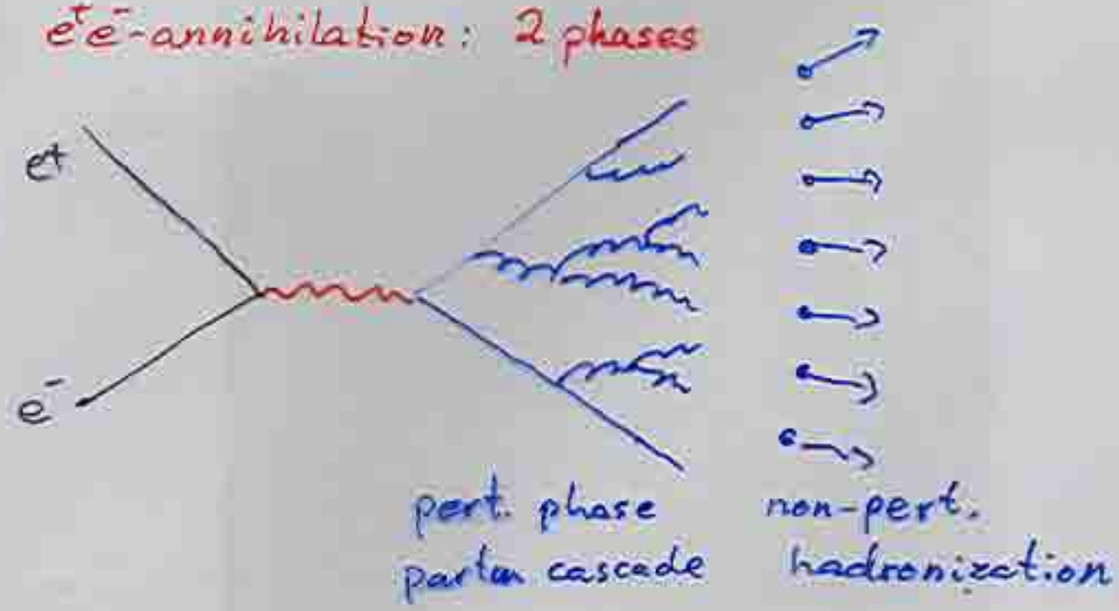
Lund Model, Multiple Scattering, Final States

G. Gustafson, Catanzaro, July '07

1. String hadronization
2. Parton cascades
3. Multiple scattering
4. Final state; underlying event

e^+e^--annihilation: 2 phases



Particle distrib. in a jet

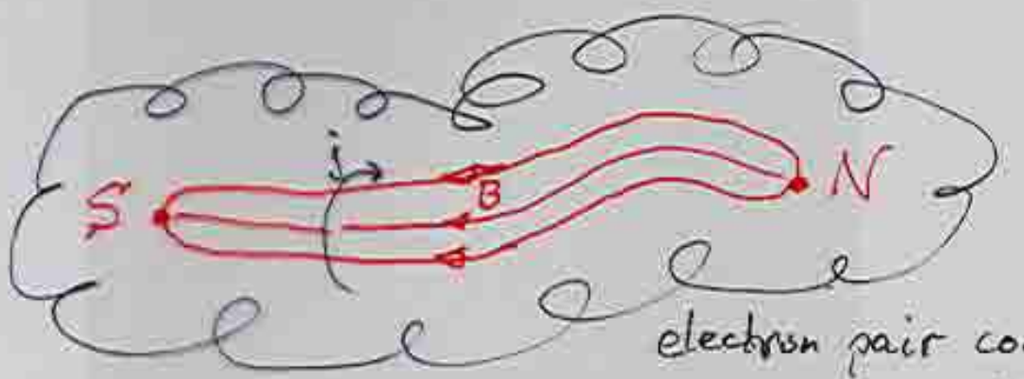
2 jet event

p_t limited

central region boost invariant

Lund String Hadronization

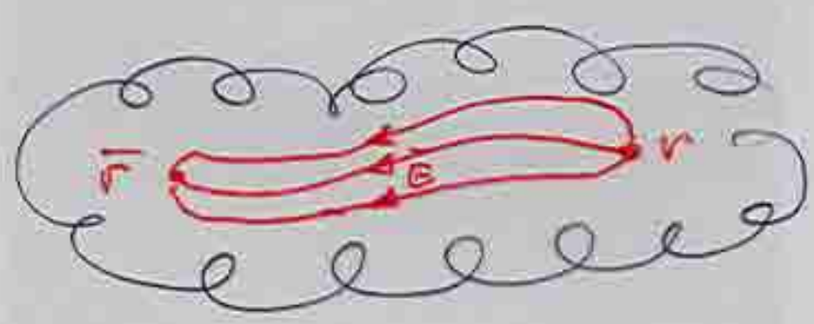
cf. vortex line in a superconductor



Magnetic field compressed to a linear structure

Energy/unit length = constant

QCD: Vacuum condensate compresses the colour electric field to stringlike structure



Energy/unit length = $\kappa \approx 1$ GeV/fm

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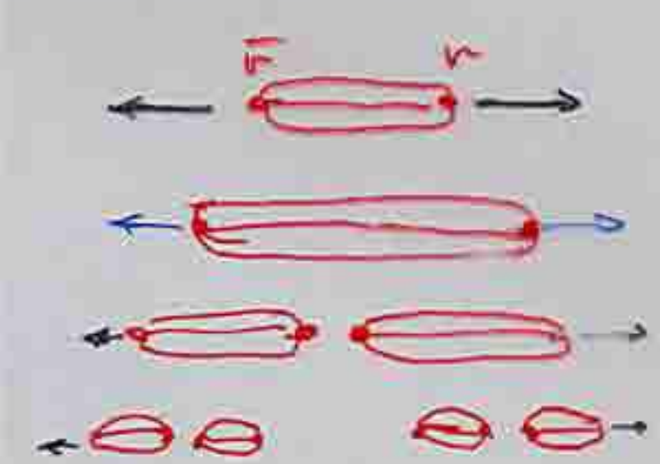
High energy $q\bar{q}$ system



Homogenous el. field invariant under longit. boosts

⇒ boost invariant particle distrib.

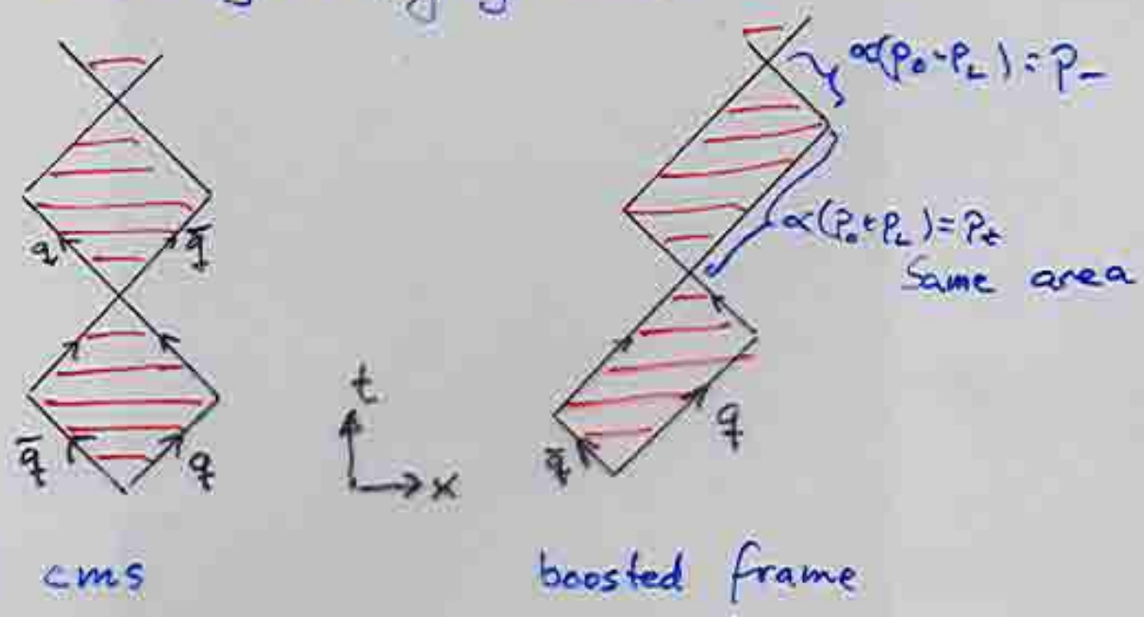
Tunneling: Prob. $\propto \exp(-\frac{\pi}{\kappa}(\mu^2 + k_\perp^2)) =$

$= \exp(-\frac{\pi}{\kappa}\mu^2) \cdot \exp(-\frac{\pi}{\kappa}k_\perp^2)$

1 dimension

Massless quarks in linear potential

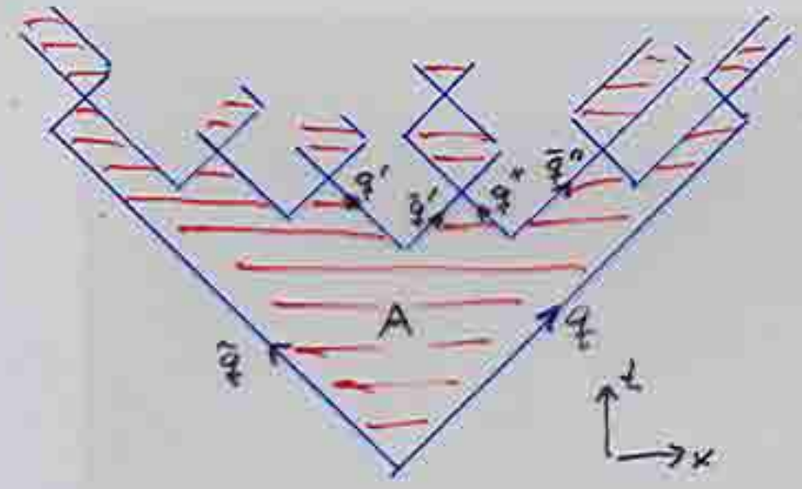
Bound system, yo-yo mode



cms

boosted frame

High energy $q\bar{q}$ system



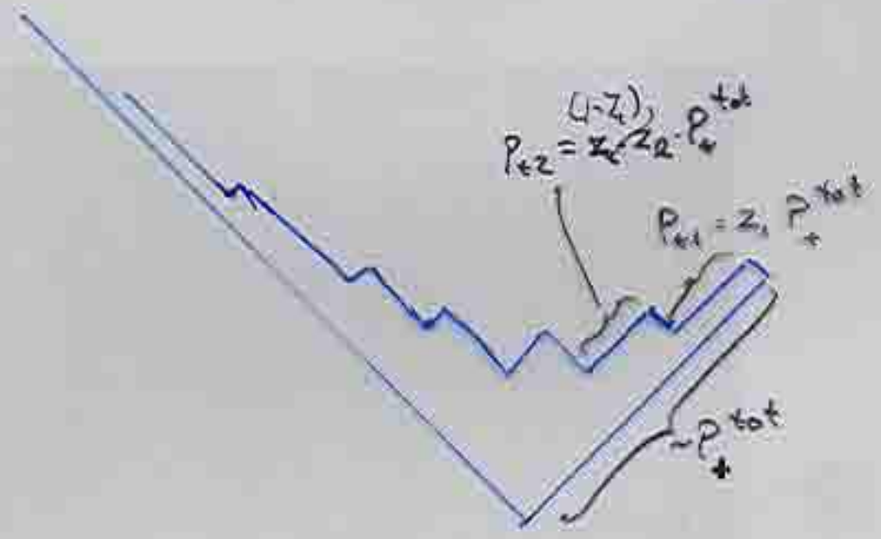
Probability for definite final state:

$$P \propto \iint \underbrace{\left\{\prod_i^M \left(N d^2 p_i\, \delta^{(2)}(p_i^2 - m^2)\right\} \delta^{(2)}\left(\sum_i p_i - P_{tot}\right)}_{\text{phase space}} \underbrace{\exp(-bA)}_{\text{Wilson loop int.}}$$

bA = Im. part of action

Boost invariant. No time ordering for the production points

Simulation



$$\frac{dP}{dz_i} = N \frac{(1-z_i)^a}{z_i} \exp\left(-b \frac{m^2}{z_i}\right)$$

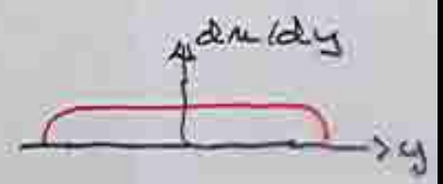
(when remaining energy is large)

N, b, a related by normaliz.

2 free parameters: (N, b) or (a, b)

<u>Multiplicity</u>: Central plateau

$$\frac{dn}{dy} \sim \frac{1+a}{b}$$



<u>Correlations</u>: $a \cdot b$ large

strong correlation

$a \cdot b$ small

weak correlation

larger fluctuations

Complications

1. Different quark species: u, d, s
   ↑ suppressed

2. Spin. $q\bar{q}$ → pseudoscalars
   vector mesons
   tensor mesons (~10%)

3. Baryons

4. Small mass systems

Computer simulations necessary

JETSET program (T. Sjöstrand)

- 2 fundamental parameters for gross features
- More extra parameters if all details have to be described.

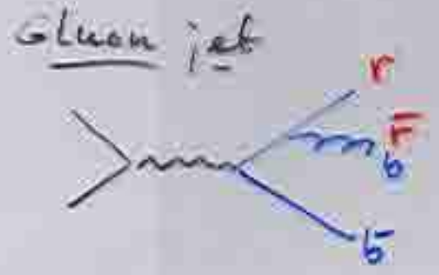
Gluon jets

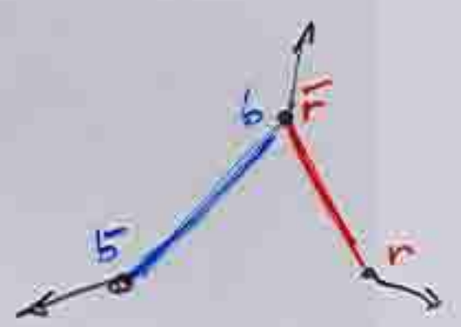
3 dim.

Extra degreas of freedom.

