Collinear improvement of the BFKL kernel in the electroproduction of two light vector mesons

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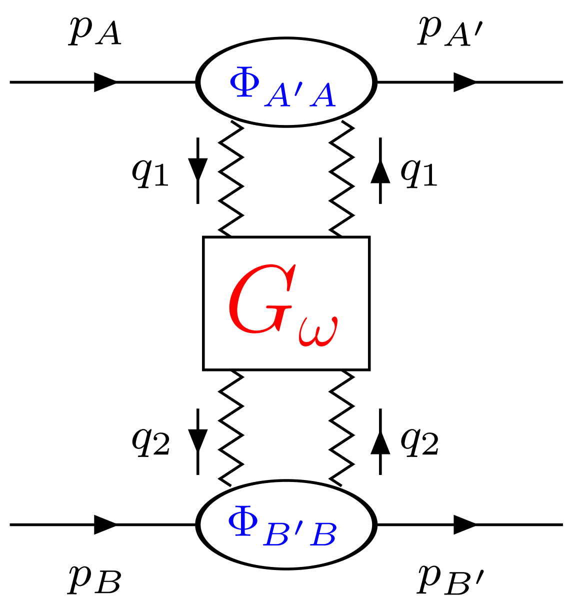
Plan of the talk

- $\boxed{\text{Introduction}}$
  - BFKL approach
  - collinear improvement of the NLA Green's function
- $\boxed{\text{NLA improved amplitude for the process } \gamma^*\gamma^* \to VV}$
  - Construction of the amplitude
  - Representations of the amplitude
- $\boxed{\text{Numerical results}}$
  - Symmetric kinematics
  - Asymmetric kinematics
- $\boxed{\text{Conclusions}}$

BFKL forward amplitude



$s \gg Q^2 \gg \Lambda^2$

$$\mathcal{I}m_s\,(\mathcal{A}) = \frac{s}{(2\pi)^2} \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} \Phi_1(\vec{q}_1, s_0)\, \Phi_2(-\vec{q}_2, s_0)\, G(\vec{q}_1, \vec{q}_2, s, s_0)$$

$$G(\vec{q}_1, \vec{q}_2, s, s_0) = \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

Troubles with the NLA corrections to the Green's function

- BFKL equation:

$$\delta^2(\vec{q}_1 - \vec{q}_2) = \omega\, G_\omega(\vec{q}_1\,, \vec{q}_2) - \int d^2\vec{q}\,\mathcal{K}(\vec{q}_1\,, \vec{q})\, G_\omega(\vec{q}, \vec{q}_2)$$

$$\boxed{\mathcal{K}(\vec{q}_1\,, \vec{q}_2) \text{ is known in the NLA}}$$

$$\int d^2\vec{q}_2 \mathcal{K}_{LLA}(\vec{q}_1\,, \vec{q}_2)\,\left(\vec{q}_2^{\,2}\right)^{\gamma-1} = \bar{\alpha_s}\chi(\gamma)\,\left(\vec{q}_1^{\,2}\right)^{\gamma-1}\;, \qquad \gamma = i\nu + 1/2$$

$$\int d^2\vec{q}_2 \mathcal{K}_{NLA}(\vec{q}_1\,, \vec{q}_2)\,\left(\vec{q}_2^{\,2}\right)^{\gamma-1} = \left(\bar{\alpha_s}(\vec{q}_1^{\,2})\chi(\gamma) + \bar{\alpha_s}^{\,2}(\vec{q}_1^{\,2})\chi^{(1)}(\gamma)\right)\,\left(\vec{q}_1^{\,2}\right)^{\gamma-1}$$

- NLA corrections to the Green's function turn out to be very large and negative

$\Longrightarrow$ Expectation: NNLA corrections large and of the opposite sign

$\Longrightarrow$ We try to infer the essential NNLA dynamics by the collinear approach

Compatibility with DGLAP

- An energy scale $s_0 = q_1 q_2$ is a natural choice in the typical BFKL kinematics

