QCD factorizations in $\gamma^* \, \gamma^* \rightarrow \rho_L^0 \rho_L^0$

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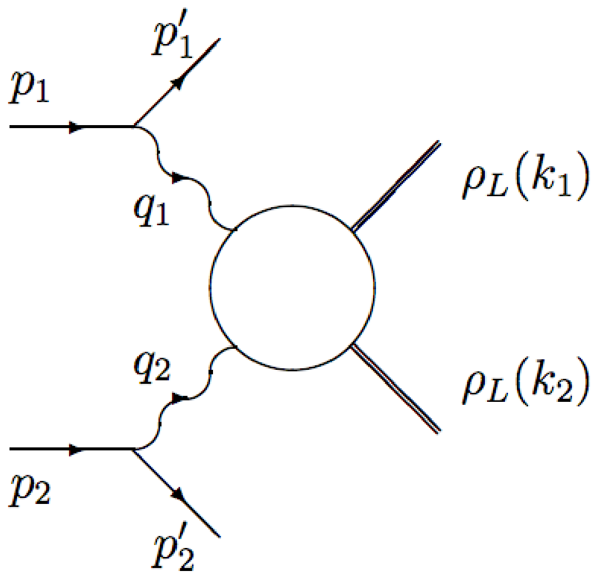
Introduction

The exclusive reaction of ρ mesons electroproduction in $\gamma^*\gamma^*$ collisions is a beautiful laboratory to study different dynamics and factorization properties in HE QCD.

It seems to be a promising probe of the BFKL effects which could be studied in the next generation of e^+e^- colliders (ILC) and at lower energy of other kind of QCD factorizations involving GDA and TDA, which could be observed at Babar or Belle.

We consider the following process :

$$e^+e^- \to e^+e^- \rho_L \rho_L$$



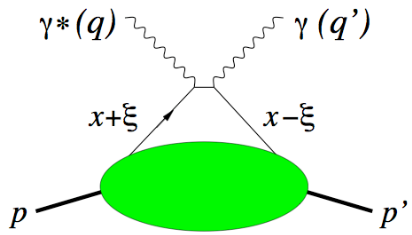
First part: Motivation and origins

- DIS : **inclusive** process t = 0

Structure function = perturbative CF ∗ PDF

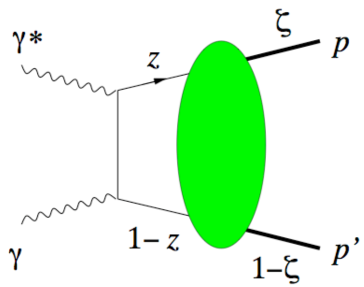
- DVCS : **exclusive** process at small t

Amplitude = perturbative CF ∗ GPD



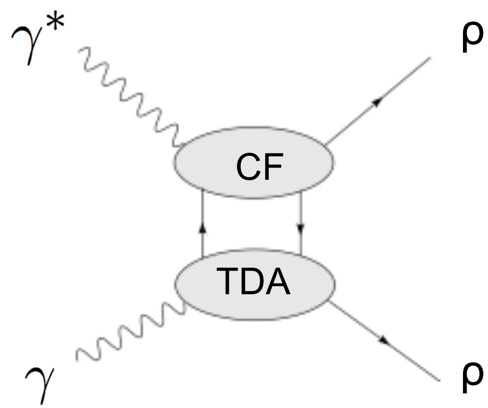
Crossed process, small cm energy

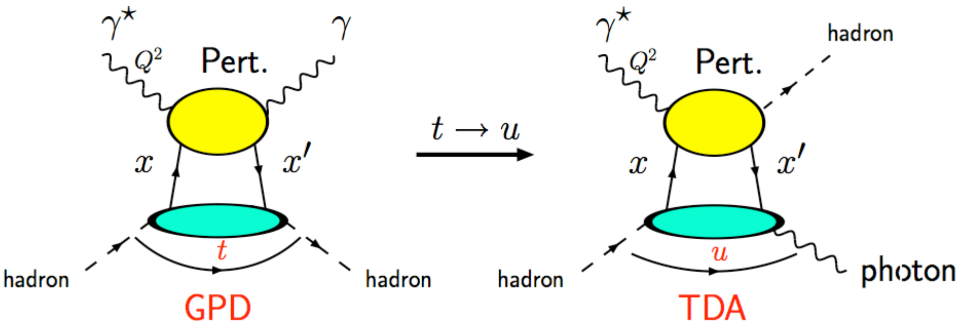
Amplitude = perturbative CF ∗ GDA



- Generalization of DVCS, small t

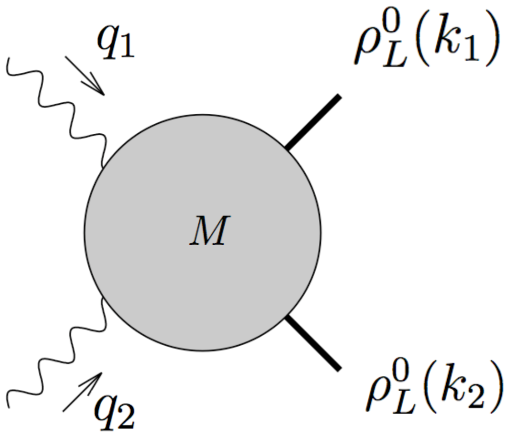
Amplitude = perturbative CF ∗ TDA





outline

$$\gamma^*(Q_1)\gamma^*(Q_2) \to \rho_L^0(k_1)\rho_L^0(k_2)$$



Exclusive reaction at Born order
quark exchange contribution

Q1, Q2 hard scales

- Direct calculation and factorization with two ρ DAs
longitudinal mesons ⟶ leading twist

Brodsky-Lepage (1981)

- Factorization with ρρ GDA for transverse γ^*
- Factorization with $\gamma^* \longrightarrow \rho$ TDA for longitudinal γ^*

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Analytic expressions in collinear factorization

The virtualities Q_1 and Q_2 of the photons supply the hard scale

Collinear approximation $\longrightarrow$ we neglect transverse relative quark momenta in the rho mesons

and in the forward kinematics

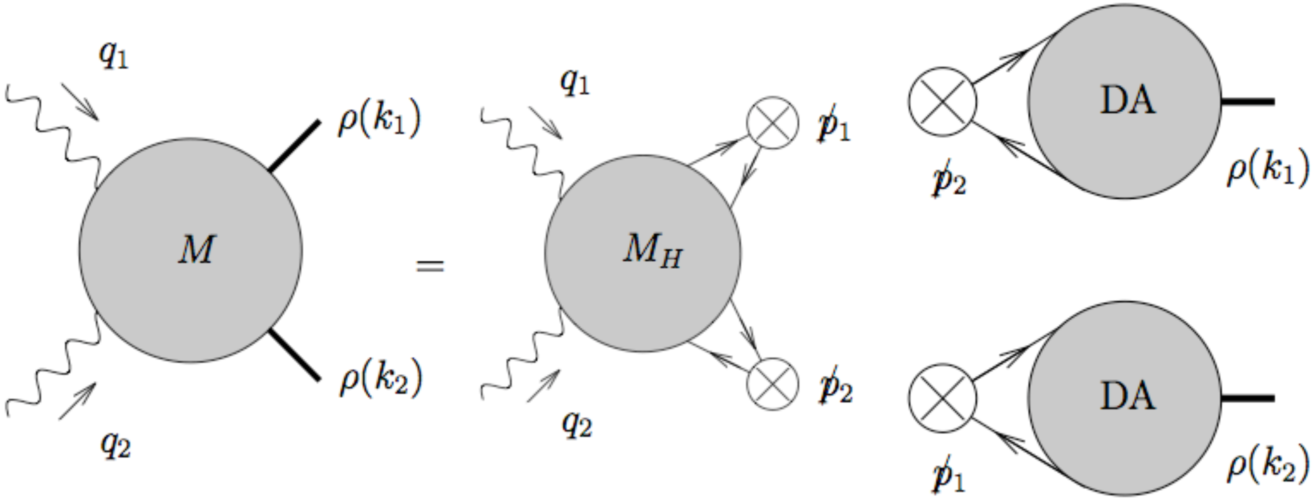
$$\ell_1 \sim z_1 k_1, \qquad \ell_2 \sim z_2 k_2$$
$$\tilde{\ell}_1 \sim \bar{z}_1 k_1, \qquad \tilde{\ell}_2 \sim \bar{z}_2 k_2$$

We use the matrix element of the non local correlator of quarks fields on the light cone to define the DA of the meson as

$$\langle \rho_L^0(k) | \bar{q}(x) \gamma^\mu q(0) | 0 \rangle = \frac{f_\rho}{\sqrt{2}} \, k^\mu \int\limits_0^1 dz \, e^{iz(kx)} \phi(z)$$

with $\phi(z) = 6z(1-z)\left(1 + \sum_{n=1}^{\infty} a_{2\,n} C_{2\,n}^{3/2}(2z-1)\right)$

Amplitude of the process in the collinear factorization with DA



The scattering amplitude reads

$$\mathcal{A} = T^{\mu\,\nu}\epsilon_\mu(q_1)\epsilon_\nu(q_2)$$

where $T^{\mu\,\nu} =$

$$\frac{1}{2} g_T^{\mu\,\nu}\,(T^{\alpha\,\beta} g_{T\,\alpha\,\beta}) + (p_1^\mu + \frac{Q_1^2}{s} p_2^\mu)(p_2^\nu + \frac{Q_2^2}{s} p_1^\nu)\,\frac{4}{s^2}(T^{\alpha\,\beta} p_{2\,\alpha}\, p_{1\,\beta})$$

