Jets and Hadronic Final States

at the School on QCD, Low-x, Saturation, Diffraction; Calabria, July, 2007

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- **Lecture I: From asymptotic freedom to jets and infrared safety**
  The quantum mechanical basis of infrared safety

- **Lecture II: Factorizations and jet cross sections**
  The physics of factorization, jet algorithms & properties

- **Lectures III,IV: Resummations**
  Threshold and k_T, event shapes, power corrections

- **Lecture I: From asymptotic freedom to jets and infrared safety**

  A. Re-enter QCD (With some general comments)
  B. How to Study a Theory with Asymptotic Freedom & Confinement?
  C. Time-evolution in Quantum Field Theory
  D. Why Jets?
  E. Infrared Safety

- **Lecture II: Factorization and jets**

  A. Summary of QCD basics
  B. Physical basis of factorization
  C. Factorization for fragmentation
  D. Algorithms and infrared safety

IA. RE-ENTER QUANTUM CHROMODYNAMICS

- **A nonabelian gauge theory built on color ($q = q_1 q_2 q_3$):**

$$\mathcal{L}_{QCD} = \sum_q \bar{q}\,(i\not\partial - g_s \not A + m_q\,)\,q - \frac{1}{4} F^2_{\mu\nu}[A]$$

(global symmetry: Han & Nambu, statistics: Greenberg)

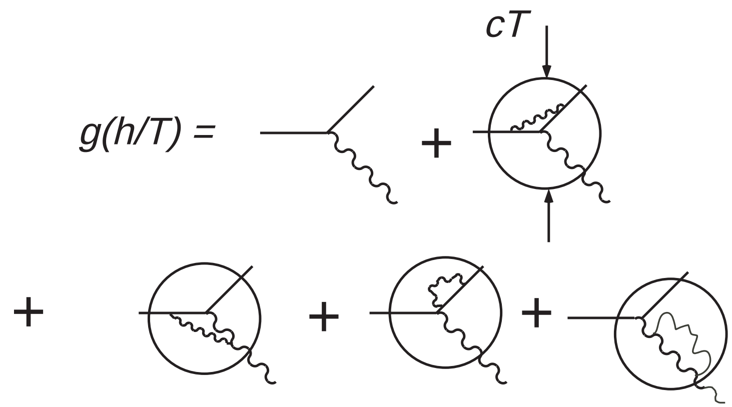
(Pati-Salam 1972, 3 . . . Bardeen, Fritzsch, Gell-Mann, Leutwyler, 1972,3)

- **Think of:** $\mathcal{L}_{EM} = K_{\rm e} + J_{EM} \cdot A + (E^2 - B^2)$

- **The Yang-Mills gauge theory of quarks (q) and gluons (A)**
  Gluons: like "charged photons". The field is a source for itself.

- **Just the right currents to couple to EM and Weak AND . . .**

- **Just the right kind of forces: QCD charge is "antishielded"**
- **Compute the T (time) -dependence of the coupling:**



and with $b_0 = 11 - 2n_{\mathrm{quarks}}/3$ we get:

$$\alpha_s(\mu') \equiv \frac{g_s^2(\mu)}{4\pi} = \frac{\alpha_s(\mu_0)}{1 + b_0 \frac{\alpha_s(\mu_0)}{4\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)} \equiv \frac{4\pi}{b_0 \ln\left(\mu^2 \,/\, \Lambda_{\mathrm{QCD}}^2\right)}$$

- This is asymptotic freedom & scaling:

$$\alpha_s(\mu = \infty) \to 0\,!$$

- Colors "line up" like magnets: depends on spin & self-interactions of the gluons

- Radiation becomes weaker as Q increases.

- Gross-Wilczek, Politzer (1973-4), Georgi
  Near a quark, coupling constant is weak

- Infrared strong coupling $\longrightarrow$ quark confinement?
  Far from a quark, coupling constant is strong

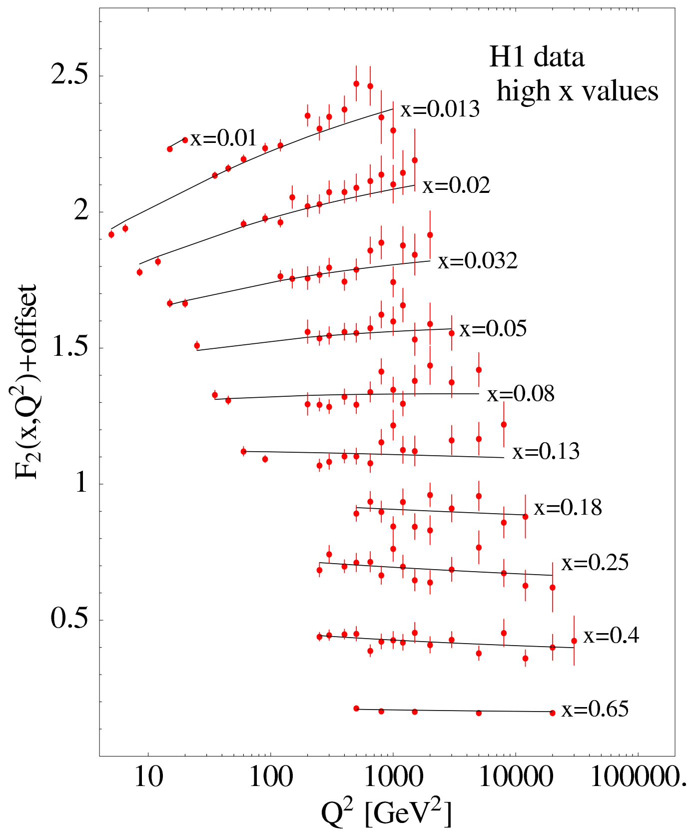
- **The template was already there . . .**
- $\gamma_N = \gamma_N^{(1)}(\alpha_s/\pi) + \ldots$ **& α_s vanishes as μ increases!**

$$\bar{\phi}_{q/H}(N,\mu) = \bar{\phi}_{q/H}(N,\mu_0)\ \exp\left[-\frac{1}{2}\int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\,\gamma(N,\alpha_s(\mu'))\right]$$

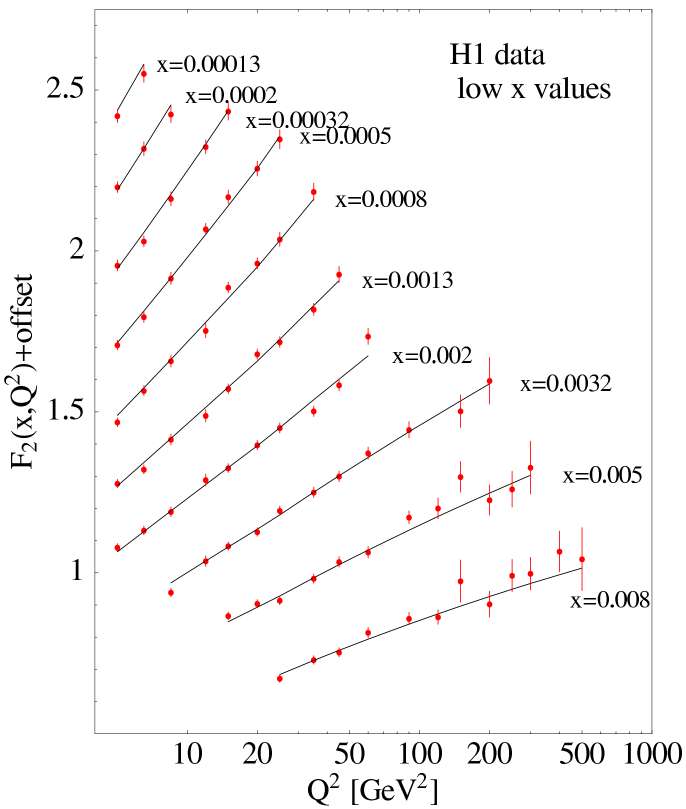
- **With** $\alpha_s(\mu) = 4\pi/b_0\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})$**, we** *now* **get:**

$$\bar{\phi}_{q/H}(N,Q)\ = \bar{\phi}_{q/H}(N,Q_0)\left(\frac{\ln(Q^2/\Lambda^2_{\mathrm{QCD}})}{\ln(Q_0^2/\Lambda^2_{\mathrm{QCD}})}\right)^{-2\gamma_N^{(1)}/b_0}$$

