Unitarity correction from the high energy QCD effective action

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Motivation

Need for corrections to the LO- BFKL-Pomeron

- NLO, NNLO accuracy
- violation of unitarity $\Rightarrow$ unitarity corrections/saturation effects etc.

many different approaches available to study those corrections

I want to use here

Gauge invariant effective action for high energy QCD [Lipatov, '95]

- an effective action for reggeon interaction

Outline

QCD at high energies

consider

- scattering of two hadrons A and B with $2p_B/s^{1/2} = n^+$ and $2p_A/s^{1/2} = n^-$
- in the Regge limit $s \to \infty$
- with a hard scale $Q^2 \gg \Lambda_{\mathrm{QCD}}$ s.t. $\alpha_s \ll 1$ and perturbation theory applies
- $\Rightarrow$ consider scattering of partons

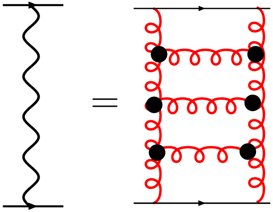
at high c.o.m. energies s

- smallness of coupling compensated by logs, $\alpha_s \ln s \sim 1$, and $(\alpha_s \ln s)^n$ need to be resummed.

The gluon reggeizes

Scattering of two partons:

- reggeized gluon/reggeon
- due to resumming leading contributions $\propto (\alpha_s \ln s)^n$
- effective degree of freedom in the t-channel at high energies

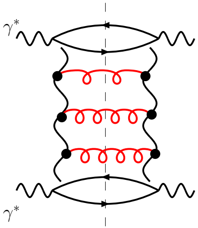


$\sim s^{1+\beta(\boldsymbol{k})}$

The reggeon

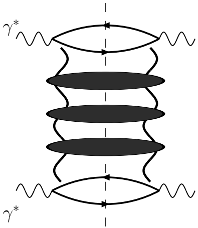
- carries quantum number of the gluon
- only transverse degrees of freedom
- has negative parity
- resummed propagator $s^{1+\beta(\boldsymbol{k})}/\boldsymbol{k}^2$
- couples (like a parton) only to single partons

The BFKL-Pomeron



Example: $\gamma^*\gamma^*$-scattering

- BFKL Pomeron $=$ bound state of two reggeons
- real s-channel gluons strongly ordered in rapidity (Multi-Regge-Kinematics)

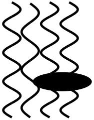


- Generalization: Quasi-Multi-Regge-Kinematics
- Clusters of particles (gluons and quarks) ordered strongly in rapidity w.r.t. each other
- idea: particle cluster local in rapidity gives interaction of reggeons; BFKL-Kernel $\equiv 2 \rightarrow 2$ reggeon transition

Unitarity corrections

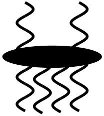
The BFKL-Pomeron violates unitarity $\Rightarrow$ need for unitarity corrections
in the present context:

exchange of more than two reggeons in the t-channel



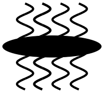
- contributions with fixed number of reggeons
  - interact to leading accuracy by $2 \rightarrow 2$ interaction kernels (BKP-equation)

- contributions where the number of reggeons fluctuates
  - need to calculate higher reggeon transitions kernels
  - so far explicit: 2-to-4 and 2-to-6



The gauge invariant effective action

want to determine

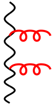


- transistion of arbitrary number of reggeons to arbitrary accuracy in $\alpha_S^m(\alpha_s \ln s)^n$

$\rightarrow$ reformulate QCD as theory of reggeons

interaction of reggeons basically given by $\mathcal{L}_{\text{QCD}}$, restricted to a certain rapidity intervall

BUT: generation and absorption of gluons is non-local in rapidity

- not only  but also 

Good news: emission from different rapidity cluster is eikonal [up to power corrections] $\Rightarrow$ universality

The gauge invariant effective action

Gauge Invariant Effective Action [Lipatov, 1995]

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}}(v_\mu, \psi) + A_+^a \partial^2 A_-^a - \text{tr}\frac{1}{g}\partial_- U(v_-)\partial^2 A_+ - \text{tr}\frac{1}{g}\partial_+ U(v_+)\partial^2 A_-$$

with $$\text{tr}\frac{1}{g}U(v_\pm)\partial^2 A_\pm = \text{tr}\mathcal{P}\exp -\frac{1}{2}\int_{-\infty}^{x^\pm} dx'^{\pm} v_\pm(x')\partial^2 A_+(x)$$