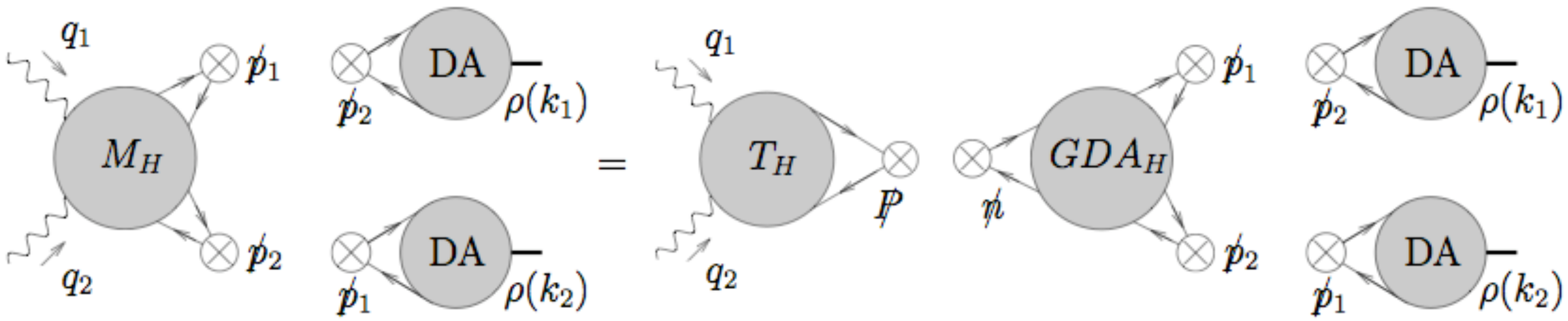
and

$$g_T^{\mu\,\nu} = g^{\mu\nu} - (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu)/(p_1.p_2)$$

Factorization of the amplitude in terms of a GDA in the generalized Bjorken limit for transverse photons

In the kinematical region where the scattering energy is smaller than the highest photon virtuality

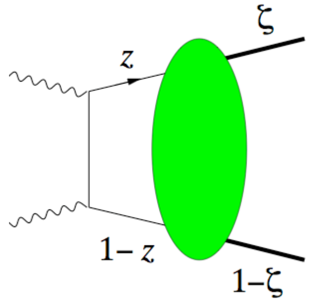
$$\frac{W^2}{Q_1^2} = \frac{s}{Q_1^2}\left(1 - \frac{Q_1^2}{s}\right)\left(1 - \frac{Q_2^2}{s}\right) \approx 1 - \frac{Q_1^2}{s} \ll 1$$

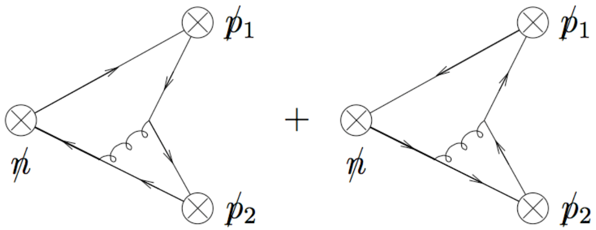


Definition of the leading twist GDA calculated in the Born order of the perturbation theory

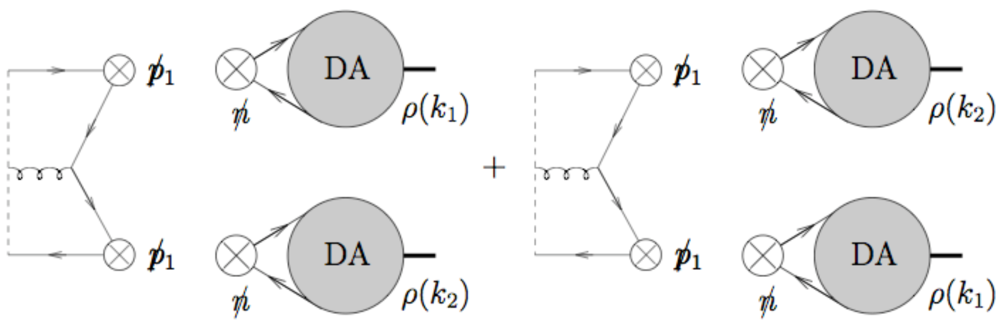
$$\langle \rho_L^0(k_1)\, \rho_L^0(k_2) | \bar{q}(-\alpha\, n/2)\, \hat{n}\ \exp\Big(ig \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} dy\, n_\nu\, A^\nu(y) \Big)\ q(\alpha\, n/2) | 0 \rangle$$

$$= \int_0^1 dz\, e^{-i(2z-1)\alpha(nP)/2} \Phi^{\rho_L \rho_L}(z, \zeta, W^2)$$



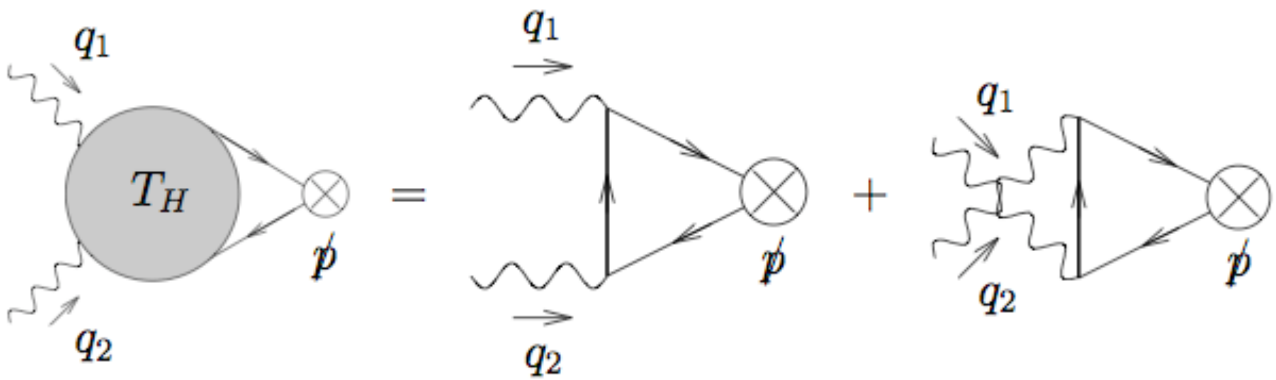


→ The QCD Wilson line vanishes



= 0

Expansion of the Hard Part at Born order



For a one flavored quarks , it equals

$$T_H(z) = -4\, e^2\, N_c\, Q_q^2 \left(\frac{1}{\bar{z} + z\, \frac{Q_2^2}{s}} - \frac{1}{z + \bar{z}\, \frac{Q_2^2}{s}} \right)$$

Summary of the factorization with GDA for transverse virtual photons

factorization of $T^{\alpha\,\beta} g_{T\,\alpha\,\beta}$ into the Hard Part and the GDA

$$T^{\alpha\,\beta} g_{T\,\alpha\,\beta} = \frac{e^2}{2}\,(Q_u^2 + Q_d^2) \int\limits_0^1 dz \left(\frac{1}{\bar{z} + z\frac{Q_2^2}{s}} - \frac{1}{z + \bar{z}\frac{Q_2^2}{s}} \right) \Phi^{\rho_L \rho_L}(z, \zeta \approx 1, W^2)$$

with $\Phi^{\rho_L \rho_L}(z, \zeta \approx 1, W^2) = -\dfrac{f_\rho^2\, g^2\, C_F}{2\, N_c\, W^2} \displaystyle\int\limits_0^1 dz_2\, \phi(z)\, \phi(z_2) \left[\frac{1}{z \bar{z}_2} - \frac{1}{\bar{z} z_2} \right]$

Limiting case of the original equation by D.Müller et al (1994)

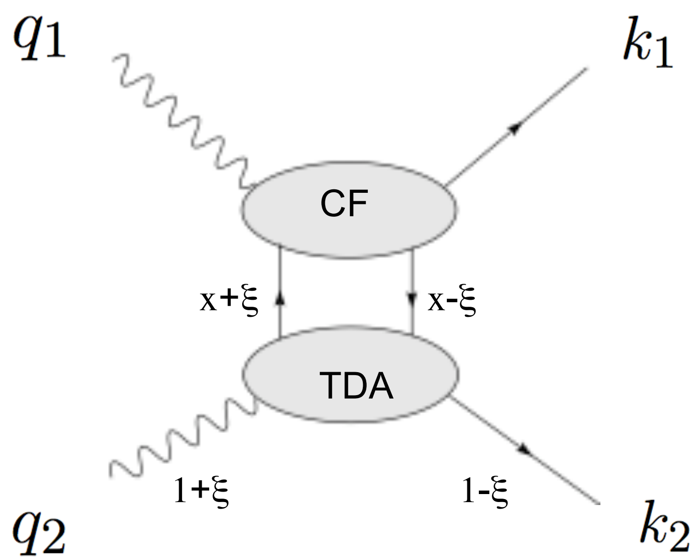
Extension of the studies of $\gamma^* \gamma \to \pi\pi$ by M.Diehl et al (2000) for virtual photons at $t = t_{min}$

Factorization of the amplitude in terms of a TDA for longitudinal virtual photons

- Kinematical regime $Q_1^2 \;>>\; Q_2^2 \;,\quad t = t_{min}$

  $\longrightarrow$ second factorization

- Amplitude = convolution of Hard Part and TDA

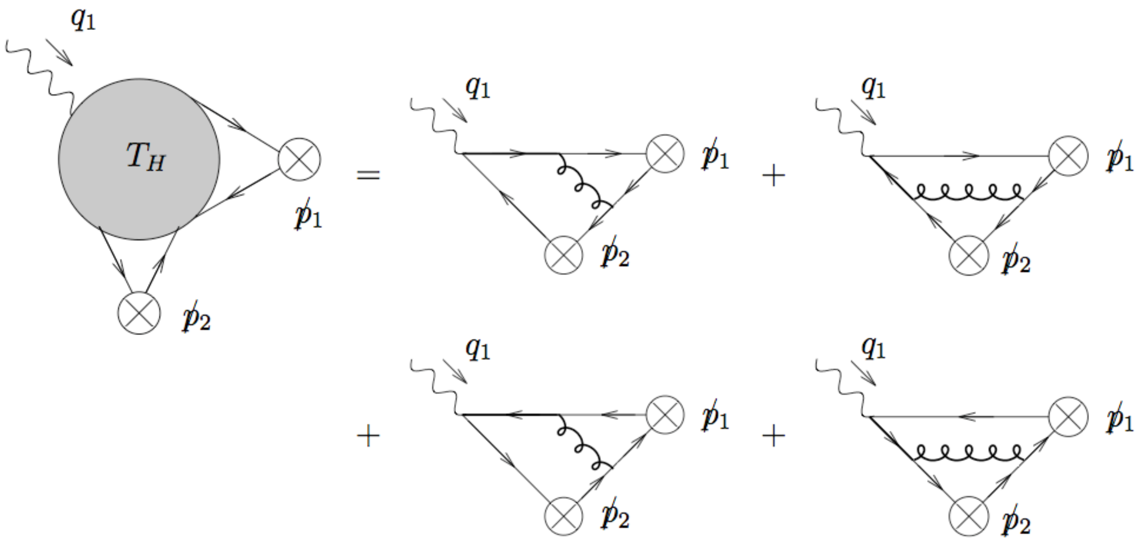


TDA kinematics = GPD kinematics

TDA $\longrightarrow \int \frac{dz^-}{2\pi} e^{-ixP^+z^-} < \rho(p_2)|\bar{q}(-z^-/2)\gamma^+ q(z^-/2))|\,\gamma(q_2) >$