- We now focus on the kinematical region in which $q_2^2 \ll q_1^2$

This region is tipical of the collinear approach and corresponds to $\gamma = 0$

- Pole structure of the kernel around $\gamma = 0$:

$$\chi(\gamma) \simeq \frac{1}{\gamma} \qquad \chi^{(1)}(\gamma) \simeq \frac{a}{\gamma} + \frac{b}{\gamma^2} - \frac{1}{2\gamma^3}$$

$$\mathrm{a} = \frac{5}{12}\frac{\beta_0}{N_c} - \frac{13}{36}\frac{n_f}{N_c^3} - \frac{55}{36}, \qquad \mathrm{b} \;=\; -\frac{1}{8}\frac{\beta_0}{N_c} - \frac{n_f}{6N_c^3} - \frac{11}{12}$$

- In the collinear kinematics the natural choice for the energy scale should be $s_0 = q_1^2$

$\Longrightarrow$ $\boxed{\text{What's happen if we shift } s_0 = q_1 q_2 \;\; \rightarrow \;\; s_0 = q_1^2 \text{ ?}}$

Compatibility with DGLAP

- Solving the BFKL equation at the leading order one obtains

$$G(\vec{q}_1,\vec{q}_2,s,s_0)=\frac{1}{\pi q_1 q_2}\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega}\int\frac{d\gamma}{2\pi i}\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{\gamma-\frac{1}{2}}\frac{1}{\omega-\omega(\gamma)}$$

$\Longrightarrow$ where $\omega(\gamma)\equiv\bar{\alpha}_s\chi(\gamma)=\bar{\alpha}_s\left(2\psi(1)-\psi\left(\gamma\right)-\psi\left(1-\gamma\right)\right)$

$$\begin{aligned}
G(\vec{q}_1,\vec{q}_2,s) &= \frac{1}{\pi q_1 q_2}\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{q_1 q_2}\right)^{\omega}\int\frac{d\gamma}{2\pi i}\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{\gamma-\frac{1}{2}}\frac{1}{\omega-\omega(\gamma)}\\
&= \frac{1}{\pi q_1 q_2}\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{q_1^2}\right)^{\omega}\int\frac{d\gamma}{2\pi i}\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{\gamma-\frac{1}{2}+\frac{\omega}{2}}\frac{1}{\omega-\omega(\gamma)}\\
&= \frac{1}{\pi q_1 q_2}\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{q_1^2}\right)^{\omega}\int\frac{d\gamma}{2\pi i}\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{\gamma-\frac{1}{2}}\frac{1}{\omega-\omega(\gamma-\frac{\omega}{2})}
\end{aligned}$$

RG-improvement of the BFKL kernel in the LLA

- In collinear limit $\gamma \sim 0 \longrightarrow \omega \sim \frac{\bar{\alpha}_s}{\gamma}$

$$\omega \sim \frac{\bar{\alpha}_s}{\gamma - \frac{\omega}{2}} \longrightarrow \omega \sim \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s^2}{2\gamma^3} + \sum_{n=2}^{\infty} \frac{(2n)!}{2^n n!(n+1)!} \frac{(\bar{\alpha}_s)^{n+1}}{\gamma^{2n+1}}$$

$\Longrightarrow$ The second term cancels with the NLA cubic pole

$\Longrightarrow$ The other subleading terms, numerically large, are not allowed by DGLAP

$\Longrightarrow$ Solution [$G.P.\,Salam, M.\,Ciafaloni,\ D.\,Colferai$]: redefine the original kernel

$$\omega^{new}(\gamma) = \omega\left(\gamma + \frac{\omega}{2}\right)$$

$$\begin{aligned} G(\vec{q}_1, \vec{q}_2, s) &= \frac{1}{\pi q_1 q_2} \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{q_1 q_2}\right)^{\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{\gamma - \frac{1}{2}} \frac{1}{\omega - \omega^{new}(\gamma)} \\ &= \frac{1}{\pi q_1 q_2} \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{q_1^2}\right)^{\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{\gamma - \frac{1}{2}} \frac{1}{\omega - \omega(\gamma)} \end{aligned}$$

RG-improvement of the BFKL kernel in the NLA

- ω-shift at the next-to-leading order [$G.P.\,Salam,\ A.\,Sabio\,Vera$]

$$\omega = \bar{\alpha}_s(1+a\bar{\alpha}_s)\left(2\psi(1)-\psi\left(\gamma+\frac{\omega}{2}-b\bar{\alpha}_s\right)-\psi\left(1-\gamma+\frac{\omega}{2}+b\bar{\alpha}_s\right)\right)$$