- **It all worked. Approximate scaling at moderate x:**



- **Pronounced evolution for smaller x:**



- **Asymptotic freedom is a big deal:**

$$\frac{\text{Scaling}}{\text{QCD}} = \frac{\text{Elliptical Orbits}}{\text{Newtonian Gravity}}$$

- **A beginning, not an end.**

  For Newtonian gravity, the three-body problem.

  For QCD . . .

IB. HOW TO STUDY A THEORY WITH CONFINEMENT?

- **The goal**

$$\frac{\text{Nuclear Physics}}{\text{QCD}} = \frac{\text{Chemistry}}{\text{QED}}$$

- **But can we**
  - **Study the particles that give the currents (quarks)?**
  - **Study the particles that the forces (gluons)?**
  - **Expand in number of gluons?** **Perturbation Theory**

- In QCD they're confined:
  observed hadrons are bound states

- Bound-state scattering:
  Complexity & strong forces

- Does this make sense at all?

- More analogies: atoms before observation of radioactivity & molecules before the explanation of Brownian motion

Learning to Calculate with the New Theory

Correlation functions $vs.$ the S-matrix

- **Correlation functions at short distances:**
  PT-friendly

$$
\begin{aligned}
\langle 0|J(x)\ J(0)|0\rangle &= C\left(x\mu, \alpha_s(\mu)\right) \\
&= C\left(1, \alpha_s(1/x)\right)
\end{aligned}
$$

  – $\mathrm{e}^+\mathrm{e}^-$ **annihilation cross section, inclusive DIS**

- **The S-matrix, even at high energy: pretty hopeless in PT**

$$
\begin{aligned}
\langle B \text{ out} | A \text{ in} \rangle &= f\left(Q/\mu, m/\mu, \alpha_s(\mu)\right) \\
&= f\left(1, m/Q, \alpha_s(Q)\right) \\
&= f\left(Q/m, 1, \alpha_s(m)\right)
\end{aligned}
$$

  – m – **mass scales:** m_π, m_p, m_q, $m_G = 0$ . . .

- **(Still, it's only the** *ratio* m/Q **that causes the problem.)**

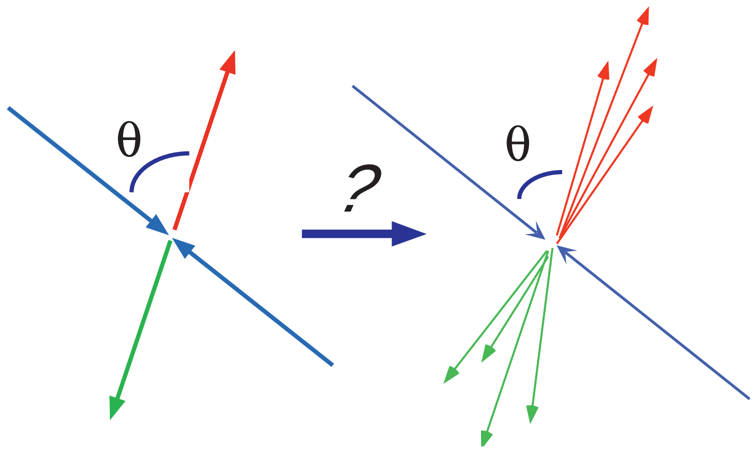
- **Were we doomed to compute only correlations of currents?**
- **Were we** $\textit{forbidden}$ **to look inside the final state?**
- **Or, could it be possible to "see" quarks and gluons?**

Structure of final states: Cosmic rays to quark pairs

- **Another strand of the story: Particle jets in cosmic rays . . .**
  - **"The average transverse momentum resulting from our measurements is p_T=0.5 BeV/c for pions . . . Table 1 gives a summary of jet events observed to date . . . "**
    – B. Edwards et al, Phil. Mag. 3, 237 (1957)

- **Limited transverse momentum in secondaries of hadron collisions**

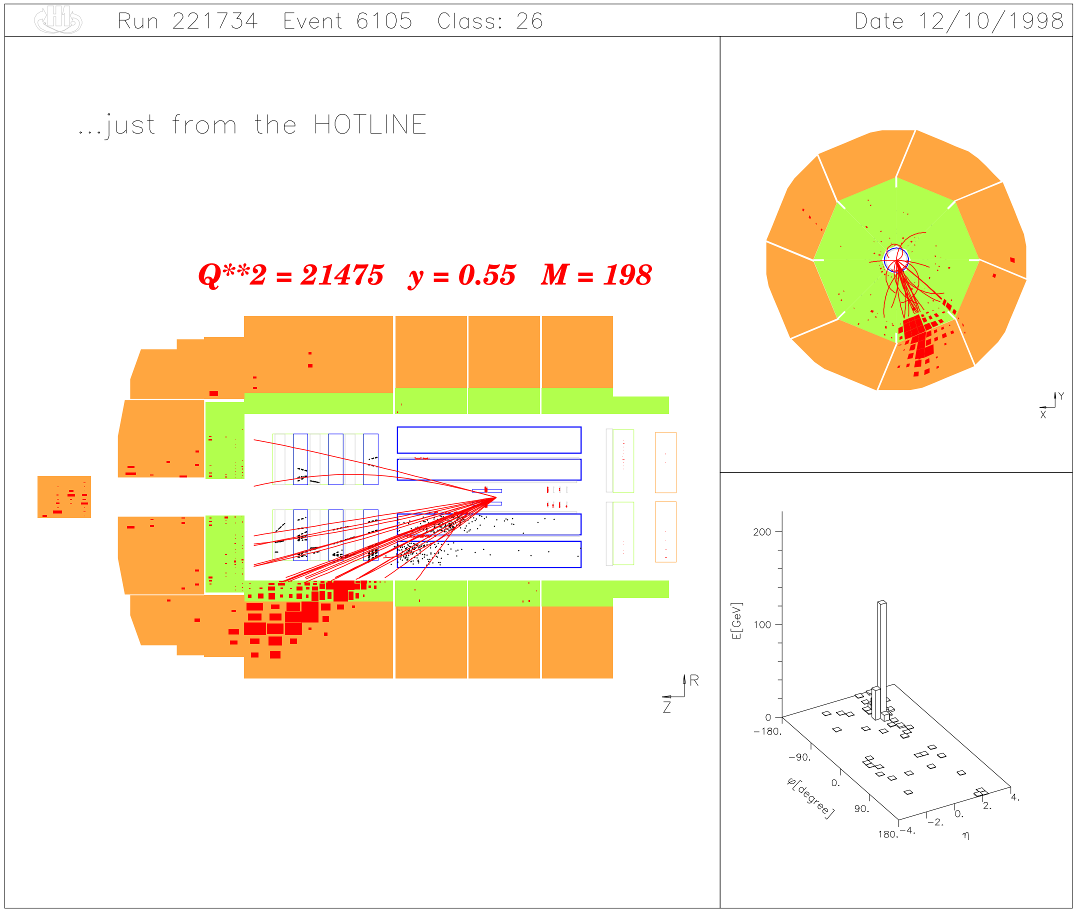
- **What about quarks produced in $\mathrm{e^+e^-}$ annihilation?**