Proof of the factorization in terms of a TDA

Hard Part



$$T_H(z_1, x) = -i\, f_\rho\, g^2\, e\, Q_q\, \frac{C_F\, \phi(z_1)}{2\, N_c\, Q_1^2} \epsilon^\mu(q_1) \left(2\xi\, n_{2\,\mu} + \tfrac{1}{1+\xi} n_{1\,\mu}\right)$$

$$\times \left[\frac{1}{z_1(x+\xi-i\epsilon)} + \frac{1}{\bar{z}_1(x-\xi+i\epsilon)} \right]$$

Proof of the factorization in terms of a TDA

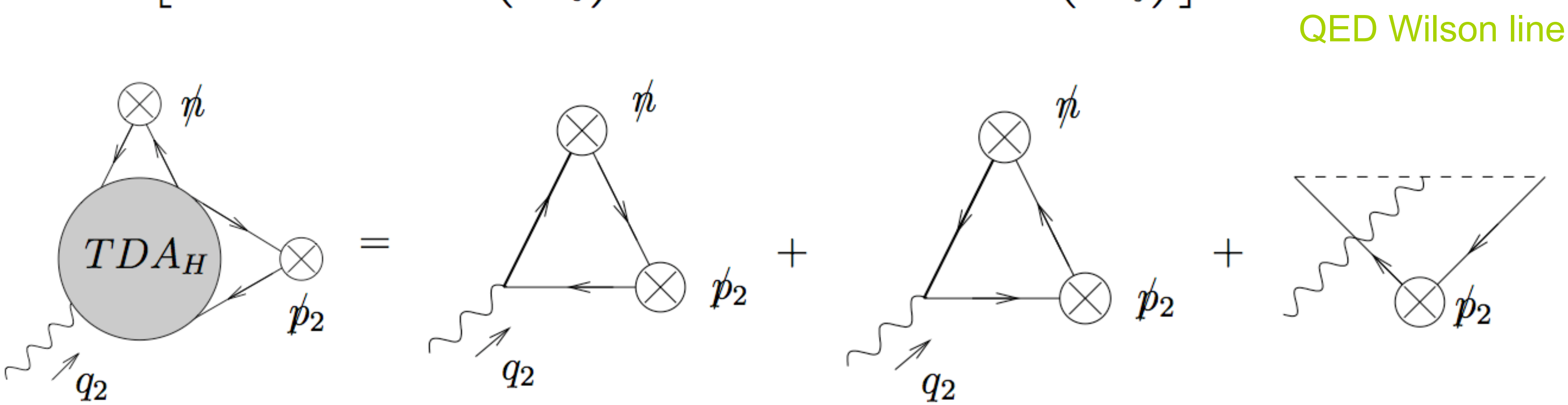
definition of the TDA

$$\int \frac{dz^-}{2\pi}\, e^{ix(P.z)}\, \langle \rho_L^q(k_2)|\bar{q}(-z/2)\hat{n}\, e^{-ieQ_q \int\limits_{z/2}^{-z/2} dy_\mu\, A^\mu(y)} q(z/2)|\gamma^*(q_2)\rangle$$

$$= \frac{e\,Q_q\,f_\rho}{P^+}\,\frac{2}{Q_2^2}\,\epsilon_\nu(q_2)\left((1+\xi)n_2^\nu + \frac{Q_2^2}{s(1+\xi)}n_1^\nu\right) T(x,\xi,t_{min})$$

with $T(x,\xi,t_{min}) \equiv$

$$N_c\left[\Theta(1 \geq x \geq \xi)\phi\left(\tfrac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1)\phi\left(\tfrac{1+x}{1-\xi}\right)\right]$$



Summary of the factorization with TDA for longitudinal virtual photons

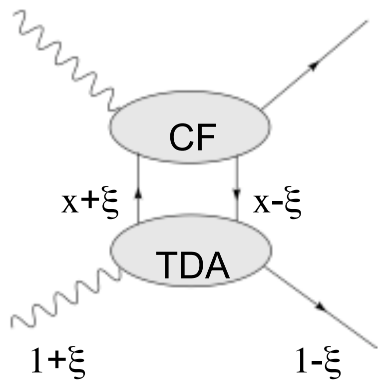
Factorized form of the amplitude involving a TDA

$$T^{\alpha\beta} p_{2\,\alpha} p_{1\,\beta} =$$
$$-i f_\rho^2 e^2 (Q_u^2 + Q_d^2) g^2 \,\frac{C_F}{8N_c} \int_{-1}^{1} dx \int_0^1 dz_1 \left[\frac{1}{\bar{z}_1(x-\xi)} + \frac{1}{z_1(x+\xi)}\right] \phi(z_1)\;\; T(x,\xi,t_{min})$$

with $\quad T(x,\xi,t_{min}) \equiv$
$$N_c \left[\Theta(1 \geq x \geq \xi)\phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1)\phi\left(\frac{1+x}{1-\xi}\right)\right]$$

Only the DGLAP part of the TDA contributes because we use the ρ-mesons DAs.

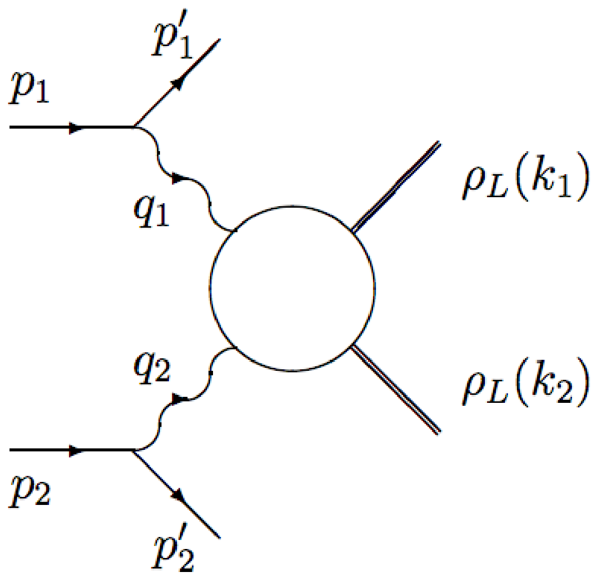
$$-\xi \geq x \geq -1$$
$$1 \geq x \geq \xi$$



We get the same kind of factorization for an opposite ordering of the photons virtualities.

Second part: diffractive mesons production with leptons tagging for studying the BFKL Pomeron at high energy

- We consider the process $e^+e^- \to e^+e^-\rho_L\rho_L$



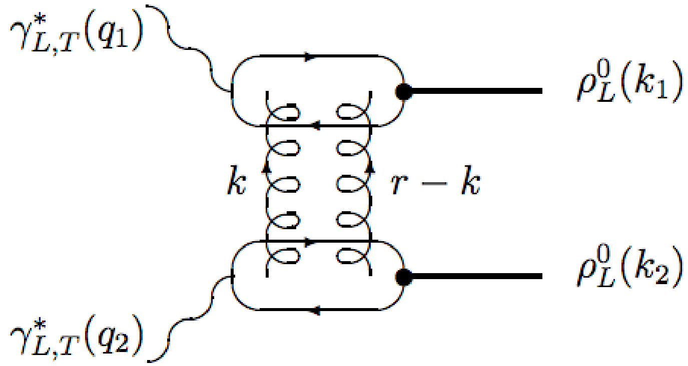
In the Regge limit, we expect to 'observe' an exchange of a BFKL Pomeron in the t-channel.

We compute the scattering amplitude in a complete analytical way at the Born order.

This process has already been studied until NLO but only in the forward case.

D.Ivanov, A.Papa

Study of the process $\gamma_L^*(q_1)\,\gamma_L^*(q_2) \to \rho_L(k_1)\,\rho_L(k_2)$



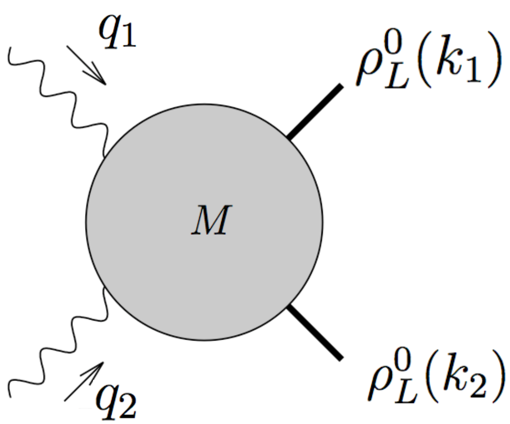
Selection of events in which two vectors ρ mesons with longitudinal polarization are produced in the final state with a big gap in rapidity.