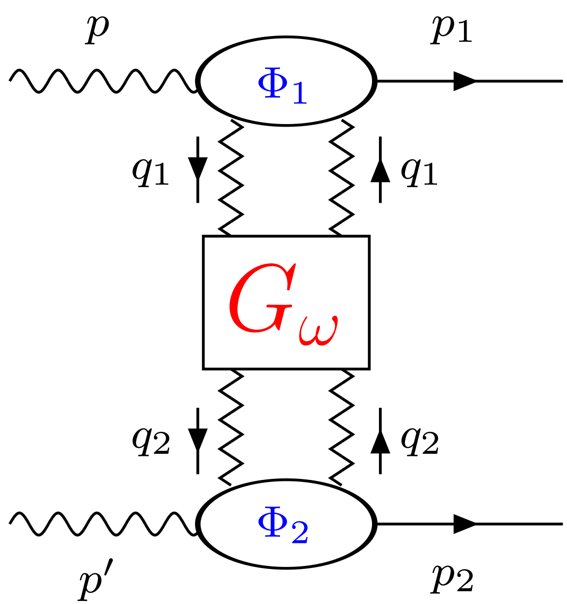
- The solution must match the NLA accuracy of the original BFKL calculation
- The esplicit solution of this equation gives these new terms

$$\begin{aligned}\chi_{RG}(\gamma) \quad &= \quad 2\Re e\left\{\sum_{m=0}^{\infty}\left[\left(\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{2^n n!(n+1)!}\frac{\left(\bar{\alpha}_s+\mathrm{a}\,\bar{\alpha}_s^2\right)^{n+1}}{(\gamma+m-\mathrm{b}\,\bar{\alpha}_s)^{2n+1}}\right)\right.\right.\\ &- \quad \left.\left.\frac{\bar{\alpha}_s}{\gamma+m}-\bar{\alpha}_s^2\left(\frac{\mathrm{a}}{\gamma+m}+\frac{\mathrm{b}}{(\gamma+m)^2}-\frac{1}{2(\gamma+m)^3}\right)\right]\right\}\end{aligned}$$

$$\Longrightarrow \quad \int d^2\vec{q}_2\mathcal{K}(\vec{q}_1,\vec{q}_2)\left(\vec{q}_2^{\,2}\right)^{\gamma-1} = \left(\bar{\alpha_s}\chi(\gamma)+\bar{\alpha_s}^2\chi^{(1)}(\gamma)+\chi_{RG}(\gamma)\right)\left(\vec{q}_1^{\,2}\right)^{\gamma-1}$$

- $\chi_{RG}(\gamma)$ is $\mathcal{O}(\bar{\alpha}_s^3)$

Amplitude for the $\gamma^*\gamma^* \to VV$ forward scattering



$$\boxed{s \gg Q_{1,2}^2 \gg \Lambda_{QCD}^2}$$

$$p_1^2 = p_2^2 = 0\ , \quad 2(p_1 p_2) = s$$

$$p = \alpha p_1 - \frac{Q_1^2}{\alpha s} p_2\ , \quad p' = \alpha' p_2 - \frac{Q_2^2}{\alpha' s} p_1$$

$$\mathcal{I}m_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^2} \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} \Phi_1(\vec{q}_1, s_0) \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} \Phi_2(-\vec{q}_2, s_0) \int\limits_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^\omega G_\omega(\vec{q}_1, \vec{q}_2)$$

Transverse momentum representation

- We define the transverse momentum representation:

$$\hat{\vec{q}}\,|\vec{q}_i\rangle = \vec{q}_i|\vec{q}_i\rangle \ ,$$

$$\langle\vec{q}_1|\vec{q}_2\rangle = \delta^{(2)}(\vec{q}_1 - \vec{q}_2) \ , \qquad \langle A|B\rangle = \langle A|\vec{k}\rangle\langle\vec{k}|B\rangle = \int d^2k A(\vec{k})B(\vec{k})$$

- The kernel operator $\hat{K}$ is

$$\mathcal{K}(\vec{q}_2, \vec{q}_1) = \langle\vec{q}_2|\hat{K}|\vec{q}_1\rangle$$