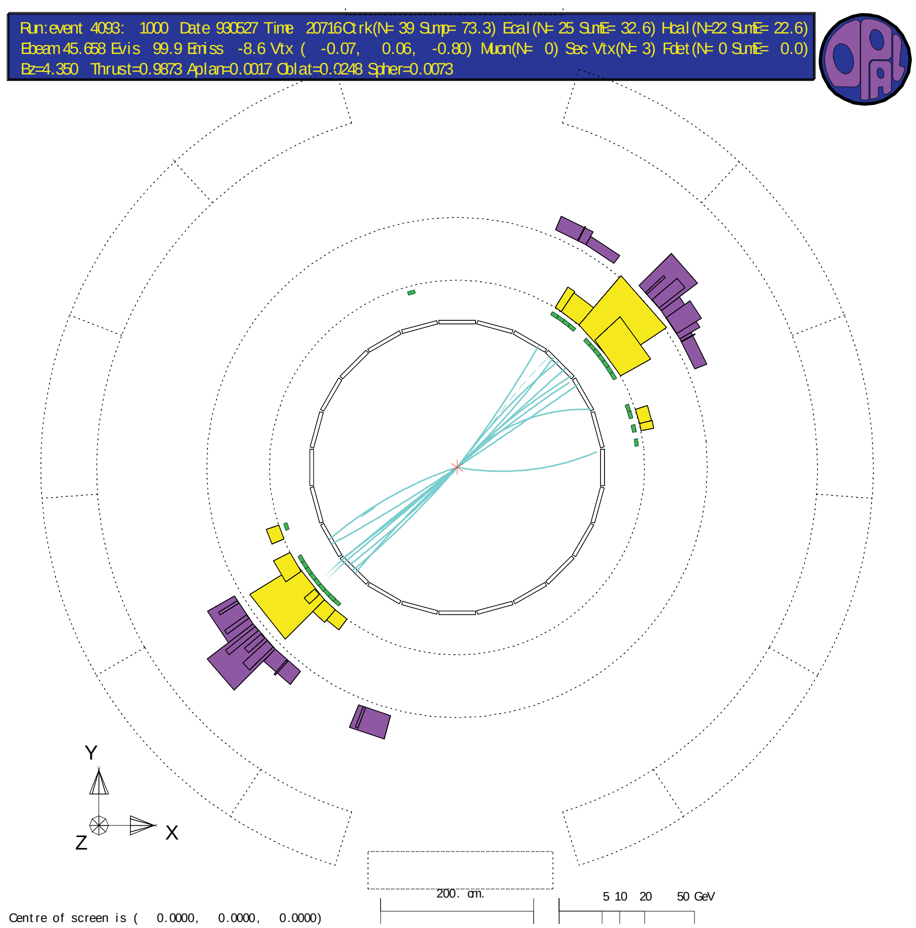
- **Extension of the parton model: q/e scattering to $\mathrm{e^+e^-} \to \mathrm{q\bar{q}}$. Conjecture "$p_T$-cutoff" relative to jet axis.**
  - **A** *prediction* **for the angular distribution in $\mathrm{e\bar{e}} \to \mathrm{q\bar{q}}$:** $1 + \cos^2\theta$
  - **"Because of our cutoff $k_{\mathrm{max}} \ll |q|$ . . . The distribution of secondaries in the colliding ring frame will look like two jets – S.D. Drell, D.J. Levy and T.-M. Yan, Phys. Rev. D1**

- In this picture, partons “fragment” into hadrons,
- *Here* was a question to ask of nature and of QCD. Would the final states look like this?
- In nature, they did:
  - G. Hansen et al, Phys. Rev. Lett. 35, 1609 (1975)

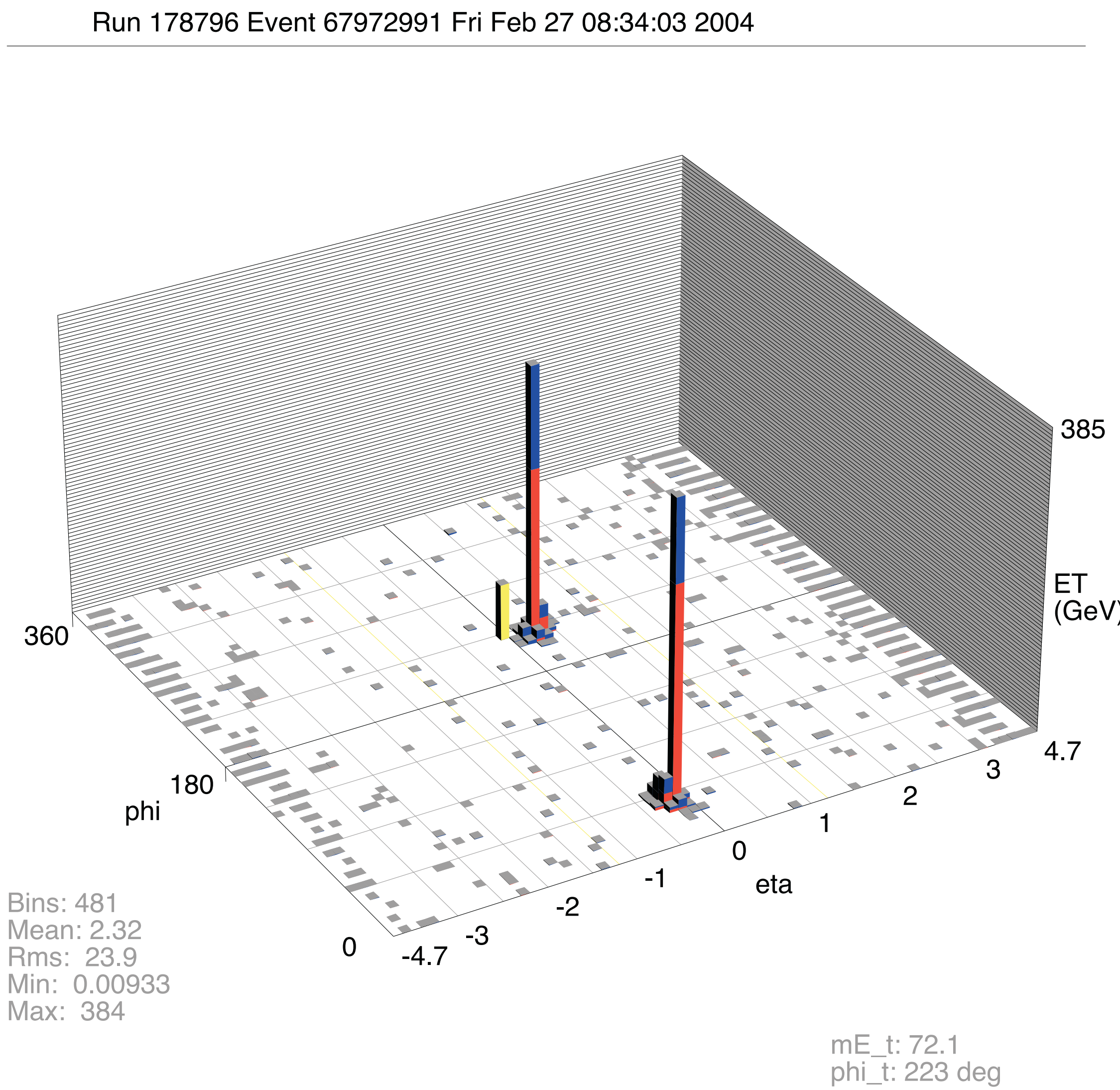
- **And that's what happens; DIS:**



- **And that's what happens;** e^+e^-**:**



- **And in hadron-hadron collisions**



Infrared Behavior and the strong coupling

- **We can observe the jets, but can we calculate with them?**
- **QED: exclusive cross sections typically have "infrared divergent" corrections**

$$\delta\sigma_{\mathrm{ee}\to\mathrm{ee}}\,(Q, m_e, m_\gamma = 0, \alpha_{\mathrm{EM}}) \sim \alpha_{\mathrm{EM}}\;\beta_{AB}(Q/m_e)\;\ln\frac{m_\gamma}{Q}$$