Colour field not determined only by position of charges

Must be possible to have transverse excitations on the field.





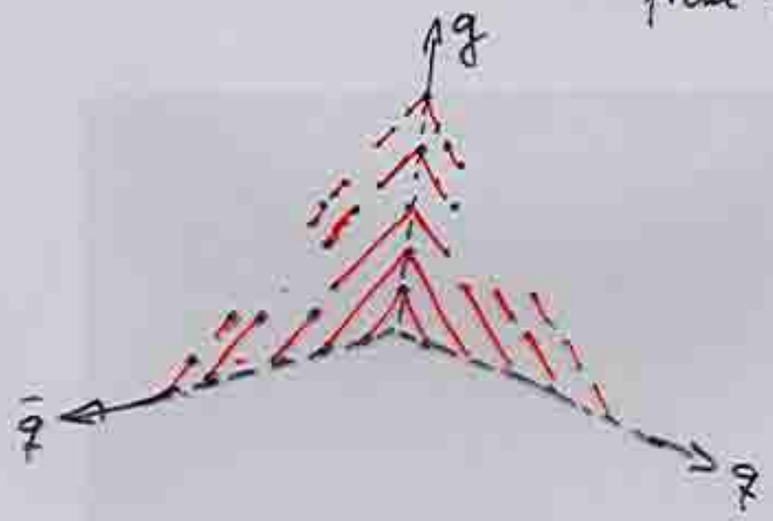
Colour field stretched from red to antired
from blue to antiblue

Approximation: Neglect transverse dim. of field

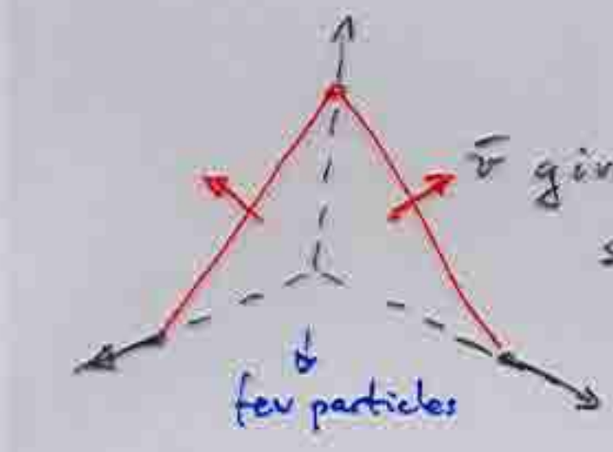
⇒ Dynamics of confined colour field =

= Massless relativistic string (Nambu-Goto string)

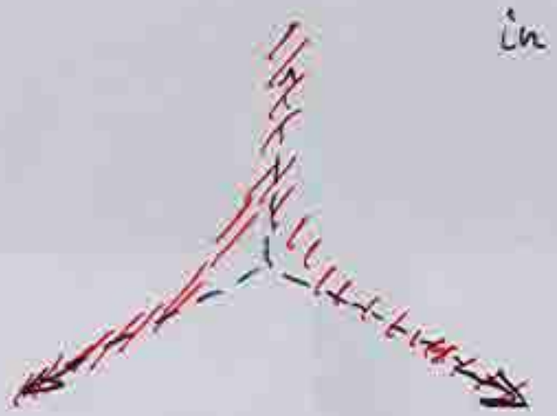
Gluon: kink on stringlike colour field from q to $\bar{q}$.



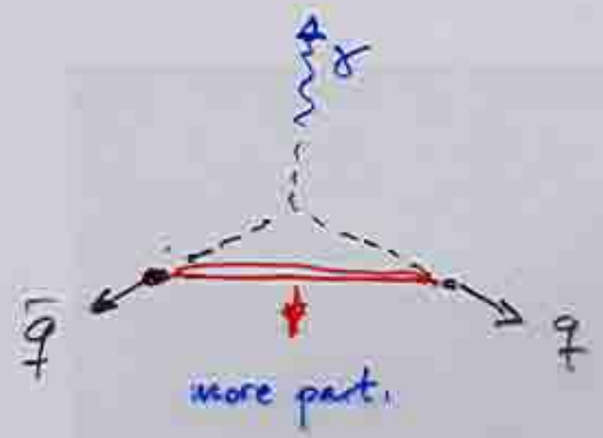
String breakup ⇒ 3 jets



Mesons produced around 2 hyperbolae in momentum space



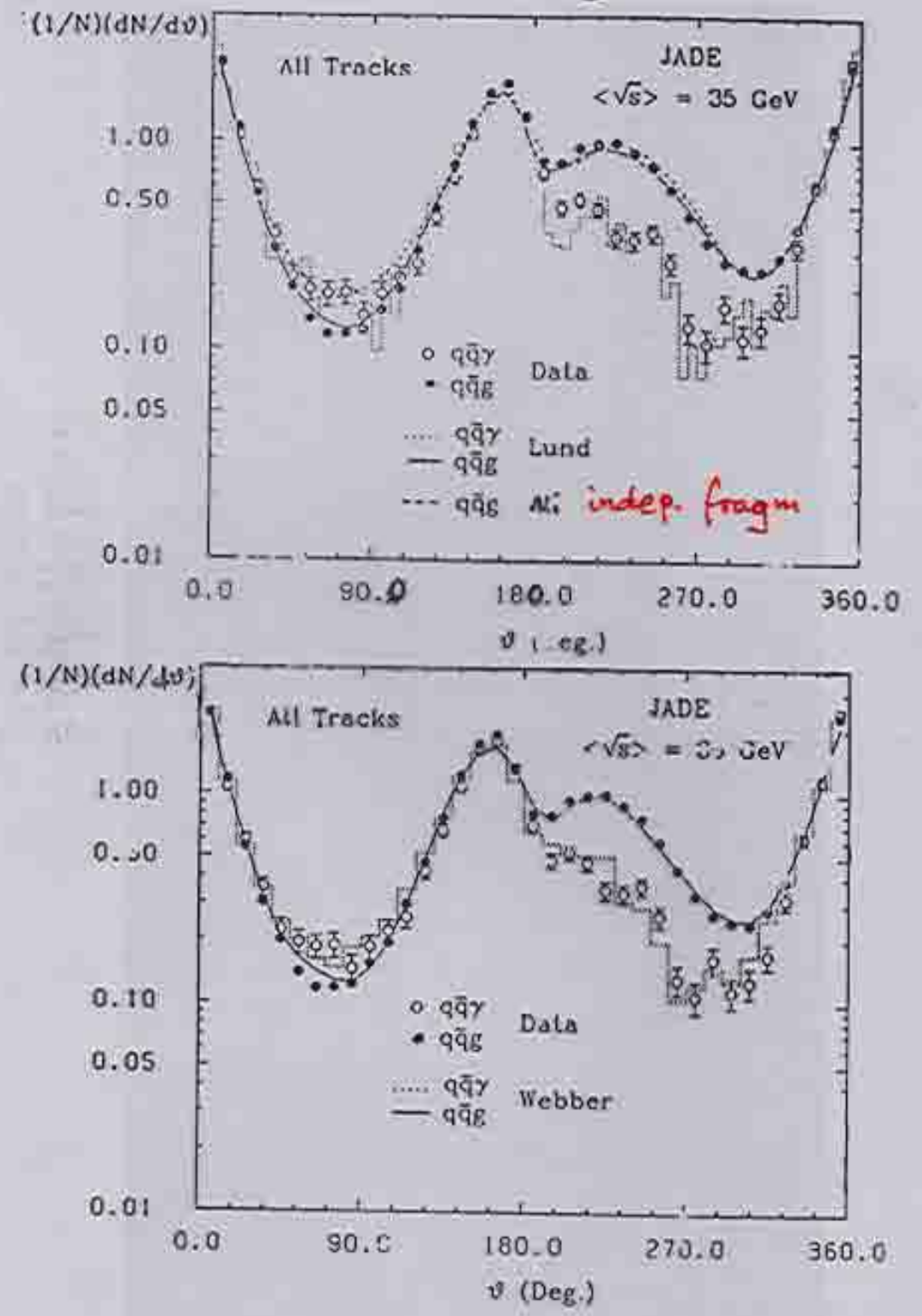
4. $e^+e^- \to q\bar{q}\gamma$



JADE collaboration; "String effect"

JADE ○ $q\bar{q}\gamma$

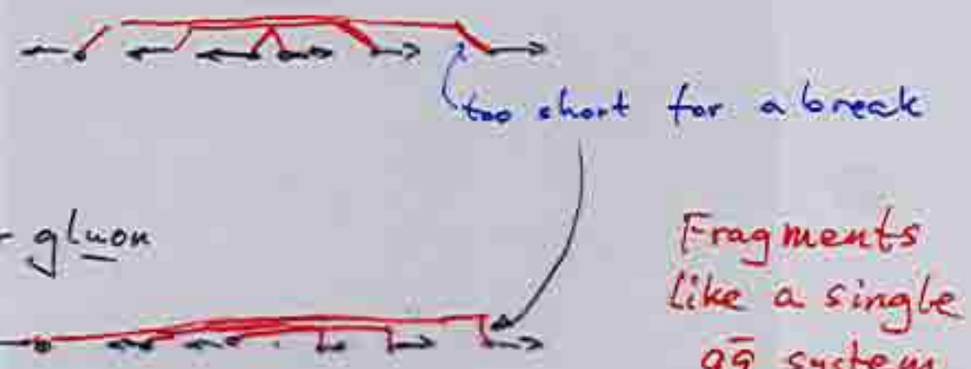
● $q\bar{q}g$



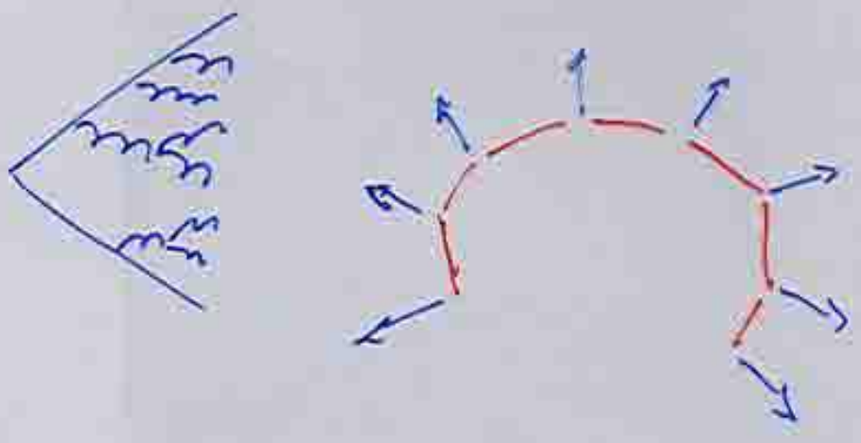
Gluon emission is singular for soft and collinear gluons.

String fragmentation is IR stable

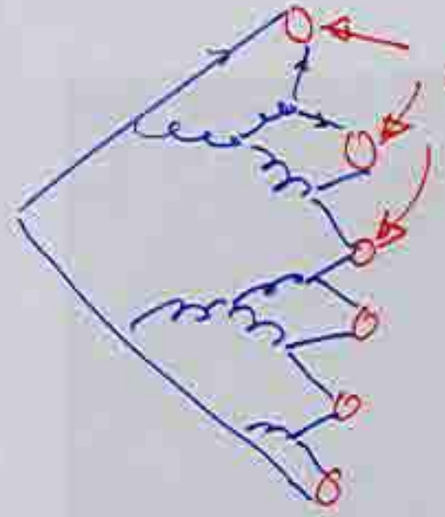
Soft gluon:



Many gluons:



Cf. Herwig cluster fragmentation

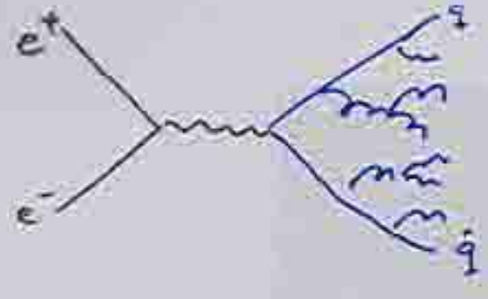


colourless clusters →
→ 2 hadrons

Further evidence in favour of string like dynamics

- In Herwig large mass clusters → stringlike decay
- $B\bar{B}$ angular correlation
- Leading cluster not isotropic decay

Parton cascades



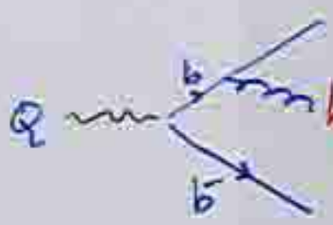
Timelike cascades

Gluon radiation has very small effect on σ_{tot}

$$\sigma_{tot} = \sigma_0 \left(1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right)$$

σ_0 ← given by EW dynamics

Single gluon emission



$$\frac{dN}{dk_1^2 dy} \approx \frac{4\alpha_s}{3\pi} \frac{dk_1^2}{k_1^2} dy$$

Colour dipole $\quad |y| \leq \ln \frac{Q}{k_1}$

Formation time (Landau - Pomeranchuk) $\sim \frac{1}{k_1}$

long cf. to initial collision time $\sim \frac{1}{Q}$

Gluon emission affects the final state

but not σ_{tot}

σ_{3jet} diverges for small $k_\perp$, but

this is compensated by virtual corr. to σ_{2jet}

σ_{2jet} =

The "hardest" gluon described by Sudakov form factors

↑ can be described in diff. ways

hardest = largest $k_\perp$ ⇒

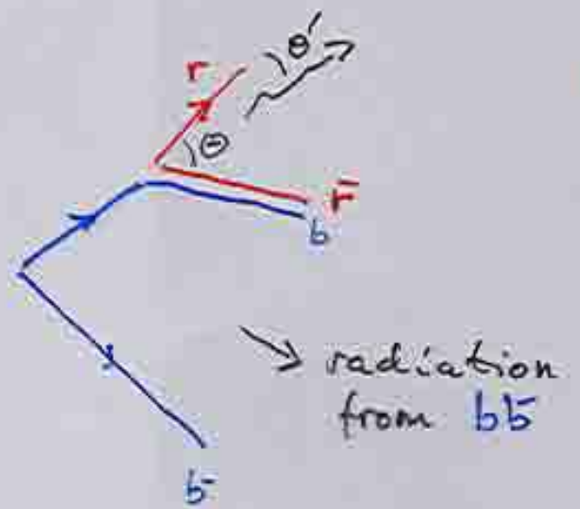
$$\frac{1}{\sigma_0} d\sigma_{hardest} = \frac{4\alpha_s}{3\pi} \frac{dk_\perp^2}{k_\perp^2} dy \cdot \exp\left(-\int_{k_\perp' > k_\perp} \frac{4\alpha_s}{\pi} \frac{dk_\perp'^2}{k_\perp'^2} dy'\right)$$