- The equation for the Green's function reads

$$\hat{1} = (\omega - \hat{K})\hat{G}_\omega$$

$\Longrightarrow$ its solution being

$$\hat{G}_\omega = (\omega - \hat{K})^{-1}$$

Application of the RG-improved kernel

$$\hat{K} = \bar{\alpha}_s \hat{K}^0 + \bar{\alpha}_s^2 \hat{K}^1 + \hat{K}_{RG}$$

$\Longrightarrow$ $\boxed{\hat{K}_{RG} \text{ includes the RG-generated terms, which are } \mathcal{O}(\bar{\alpha}_s^3)}$

- The solution for the $\hat{G}_\omega$ with NLA accuracy is

$$\hat{G}_\omega = (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} \left(\bar{\alpha}_s^2 \hat{K}^1 + \hat{K}_{RG} \right) (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + \mathcal{O}\left[\left(\bar{\alpha}_s^2 \hat{K}^1 \right)^2 \right]$$

- Basis of eigenfunctions of the LLA kernel

$$\hat{K}^0 |\nu\rangle = \chi(\nu) |\nu\rangle \,, \qquad \langle \vec{q}\, | \nu \rangle = \frac{1}{\pi \sqrt{2}} \left(\vec{q}^{\,2} \right)^{i\nu - \frac{1}{2}}$$

$$\langle \nu' | \nu \rangle = \int \frac{d^2 \vec{q}}{2\pi^2} \left(\vec{q}^{\,2} \right)^{i\nu - i\nu' - 1} = \delta(\nu - \nu')$$

Application of the RG-improved kernel

- The action of the modified BFKL kernel on these functions may be expressed as follows:

$$\begin{aligned}\hat{K}|\nu\rangle &= \bar{\alpha}_s(\mu_R)\chi(\nu)|\nu\rangle + \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(\nu) + \frac{\beta_0}{4N_c}\chi(\nu)\ln(\mu_R^2)\right)|\nu\rangle \\ &+ \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(\nu)\left(i\frac{\partial}{\partial\nu}\right)|\nu\rangle + \chi_{RG}(\nu)|\nu\rangle\end{aligned}$$

$\Longrightarrow$ where we have taken into account the running of the coupling constant

$\boxed{\chi_{RG}(\nu) \text{ is the solution of the } \omega\text{-shift equation}}$

Impact factors

- Expansion in α_s of the impact factors [$D.Yu.\,Ivanov,\ M.I.\,Kotsky,\ A.\,Papa$]:

$$\Phi_{1,2}(\vec{q}) = \alpha_s\, D_{1,2}\left[C^{(0)}_{1,2}(\vec{q}^{\,2}) + \bar{\alpha}_s C^{(1)}_{1,2}(\vec{q}^{\,2})\right]\,, \quad D_{1,2} = -\frac{4\pi e_q f_V}{N_c Q_{1,2}}\sqrt{N_c^2-1}$$

- $|\nu\rangle$ representation for the impact factors:

$$c_1(\nu) = \int d^2\vec{q}\, C_1^{(0)}(\vec{q}^{2})\frac{\left(\vec{q}^{2}\right)^{i\nu-\frac{3}{2}}}{\pi\sqrt{2}}\,, \qquad c_2(\nu) = \int d^2\vec{q}\, C_2^{(0)}(\vec{q}^{2})\frac{\left(\vec{q}^{2}\right)^{-i\nu-\frac{3}{2}}}{\pi\sqrt{2}}\,,$$

$$c_1^{(1)}(\nu) = \int d^2\vec{q}\, C_1^{(1)}(\vec{q}^{2})\frac{\left(\vec{q}^{2}\right)^{i\nu-\frac{3}{2}}}{\pi\sqrt{2}}\,, \qquad c_2^{(1)}(\nu) = \int d^2\vec{q}\, C_2^{(1)}(\vec{q}^{2})\frac{\left(\vec{q}^{2}\right)^{-i\nu-\frac{3}{2}}}{\pi\sqrt{2}}\,,$$