- **Energy resolution ϵQ (Bloch-Nordsieck)**
  $\longrightarrow$ IR finiteness (sum over $E_\gamma \le \epsilon Q$)

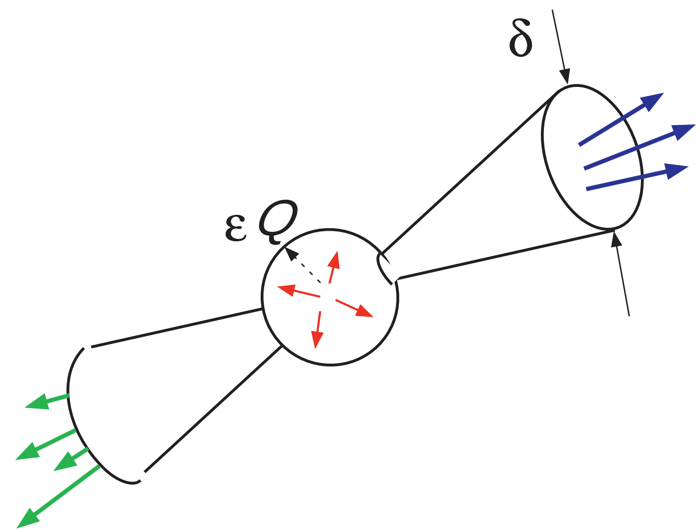
$$\delta\overline{\sigma}_{\mathrm{ee}\to\mathrm{ee}+X(\epsilon)}\,(Q, m_e, \epsilon Q, \alpha_{\mathrm{EM}}) \sim \alpha_{\mathrm{EM}}\;\beta_{AB}(Q/m_e)\;\ln\frac{1}{\epsilon}$$

- **Correction is small if $\alpha_{\mathrm{EM}}\ln(1/\epsilon)$ is small**

- **Impossibility of observing arbitrarily soft γ**
  $\leftrightarrow$ **radiation of accelerated charges in the classical limit**

- **Could something like this happen:**
  - **For QED with $m_e = 0$?**
  - **For QCD with $m_q = 0$?**
  - **Kinoshita, Lee-Nauenberg**
  - **Can we find observables that have no factors $\ln(m/Q)$, only at worst $(m/Q)\ln(m/Q)$?**

- **We'll see that:**
  - ϵ **not enough . . . but an extra** *angular* **resolution works**
  - **Impossibility of resolving collinear massless particles**



- **No factors** Q/m **or** $\ln(Q/m)$ **Infrared Safety (GS 1975, Politzer 1977, GS Weinberg 1977)**

- **Now:** *Trade high-energy for zero-mass limit*

- **Perfect for QCD: asymptotic freedom** $\rightarrow \alpha_s(Q)$ **decreases with** Q

- **New class of observables: Jet Cross sections**

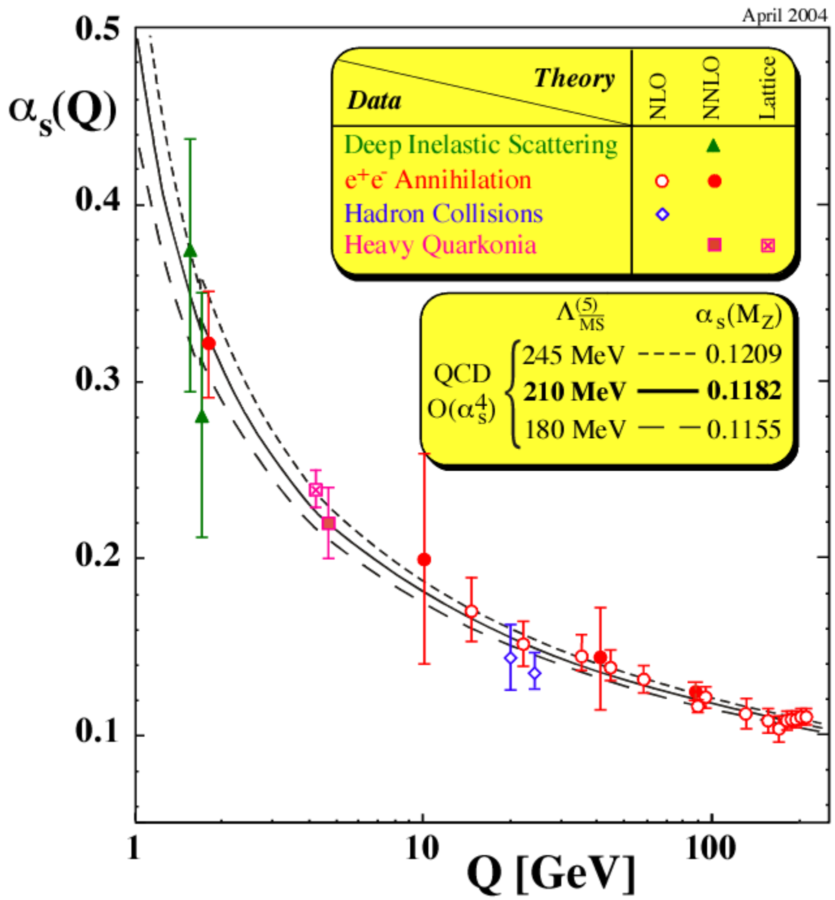
$$\sigma\left(Q/\mu, \alpha_s(\mu)\right) = \sigma\left(1, \alpha_s(Q)\right)$$

- **No need for a transverse momentum cutoff**
  - **IR finiteness** $\rightarrow$ **high-**p_T **suppressed by** $\alpha_s(p_T)$

- **Determination of** α_s **from an infrared safe cross section**

$$\hat{\sigma}(\alpha_s) = \sum_{n=0}^{n_{\max}} C_n(\mu)\alpha_s^n(\mu) + \Delta \quad \rightarrow \quad \alpha_s(\mu) = f\left(\sigma(\mu), C_n(\mu), \Delta\right)$$

- **And here's what it looks like now:**



Through asymptotic freedom, QCD has become

- **A window to the shortest distances/highest energies**
- **A signal through jets when heavy particles decay**
  Higgs, supersymmetry, technicolor . . .
- **A signal from the** *absence* **of jets which remains eloquent:**
  Jet quenching, mini black hole production
- **A testing-ground for string theory**
- **Still retains mysteries: strong CP, confinement, χSB**
- **In many ways, the exemplary quantum field theory**

IC. TIME-EVOLUTION IN QFT

Infrared divergences have their origin in long-time, or low virtuality). We can learn about them by studying the systematics.