IR safe probes: double tagging of final leptons (➔photons polarizations) and Cut off over soft photons. The highly virtual photons $Q_1^2\,, Q_2^2 \gg \Lambda^2_{QCD}$ give the hard scales on both sides of the *t*-channel exchanged state ➔ fully perturbative process (except for the final mesons)

$Q_1^2 \sim Q_2^2$ ➔ neglect DGLAP partonic evolution

In the Regge limit $s \gg -t,\, Q_1^2\,, Q_2^2$, the process is dominated by BFKL evolution.

kinematics



Q_1 , Q_2 $\longrightarrow$ hard scales

- Sudakov decomposition : two light-cone vectors

momentum transfer

$$t \sim -\frac{Q_1^2\,Q_2^2}{s} - \underline{r}^2\left(1 + \frac{Q_1^2}{s} + \frac{Q_2^2}{s} + \frac{\underline{r}^2}{s}\right)$$

photons momenta

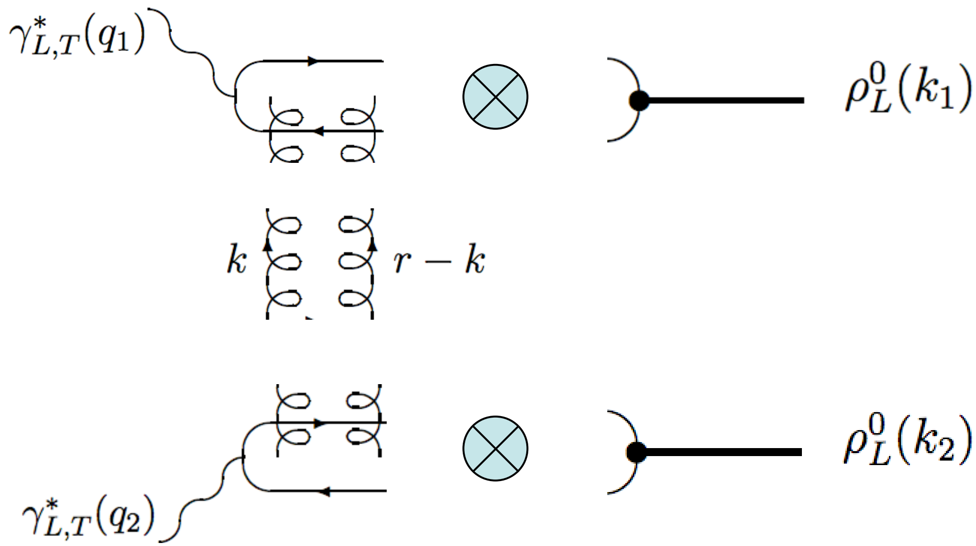
$$q_1 = q_1' - \frac{Q_1^2}{s} q_2' \qquad\qquad q_2 = q_2' - \frac{Q_2^2}{s} q_1'$$

mesons momenta

$$k_1 = \alpha(k_1)\,q_1' + \frac{\underline{r}^2}{\alpha(k_1)\,s} q_2' + r_\perp$$

$$k_2 = \beta(k_1)\,q_2' + \frac{\underline{r}^2}{\beta(k_1)\,s} q_1' - r_\perp$$

Amplitude of the process at the Born order



Integration over the internal moments :

◆Sudakov basis $\quad k = \alpha q_1' + \beta q_2' + k_\perp \qquad q_1'^{\,2} = q_2'^{\,2} = 0$

◆ In the BFKL dynamics the longitudinal momenta of the gluons are strongly ordered.

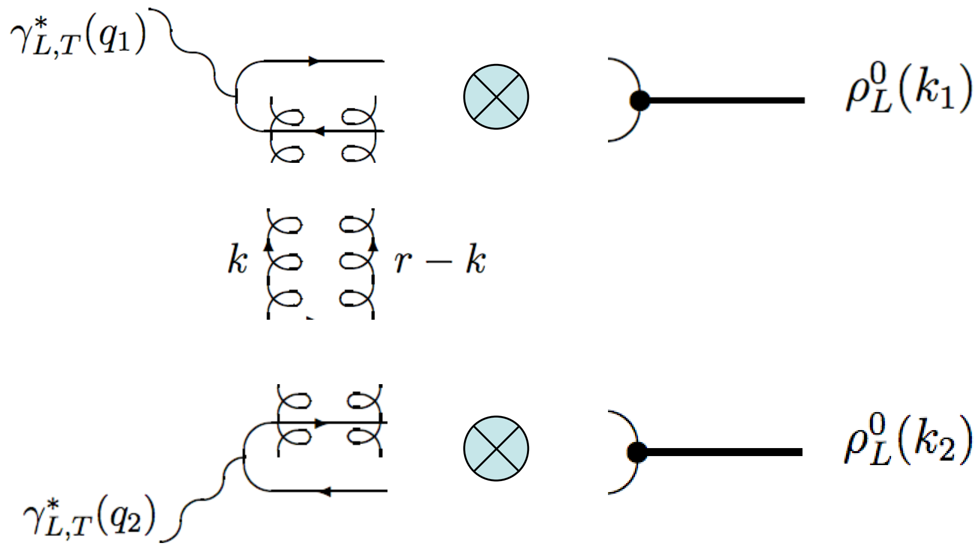
➔ kT-factorization in transverse momentum $\quad$ cf. $\quad \int d^4k = \int d\alpha d\beta d\underline{k}^2$

◆ Collinear approximation ➔ we neglect transverse relative momentum of quark inside the mesons.

Impact representation of the amplitude

$$\mathcal{J}^{\gamma^*_{L,T}(q_1)\to\rho^0_L(k_1)} \longrightarrow$$

$$\mathcal{J}^{\gamma^*_{L,T}(q_2)\to\rho^0_L(k_2)} \longrightarrow$$



$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^4 \underline{k}^2\,(\underline{r}-\underline{k})^2} \mathcal{J}^{\gamma^*_{L,T}(q_1)\to\rho^0_L(k_1)}(\underline{k},\underline{r}-\underline{k})\;\mathcal{J}^{\gamma^*_{L,T}(q_2)\to\rho^0_L(k_2)}(-\underline{k},-\underline{r}+\underline{k})$$

Every impact factor $\mathcal{J}^{\gamma^*_{L,T}(q_1)\to\rho^0_L(k_1)}$ is written as a convolution of the DA of the meson with the more simple impact factor corresponding to the quark-antiquark opened pair production from one polarized photon with two reggeized gluons exchanged in the t channel.

We use the matrix element of the non local correlator of quarks fields on the light cone to define the DA of the meson as :

$$\langle\rho(k_2)|\bar{q}(-\frac{z}{2})\;\gamma^\mu\;q(\frac{z}{2})|0\rangle = f_\rho\,k_2^\mu \int_0^1 du e^{i(1-2u)(k_2\frac{z}{2})}\phi(u)$$

- In the case of longitudinaly polarized photons, they read :

$$\mathcal{J}^{\gamma_L^*(q_i)\to\rho_L(k_i)}(\underline{k},\underline{r}-\underline{k})$$

$$= 8\pi^2\alpha_s \frac{e}{\sqrt{2}}\frac{\delta^{ab}}{2N_c} Q_i\, f_\rho \alpha(k_i) \int_0^1 dz_i z_i\, \bar{z}_i\, \phi(z_i) \mathrm{P_P}(\mathrm{z_i}, \underline{\mathrm{k}}, \underline{\mathrm{r}}, \mu_\mathrm{i})$$

with $$\mathrm{P_P}(\mathrm{z_i}, \underline{\mathrm{k}}, \underline{\mathrm{r}}, \mu_\mathrm{i}) = \frac{1}{\mathrm{z_i^2}\underline{\mathrm{r}}^2 + \mu_\mathrm{i}^2} + \frac{1}{\bar{\mathrm{z}}_\mathrm{i}^2\underline{\mathrm{r}}^2 + \mu_\mathrm{i}^2} - \frac{1}{(\mathrm{z_i}\underline{\mathrm{r}} - \underline{\mathrm{k}})^2 + \mu_\mathrm{i}^2} - \frac{1}{(\bar{\mathrm{z}}_\mathrm{i}\underline{\mathrm{r}} - \underline{\mathrm{k}})^2 + \mu_\mathrm{i}^2}$$

where $\mu_i^2 = Q_i^2\, z_i\, \bar{z}_i + m^2$

- For transversely polarized photons, one obtains :

$$\mathcal{J}^{\gamma_T^*(q_i)\to\rho_L(k_i)}(\underline{k},\underline{r}-\underline{k})$$

$$= 4\pi^2\alpha_s \frac{e}{\sqrt{2}}\frac{\delta^{ab}}{2N_c} f_\rho \alpha(k_i) \int_0^1 dz_i\,(z_i - \bar{z}_i)\,\phi(z_i)\,\underline{\epsilon}\cdot\underline{Q}(z_i, \underline{k}, \underline{r}, \mu_i)$$

with $$\underline{Q}(z_i, \underline{k}, \underline{r}, \mu_i) = \frac{z_i\,\underline{r}}{z_i^2\underline{r}^2 + \mu_i^2} - \frac{\bar{z}_i\,\underline{r}}{\bar{z}_i^2\underline{r}^2 + \mu_i^2} + \frac{\underline{k} - z_i\,\underline{r}}{(z_i\underline{r} - \underline{k})^2 + \mu_i^2} - \frac{\underline{k} - \bar{z}_i\underline{r}}{(\bar{z}_i\,\underline{r} - \underline{k})^2 + \mu_i^2}$$

Both Impact factor vanish when $\underline{k} \to 0$ or $\underline{r} - \underline{k} \to 0$ due to QCD gauge invariance (probes are colorless)

To compute the scattering amplitude $M_{\lambda_1\lambda_2}$ we have to perform analytically the 2D integration over the transverse momentum.

Analytical computation of the 2D integrals involved is performed after the use of conformal transformations in the transverse momentum space. (method inspired by Vassiliev in 2-d coordinate space)

This reduces the number of propagators .