Prob. for no emission above $k_\perp$, when these emissions are uncorrelated

2 gluon emission

Soft gluon emission factorizes,

but currents are modified by the first emission



→ radiation from $b\bar{b}$

Colour coherence ⇒

Angular ordering:

Radiation from $r\bar{r}$ dipole only with

$\theta' < \theta$

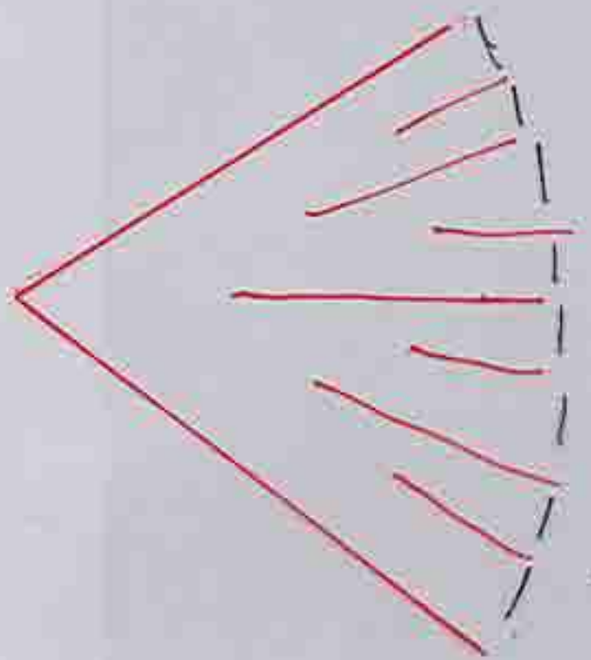
2nd gluon emitted from 2 dipoles

3rd " " 3 dipoles etc.

Dipole cascade model

(G.G., V. Pettersson, '88)

(in momentum space ≠ Mueller's cascade in transv. coord. space)



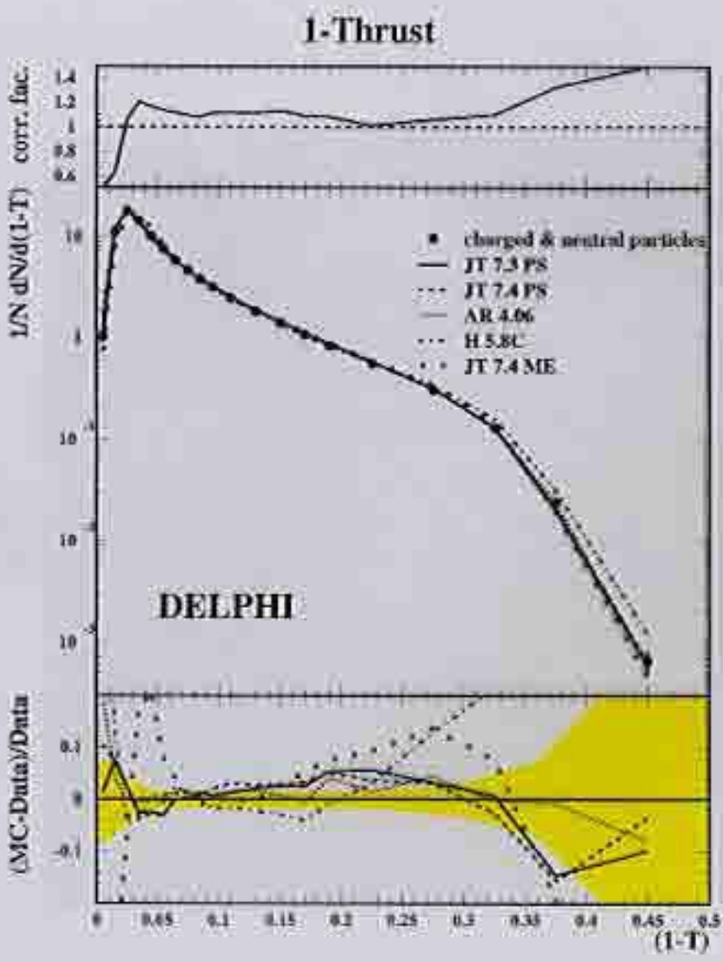
Gluons emitted by coherent emission from colour + anticolour in parent dipole.

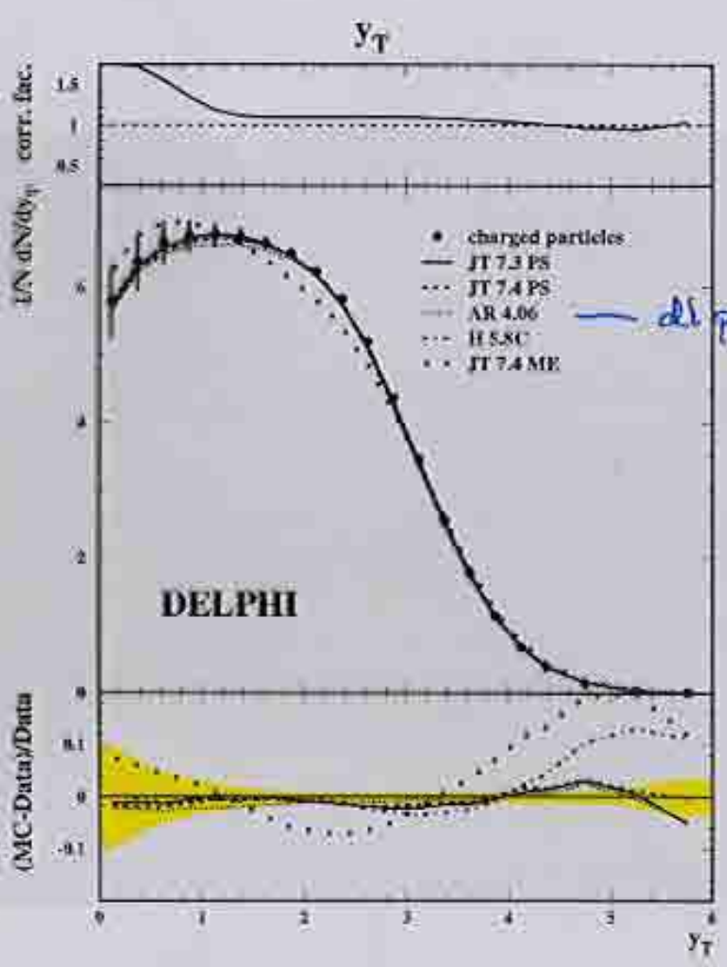
⇒ 1 dipole splits in 2 dipoles

Implemented in the ARIADNE MC

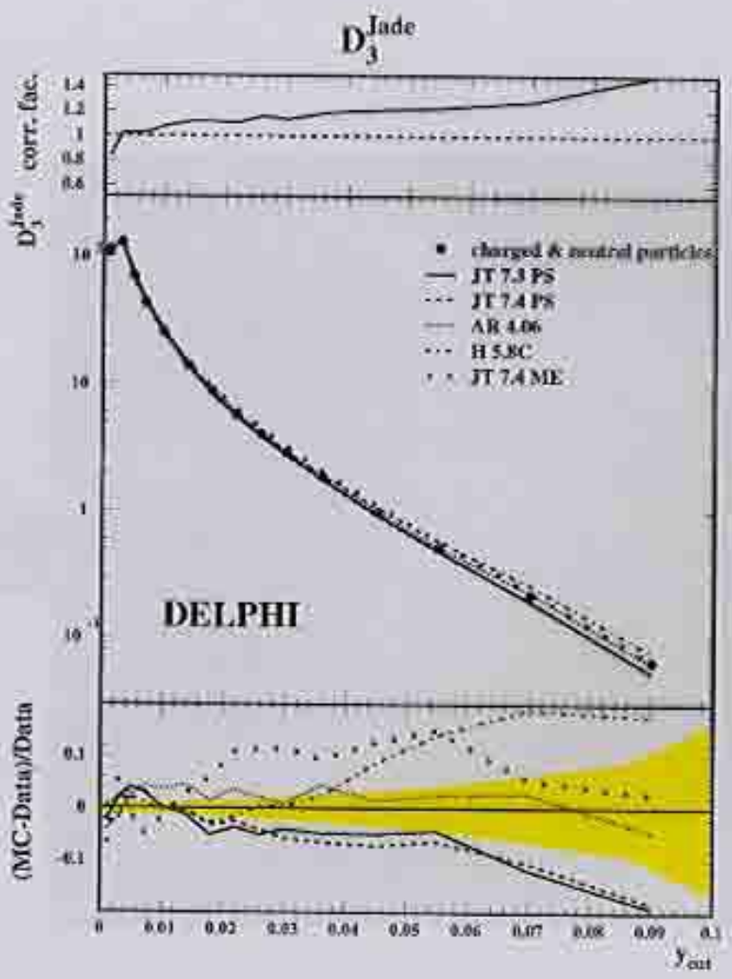
(L. Lönnblad)

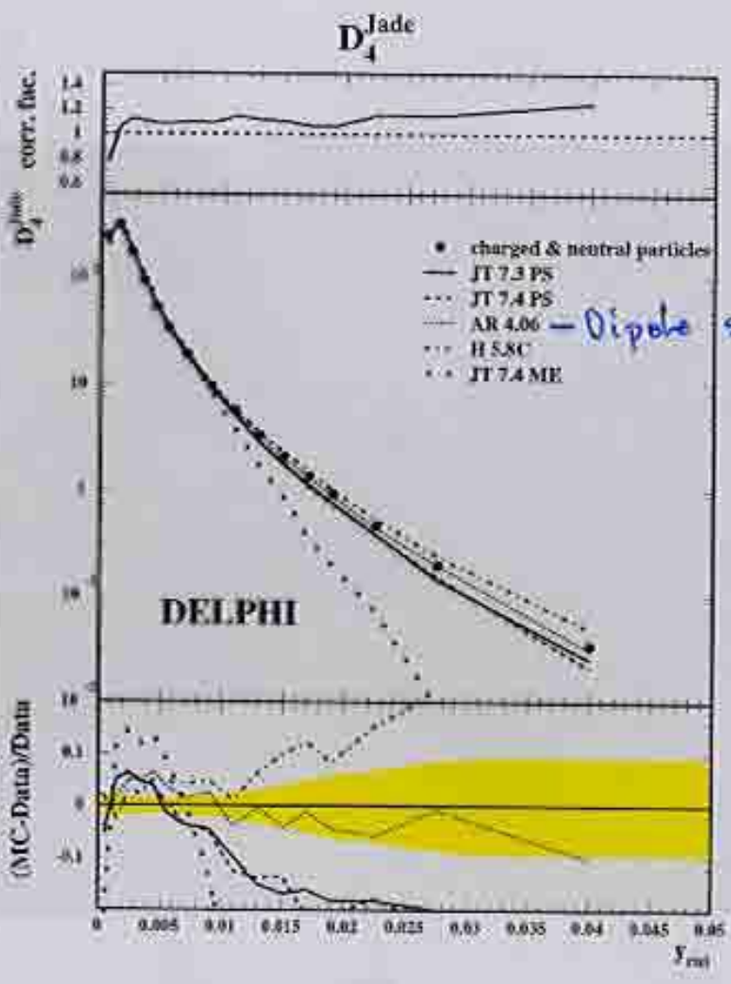
Note: The dipole chain joins nicely onto string hadronization (in leading $1/N_c$)





— dipole scheme

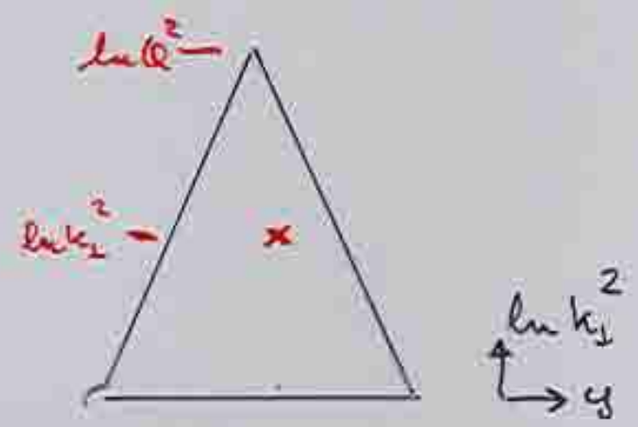

D_3^{Jade}
D_3^{Jade} corr. fac.
charged & neutral particles
JT 7.3 PS
JT 7.4 PS
AR 4.06
H 5.8C
JT 7.4 ME
DELPHI
(MC-Data)/Data
y_{cut}


D_4^{Jade}
D_4^{Jade} corr. fac.
charged & neutral particles
JT 7.3 PS
JT 7.4 PS
AR 4.06 — Dipole scheme
H 5.8C
JT 7.4 ME
DELPHI
(MC-Data)/Data
y_{cut}

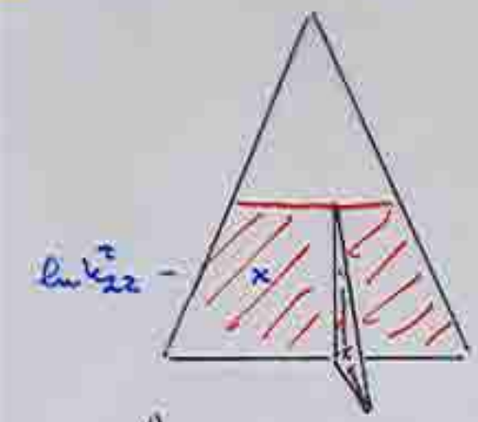
Final state

1st emission: $dN = \bar{\alpha}\,\frac{dk_\perp^2}{k_\perp^2}\,dy = \bar{\alpha}\cdot d\ln k_\perp^2 \cdot dy$