Two Approaches to Infrared Behavior

- **Old-fashioned Perturbation Theory**
- **Covariant (Feynman) Perturbation Theory**
- **We'll take the first approach. A little unconventional, but sheds new light on the origin of jets.**

From $H^{(0)}$ to Free Quark-Gluon States

- **Degrees of Freedom:** $q(x),\ A^\mu(x) \rightarrow \tilde{q}(\vec{k}, x^0),\ \tilde{A}^\mu(\vec{k}, x^0)$

- **Hamiltonian:** $H = H^{(0)}(q, A_\mu) + V(q, A_\mu)$

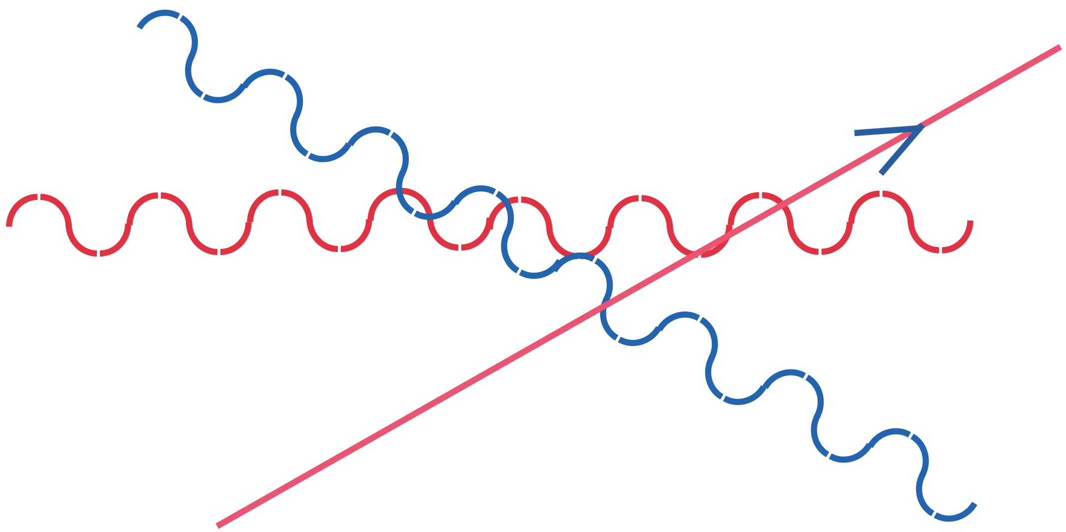
- **"Free Hamiltonian"** $H^{(0)}$ **quadratic** $\rightarrow$ **wave equation**
  $\rightarrow$ **superposition**

- **"Free" states:** $|m >= |\{k_i\}, \{q_j\} >$
  $\rightarrow$ **no scattering**
- **Energies:** $H^{(0)}|m >= \left(\sum_i \omega_i(\vec{k}) + \sum_j \omega_j(\vec{q})\right)|m >\equiv S_m|m >$

$$\omega(k) = \sqrt{k^2 + m_q^2} = \frac{1}{\hbar}\sqrt{p^2c^2 + m_q^2c^4}$$

$$\omega(q) = |\vec{q}| = \frac{1}{\hbar}E(q)$$

- **Picture: Independent Waves:**



TOPT: The Interaction Mixes the Free States

- **Schrödinger equation for Interacting states :**

$$i\hbar \frac{\partial}{\partial t}|\psi(t) >= \left(H^{(0)} + V\right)|\psi(t) >$$

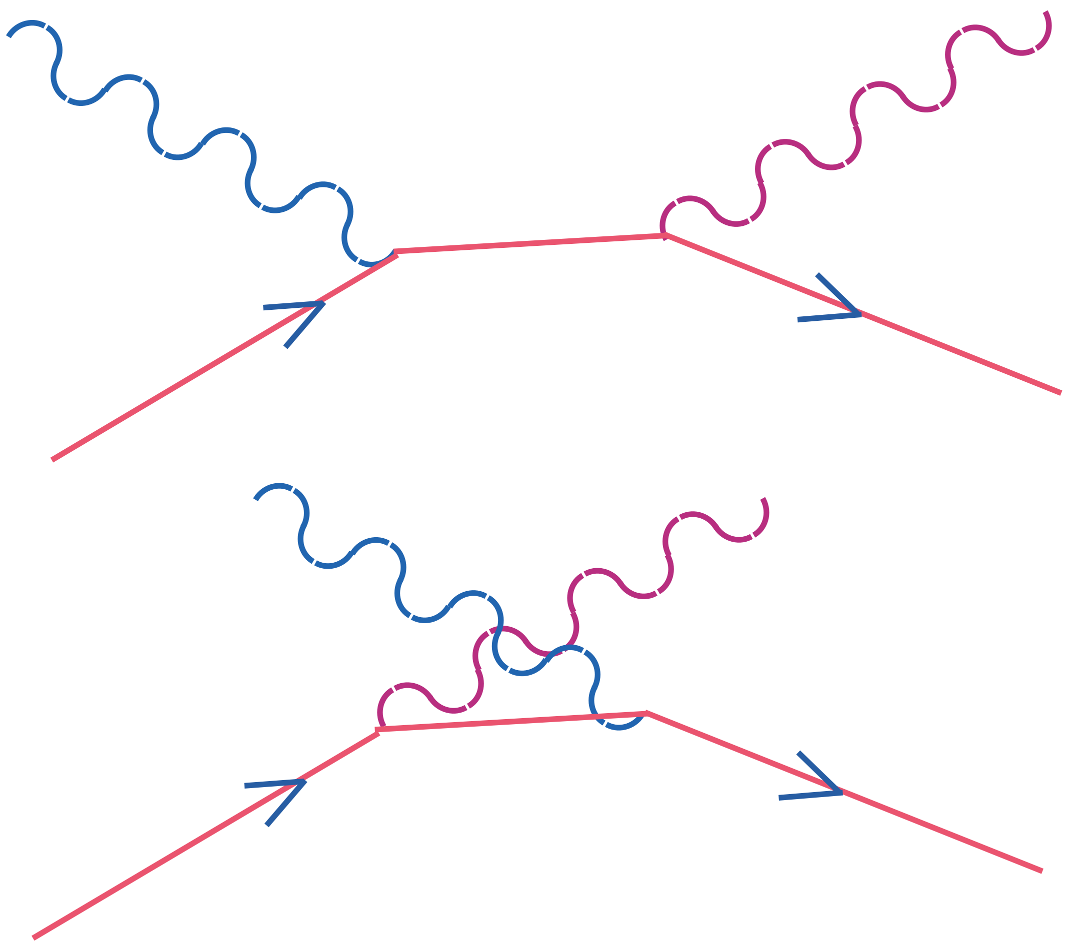
- **With free-state BC :** $|\psi(t = -\infty) >= |m_0 >$

- **Notation :** $V_{ji} = \langle m_j|V|m_i\rangle$

- **Solution :**

$$
\begin{aligned}
< m_n|\psi_0(t) > \;=\; & \sum_{n=0}^{\infty} \sum_{m_1 \ldots m_n} \mathrm{e}^{-iS_n t}\,(-i)^n\, V_{n,n-1}\, V_{n-1,n-2} \\
& \times \cdots \times V_{1,0} \\
& \times \int_{-\infty}^{t} d\tau_n\, e^{-i(S_{n-1}-S_n)\tau_n} \int_{-\infty}^{\tau_n} d\tau_{n-1} e^{-i(S_{n-2}-S_{n-1})\tau_{n-1}} \\
& \times \cdots \times \int_{-\infty}^{\tau_2} d\tau_1 e^{-i(S_0-S_1)\tau_1}
\end{aligned}
$$