"Exponentiated" representation of the amplitude

- We convolute the improved Green's function with the impact factors

$$
\begin{aligned}
\frac{\mathcal{I}m_s\left(\mathcal{A}\right)}{D_1 D_2} &= \frac{s}{(2\pi)^2}\int\limits_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(\nu)+\bar{\alpha}_s^2(\mu_R)\left(\bar{\chi}(\nu)+\frac{\beta_0}{8N_c}\chi(\nu)\left[-\chi(\nu)+\frac{10}{3}\right]\right)+\chi_{RG}(\nu)} \\
&\times \quad \alpha_s^2(\mu_R) c_1(\nu) c_2(\nu) \left[1+\bar{\alpha}_s(\mu_R)\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)}+\frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right)\right. \\
&+ \quad \left.\bar{\alpha}_s^2(\mu_R)\ln\left(\frac{s}{s_0}\right)\frac{\beta_0}{8N_c}\chi(\nu)\left(i\frac{d\ln\left(\frac{c_1(\nu)}{c_2(\nu)}\right)}{d\nu}+2\ln(\mu_R^2)\right)\right]
\end{aligned}
$$

where $\quad c_1(\nu)c_2(\nu) = \frac{1}{Q_1 Q_2}\left(\frac{Q_1^2}{Q_2^2}\right)^{i\nu} \frac{9\,\pi^3(1+4\nu^2)\sinh(\pi\nu)}{32\,\nu\,(1+\nu^2)\cosh^3(\pi\nu)}$

Series representation of the amplitude

- Another possible representation of the amplitude closer to the original idea of the BFKL approach, is the “series" representation

$$
\begin{aligned}
\frac{Q_1 Q_2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s} &= \frac{1}{(2\pi)^2}\alpha_s(\mu_R)^2 \\
&\times \left\{ b_0 + a_0 \ln\left(\frac{s}{s_0}\right) + \sum_{n=1}^{\infty}\bar{\alpha}_s(\mu_R)^n \left[a_n \ln\left(\frac{s}{s_0}\right)^{n+1} \right.\right. \\
&+ \left.\left. b_n \left(\ln\left(\frac{s}{s_0}\right)^n + d_n(s_0,\mu_R)\ln\left(\frac{s}{s_0}\right)^{n-1}\right)\right]\right\},
\end{aligned}
$$

$$
\frac{b_n}{Q_1 Q_2} = \int_{-\infty}^{+\infty} d\nu\, c_1(\nu) c_2(\nu)\frac{\chi^n(\nu)}{n!}, \qquad \frac{a_n}{Q_1 Q_2} = \int_{-\infty}^{+\infty} d\nu\, c_1(\nu) c_2(\nu)\chi_{RG}(\nu)\frac{\chi^n(\nu)}{n!}
$$

$\Longrightarrow$ we stress that χ_{RG} is $\mathcal{O}(\bar{\alpha}_s^3)$

Series representation of the amplitude

$\Longrightarrow$ NLA coefficients

$$d_n = n \ln\left(\frac{s_0}{Q_1 Q_2}\right) + \frac{\beta_0}{4N_c}\left((n+1)\frac{b_{n-1}}{b_n}\ln\left(\frac{\mu_R^2}{Q_1 Q_2}\right) - \frac{n(n-1)}{2}\right.$$

$$\left. + \frac{Q_1 Q_2}{b_n}\int\limits_{-\infty}^{+\infty} d\nu\,(n+1) f(\nu) c_1(\nu) c_2(\nu) \frac{\chi^{n-1}(\nu)}{(n-1)!}\right)$$

$$+ \frac{Q_1 Q_2}{b_n}\left(\int\limits_{-\infty}^{+\infty} d\nu\, c_1(\nu) c_2(\nu) \frac{\chi^{n-1}(\nu)}{(n-1)!}\left[\frac{\bar{c}_1^{(1)}(\nu)}{c_1(\nu)} + \frac{\bar{c}_2^{(1)}(\nu)}{c_2(\nu)} + (n-1)\frac{\bar{\chi}(\nu)}{\chi(\nu)}\right]\right)$$

$\Longrightarrow$ $\bar{c}_{1,2}^{(1)}(\nu)$ represent the contribution without the terms depending on s_0 and on β_0

$$f(\nu) = \frac{5}{3} + \psi(3+2i\nu) + \psi(3-2i\nu) - \psi\left(\frac{3}{2}+i\nu\right) - \psi\left(\frac{3}{2}-i\nu\right)$$