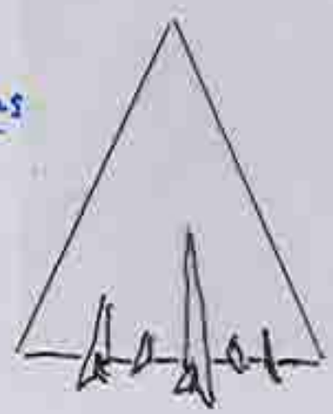
Phase space $|y| \lesssim \frac{1}{2}\ln\frac{Q^2}{k_\perp^2}$



2nd emission from 2 dipoles. Increased phase space



Many gluons



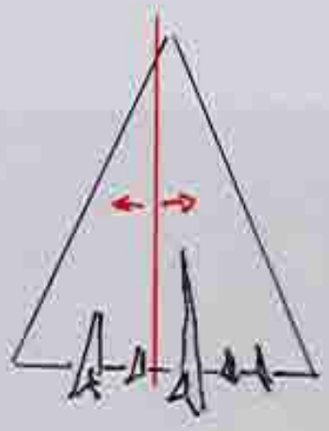
Study the state with improved resolution
⇒ more gluons resolved

Fractal dimension $= \sqrt{\frac{6\alpha_s(k_\perp^2)}{\pi}}$

Hadron mult. ~ length of base line

Total mult. ~ $\exp(2\sqrt{\bar{\alpha}\ln Q^2})$ (fixed α_s)

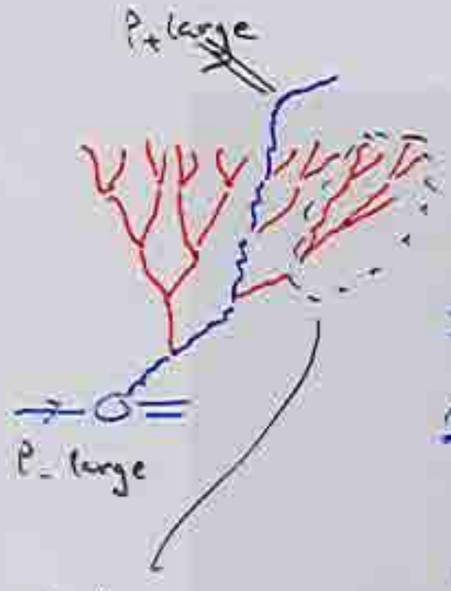
Cf. Herwig



Covers the same area "sideways"

Main differens – recoils

Spacelike Cascades



Virtual
must get P_+ from
the projectile

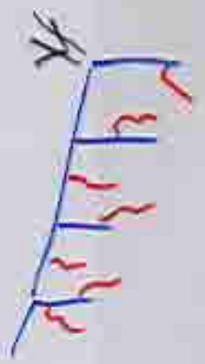
Gluon cascade increases the cross section.

2 variables: Q^2, W^2

2 separate problems:

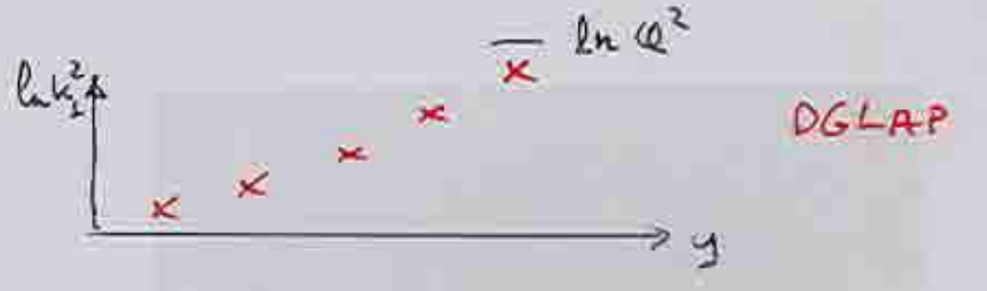
- Cross section
- Final states

Final state



Add final state radiation in angular ordered regions.

Q^2 large: $k_\perp$-ordered emissions dominate



x small, Q^2 limited: $k_\perp$-nonordered emissions impotent

Interpolation: CCFM (Catani, Ciafaloni, Fiorani, Marchesi

$\Rightarrow$ LDC (Linked Dipole Chains) model (Lund)

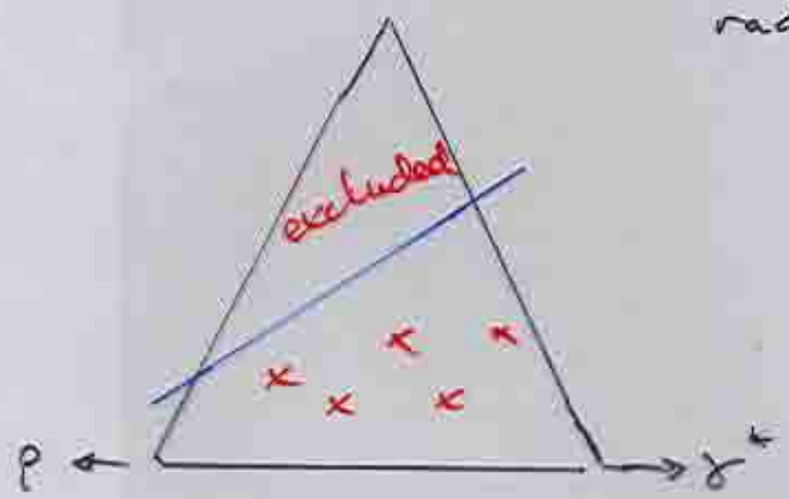
$\ln k_{\perp i}^2 \gtrsim \ln k_{\perp i-1}^2 - 1$

DGLAP evol. OK if soft input down to small x

Soft radiation model = CDM

implemented in ARIADNE MC

Determines phase space for gluon radiation



Non-ordered emissions à la BFKL

Complications: 1) Photon-gluon fusion

2) Q^2 small ⇒ Also the photon extended

Conclusions

Hadronization: Data well fitted by stringlike confining field

Gluon = transverse excitation on the string

IR stable

Parton cascades

Timelike: Leading $1/N_c$ cascade works well

(Why no colour reconnection?)

Spacelike: 2 problems:

- cross section
- final states

More to do!

Multiple Scattering Underlying Event, Minimum Bias

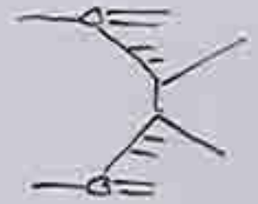
lecture II

- Experimental situation
  - Mult. hard subcollisions
  - Underlying event
  - CDF data vs. PYTHIA MC

- Multiple collisions and unitarity
  - AGK
  - $\mathbb{P}$ loops
  - Dipole cascades
  - Diffraction

Minijet cross section

Collinear factorization



! subcollision = 2 jets

$$\frac{d\sigma^{subcoll}}{dp_\perp^2} \sim \int dx_1\, dx_2\, f(x_1, p_\perp^2)\, f(x_2, p_\perp^2)\, \frac{d\hat{\sigma}(\hat{s}=x_1 x_2 s, p_\perp^2}{dp_\perp^2}$$

$$\sim \frac{1}{p_\perp^4}$$

Cutoff needed: $p_{\perp min}$

Total subcollision cross section $\propto \frac{1}{p_{\perp min}^2}$

Sjöstrand - Skands

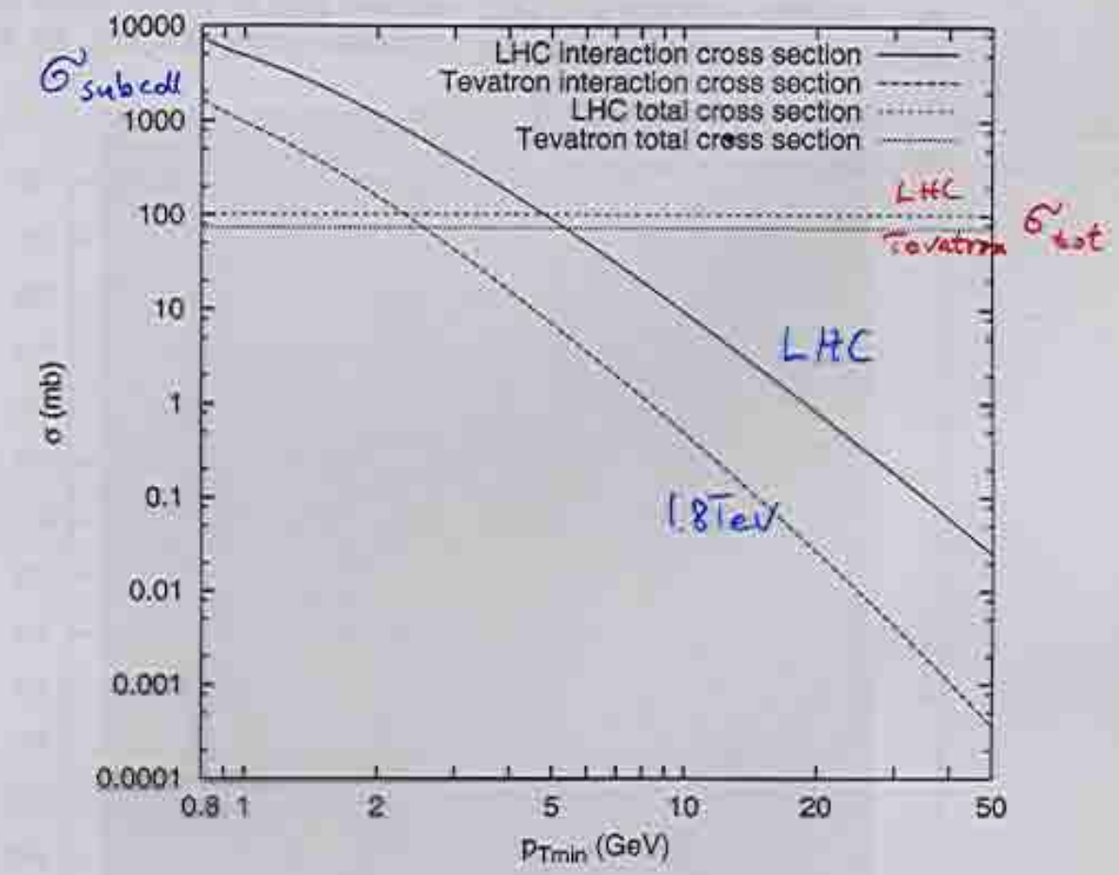
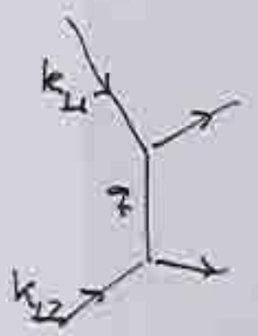


Figure 2: The integrated interaction cross section $\sigma_{\rm int}$ above $p_{\perp\rm min}$ for the Tevatron, with 1.8 TeV $p\bar{p}$ collisions, and the LHC, with 14 TeV pp ones. For comparison, the flat lines represent the respective total cross section.

Fit to data : $p_{\perp min} \sim 2$ GeV at the Tevatron

Slowly growing with energy (Sjöstrand-v.Zijl)

$k_\perp$-factorization



Dynamic cutoff when

$q_\perp < k_{\perp 1}, k_{\perp 2}$ (G.G.-G. Miu '01)

Similar effect

Result : $\sigma_{subcoll} >> \sigma_{tot}$

=> On average several subcoll./event

Early suggestion: The increase in σ_{tot} is driven by hard parton-parton subcollisions

UA1

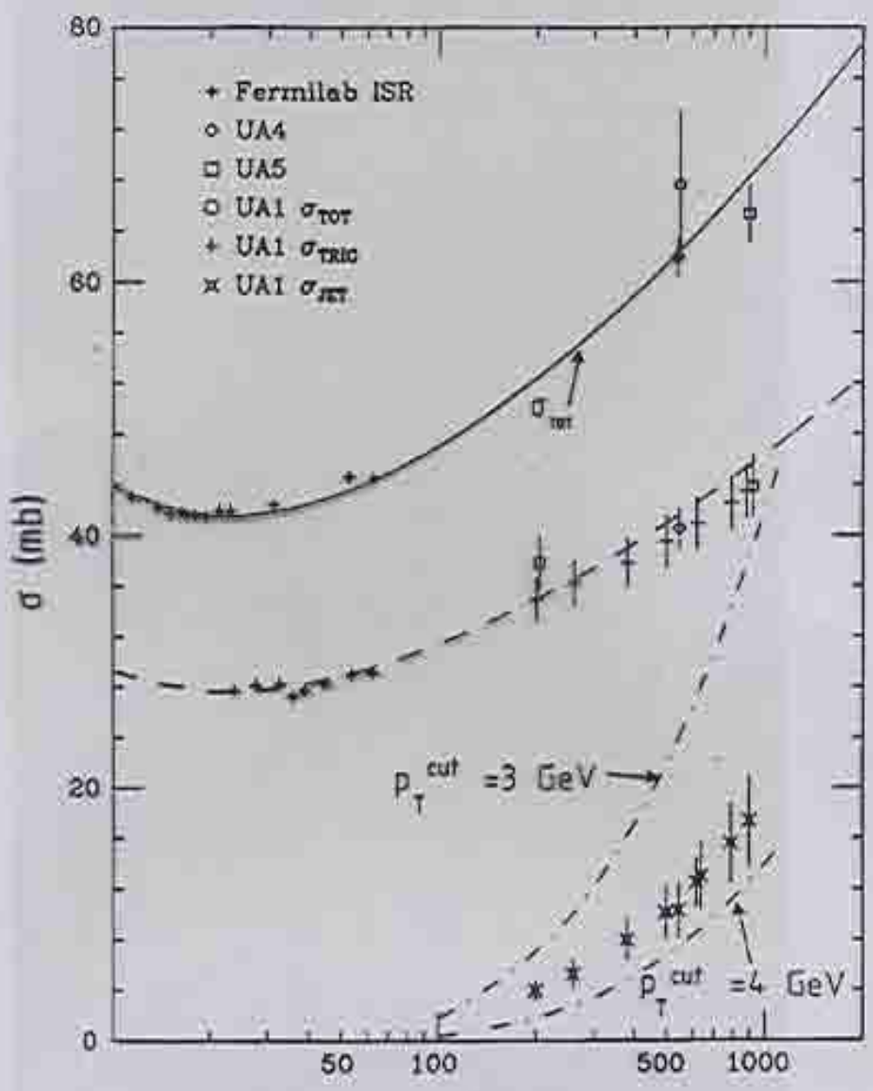
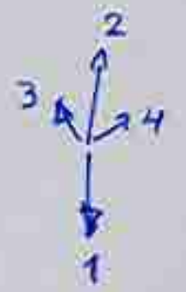

80
60
40
20
0
σ (mb)
Fermilab ISR
UA4
UA5
UA1 σ_TOT
UA1 σ_TRIG
UA1 σ_JET
σ_TOT
p_T^cut =3 GeV
p_T^cut =4 GeV
50
100
500
1000
√s (GeV)

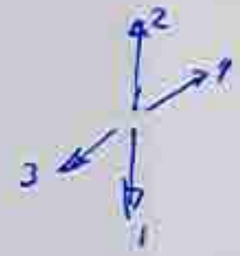
FIG. 10

Experimental evidence for multiple subcollisions

1. Multijet events

Cf. double bremsstrahlung – double parton scatt.