- **Picture: Time-Ordered Diagrams:**



“Old-Fashioned Perturbation Theory”: Energy Deficits

$$< m_n|m_0(t) > \;=\; \sum_{n=0}^{\infty} \sum_{m_1 \ldots m_n} \mathrm{e}^{-iS_n t}\,(-i)^n\, V_{n,n-1}\, V_{n-1,n-2} \cdots \times V_{1,0}$$

$$\times \int_{-\infty}^{t} d\tau_n\; e^{-i(S_{n-1}-S_n)\tau_n} \int_{-\infty}^{\tau_n} d\tau_{n-1} e^{-i(S_{n-2}-S_{n-1})\tau_{n-1}}$$

$$\times \cdots \times \int_{-\infty}^{\tau_2} d\tau_1 e^{-i(S_0-S_1)\tau_1}$$

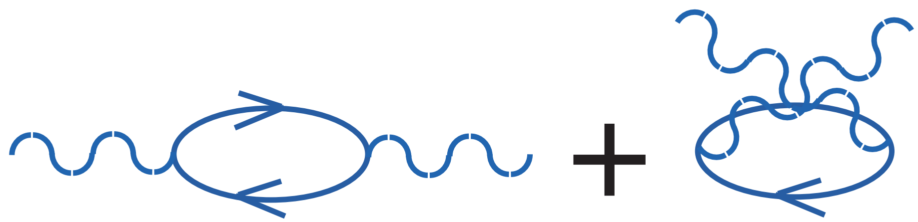
- **We can do these time integrals!**

• Time Integrals Give Time-Ordered ("Old Fashioned") Perturbation Theory. Γ_i **denotes ordering** i **of** V**'s:**

$$\Gamma_i(p) = -i \sum_{\text{states in i}} \prod_{\text{states } a} \frac{1}{E_a - S_a + i\epsilon} N_{\text{spin}}(p, \ell_i)$$

$$\sum_{\text{states}} \equiv \prod_{\text{loops } i} \int \frac{d^3\ell_i}{(2\pi)^3} \prod_{\text{lines } j} \frac{1}{2\omega_j(p, \ell_i)}$$

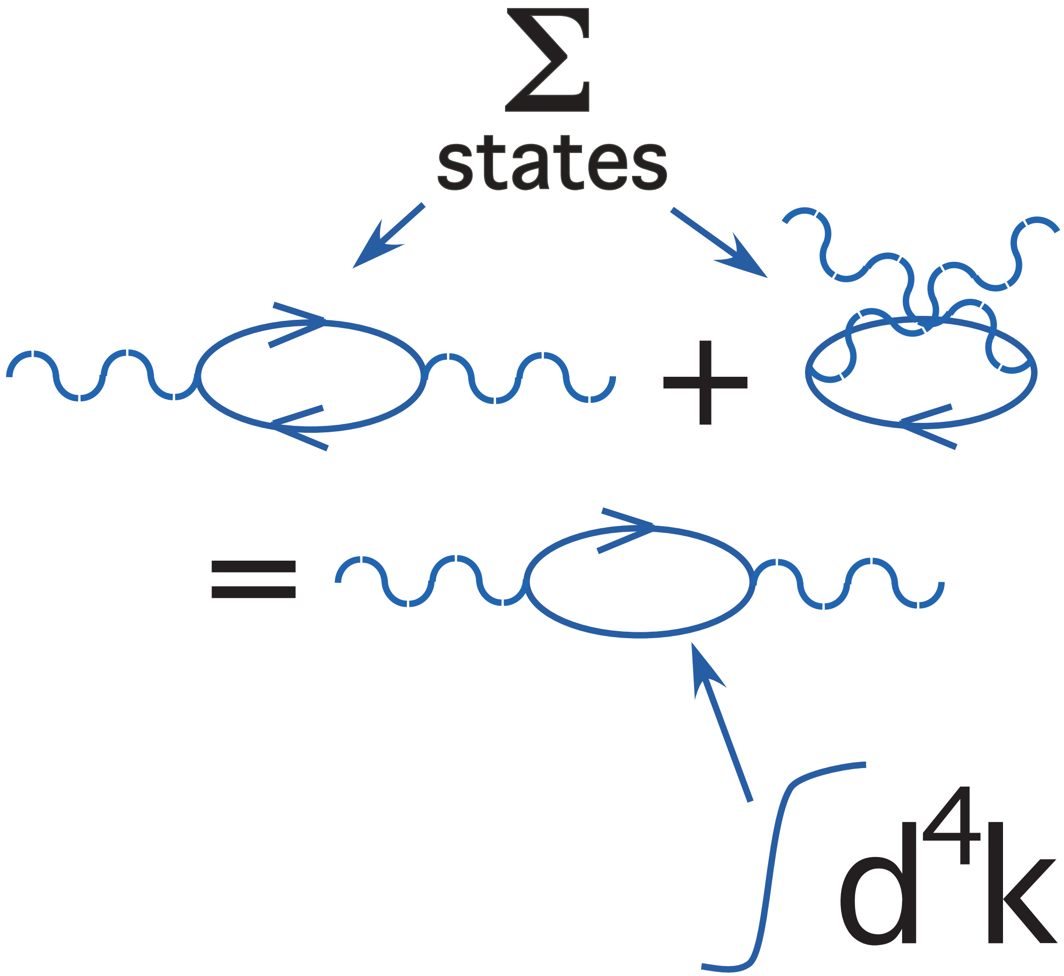
- **Example:**

$p \Rightarrow$ 

$$\Gamma_1 + \Gamma_2 = \sum_{2-\text{particle states}} \left(\frac{1}{p_0 - \omega(k_1) - \omega(k_2)} + \frac{1}{-p_0 - \omega(k_1) - \omega(k_2)} \right)$$

Sum Over Time-Orders $\rightarrow$ Feynman (Covariant) Diagrams:

$$G(p) = \sum_i \Gamma_i(p) = \prod_{\text{loops } i} \int \frac{d^4 \ell_i}{(2\pi)^4} \prod_{\text{lines} j} \frac{i}{k_j^2(p, \ell_i) - m_j^2} \tilde{N}_{\text{spin}}(p, \ell)$$



$$\Gamma_1 + \Gamma_2 \;=\; \int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2 - m^2}\,\frac{1}{(p-k)^2 - m^2}$$

ID. NARROWING DOWN THE TIME: INFRARED SAFETY AND THE ORIGIN OF JETS

- **Look for Physical Quantities that *Don't* Depend on Large Times**

- ***Infrared Safety***

- **Central Role of *Unitarity*:** $\sum_n P_n = 1$ **(KLN Theorem)**

- **Generalized Unitarity** $\rightarrow$ ***Jets*** **from Quarks and Gluons**

- ***Parton-Hadron Duality*; Structure of the Final State**

Large Times and Physical Pictures

- **The** ***Only*** **Time Integral:**

$$\int_{-\infty}^{t} d\tau_n \, e^{-i(S_{n-1}-S_n)\tau_n} \int_{-\infty}^{\tau_n} d\tau_{n-1} e^{-i(S_{n-2}-S_{n-1})\tau_{n-1}} \ldots \times \int_{-\infty}^{\tau_2} d\tau_1 e^{-i(S_0-S_1)\tau_1}$$