For example , we have to compute this kind of integrals with 3 propagators (1 massive) :

$$J_{3\mu}(a) = \int \frac{d^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{r})^2}\left[\frac{1}{(\underline{k}-\underline{r}a)^2+\mu^2} - \frac{1}{a^2\underline{r}^2+\mu^2} + (a \leftrightarrow \bar{a})\right]$$

Inversion on the integration variable and vector parameter

$$\underline{k} \to \frac{\underline{K}}{\underline{K}^2}, \quad \underline{r} \to \frac{\underline{R}}{\underline{R}^2}, \quad m \to \frac{1}{M}$$

$$= R^2 \int \frac{d^2\underline{K}}{(\underline{K}-\underline{R})^2}\left(\frac{K^2R^2}{(\underline{R}-a\underline{K})^2+\frac{K^2R^2}{M^2}} - \frac{1}{a^2r^2+m^2} + (a \leftrightarrow \bar{a})\right)$$

Then we perform the shift of variable $\underline{K} = \underline{R} + \underline{k}'$

And an other inversion

And we obtain an integral with 3 propagators (1 massive) :

$$J_{3m} = \frac{1}{r^2} \int \frac{d^2\underline{k}}{\underline{k}^2} \left[\frac{(\underline{r}+\underline{k})^2}{(r^2a^2+m^2)\left(\left(\underline{k} - \underline{r}\frac{r^2 a\,\underline{a} - m^2}{r^2\underline{a}^2+m^2}\right)^2 + \frac{m^2 r^4}{(r^2\underline{a}^2+m^2)^2}\right)} - \frac{1}{a^2r^2+m^2} + (a \leftrightarrow \bar{a}) \right]$$

UV and IR finite

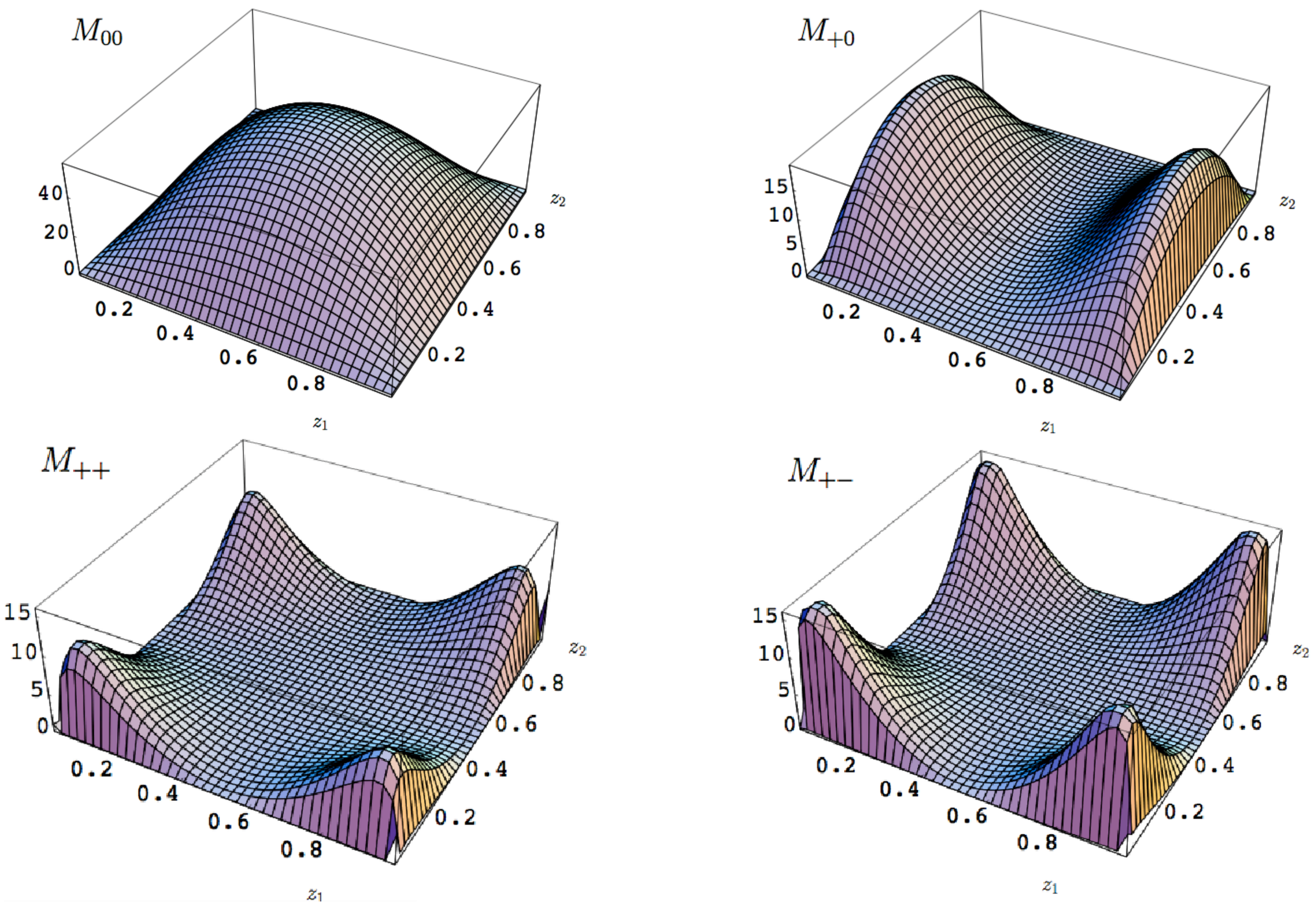
It is now possible to compute this integral by using standard technique

$$J_{3m} = \frac{2\pi}{r^2} \left\{ \left(\frac{1}{r^2a^2+m^2} - \frac{1}{r^2\bar{a}^2+m^2} \right) \ln \frac{r^2a^2+m^2}{r^2\bar{a}^2+m^2} \right.$$
$$\left. + \left(\frac{1}{r^2a^2+m^2} + \frac{1}{r^2\bar{a}^2+m^2} + \frac{2}{r^2a\bar{a}-m^2} \right) \ln \frac{(r^2a^2+m^2)(r^2\bar{a}^2+m^2)}{m^2r^2} \right\}$$

We deduce the differential cross-section in the large s limit :

$$\frac{d\sigma^{\gamma^*_{\lambda_1}\gamma^*_{\lambda_2}\to\rho^0_L\rho^0_L}}{dt} = \frac{|\mathcal{M}_{\lambda_1\lambda_2}|^2}{16\,\pi\,s^2}$$

Shape of the k_T-integrated amplitudes in the z_i plane



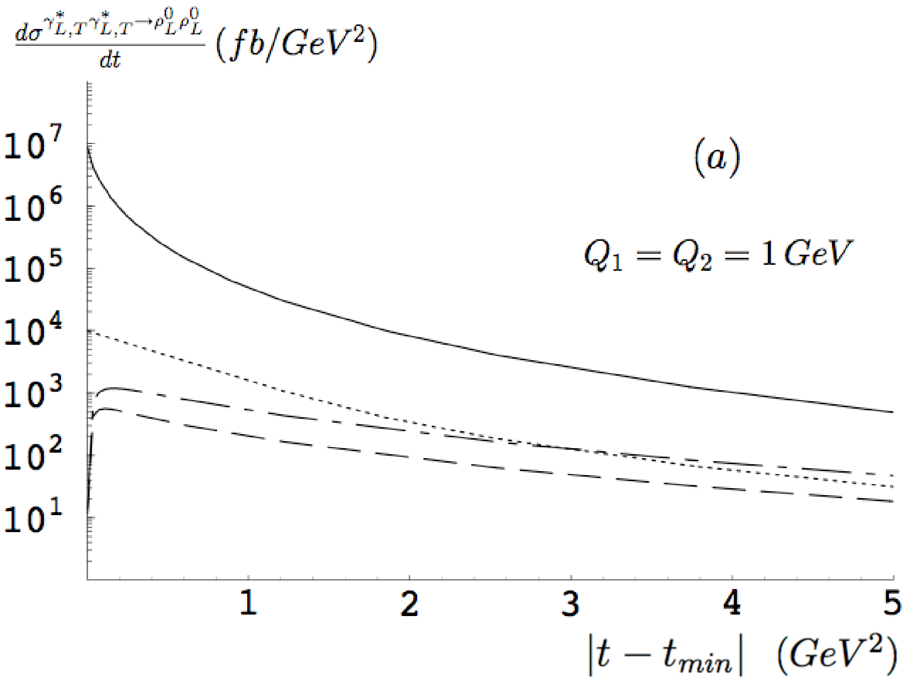
$$\overline{\mathrm{M}}_{00} = z_1\,\bar{z}_1\,\phi(z_1)\,z_2\,\bar{z}_2\,\phi(z_2)\,\mathrm{M}_{00}(z_1,\,z_2)$$

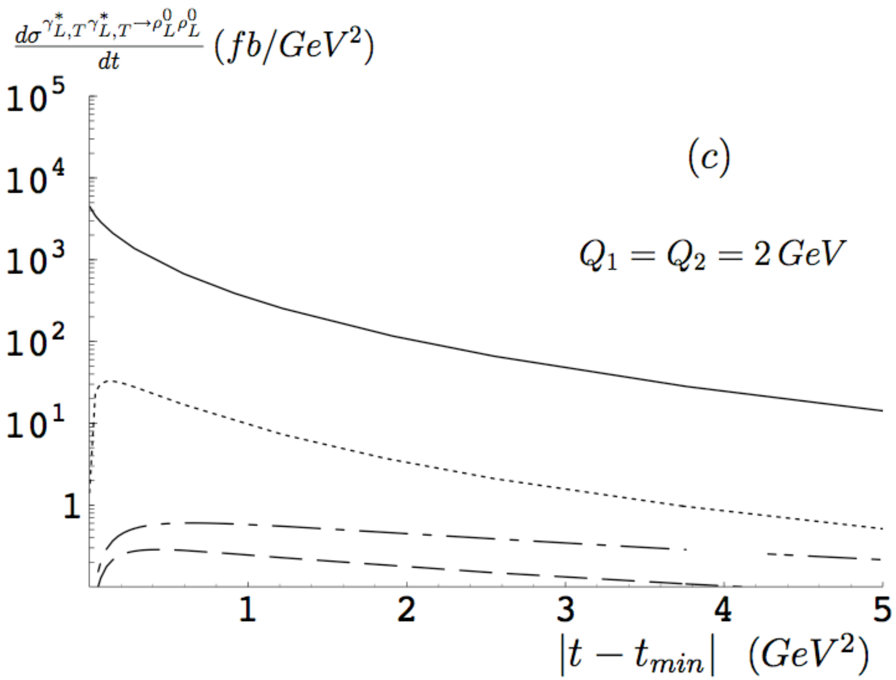
$$\overline{\mathrm{M}}_{\lambda_1 0} \;=\; (z_1 - \bar{z}_1)\,\phi(z_1)\,z_2\,\bar{z}_2\,\phi(z_2)\,\mathrm{M}_{\lambda_1 0}(z_1,\,z_2)$$

$$\overline{\mathrm{M}}_{\lambda_1 \lambda_2} \;=\; (z_1 - \bar{z}_1)\,\phi(z_1)\,(z_2 - \bar{z}_2)\,\phi(z_2)\,\mathrm{M}_{\lambda_1 \lambda_2}(z_1,\,z_2)$$