- $A_\pm^a(x)$ are unresummed reggeon fields: propagator $\frac{i}{2\boldsymbol{k}^2}$
- connect cluster of different rapidities
- kinematical constraint $\partial_+ A_- = 0 = \partial_- A_+$: guarantees gauge invariance of the action
- action is formulated for central rapidity region $\ln s > Y > 0$ (keep that in mind!) and is local in rapidity.
- Lagrangian is hermitian $\Rightarrow$ expect unitary S-Matrix

The gauge invariant effective action

- Action contains all possible transitions to arbitrary accuracy (up to power suppressed contributions)
- Action is gauge invariant $\Rightarrow$ no restriction certain gauge
- should respect unitarity

● But it is still 4-dimensional and integrations over plus- and minus components still unperformed ($\rightarrow$ logarithms !).

● To make use of the action $\Rightarrow$ perform longitudinal integrations $\equiv$ put the transition vertices into a 2-dimensional form.

●Then:
Can use them for further phenomenological and theoretical investigations + compare them to other results (BFKL, 2-4-Vertex, BK, JIMWLK, ...)

At the moment, it is not clear, how this shall be done in general

Longitudinal integrations

Our strategy:

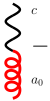
- derive explicit Feynman-rules from the action
- calculate 4-dimensional transition-kernels
- perform (or rather find a description how to perform) integrations over plus and minus components

In particular want to reproduce known results to have a check on the correctness, in detail we want to calculate

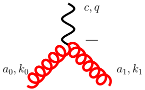
- the gluon-trajectory
- the 2-to-2 transition kernel (real part of the BFKL-kernel)
- vanishing of a transition 2-to-3 (required by parity)
- the 2-to-4 reggeon vertex

Feynman rules

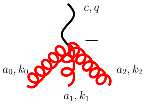
obtain usual QCD Feynman-rules + induced vertices describing emission from different rapidity cluster



$$iq^2 n^-_\nu \delta^{a_0 c} \qquad k_0^- = 0$$



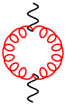
$$-gq^2 n^-_{\nu_0} n^-_{\nu_1} \left(\frac{f_{a_0 a_1 c}}{k_0^-}\right) \qquad k_0^- + k_1^- = 0$$



$$-ig^2 q^2 n^-_{\nu_0} n^-_{\nu_1} n^-_{\nu_2} \left(\frac{f_{a_2 a_1 a} f_{a_0 a c}}{k_0^- k_2^-} + \frac{f_{a_2 a_0 a} f_{a_1 a c}}{k_1^- k_2^-}\right) \qquad k_0^- + k_1^- + k_2^- = 0$$

and infinite many more ....

The gluon trajectory



$$= 2g^2 N_c \int \frac{dl^+ dl^-}{2\pi} \int \frac{d^2\boldsymbol{l}}{(2\pi)^3} \frac{\boldsymbol{k}^2}{l^-} \frac{\boldsymbol{k}^2}{l^+} \frac{-i}{l^+l^- - \boldsymbol{l}^2 + i\epsilon} \frac{-i}{l^+l^- - (\boldsymbol{l} - \boldsymbol{k})^2 + i\epsilon}$$

- interpret now $\frac{1}{l^+} = \frac{1}{2}\left(\frac{1}{l^+ + i\epsilon} - \frac{1}{-l^+ + i\epsilon}\right)$ as principal value
- and close contour for l^+ at infinity
- this treatment coincides with the analog calculation in QCD

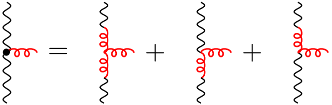
find

$$(-i2\boldsymbol{k}^2) \ln s g^2 \frac{N_c}{2} \int \frac{d^2\boldsymbol{l}}{(2\pi)^3} \frac{-\boldsymbol{k}^2}{\boldsymbol{l}^2(\boldsymbol{k} - \boldsymbol{l})^2} = \beta(\boldsymbol{k})(-i2\boldsymbol{k}^2)$$

the gluon trajectory

The BFKL kernel (2-to-2 transition vertex)