Imbalance parameter

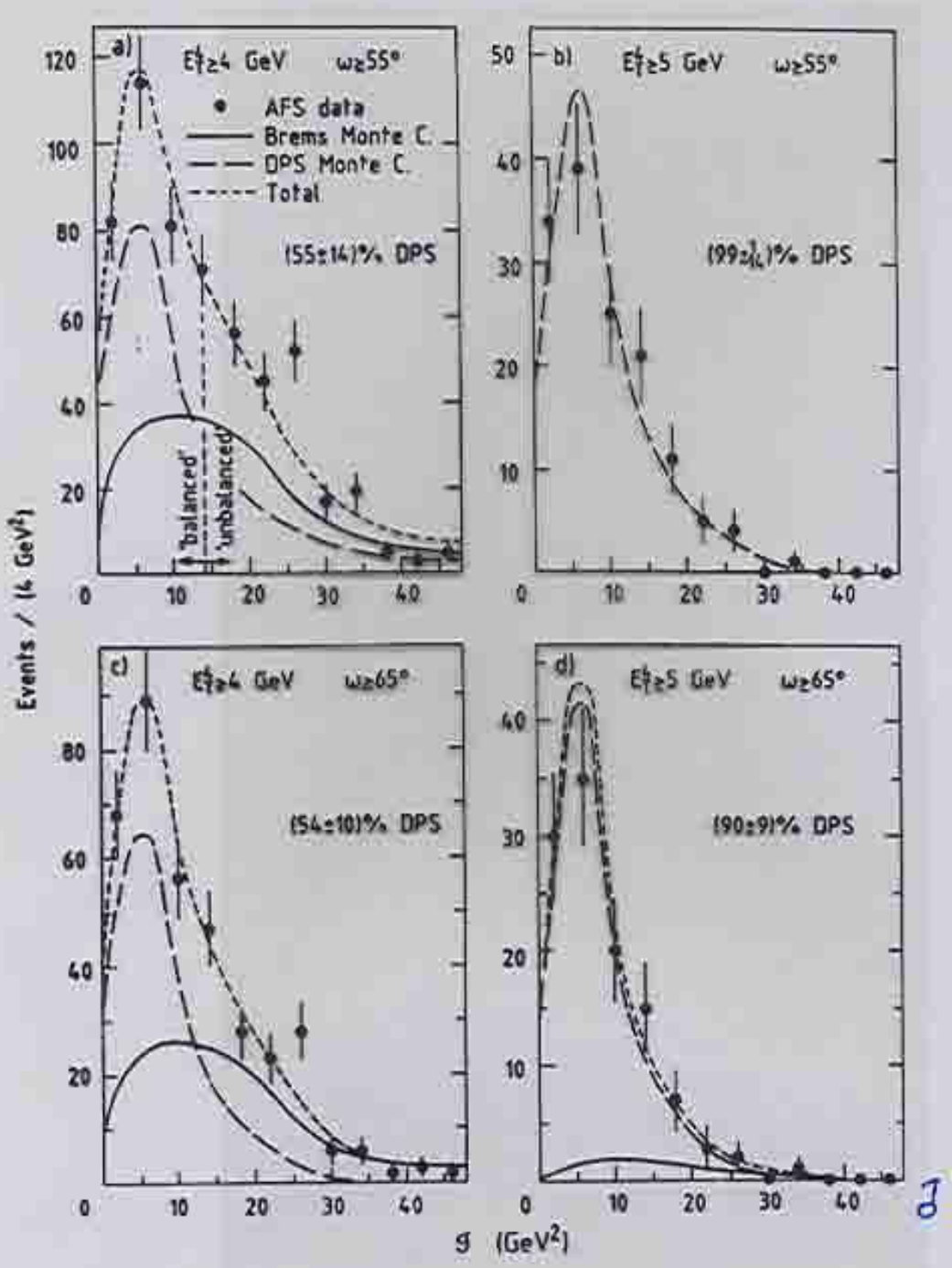
$$J = \frac{1}{2}\left[(\vec{p}_{\perp 1} + \vec{p}_{\perp 2})^2 + (\vec{p}_{\perp 3} + \vec{p}_{\perp 4})^2\right]$$

AFS (fig)

Also CDF, DØ 4jet ev.

CDF $3j + \gamma$ (fig)

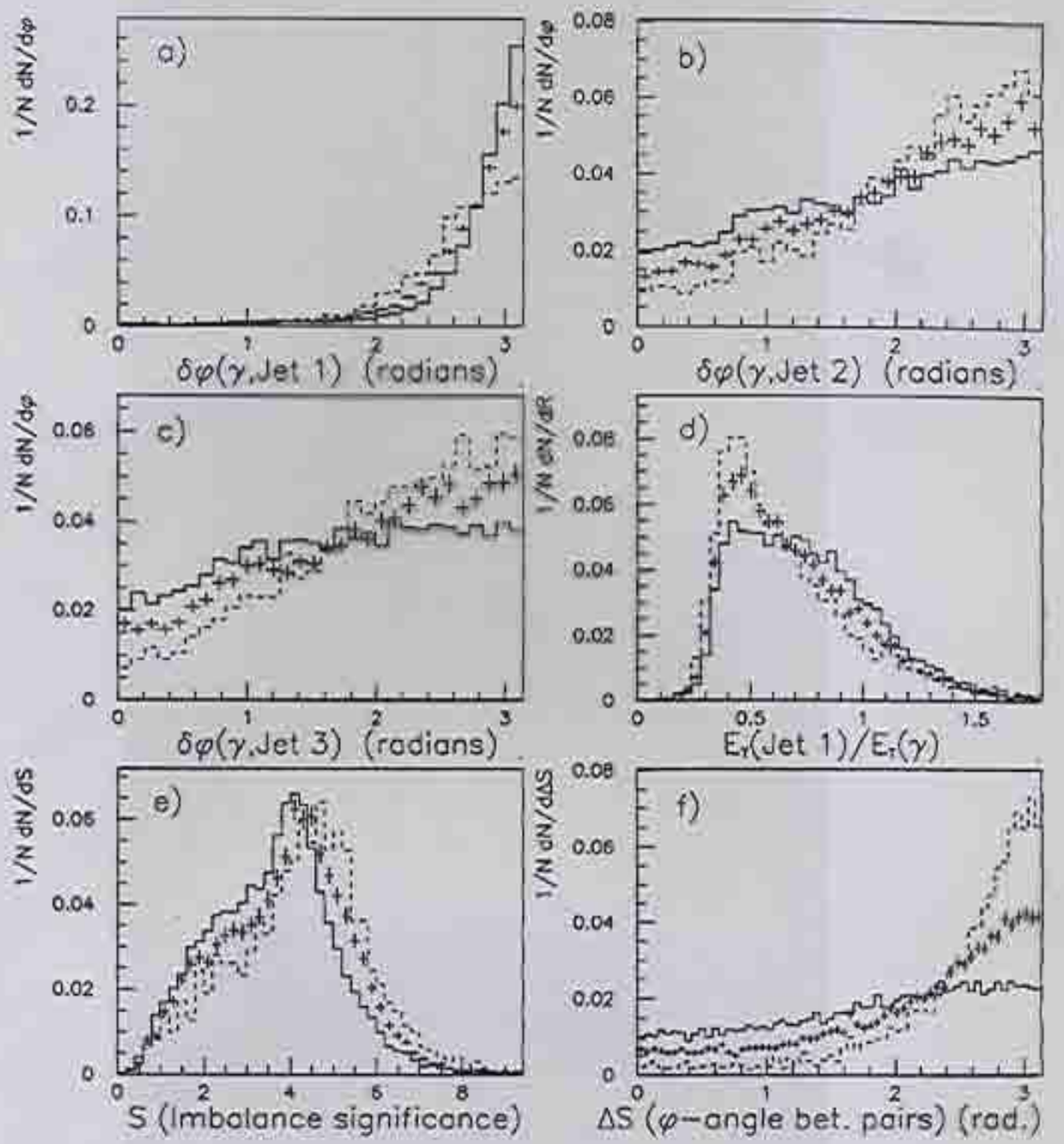
Zeus photoprod.



AFS at ISR
63 GeV

Fig. 3a–d. The distribution of the imbalance variable $\mathcal{J}$ for our (uncorrected) data and the predictions of the two model calculations including full detector simulation etc. Figures **a–d** correspond to various cuts on the E_T of jet 4(E_T^4) and interjet angle (ω) as indicated. The data are fitted to a sum of the two components. The results of the fit are the fractions of DPS indicated, and the model curves

CDF : 3jets + γ
--- with mult_int.
——— without "



Underlying event

Is a high $p_\perp$ event = 2 jets + min. bias ?

Correlations

Uncorrelated subcollisions ⇒

Prob. (n subcollisions) = Poisson distrib.

$\therefore \quad P(2) = \frac{1}{2} P_{(1)}^2$

↕ double counting

$$P(2) = \frac{\sigma_2}{\sigma_{inel,nd}} \quad ; \quad P(1) = \frac{\sigma_1}{\sigma_{inel,nd}}$$

$$\therefore \quad \sigma_2 = \frac{1}{2} \frac{\sigma_1^2}{\sigma_{inel,nd}}$$

Exp. notation: $\sigma_2 = \frac{1}{2} \frac{\sigma_1^2}{\sigma_{eff}}$

- ISR: $\sigma_{eff} \sim 5$ mb ($p_\perp > 4$ GeV) $\sigma_{nd} \sim 30$ mb
- CDF: $\sigma_{eff} \sim 12$ mb ($p_\perp > 25$ GeV) $\sigma_{nd} \sim 50$ mb
- CDF $3j + \gamma$ $\sigma_{eff} \sim 14$ mb

$\sigma_{eff} \ll \sigma_{nd}$ ⇒ Correlated mult. coll.

1 hard coll. ⇒ more likely to have another one.

Interpretation:

{ Central coll. ⇒ many hard sub coll.
{ Peripheral coll. ⇒ few " "

Pedestal effect

High $p_\perp$ jet ⇒ Underlying event grows

UA1

Fig

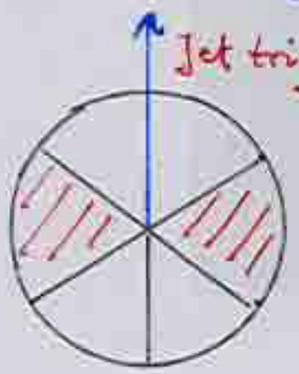
H1 resolved γ-prod

CDF Rick Field

Tuned PYTHIA MC to CDF data

Tune A, tune DW

give good fits to "all" data

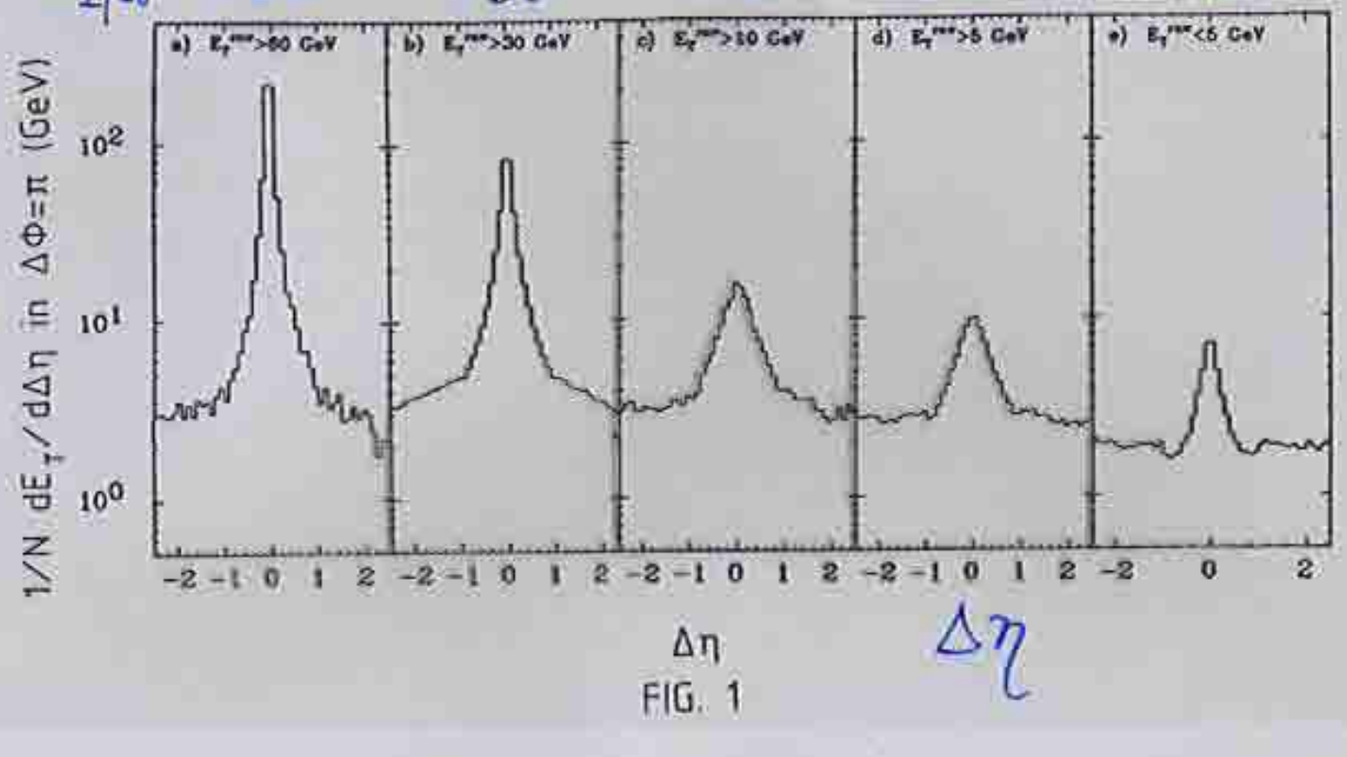


$E_\perp$ & charged mult.