Differential cross section for the different polarizations of the virtual photons

The integration over momentum fractions z_1 and z_2 are performed numerically
we use Q_1Q_2 as a scale for $\alpha_s(\sqrt{Q_1Q_2})$ running at 3 loops





solid curve : LL mode
dotted curve : LT mode
dashed and dashed-dotted curves :TT mode

strong decrease with Q

any cross-section with at least one tranverse photon vanishes in the forward case

Non-forward cross-sections at ILC for

$$e^+e^- \to e^+e^-\rho_L^0\,\rho_L^0$$

We use the same Sudakov basis

and the equivalent photon approximation

Weizsacker-Wiliams

$$\frac{d\sigma(e^+e^- \to e^+e^-\rho_L\rho_L)}{dy_1 dy_2 dQ_1^2 dQ_2^2}$$
$$= \frac{1}{y_1 y_2\, Q_1^2 Q_2^2}\left(\frac{\alpha}{\pi}\right)^2 [l_1(y_1)\, l_2(y_2)\sigma(\gamma_L^*\gamma_L^* \to \rho_L\rho_L) + t_1(y_1)\, l_2(y_2)\,\sigma(\gamma_T^*\gamma_L^* \to \rho_L\rho_L)$$
$$+\ l_1(y_1)\, t_2(y_2)\,\sigma(\gamma_L^*\gamma_T^* \to \rho_L\rho_L) + t_1(y_1)\, t_2(y_2)\,\sigma(\gamma_T^*\gamma_T^* \to \rho_L\rho_L)]\ .$$

with the usual photons flux factors given by $t_i = \frac{1+(1-y_i)^2}{2}, \quad l_i = 1 - y_i$

$y_i\ (i = 1, 2)$ are the longitudinal momentum fractions of the bremsstrahlung photons with respect to the incoming leptons

$$s_{\gamma^*\gamma^*} \sim y_1\, y_2\, s_{e^+e^-}$$

$\Rightarrow\ \sigma^{e^+e^- \to e^+e^-\rho_L\rho_L}$ is peaked in the low y and Q^2 region

Kinematical constraints coming from experimental features of the ILC collider are used to perform the phase-space integration.

Photons momentum fractions $y_i = \dfrac{E - E'_i \cos^2(\theta_i/2)}{E}$ In the cms frame

and virtualities $Q_i^2 = 4EE'_i \sin^2(\theta_i/2)$

kinematical constraints coming from the minimal detection angle around the beampipe and from the conditions on the energies of the scattered leptons and the Regge limit.

$$y_{i\,max} = 1 - \frac{E_{min}}{E}$$

$$y_{1\,min} = \max\left(f(Q_1), 1 - \frac{E_{max}}{E}\right)$$

$$y_{2\,min} = \max\left(f(Q_2), 1 - \frac{E_{max}}{E}, \frac{c\,Q_1\,Q_2}{s\,y_1}\right)$$

with $f(Q_i) = 1 - \dfrac{Q_i^2}{s\tan^2(\theta_{min}/2)}$

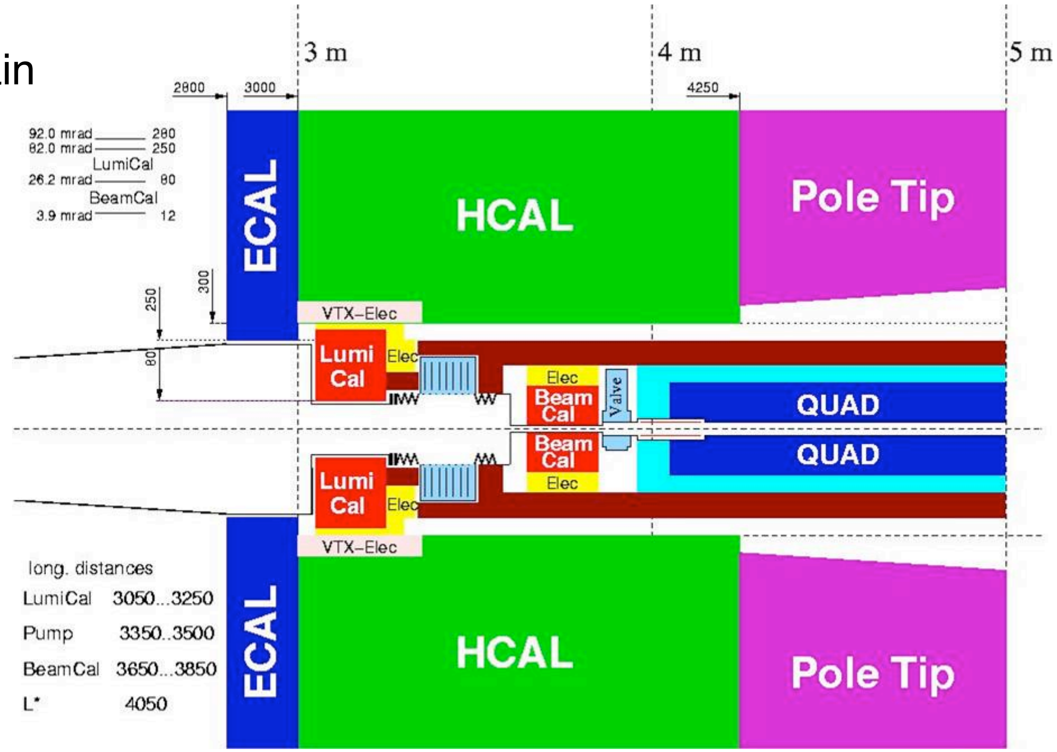
$$\frac{d\sigma^{e^+e^-\to e^+e^-\rho_L\rho_L}}{dt} = \int_{Q^2_{1min}}^{Q^2_{1max}} dQ_1^2 \int_{Q^2_{2min}}^{Q^2_{2max}} dQ_2^2 \int_{\epsilon}^{y_{max}} dy_1 \int_{\frac{Q_1Q_2}{sy_1}}^{y_{max}} dy_2 \frac{d\sigma^{e^+e^-\to e^+e^-\rho_L\rho_L}}{dt\,dy_1\,dy_2\,dQ_1^2\,dQ_2^2}$$

Experimental features of the ILC collider
design of the detector

Each design of detector for ILC project involves a very forward electromagnetic calorimeter for luminosity measurement, with tagging angle for outgoing leptons down to 5 mrad which is an ideal tool for diffractive physics whose cross-sections are sharply peaked in the very forward region

The high luminosity will allow to obtain sufficient statistics to measure exclusive events

European LDC collaboration



ECAL, HCAL : hadron calorimeters

LumiCal, BeamCal : electromagnetic calorimeters

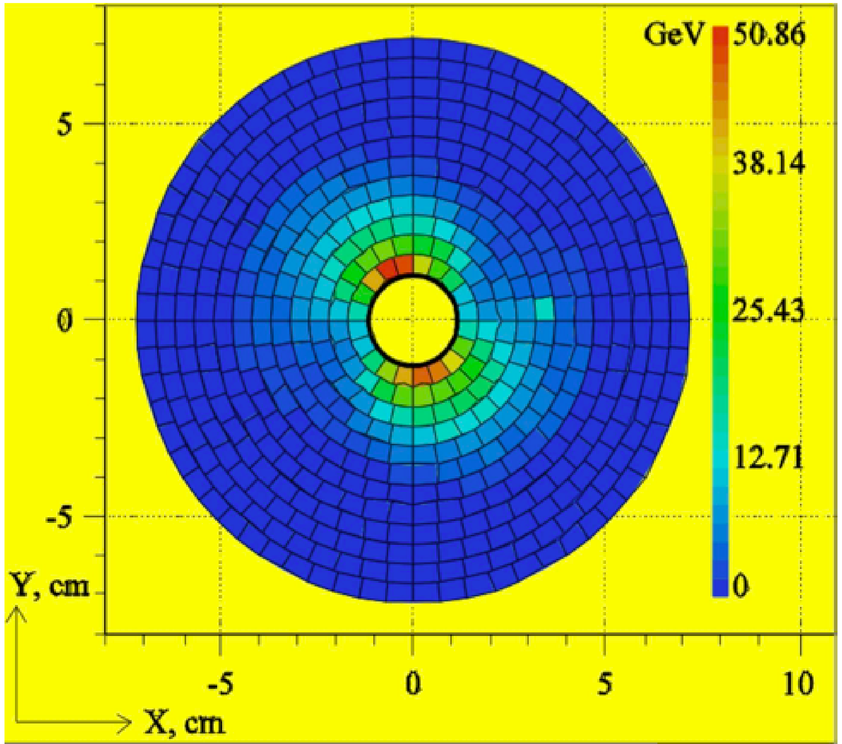
Experimental features of the ILC collider

Foreseen cms energy $\sqrt{s} = 2E = 500\,GeV$

LDC detector :
emc BeamCal around the beampipe at 3.65 m from the vertex and devoted to the luminosity measurement

It can be used for diffractive physics

Simulation of the energy density of beamstrahlung remnants (photons..) per bunch crossing at the front face of the BeamCal