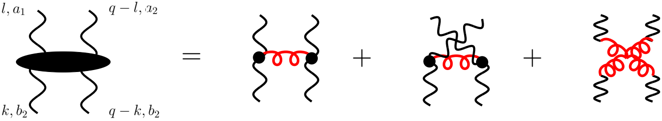
at first construct the Lipatov vertex:



$\rightarrow$ gauge invariant

$\cdot k_\mu = 0$

$\Rightarrow$ connected part of LO-BFKL-Kernel



$$= 4ig^2 \int \frac{dl^+ k^-}{(2\pi)^2}\Big[\frac{f^{a_1b_1c}f^{cb_2a_2}}{l^+k^- - (\boldsymbol{k}-\boldsymbol{l})^2 + i\epsilon}\Big(-\boldsymbol{q}^2 - \frac{(\boldsymbol{l}-\boldsymbol{q})^2\boldsymbol{k}^2}{l^+k^-} - \frac{(\boldsymbol{k}-\boldsymbol{q})^2\boldsymbol{l}^2}{l^+k^-}\Big)$$

$$+ \frac{f^{a_2b_1c}f^{cb_2a_1}}{l^+k^- - (\boldsymbol{q}-\boldsymbol{k}-\boldsymbol{l})^2+i\epsilon}\Big(-\boldsymbol{q}^2 + \frac{\boldsymbol{l}^2\boldsymbol{k}^2}{l^-k^-} + \frac{(\boldsymbol{k}-\boldsymbol{q})^2(\boldsymbol{l}-\boldsymbol{q})^2}{l^+k^-}\Big)\Big]$$

NOTE: This is (partially) logarithmically divergent in μ^2!

The 2-2 transition in the colour singlet/symmetric case

Consider the case where the two reggeons from above are in the colour singlet i.e. $a_1 = a_2$ (general: consider symmetric state)

Colour factor reduces for both terms to $N_c \delta^{b_1 b_2}$. The divergent part is $(\mu^2 = -k^- l^+)$

$$\sim \delta^{b_1 b_2} \boldsymbol{q}^2 \int^{s^{1/2}} \frac{dl^+}{|l^+|} \int \frac{d\mu^2}{(2\pi)^2} \Big(\frac{1}{\mu^2 - (\boldsymbol{k}-\boldsymbol{l})^2 + i\epsilon} + \frac{1}{-\mu^2 - (\boldsymbol{q}-\boldsymbol{k}-\boldsymbol{l})^2 + i\epsilon} \Big)$$

This is now convergent for large μ^2 as the divergencies cancel! Obtain therefore (toghether with the convergent parts)

$$2^4 \frac{g^2}{2\pi} \ln s \Big[-\boldsymbol{q}^2 + \frac{1}{2} \frac{(\boldsymbol{l}-\boldsymbol{q})^2(\boldsymbol{k}-\boldsymbol{q})^2 + \boldsymbol{l}^2\boldsymbol{k}^2}{(\boldsymbol{k}+\boldsymbol{l}-\boldsymbol{q})} + \frac{1}{2} \frac{\boldsymbol{l}^2(\boldsymbol{k}-\boldsymbol{q})^2 + (\boldsymbol{l}-\boldsymbol{l})^2\boldsymbol{k}^2}{(\boldsymbol{k}-\boldsymbol{l})} \Big]$$

the real part of the BFKL-Kernel [also for general symmetric case $\sim \boldsymbol{8}_S$ state]

The 2-2 transition in the colour octet/anti-symmetric case

Consider now the antisymmetric (i.e. colour octet $\sim f^{a_1 a_2 d}$) state . The divergent part is there

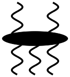
$$\sim \boldsymbol{q}^2 \int^{s^{1/2}} \frac{dl^+}{|l^+|} \int \frac{d\mu^2}{(2\pi)^2} \left(\frac{1}{\mu^2 - (\boldsymbol{k}-\boldsymbol{l})^2 + i\epsilon} - \frac{1}{-\mu^2 - (\boldsymbol{k}-\boldsymbol{l})^2 + i\epsilon} \right)$$

We define now $\int d\mu^2 := \lim_{M^2 \to \infty} \int_{-M^2}^{M^2} d\mu^2 \Rightarrow$ integral vanishes (principal value description). This is in accordance with the fact, that the effective theory is formulated only for central rapidities.

Q: Also the convergent parts vanish in the anti-symmetric case: How come? In QCD the 2-2 transition in the $\mathbf{8}_A$ channel does not vanish, but gives reggeization of the gluon.