in transverse region

UA1

E_{Jet} >60 >30 >10 >5 <5



Δη

FIG. 1

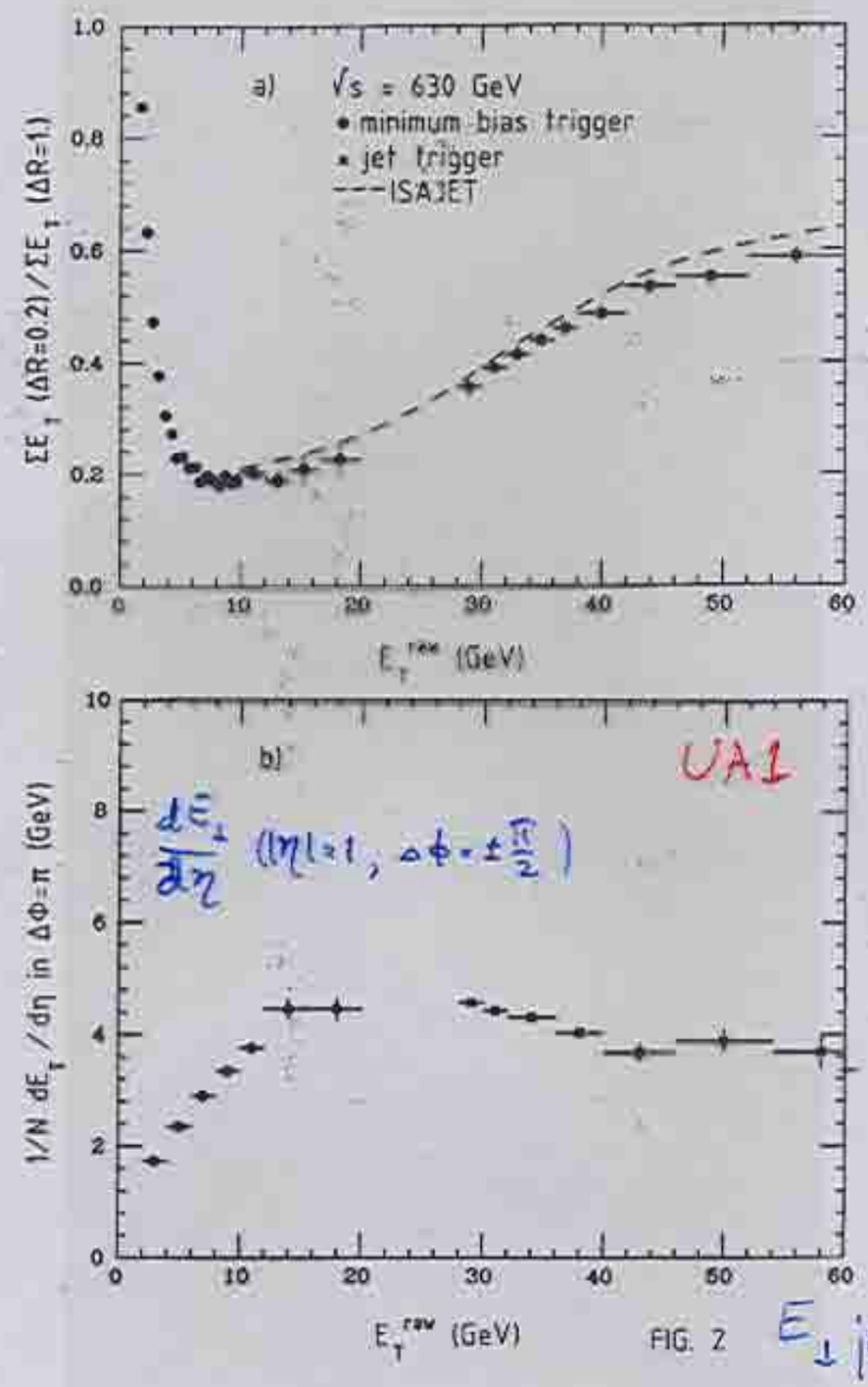
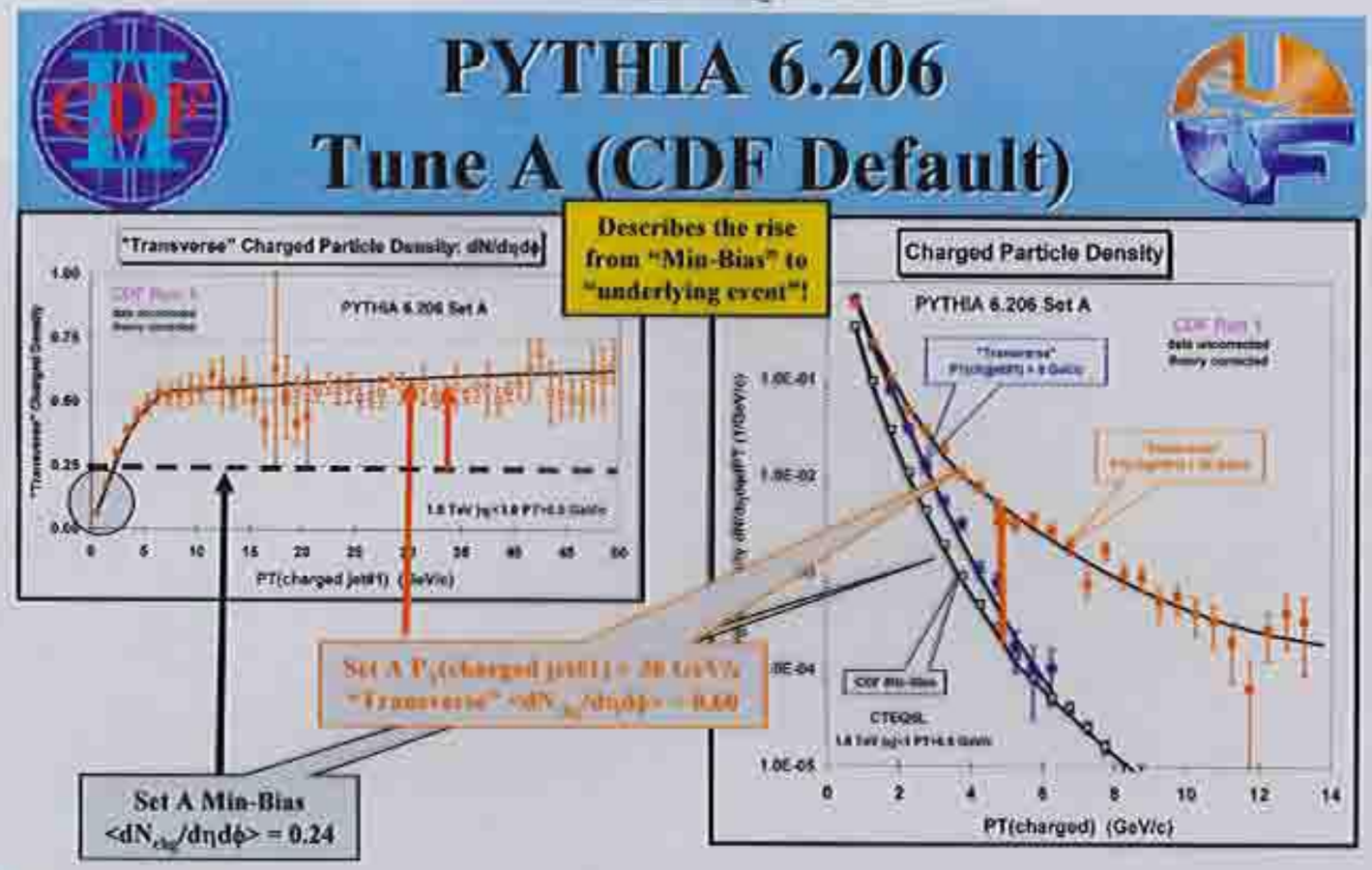

a)
√s = 630 GeV
• minimum bias trigger
• jet trigger
---ISAJET
ΣE_T (ΔR=0.2)/ΣE_T (ΔR=1.)
E_T^raw (GeV)
b)
UA1
dE_T/dη (|η|<1, Δφ = ±π/2)
1/N dE_T/dη in ΔΦ=π (GeV)
E_T^raw (GeV)
E_⊥ jet

FIG. 2

Rick Field

PYTHIA 6.206
Tune A (CDF Default)



- **Compares the average "transverse" charge particle density ($|\eta|<1$, $P_T>0.5$ GeV) versus P_T(charged jet#1) and the P_T distribution of the "transverse" and "Min-Bias" densities with the QCD Monte-Carlo predictions of a tuned version of PYTHIA 6.206 (P_T(hard) > 0, CTEQ5L, Set A). Describes "Min-Bias" collisions! Describes the "underlying event"!**

Early Sjöstrand - v. Zijl model implemented in PYTHIA

Assume: High energy collisions are dominated by parton-parton subcoll.

Min. bias: At least one parton-parton subcoll.

Parton distrib.: ~ Double Gaussian

Fixed b: Poisson distrib.

$\langle \#\text{ subcoll.} \rangle \propto$ overlap

$\Rightarrow P(n) \approx$ Geometric distrib.

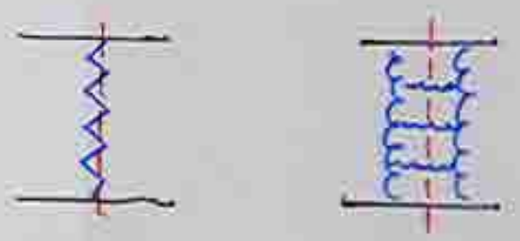
Wider than Poisson

Note: Diffraction is not included

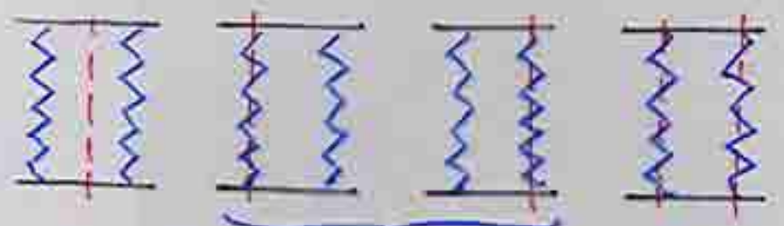
Multiple collisions and unitarity

AGK cutting rules

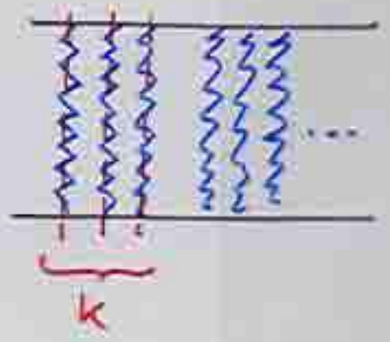
$\mathbb{P}$ = gluon ladder



Double $\mathbb{P}$ exch.



1 : −4 : 2



k cut $\mathbb{P}$. Arbitrary # uncut $\mathbb{P}$ $\Rightarrow$

Poisson distrib.: $e^{-\bar{k}} \frac{\bar{k}^k}{k!}$ $(k \geq 0)$

Cf. Pythia

16 ε

Problems

Relation $E_\perp - n_{ch}$ not as expected.

Rick Field's tunes fit both $E_\perp$ flow and particle flow, but pay a price.