Numerical study

- We have studied the s-dependence of the $\mathcal{I}m_s(\mathcal{A})Q_1Q_2/(s\,D_1D_2)$ using the PMS method

  $\Longrightarrow$ $\boxed{\text{minimal sensitivity to the change of } \mu_R \text{ and } s_0}$

- In the previous determinations [$D.Yu.\,Ivanov,\ A.\,Papa$] the optimal choice for μ_R and s_0 turned out to be very far from the kinematical scales of the process!

  $\Longrightarrow$ $\boxed{\text{Does the RG-improvement lead to more "natural" values for } \mu_R \text{ and } s_0 \text{ ?}}$

- If true, this would demonstrate that the RG-generated terms catch the essential subleading dinamics

  $\Longrightarrow$ $\boxed{\text{the perturbative series of the amplitude should be more stable}}$

Numerical results: symmetric kinematics

- We consider here $Q_1^2 = Q_2^2 \equiv Q^2$=24 GeV^2 , $n_f = 5$
- We start with the "exponentiated" representation, putting

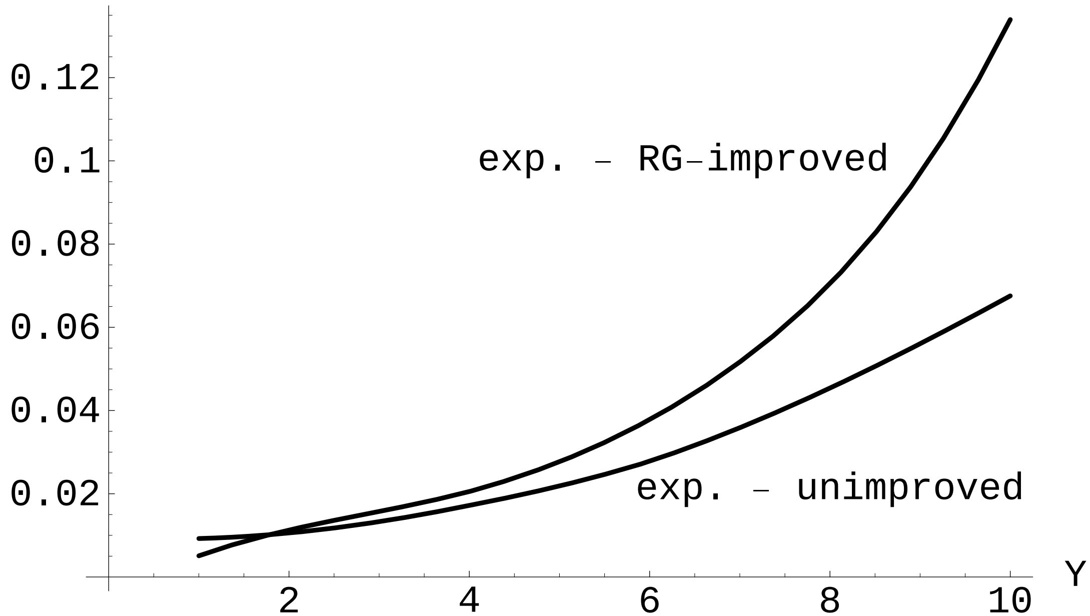
$$\ln\left(\frac{s}{s_0}\right) = Y - Y_0 \qquad Y = \ln(s/Q^2) \qquad Y_0 = \ln(s_0/Q^2)$$

- In practice, for each fixed value of Y we have determined the optimal choice of μ_R and Y_0 for which the amplitude is the least sensitive to their variation

$\Longrightarrow$ $\boxed{\text{typically } Y_0 \simeq 2 \text{ and } \mu_R \simeq 3Q}$

- In the previous determinations $Y_0 \simeq 2$ and $\mu_R \simeq 10Q$
- We see that there is a remarkable move toward "naturalness" for μ_R!