A: In the effective theory, reggeization is taken into account already by the gluon 'bubbles' of the reggeon. Furthermore the reggeized gluon has negative parity and therefore an antisymmetric two reggeon-state cannot exist.

The 2-to-3 transition

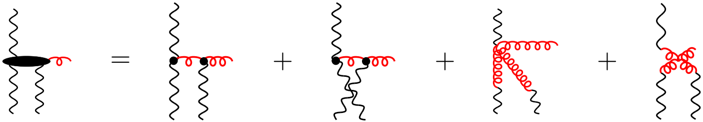


Recall: Transition of 2 to 3 reggeons has to vanish due to parity conservation
- want to verify this

to construct the transition, need the RPP - vertex:

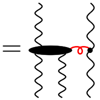
to construct the RRRP-vertex

RRRP-vertex:



Like the Lipatov-Vertex, the RRRP-Vertex is gauge invariant

allows me to construct the connect part of the transition



+ permutations

The 2-to-3 transition

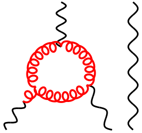
evaluating this, one finds with $\mu_1^2 = -l^+ k_1^-$ and $\mu_2^2 = -l^+ k_2^-$

$$2^5 g^3 f^{a_1 b_1 c_1} f^{c_1 b_2 c_2} f^{c_2 b_3 a_2} \int \frac{dl^+}{l^+} \int \frac{d\mu_1^2 d\mu_2^2}{(2\pi)^3} \Big[\Big(-\boldsymbol{q}^2 + \frac{(\boldsymbol{q}-\boldsymbol{l})^2 (\boldsymbol{k_1}+\boldsymbol{k_2})^2}{\mu_1^2 + \mu_2^2} + \frac{\boldsymbol{l}^2 (\boldsymbol{k_2}+\boldsymbol{k_3})^2}{\mu_1^2} - \frac{\boldsymbol{l}^2 \boldsymbol{k_2}^2 (\boldsymbol{q}-\boldsymbol{l})^2}{\mu_1^2 (\mu_1^2 + \mu_2^2)} \Big) \frac{1}{(\mu_1^2 - (\boldsymbol{l}-\boldsymbol{k_1})^2 + i\epsilon)(\mu_1^2 + \mu_2^2 - (\boldsymbol{l}-\boldsymbol{k_1}-\boldsymbol{k_2})^2 + i\epsilon)}$$
$$+ \frac{\boldsymbol{l}^2 \boldsymbol{k_3}^2}{\mu_1^2 (\mu_1^2 + \mu_2^2)} \frac{1}{\mu_1^2 - (\boldsymbol{l}-\boldsymbol{k_1})^2 + i\epsilon} + \frac{(\boldsymbol{q}-\boldsymbol{l})^2 \boldsymbol{k_1}^2}{\mu^2 (\mu_1^2 + \mu_2^2)} \frac{1}{\mu_1^2 + \mu_2^2 - (\boldsymbol{l}-\boldsymbol{k_1}-\boldsymbol{k_2})^2 + i\epsilon} \Big]$$

Note: The overall integral $\int \frac{dl^+}{l^+}$ vanishes with the previous principal-value description (central rapidities argument).

The 2-to-3 transition

There are also virtual/disconnected contributions to the 2-to-3 transistion.

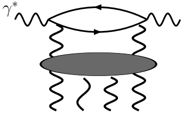


- They have been shown to vanish by performing first the k_i^- (the μ_i^2)-integrals and after that the l^+-integral.
- The argument used for the connected parts (vanishing by the l^+-integrations) works here in principel as well.

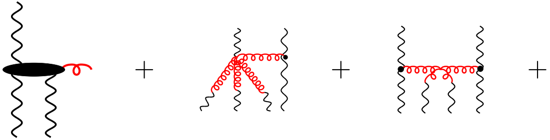
Note, that vanishing of the 2-to-3 tranisiton coincides with an earlier result by Bartels and Wüsthoff for 3-reggeon-amplitude

The 2-to-4 transition vertex

- Transition allowed by reggeon-parity
- There exist a previous result by Bartels and Wüsthoff on this transition → have an independent check on the results
- Possibility to relax restriction to the overall colour singlet state
- Possibility to disentangle 4-reggeon state coupled to photon and 2-to-4 transition (if there is something to disentangle)