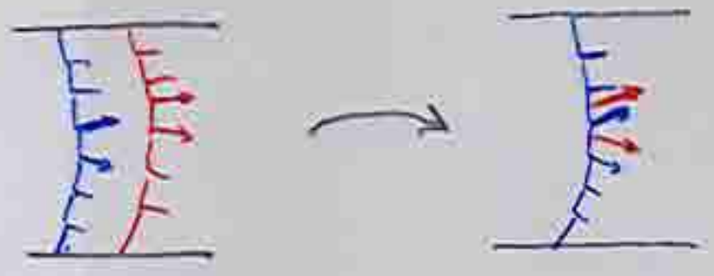
1 cut ℙ :

2 cut ℙ :

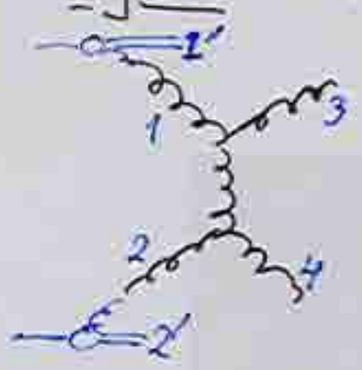
Double particle density expected

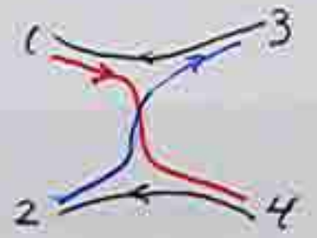
CDF data : $E_\perp$ grows more than the multiplicity

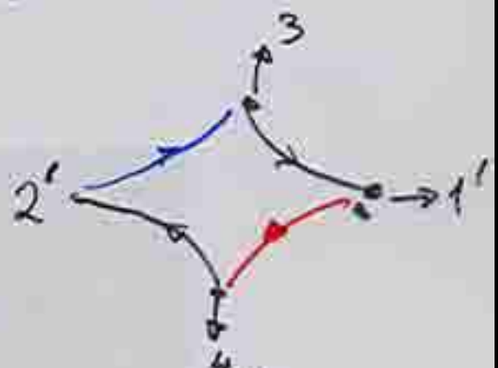
Rick's tunes: Color rearrangement

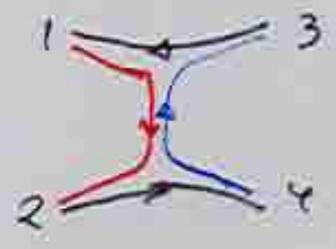


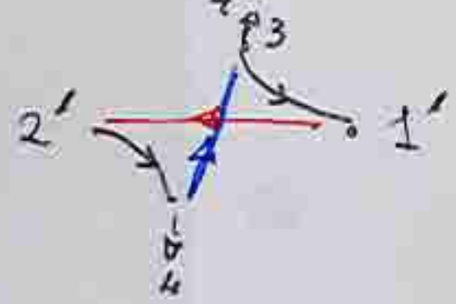
Pythia: 1 hard collision



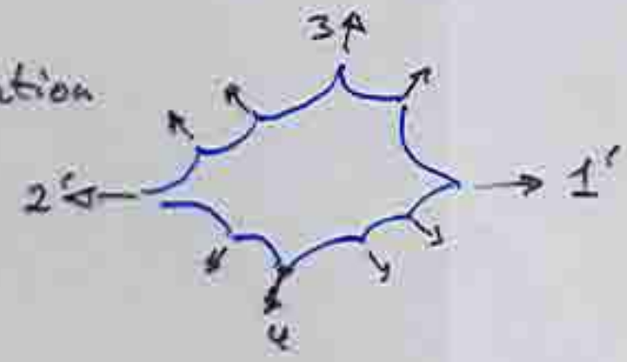






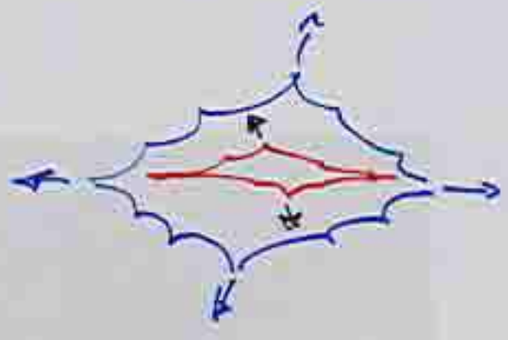


+ initial state radiation

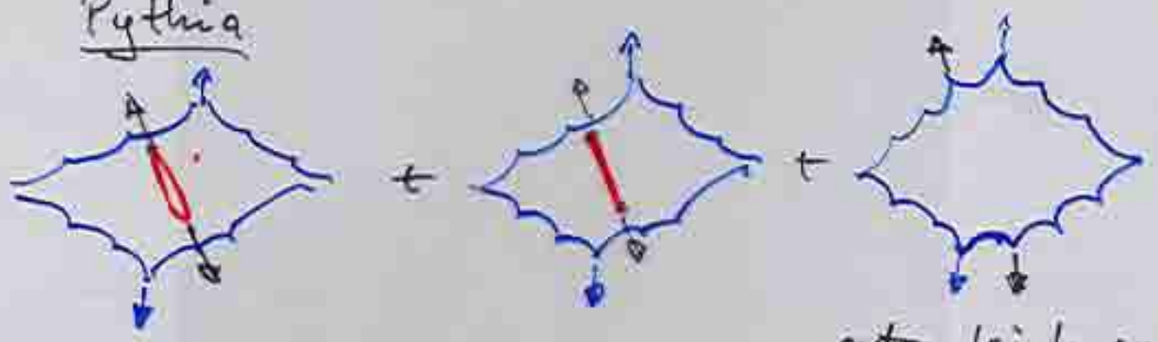


2nd hard subcoll.

à la AGK



Pythia



extra kink on earlier string

default 1/3 : 1/3 : 1/3

Tune A: 5% : 5% : 90%

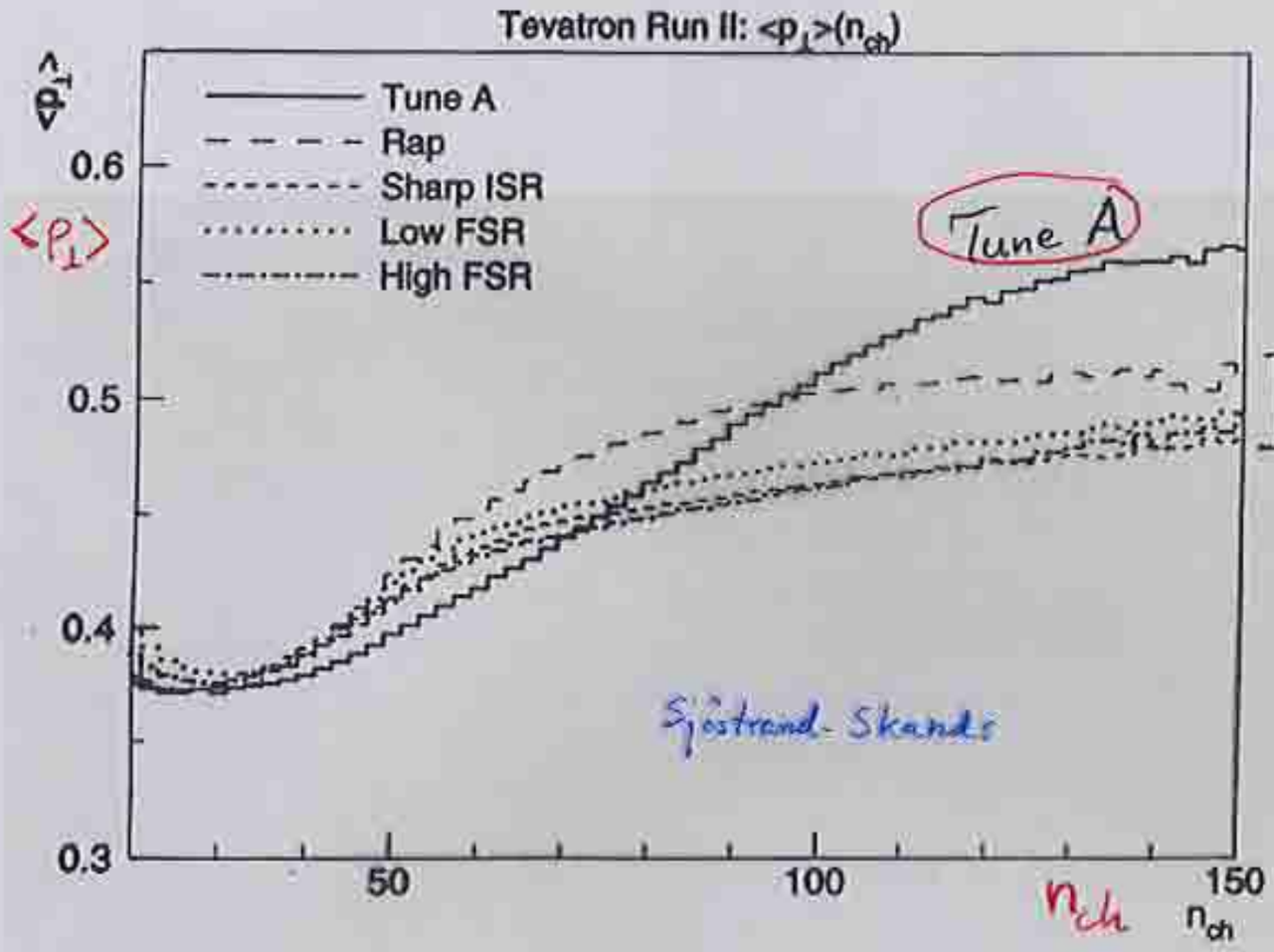
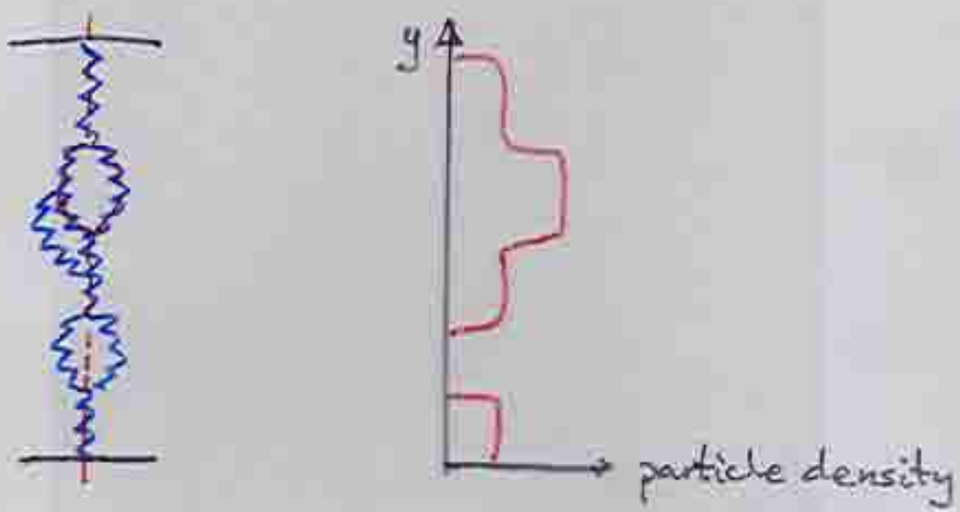


Figure 6: Average $p_\perp$ as a function of charged multiplicity, $\langle p_\perp \rangle (n_{\rm ch})$, for 1.96 TeV $p\bar{p}$ minimum-bias events. Note that the origo of the plot is *not* at (0,0).

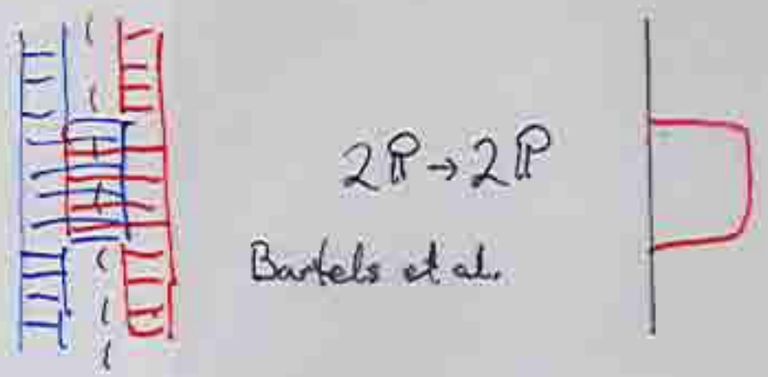
What could be missing?

$I\!P$ interactions

1. AGK (before QCD): $I\!P$ vertices



Renormalized $I\!P$: gaps and bumps



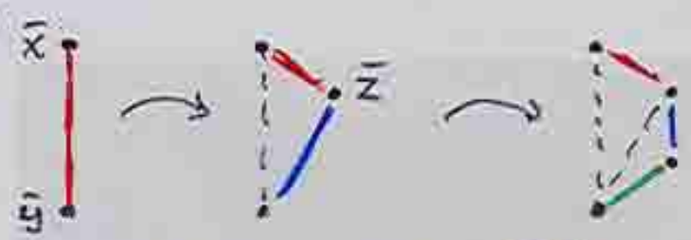
Can the dipole model help ?

transv. coord.

Up to now: σ_{tot}, but not exclusive final states

Dipole cascade model

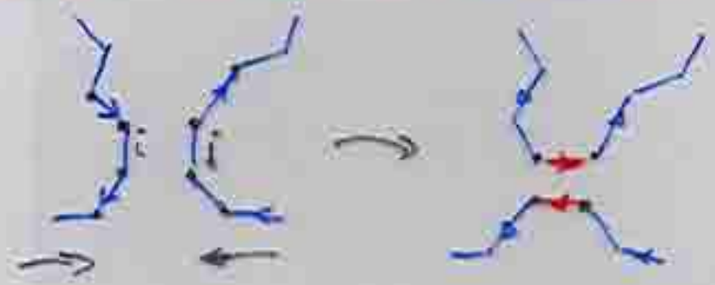
Mueller:



Large N_c. Prob. $\propto \bar{\alpha} = \frac{N_c \alpha_s}{\pi}$

Reproduces BFKL evolution

Collisions between 2 cascades



Dipole-dipole scattering $f_{ij} \sim \alpha_s^2 = \frac{\pi^2 \bar{\alpha}^3}{N_c^2}$

Formally color suppressed.