We cut-off the cells for leptons tagging with

→ $E_{min} = 100\,GeV$

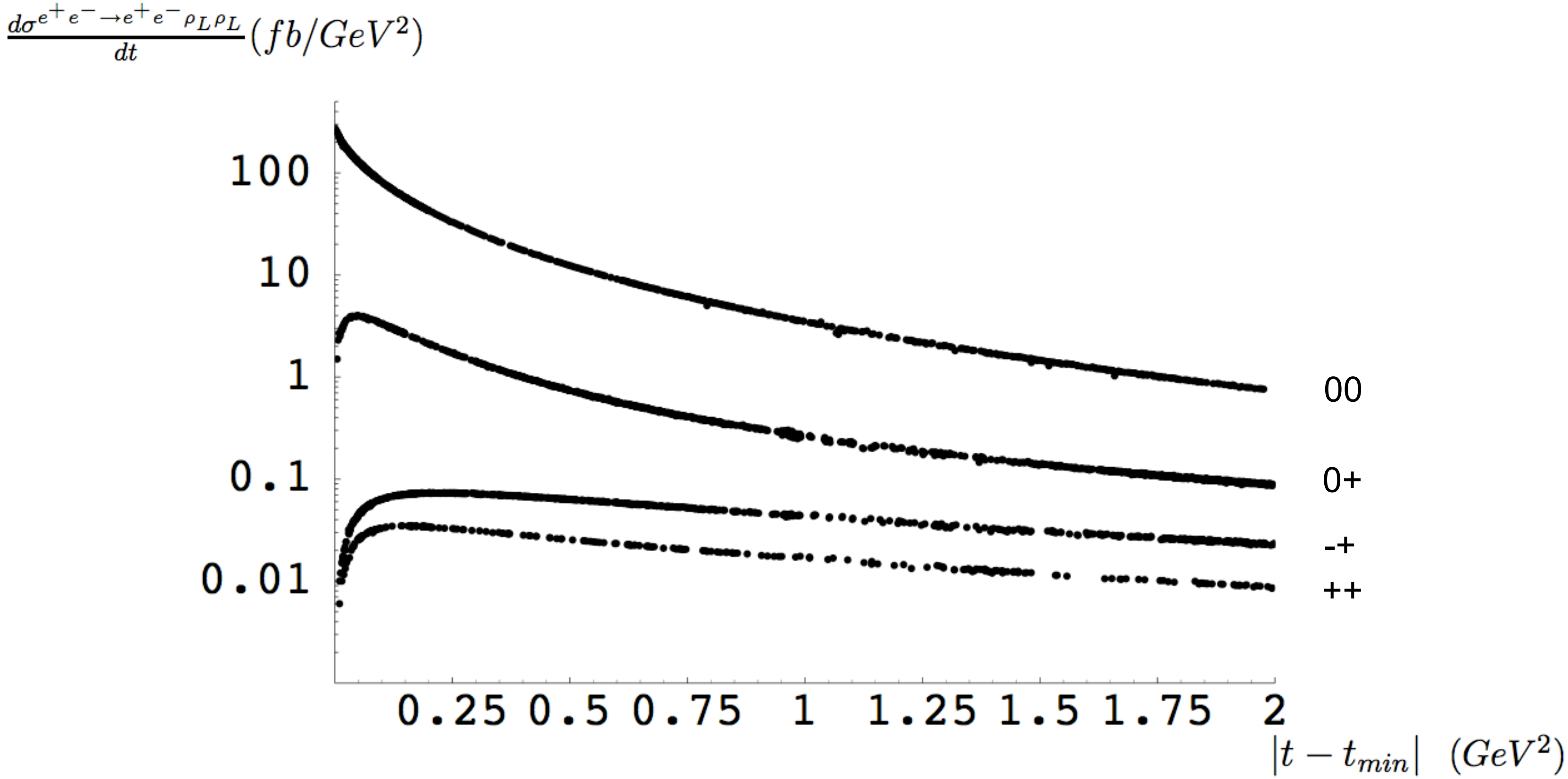
→ $\theta_{min} = 4\ \mathrm{mrad}$

⇒ access to the (very) forward region

$$\frac{d\sigma^{e^+e^- \to e^+e^- \rho_L \rho_L}}{dt} = \int_{Q^2_{1min}}^{Q^2_{1max}} dQ_1^2 \int_{Q^2_{2min}}^{Q^2_{2max}} dQ_2^2 \int_{\epsilon}^{y_{max}} dy_1 \int_{\frac{Q_1 Q_2}{s y_1}}^{y_{max}} dy_2 \frac{d\sigma^{e^+e^- \to e^+e^- \rho_L \rho_L}}{dt\, dy_1\, dy_2\, dQ_1^2\, dQ_2^2}$$

Results for non-forward cross-sections at ILC for

$$e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$$



$$\sigma^{LL} = 32.4\, fb$$
$$\sigma^{LT} = 1.5\, fb$$
$$\sigma^{TT} = 0.2\, fb$$

$$\sigma^{Total} = 34.1\, fb$$

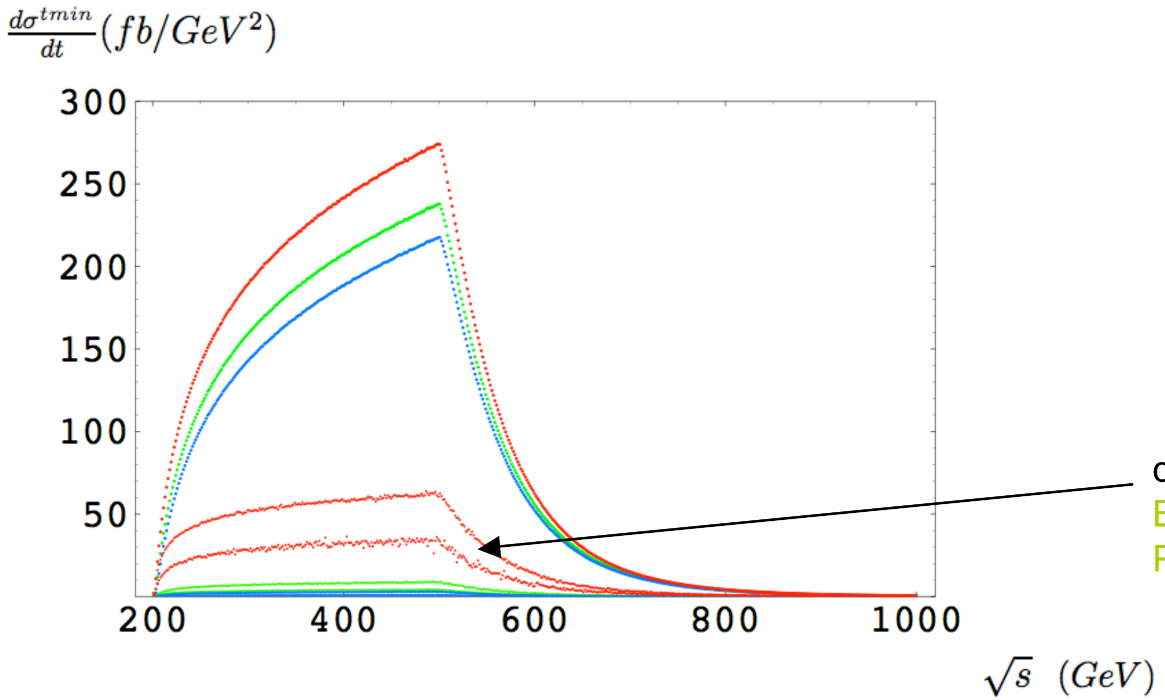
with $\alpha_s(\sqrt{Q_1 Q_2})$ running at three loops

$\sqrt{s} = 500\text{GeV}$

$c = 1$

$\longrightarrow\ 4.26\, 10^3$ events per year with foreseen luminosity

Effects of parameters and quark exchange contribution to the non-forward cross-sections for $e^+e^- \rightarrow e^+e^-\rho_L^0\,\rho_L^0$ at t_{min}



red curves: c = 1 $\longrightarrow$ $\sigma^{Total} = 34.1\,fb$

green curves: c = 2 $\longrightarrow$ $\sigma^{Total} = 29.6\,fb$

blue curves: c = 3

quarks exchange
B.Pire, M.S., L.Szymanowski, S.Wallon
Phys.Lett.B hep-ph/0605320

quarks contribution are indeed negligible. This is related to c through $s_{\gamma^*\gamma^*} > c\,Q_1\,Q_2$

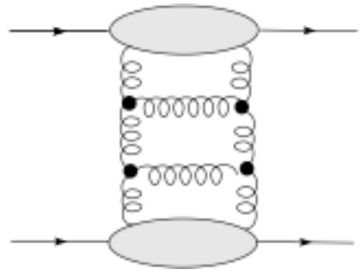
a more drastic Regge limit with c=10 reduces the total cross-section by 40% which is still measurable

effects of radiative corrections (order of loops) for $\alpha_s(\sqrt{Q_1 Q_2})$ are negligible

The strong suppression beyond 500 GeV comes from the detector and kinematical constraints

Forward cross-section for $e^+e^- \to e^+e^-\rho_L^0\,\rho_L^0$ with LO BFKL evolution

$$A(s,t) = is \int \frac{d\omega}{2\pi i} e^{\omega Y} f_\omega(r^2)$$



$$f_\omega(0) = \frac{1}{2}\int \frac{dk^2}{k^3}\frac{dk'^2}{k'^3}\Phi_1^{ab}(k)\Phi_2^{ab}(k') \int_{-\infty}^{\infty} d\nu \, \frac{1}{\omega-\omega(\nu)} \left(\frac{k^2}{k'^2}\right)^{i\nu}$$