Numerical results: symmetric kinematics



- Good agreement at the lower energies but not for large values of Y
- $\bar{\alpha}_s(\mu_R)Y \sim 1 \quad \Longrightarrow \quad Y \sim 6$

$\boxed{\text{around } Y \sim 6 \text{ the discrepancy is not so pronounced}}$

Numerical results: symmetric kinematics

- We consider the “series" representation always with Q^2=24 GeV^2 , $n_f = 5$

PMS $\Longrightarrow$ $\boxed{\text{typically } Y_0 \simeq 3 \text{ and } \mu_R \simeq 3Q}$

$$b_0 = 17.0664 \quad b_1 = 34.5920 \quad b_2 = 40.7609 \quad b_3 = 33.0618 \quad b_4 = 20.7467$$

$$d_1 = 0.674275 \quad d_2 = -1.73171 \quad d_3 = -7.46518 \quad d_4 = -15.927$$

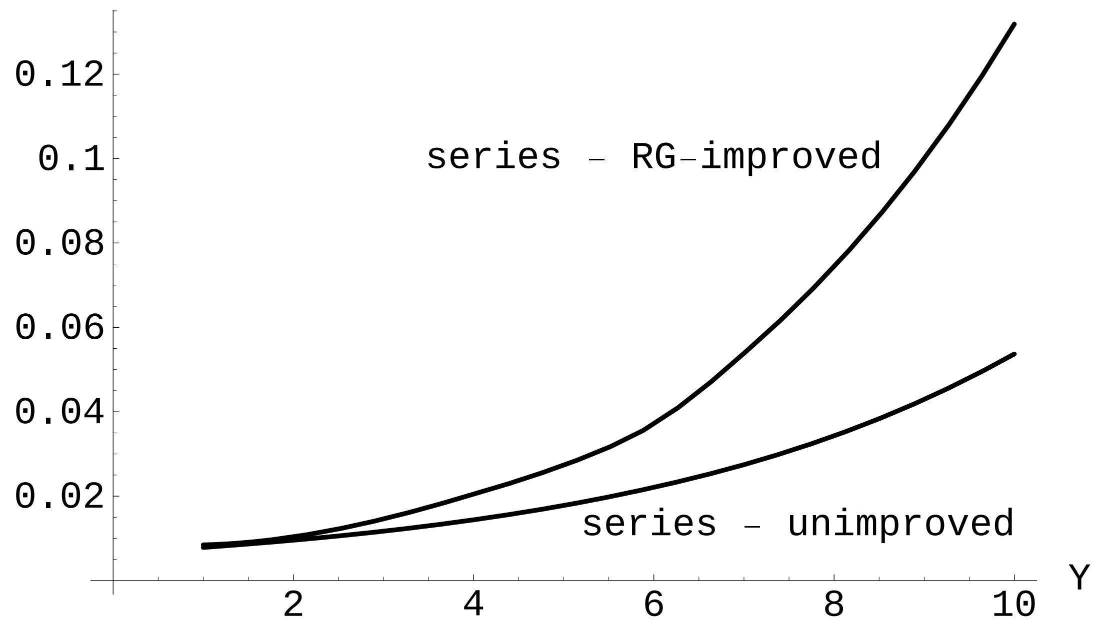
$$a_1 = 5.52728 \quad a_2 = 7.30295 \quad a_3 = 6.42149 \quad a_4 = 4.24011$$

$\Longrightarrow$ $\boxed{a_n \text{ coefficients cure the bad behavior of the BFKL series}}$

- a_n appear in the amplitude with two more powers of the energy logarithm!

$\Longrightarrow$ $\boxed{\text{their effect is not limited to low energies}}$

Numerical results: symmetric kinematics



- The curves for “exponentiated” and the “series” representations with collinear improvement fall almost on top of each other
- Without the collinear improvement there was an evident discrepancy

$\Longrightarrow$ $\boxed{\text{This indicate a better stability, induced by the collinear improvement}}$

Numerical results: asymmetric kinematics

- When the virtualities of the photons are strongly ordered we enter the "DGLAP" regime

  $\Longrightarrow$ $\boxed{\text{collinear effects should come heavily into the game}}$

- Previous attempts to numerically determine the amplitude using unimproved kernels were unsuccessful due to severe instabilities

