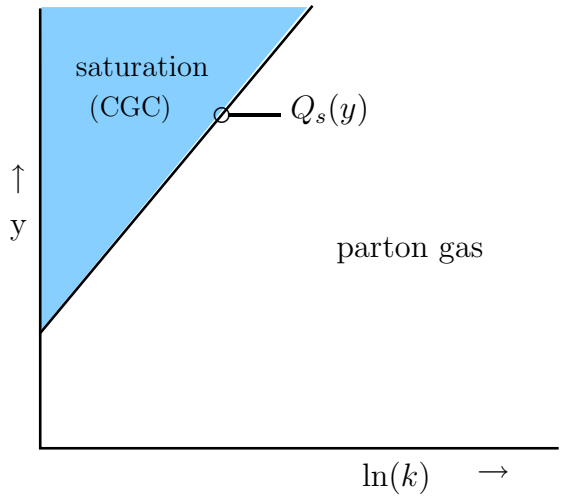
- Numerical result [Rummukainen, Weigert 04]: $\langle T\rangle_Y^{\text{Kovchegov}} \approx \langle T\rangle_Y^{\text{B-JIMWLK}}$

Results from "Mean Field Equations"

[Mueller, Triantafyllopoulos 2002],[Munier, Peschanski 2004]

- Physical picture:

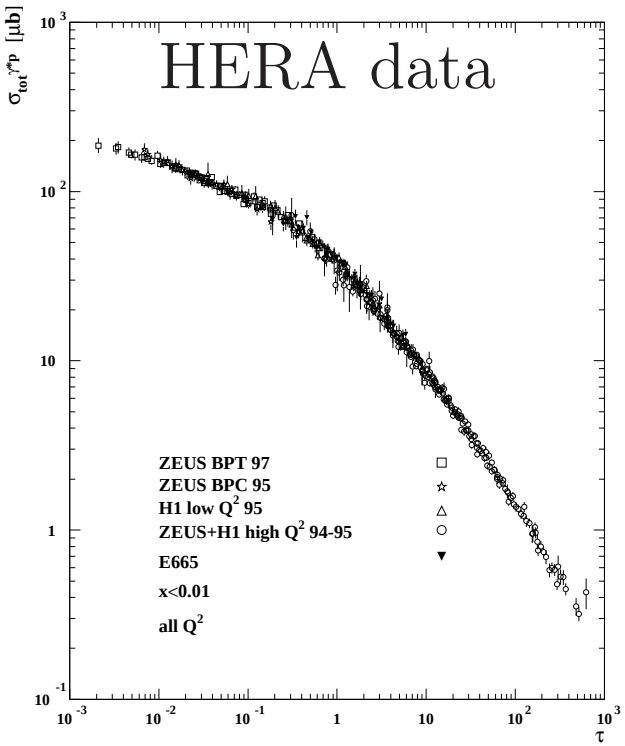




- Geometric scaling:

$$T(r,Y) = C\ [Q_s^2(Y)\ r^2]^{1-\lambda_0}\ [\ln(\frac{1}{Q_s^2(Y)\ r^2}) + c]$$

$$\sigma^{\gamma^* p}(Q, x_{Bj}) \Rightarrow \sigma^{\gamma^* p}(Q/Q_s(Y))!$$



- Saturation momentum:

$$Q_s(x_{Bj}) = Q_0^2(1/x_{Bj})^\lambda,$$

Experiment: $\lambda \approx 0.3$

Theory [Triantafyllopoulos 02]: λ close to 0.3

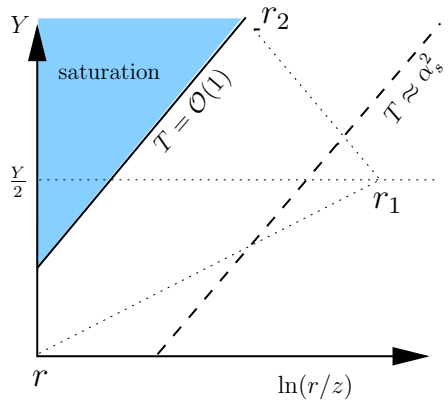
Beyond the Mean Field Approximation

- Shortcomings of the Kovchegov equation [Mueller, Shoshi 2004]:

  Completeness relation:

$$\underbrace{\frac{1}{r_2^2}\,T(r,r_2,Y)}_{\approx\,\mathcal{O}(1)} \approx \frac{1}{\alpha^2}\int d^2\rho\,\underbrace{\left(\frac{1}{r_1^2}\,T(r,r_1,Y/2)\right)}_{<\,\alpha^2}\,\underbrace{\left(\frac{1}{r_2^2}\,T(r_1,r_2,Y/2)\right)}_{>\,1}$$

- Extension: Kovchegov equation
  + absorbtive boundary



- Results for asymptotic Y:

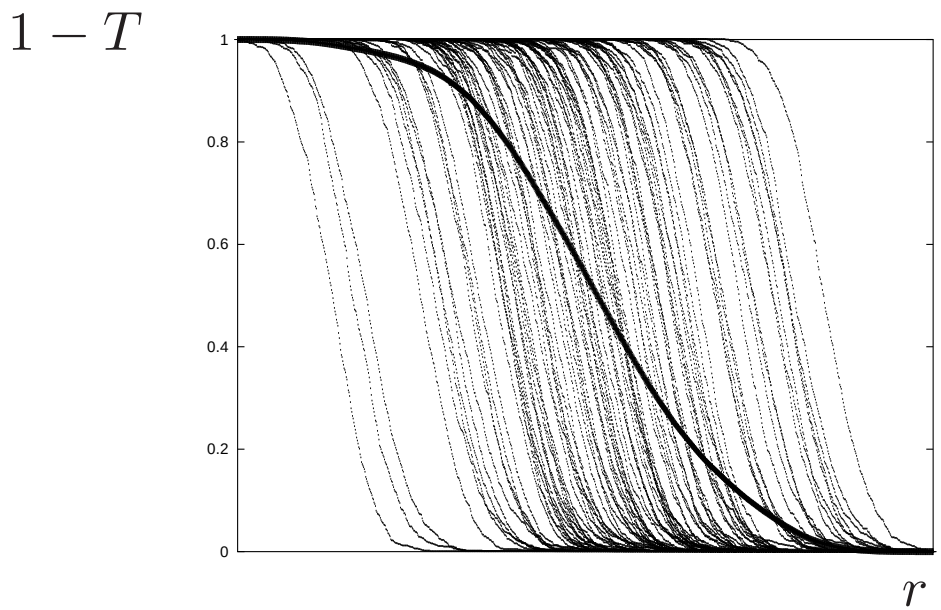
  No geometric scaling: $$T\left(\frac{\ln(Q_s^2(Y)\,r^2)}{[\alpha_s Y/\ln^3(1/\alpha_s^2)]}\right)$$

  saturation momentum: $$Q_s^2(x_{Bj}) = Q_0^2\,\left(\frac{1}{x_{Bj}}\right)^{\lambda}, \quad \lambda = \lambda_0 - \frac{C}{\ln^2(1/\alpha_s^2)}$$

QCD - Statistical Physics Correspondence

[Iancu,Mueller,Munier 2004]

- Event-by-event picture:



Average over all events: $$\overline{T}(r, Q_s(Y)) = \int dQ_s \; P(Q_s) \; T(r, Q_s(Y))$$