Eikonal approx. $1 - \prod_{ij} (1 - f_{ij})$ =>

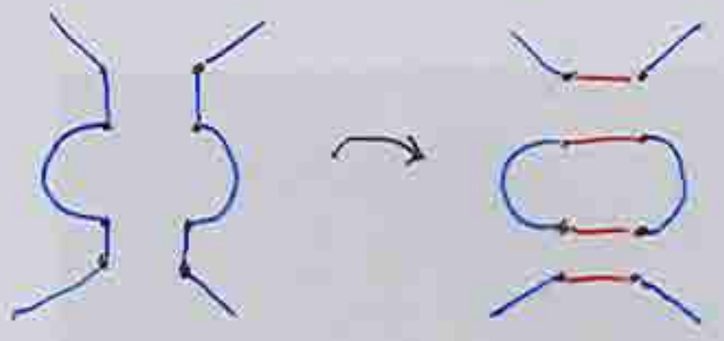
⇒ Unitarity. Scatt. prob. ≤ 1

Improvements: (E. Avsar - G.G. - L. Lönnblad)

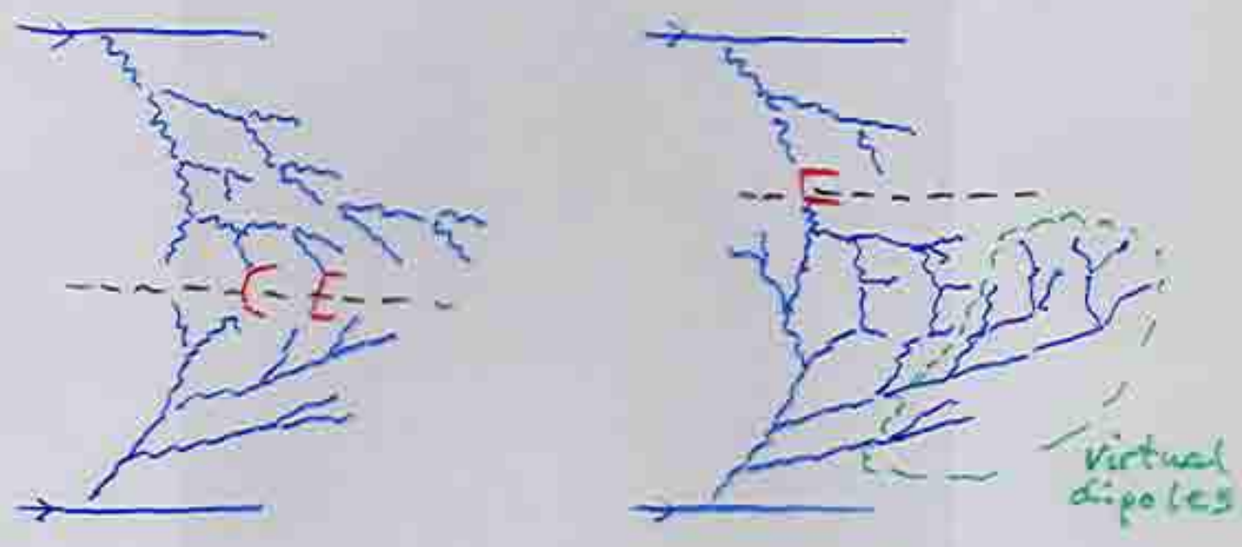
1. Energy-mom. conservation
   Reduces the growth at small x
   Makes MC working

2. Frame independence
   "Dipole swing"

Mult. coll. ⇒ Color loops ~ $\mathbb{P}$ loops



2) Frame independence



cms

different frame
gluons join

Different approaches:

1. Express the evolution in terms of interacting dipoles ⇒ # dipoles can be reduced.
   2 dipoles → 1 dipole. Evol. dep. on target.
2. Eliminate noninteracting dipoles afterwards. Evol. independent of target. No need to reduce # dipoles in the evol.. Color loops ($\mathbb{P}$ loops) formed by "swing".

Dipole swing

Finite N_c corrections

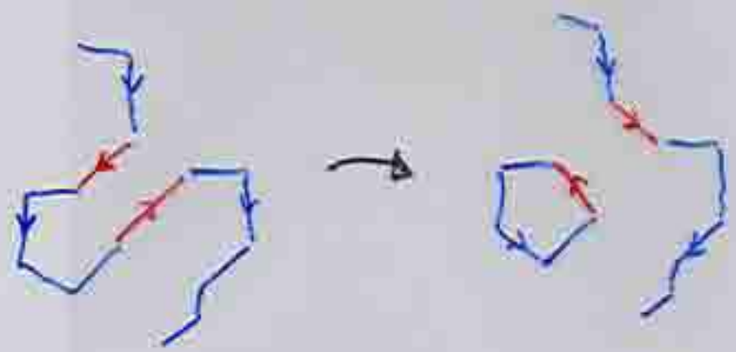
– Identical colors → color multiplets

Quadrupole better approximated by recoupled dipoles

– Gluon exchange. Color suppressed $\sim \alpha_s^2 \sim \frac{\bar{\alpha}^2}{N_c^2}$

like the dipole scatt. f_{ij}

Both mechanisms described by "swing"



Prob. $\sim \frac{1}{N_c^2}$

Inserted in MC ⇒ Approximately frame indep.

E. Avsar – G.G. – L. Lönnblad
JHEP 0701:012, 2007 ; hep-ph/0601589

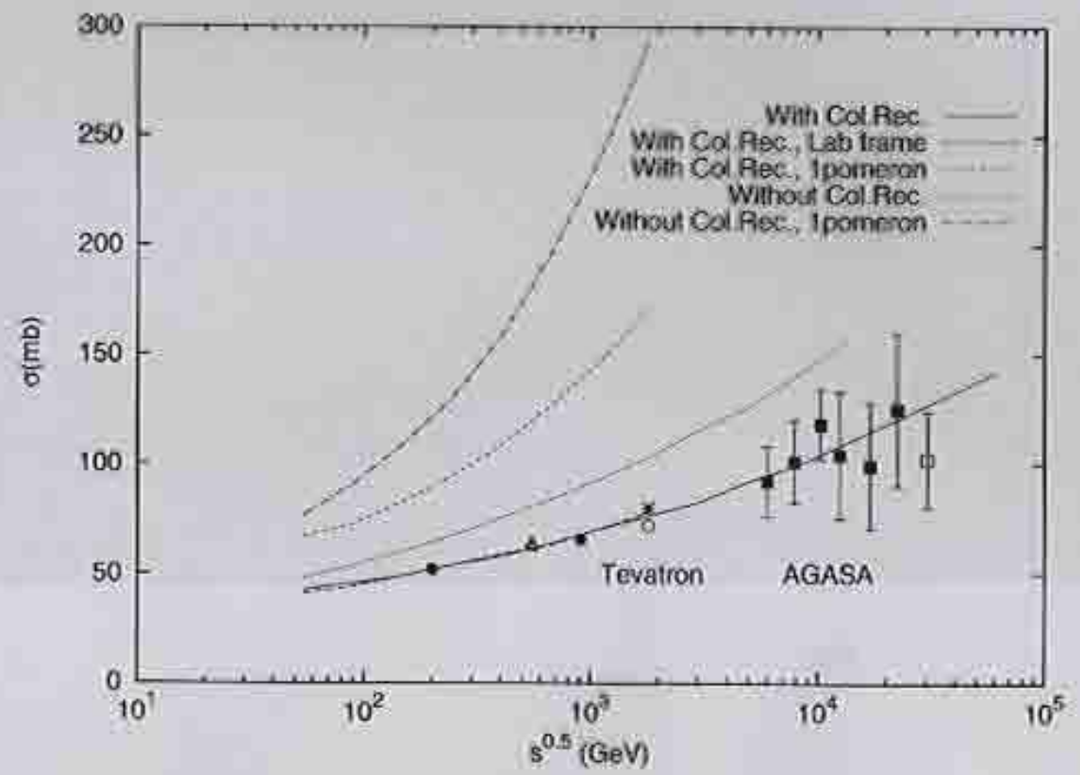
Describes F_2 and pp cross section

p represented by 3 dipoles

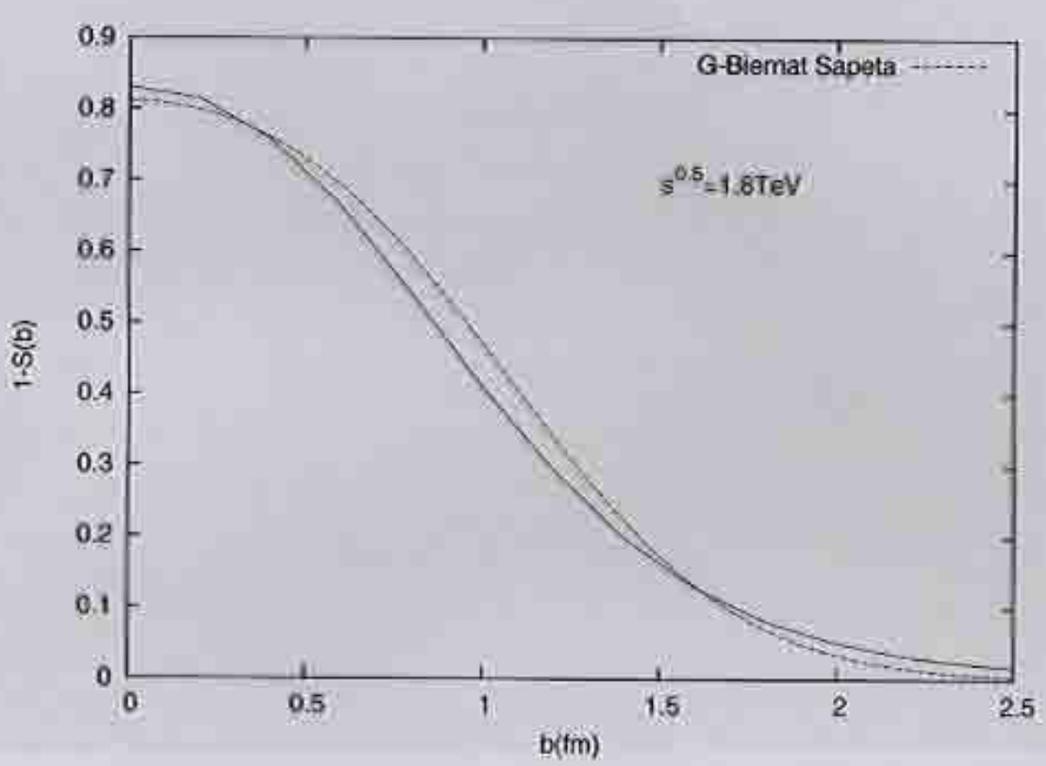
pp Total Cross Section

Saturation very important (small effect at HERA).

Dipole swing $\Longrightarrow$ Near frame independence.

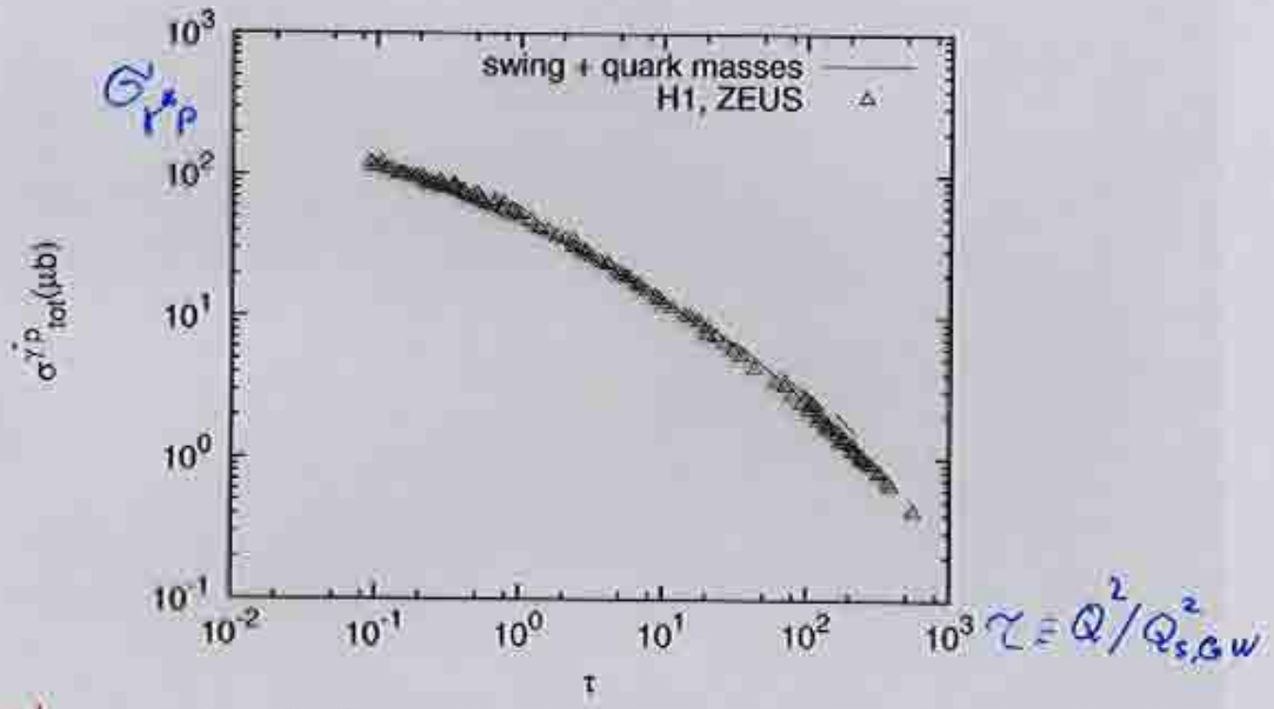


pp Impact Parameter Profile



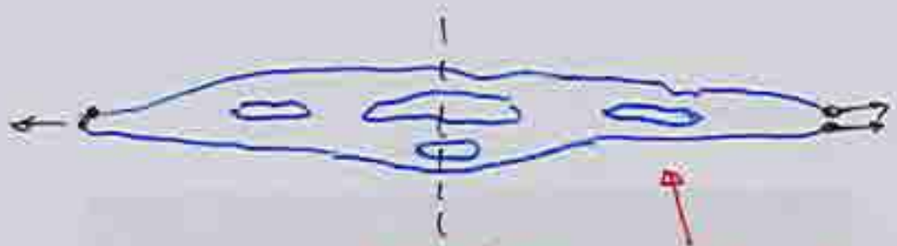
S =exp($-\sum_{ij} \mathcal{T}_{ij}$). GB-S = two parameter fit to data.

Full Results



Note: The result satisfies geometric scaling

- $\tau \equiv Q^2/Q_s^2(x)$, $Q_s^2(x) = (x_0/x)^{0.3}$, $x_0 = 3 \cdot 10^{-4}$. Effective light quark mass = 60MeV and m_c =1.4GeV.



Tevatron

$\langle N_{loops} \rangle$ cms: $0.65 + 2.2 + 0.65 = 3.5$

Asymmetric frame

$Y = 4.5 + 10.5$ $\quad 0.15 + 2 + 1.35 = 3.5$

LHC $\langle N_{loops} \rangle = 5.0$

Dipole model inclusive $\Rightarrow \sigma_{tot}$

Eikonal approx. / AGK $\Big\} \Rightarrow$ Diffraction and elastic scatt. / Rapidity gaps

Work in progress

Aim: Bridge the gap between dipole cascades, AGK and trad. MC generators

$\Rightarrow$ Event Generator fully compatible with unitarity and AGK.

24th

Conclusions

- Mult. coll. present in data

- Hard subcoll. correlated
  Underlying event ≠ min. bias

- R. Field's tunes of PYTHIA fit Tevatron data well.

- Relation E_T-n_{ch} not understood
  - Color rearrangement ?
  - Renormalized $I\!P$?

- Unitarity:
  Dipole formalism ⇒ $I\!P$ loops, Diffraction
  + AGK ⇒ gaps

Future: Combine dipole formalism and trad. MC to obtain event generators which include diffraction and are compatible with unitarity and AGK.