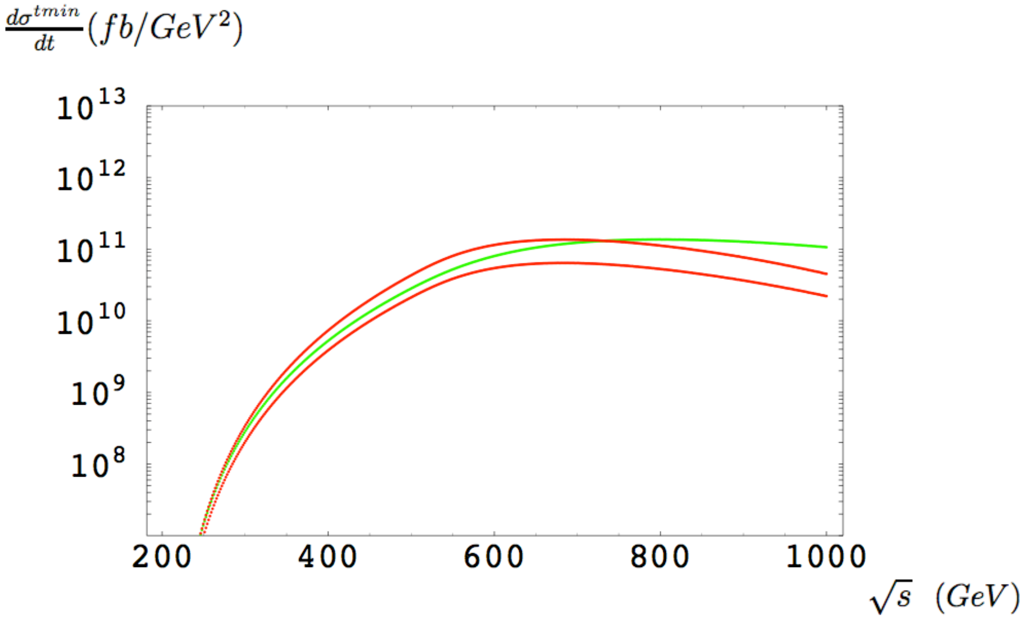
$\omega(\nu) = \bar{\alpha}_s \chi(\nu)$ BFKL characteristic function

forward case LO BFKL amplitude in the saddle-point approximation:

$$A(s,t=t_{min},Q_1,Q_2) \sim is \;\; \pi^5\sqrt{\pi}\,\frac{9(N_c^2-1)}{4N_c^2}\,\frac{\alpha_s^2\alpha_{em}f_\rho^2}{Q_1^2Q_2^2}\,\frac{e^{4\ln 2\,\bar{\alpha}_s Y}}{\sqrt{14\bar{\alpha}_s\zeta(3)Y}}\exp\left(-\frac{\ln^2 R}{14\bar{\alpha}_s\zeta(3)Y}\right)$$

with: $Y = \ln\left(\frac{c'\,s\,y_1\,y_2}{Q_1Q_2}\right)$ $\quad \bar{\alpha}_s = \frac{N_c}{\pi}\alpha_s(\sqrt{Q_1Q_2})$ and $R = \frac{Q_1}{Q_2}$

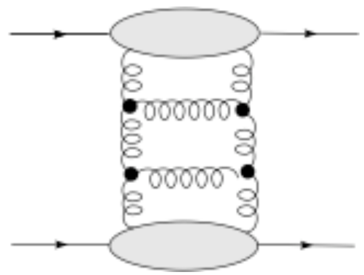
Forward cross-section for $e^+e^- \to e^+e^-\rho_L^0\,\rho_L^0$ with LO BFKL evolution



upper red curve: $\alpha_s(\sqrt{Q_1Q_2})$ running at one loop

lower red curve: $\alpha_s(\sqrt{Q_1Q_2})$ running at three loops

green curve: fixed value of $\alpha_s = 0.46$



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with: $Y = \ln(\frac{c'\,s\,y_1\,y_2}{Q_1Q_2})$ $\quad \bar{\alpha}_s = \frac{N_c}{\pi}\alpha_s(\sqrt{Q_1Q_2})$ and $R = \frac{Q_1}{Q_2}$

flat curve to be compared with strongly decreasing curve at Born level, between 500GeV and 1TeV

Forward cross-section for $e^+e^- \to e^+e^-\rho_L^0\,\rho_L^0$ with NLO BFKL evolution (in progress)

work based on resummed BFKL (Khoze, Martin, Ryskin, Stirling, and Ciafaloni, Colferai, Salam) with LL impact factor and BLM scale fixing (Enberg, Pire, Szymanowski, Wallon).

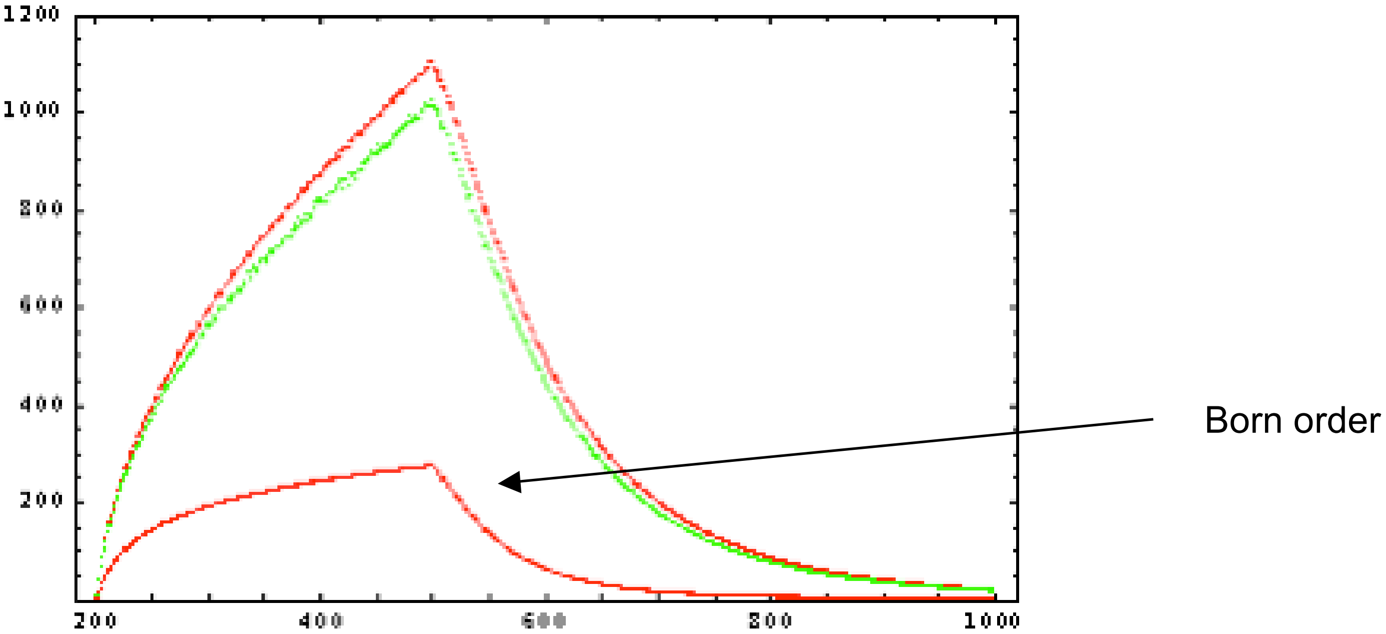
$$\omega = \bar{\alpha}_s \chi(\gamma, \omega) \qquad \chi(\gamma,\omega) = \chi_0(\gamma) + \bar{\alpha}_s \chi_1(\gamma,\omega) \qquad \gamma = \tfrac{1}{2} + i\nu$$

saddle-point approximation

$$\begin{cases} \left.\dfrac{\partial \chi(\gamma,\omega_s)}{\partial \gamma}\right|_{\gamma=\gamma_s} = 0, \\ \omega_s = \bar{\alpha}_s \chi(\gamma_s, \omega_s). \end{cases}$$

$$A(s, t = t_{min}, Q_1, Q_2) \sim is\ \pi^5 \sqrt{2\pi}\, \frac{9(N_c^2-1)}{4N_c^2} \frac{\alpha_s^2 \alpha_{em} f_\rho^2}{Q_1^2 Q_2^2}\ \frac{e^{\omega_s Y}}{\sqrt{\omega_s'' Y}} \frac{\exp\left(-\frac{2\ln^2 R}{\omega_s'' Y}\right)}{1-\dot{\omega}_s}$$

Forward cross-section for $e^+e^- \to e^+e^-\rho_L^0\ \rho_L^0$ with NLO BFKL evolution (in progress)



$$A(s,t=t_{min},Q_1,Q_2) \sim is\ \pi^5\sqrt{2\pi}\,\frac{9(N_c^2-1)}{4N_c^2}\,\frac{\alpha_s^2\alpha_{em}f_\rho^2}{Q_1^2Q_2^2}\ \frac{e^{\omega_s Y}}{\sqrt{\omega_s''Y}}\,\frac{\exp\left(-\frac{2\ln^2 R}{\omega_s''Y}\right)}{1-\dot{\omega}_s}$$

Conclusions

At low energy, quark exchange processes appear at lower order in α_s

The perturbative analysis of $\gamma^* \gamma^* \to \rho_L^0 \rho_L^0$ he Born order leads to two different types of factorization.

Not only kinematics but polarization states also dictate the type of the factorization.

Usually they are applied for two different kinematics but the arbitrariness in choosing the photons virtualities shows that there may exist an intersection region.

Generalization for transverse mesons, non-forward kinematics and charged mesons pair.

At high energy, we gave a precise estimation of the two gluons t-channel exchange which dominates, in the exclusive production of rho meson pairs at the ILC.

This evaluation corresponds to the BFKL background.

Since the impact factor are completely known in a pertubative way, not only the behaviour with energy but the complete amplitude can be analytically computed.

$\longrightarrow$ Clean test of the BFKL resummation scheme at ILC.

We demonstated the measurability of this process at the level of $e^+e^- \to e^+e^- \rho_L \rho_L$ within LDC detector and with a emc located in the forward region.

Born order evaluation $\longrightarrow$ resummed BFKL or NLO BFKL evolution fot any t.

Possibility of entering in the saturation regime when increasing the cms energy from 500 GeV ($\longrightarrow Q_{sat} \sim 1.1$ GeV) to 1 TeV ($\longrightarrow Q_{sat} \sim 1.4$ GeV)