- **Oscillating t-Dependence Suppresses Large Times**
- **Stationary Phase $\rightarrow$ Long Time Dependence**

$$\begin{aligned}
\text{PHASE} &= \sum_{\text{states } m=1}^{n} S_m(\tau_m - \tau_{m-1}) \\
&= \sum_{\text{states } m=1}^{n} \left(\sum_{\text{particle j in m}} \omega(\vec{p}_j) \right) (\tau_m - \tau_{m-1}) \\
&= \text{FREE} - \text{PARTICLE ACTION}
\end{aligned}$$

STATIONARY PHASE $\rightarrow$ STATIONARY ACTION

$$\downarrow$$

FREE PARTICLE PROPAGATION IN SPACE-TIME

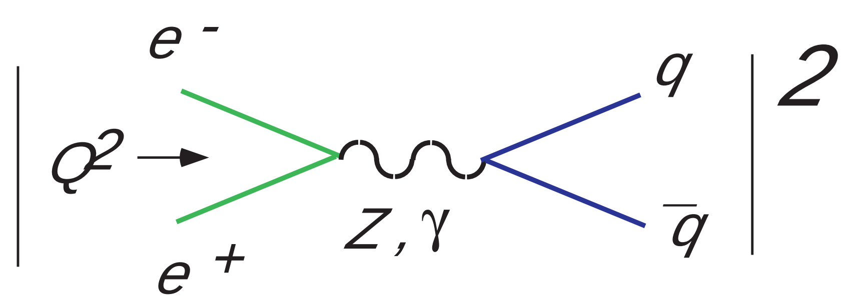
Quarks and gluons are OK in PT if they don't interact like free particles.

Must seek physical quantities for which quantum histories that coincide with classical histories are suppressed.

Example: One-loop decay of the Z

(Collinear and Soft Gluons)

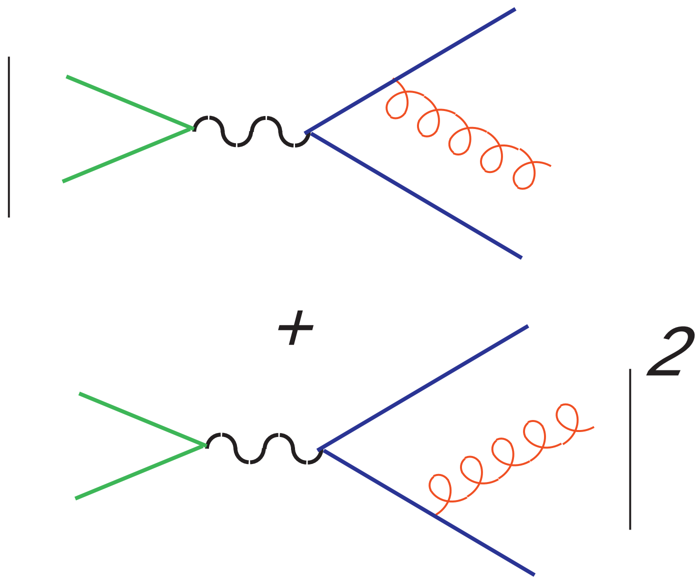
$m_{\mathrm{Z}} \sim 90$ **GeV; couples to quarks via weak interactions**

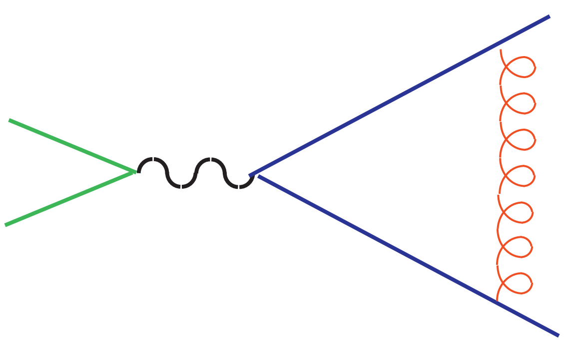


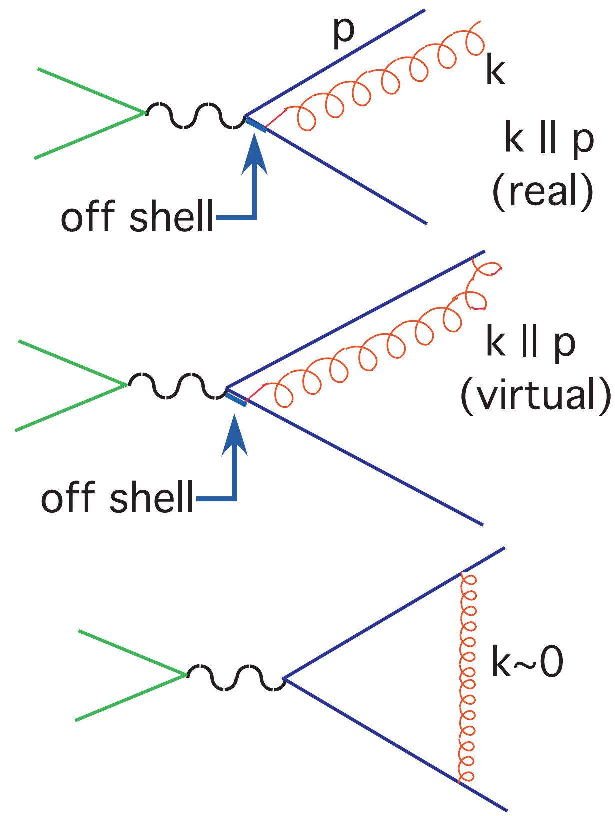
(lowest order)

- $A(\mathrm{Z} \to q\bar{q})$ **at order** g^2

Reduced diagrams
(Contract off-shell lines to points)



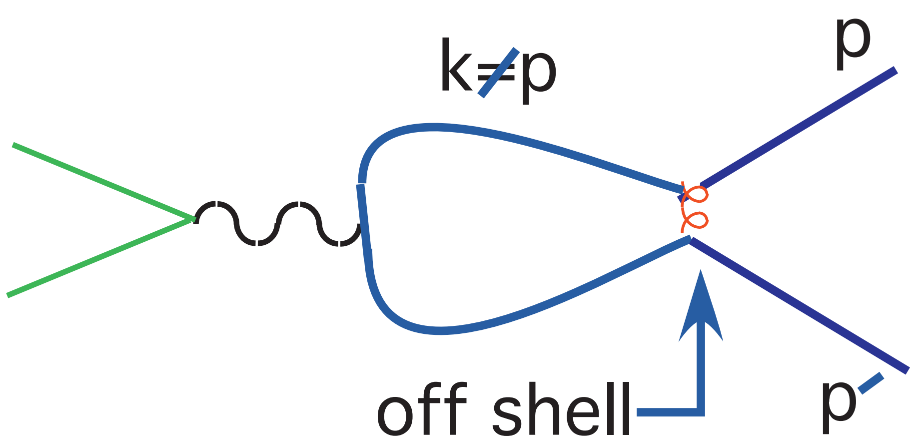




Physical Pictures for Massless Particles ("Why jets?")