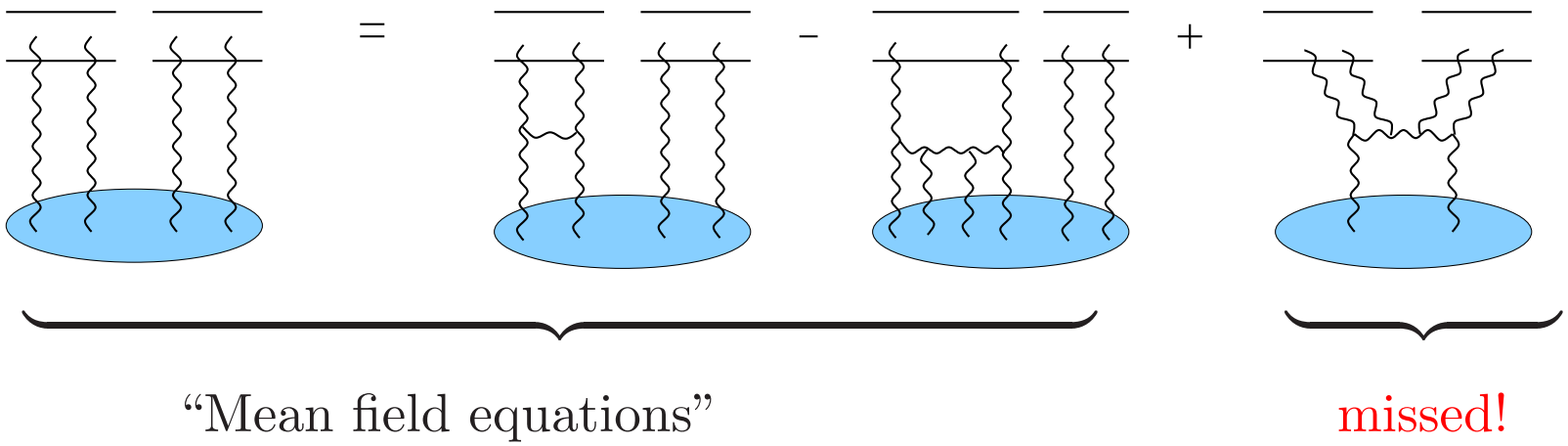
Result for $Y \to \infty$: $$\overline{T}\left(\frac{\ln(Q_s^2(Y)\, r^2)}{\sqrt{\alpha_s Y / \ln^3(1/\alpha_s^2)}} \right)$$

Flutuations $\Rightarrow$ geometric scaling violation!

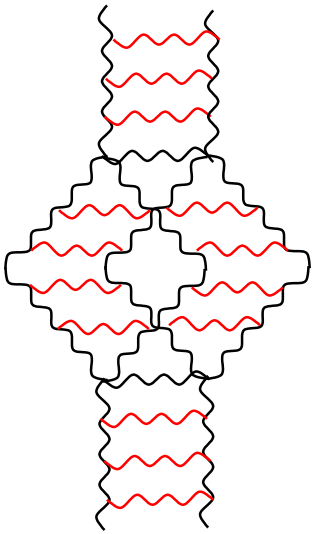
- $Q_S(Y)$ as in [Mueller,Shoshi 2004] (see talks by E. Iancu, S. Munier)

Shortcoming of "Mean Field Equations": Pomeron Loops

- Two dipoles scattering off a target:



- New element: Pomeron Loops



"Pomeron Loop Equations"

- Extended JIMWLK equation [Mueller,Shoshi,Wong 2005]:

$$\frac{\partial}{\partial Y} W_Y[\alpha] \propto \left(\alpha^2 \left(\frac{\delta}{\delta\alpha}\right)^2 + \sum_{n>1} \alpha^{2n} \left(\frac{\delta}{\delta\alpha}\right)^2 + \left(\frac{\delta}{\delta\alpha}\right)^4 \alpha^2 \right) W_Y[\alpha]$$

$\Rightarrow$ Langevin equations contains fourth order and offdiagonal noises

- Extended Balitsky equations [Iancu,Triantafyllopoulos 2005]:

$$\begin{aligned} \frac{\partial}{\partial Y} \langle T \rangle_Y &\propto \bar{\alpha}_s \left[\langle T \rangle_Y - \langle T\ T \rangle_Y \right] \\ \frac{\partial}{\partial Y} \langle T\ T \rangle_Y &\propto \bar{\alpha}_s \left[\langle T\ T \rangle_Y - \langle T\ T\ T \rangle_Y + \alpha_s^2\ \langle T \rangle_Y \right] \end{aligned}$$

$\Rightarrow$ Langevin equation contains offdiagonal noise

- (See talks by Levin, Kovner, Lublinsky, Hatta ..)

Pomeron loops in zero transverse dimensions

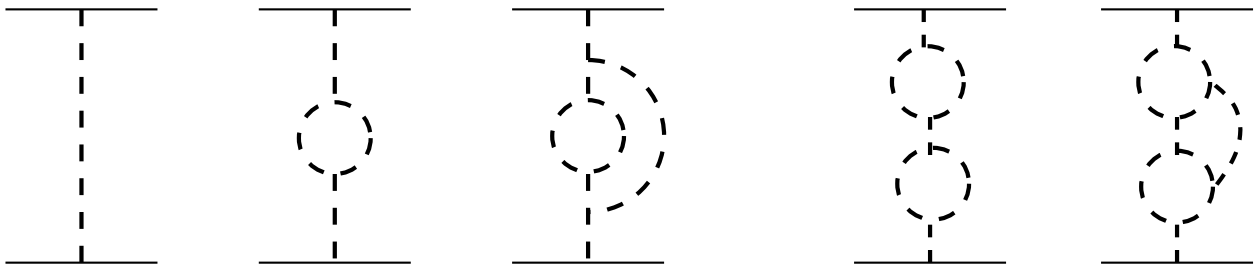
[Shoshi, Xiao 2005]