Step 1: Construct the connected parts of the transition kernel



4!-1 permutations of the four reggeons

The connected part

$$
\begin{aligned}
&2^6 i g^4 f^{a_1 b_1 c_1} f^{c_1 b_2 c_2} f^{c_2 b_3 c_3} f^{c_3 b_4 a_2} \int \frac{dl^+}{2|l^+|} \int \frac{d\mu_1^2 d\mu_2^2 d\mu_4^2}{(2\pi)^3} \Big[\Big(\boldsymbol{q}^2 - \frac{(\boldsymbol{q}-\boldsymbol{l})^2 (\boldsymbol{q}-\boldsymbol{k_4})^2}{-\mu_4^2} \\
&- \frac{\boldsymbol{l}^2 (\boldsymbol{q}-\boldsymbol{k_1})^2}{\mu_1^2} + \frac{\boldsymbol{l}^2 (\boldsymbol{k_2}+\boldsymbol{k_3})^2 (\boldsymbol{q}-\boldsymbol{l})^2}{\mu_1^2 (-\mu_4^2)} \Big) \frac{1}{(\mu_1^2 - (\boldsymbol{l}-\boldsymbol{k_1})^2 + i\epsilon)(\mu_1^2+\mu_2^2-(\boldsymbol{l}-\boldsymbol{k_1}-\boldsymbol{k_2})^2 + i\epsilon)} \\
&\frac{1}{-\mu_4^2 - (\boldsymbol{q}-\boldsymbol{l}-\boldsymbol{k_4})^2 + i\epsilon} - \frac{(\boldsymbol{q}-\boldsymbol{l})^2}{(\mu_1^2+\mu_2^2)(\mu_4^2)} \Big((\boldsymbol{k_1}+\boldsymbol{k_2})^2 - 2 \frac{\boldsymbol{l}^2 \boldsymbol{k_2}^2}{\mu_1^2} \Big) \frac{1}{\mu_1^2 - (\boldsymbol{l}-\boldsymbol{k_1})^2 + i\epsilon} \\
&\frac{1}{\mu_1^2+\mu_2^2-(\boldsymbol{l}-\boldsymbol{k_1}-\boldsymbol{k_2})^2 + i\epsilon} - \frac{\boldsymbol{l}^2}{\mu_1^2(\mu_1^2+\mu_2^2)} \Big((\boldsymbol{k_3}+\boldsymbol{k_4})^2 - 2 \frac{(\boldsymbol{q}-\boldsymbol{l})^2 \boldsymbol{k_3}^2}{-\mu_4^2} \Big) \\
&\frac{1}{(\mu_1^2+\mu_2^2-(\boldsymbol{l}-\boldsymbol{k_1}-\boldsymbol{k_2})^2 + i\epsilon)(-\mu_4^2 - (\boldsymbol{q}-\boldsymbol{l}-\boldsymbol{k_4})^2 + i\epsilon)} + 2 \frac{(\boldsymbol{q}-\boldsymbol{l})^2 \boldsymbol{k_1}^2}{\mu_1^2(\mu_1^2+\mu_2^2)(-\mu_4^2)} \\
&\frac{1}{\mu_1^2 - (\boldsymbol{l}-\boldsymbol{k_1})^2 + i\epsilon} + 2 \frac{\boldsymbol{l}^2 \boldsymbol{k_4}^2}{\mu_1^2(\mu_1^2+\mu_2^2)(-\mu_4^2)} \frac{1}{-\mu_4^2 - (\boldsymbol{q}-\boldsymbol{l}-\boldsymbol{k_4})^2 + i\epsilon} \Big]
\end{aligned}
$$

The 2-to-4 transition - performing integrations

- The task is now - similar to the case of the 2-to-2 transition - to find by intelligent combinations of the 24 permutations a convergent set.
- Moreover the remaining (non-convergent) terms, have to be shown to vanish
- Work is in progress ....

Conclusions and Outlook

Using Lipatov's gauge invariant effective action

- we obtained the gluon trajectory
- and the 2-to-2 transition/BFKL-kernel for the colour singlet/symmetric case.
- In the anti-symmetric/coulor octet channel, this transition is absent in agreement with parity conservation.
- Again due to parity conservation, the 2-to-3 transition is absent.
- The four dimensional expression for the connected part of the $2 \to 4$ reggeon transition-kernel have been obtained; longitudinal integrations still need to be performed.