- We use the "exponentiated" representation, and we define

$$Y = \ln(s/Q_1 Q_2) \qquad Y_0 = \ln(s_0/Q_1 Q_2)$$

Numerical results: asymmetric kinematics

- We choose Q_1=2 GeV, Q_2=12 GeV $\Longrightarrow$ $Q_1 Q_2$=24 GeV^2
- The amplitude is still quite stable under the variation of the energy parameters

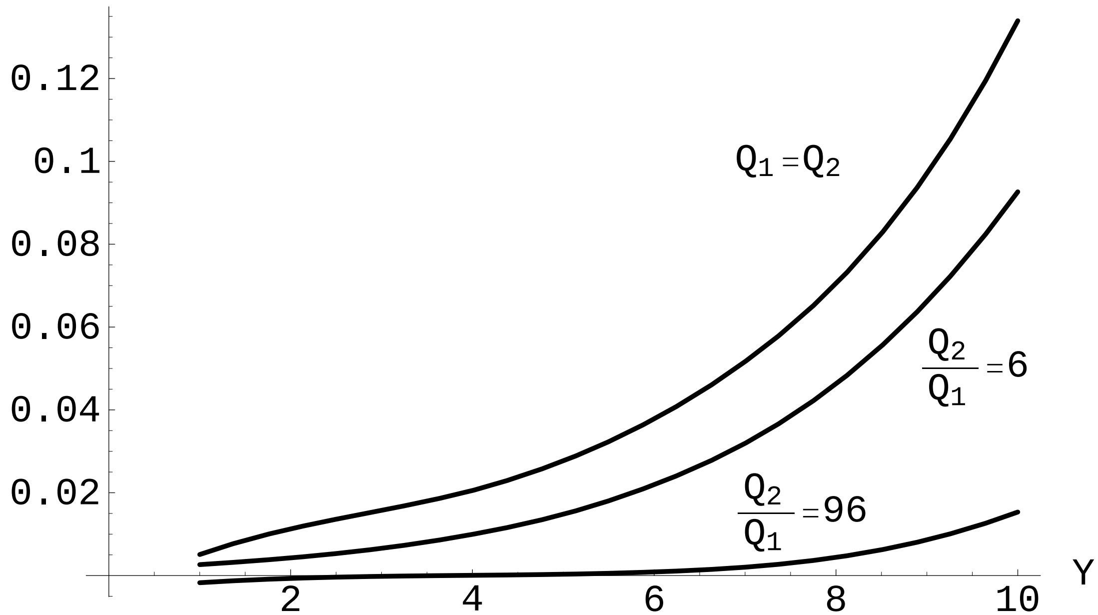
PMS $\Longrightarrow$ $\boxed{\text{typically } Y_0 \simeq 2 \text{ and } \mu_R \simeq 4\sqrt{Q_1 Q_2}}$

- We choose Q_1=0.5 GeV, Q_2=48 GeV $\Longrightarrow$ $Q_1 Q_2$=24 GeV^2
- The amplitude is still quite stable but the optimal values depend strongly on Y

PMS $\Longrightarrow$ $\boxed{\text{for } Y = 6 \text{ we find } Y_0 \simeq 7 \text{ and } \mu_R \simeq 3\sqrt{Q_1 Q_2}}$

- If we ridefine $Y_0' = \ln(s_0/Q_2^2)$ $\Longrightarrow$ $Y_0' \simeq 2.5$

Numerical results: asymmetric kinematics



- The amplitude becomes smaller and smaller when Q_2/Q_1 increases

$\Longrightarrow$ due to the presence of the factor $\cos(\nu \log(Q_2^2/Q_1^2))$ in the integration over ν

Conclusions

- We have applied a RG-improved kernel to determine the amplitude for the forward transition $\gamma^*\gamma^* \to VV$
- We considered both equal and strongly ordered photons' virtualities
- The PMS method has led to results stable in the considered energy interval which allow to predict the energy behavior of the forward amplitude
- The optimal choice of s_0 and μ_R are much closer to the kinematical scales of the problem than in previous determinations based on unimproved kernels

$\Longrightarrow$ the extra-terms coming from collinear improvement, which are subleading to the NLA, catch an important fraction of the NNLA order dynamics

$\Longrightarrow$ the use of the improved kernel has allowed to obtain the energy behavior of the forward amplitude in the case of strongly ordered photons' virtualities