- Evolution equations:

$$\frac{dn^{(k)}}{dy} = k\alpha n^{(k)} + k(k-1)\alpha n^{(k-1)} - k\beta n^{(k+1)}$$

where $n^{(k)} = \langle n(n-1)\cdots(n-k+1)\rangle$ and $\frac{\beta}{\alpha} = \alpha_s^2$

- Graphs:



LO graphs NLO graphs

- Result for dipole-dipole scattering:

$$S(Y) \simeq \frac{1}{\alpha_s^2 e^{\alpha Y}} \ln\left(\alpha_s^2 e^{\alpha Y}\right)\left\{1 + 4\alpha_s^2 \alpha Y + \text{corrections}\right\}$$

* Philosophy: treat $k\beta n^{(k+1)}$ as a perturbation

* Initial conditions: $n^{(1)}|_{y=0} = N$ and $n^{(k)}|_{y=0}^{k>1} = 0$

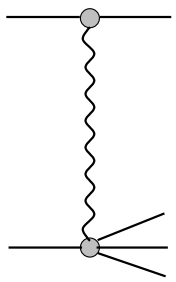
- LO result valid until $Y \sim \frac{1}{\alpha_s^2 \alpha}$! (LO result agrees with [Mueller, Salam 96])

Diffractive Scattering Processes

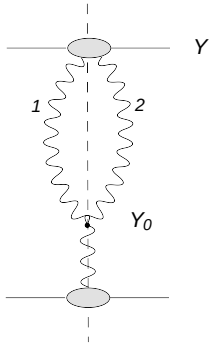
[Shoshi, Xiao 2006]

- Single diffractive scattering:

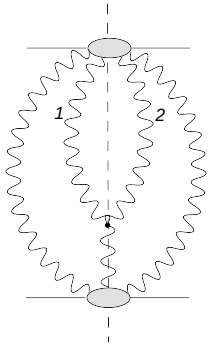
$$\frac{d\sigma_{SD}\left(Y,Y_0\right)}{dY_0} = \sum_{n=2}^{\infty} (-1)^n \, 2^{n-1} F^n \left(Y,Y_0\right)$$



,



A

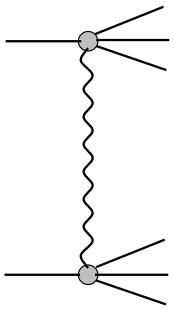


B

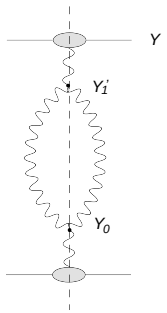
$$\lim_{Y\to\infty} \frac{d\sigma_{SD}\left(Y,Y_0\right)}{dY_0} \approx \frac{1}{2}\alpha e^{-\alpha Y_0} \qquad \text{when} \quad Y_0 = \ln\frac{M_x^2}{m_p^2} \quad \text{and} \quad Y - Y_0 \quad \text{large !}$$

- Double diffractive scattering:

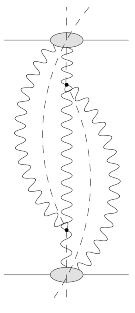
$$\lim_{Y\to\infty} \frac{d\sigma_{DD}\left(Y,Y_0,Y_1\right)}{dY_0 dY_1} \approx \frac{1}{4}\alpha^2 e^{-\alpha Y_0 - \alpha Y_1}$$



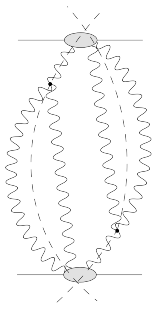
,



I



II



III

$$\text{when} \quad Y_0 = \ln\frac{M_x^2}{m_p^2}\ , \ Y_1 = \ln\frac{M_{x_1}^2}{m_p^2} \quad \text{and} \quad Y - Y_0 - Y_1 \quad \text{large !}$$

- Diffractive cross sections unitarize at $Y \to \infty$!