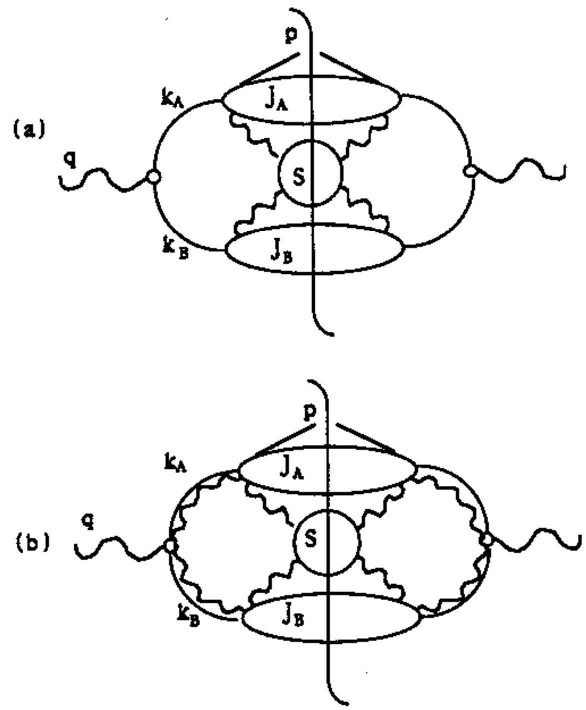
- **Collinear: particles travel side-by-side with $0 < x < 1$ (partonic)**
- **Soft: 'infinite' wavelengths**

- **A reduced diagram that does *not* give long-distance behavior:**



Generalization: *All-Order* Decays of the Z: Jets

The Only sources of long-time behavior **(cut diagrams)**

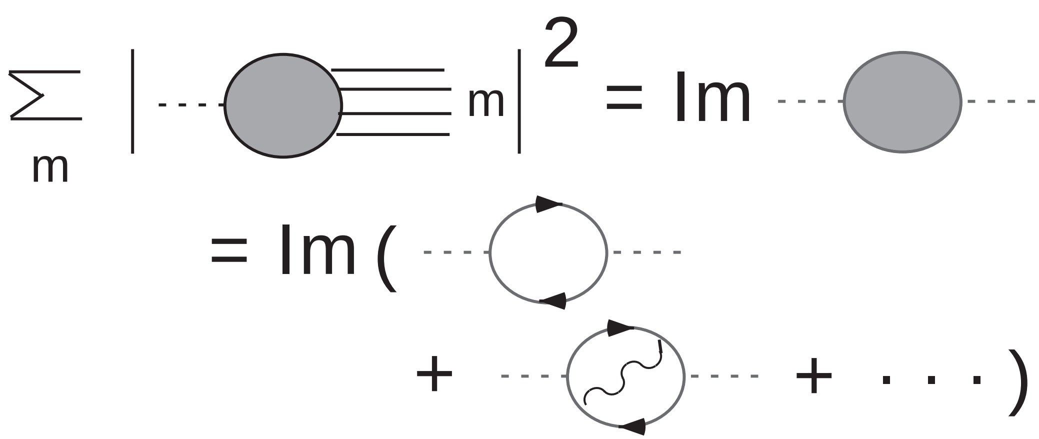


(a) Physical gauges **(b) Covariant gauges [include $\epsilon^\mu(k) \sim k$]**

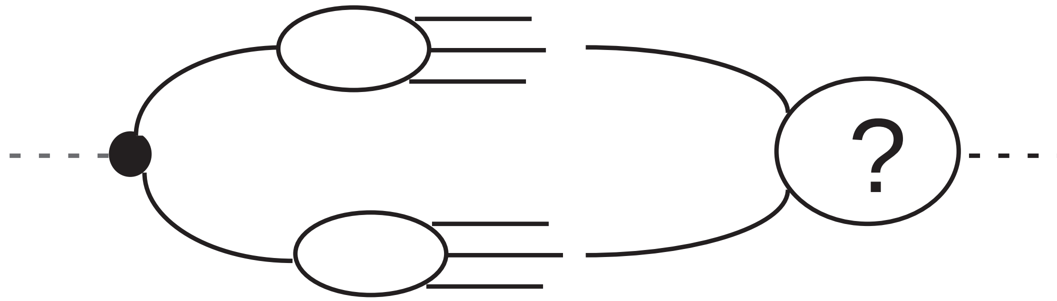
Infrared Safety of Γ_Z, $\sigma_{\rm tot}^{(e^+e^-)}$

$$\Gamma_Z = \mathrm{Im}(-i\Pi_Z(q^2 = m_Z^2))$$

- **Optical Theorem for the Total Decay Rate:**



- **Physical Pictures for $\Pi(Q^2)$? There are none! q.e.d.**



- $\Gamma_{\rm Z}$ **an expansion in $\alpha_s(M_{\rm Z})$,**

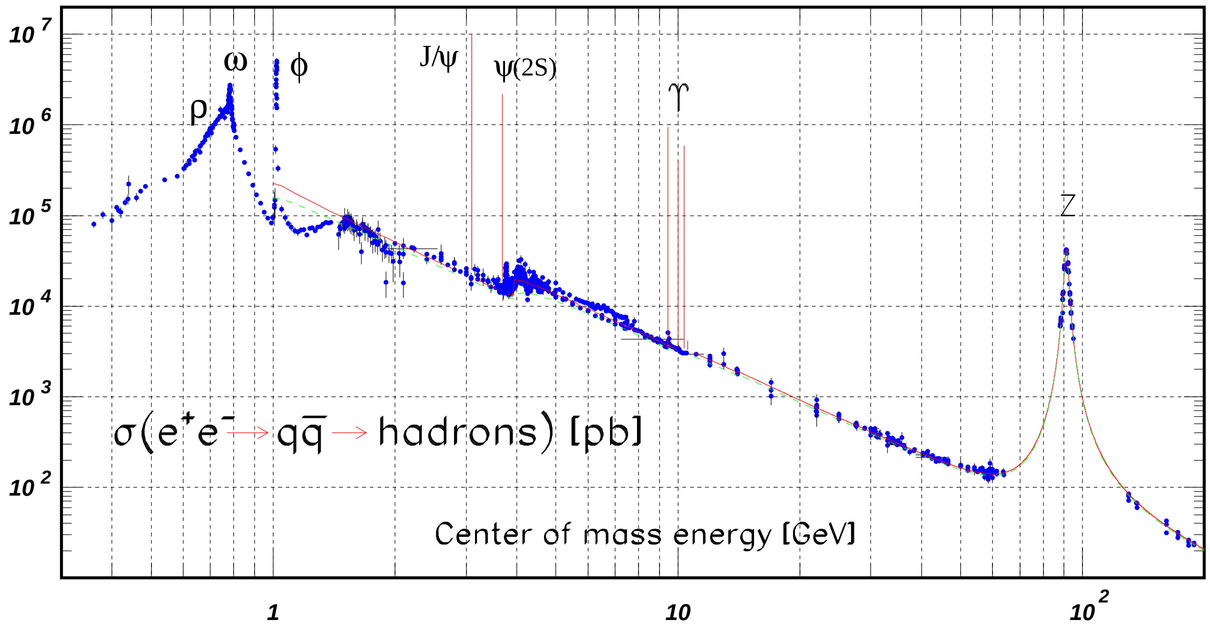
$$\Gamma_{\rm Z} = \Gamma_{\rm Z}^{(\rm EW)} \sum_{n=0}^{\infty} c_n \alpha_s^n(M_{\rm Z}) \, ,$$

with finite coefficients (c_n).

- **Same applies to** $\sigma_{\rm tot}^{(\rm e^+e^-)}$**, with** $\alpha_s(Q)$**,**

$$\begin{aligned}
\sigma_{\rm tot}^{(\gamma^*)} &= \left(\frac{4\pi\alpha^2}{3s}\right) \sum_q Q_q^2 \sum_{n=0}^{\infty} c_n \alpha_s^n(Q) \\
&= \left(\frac{4\pi\alpha^2}{3s}\right) \sum_q Q_q^2 \left[1 + \frac{\alpha_s(Q)}{\pi} + (1.986 - 0.115 n_q \left(\frac{\alpha_s(Q)}{\pi}\right)^2\right. \\
&\left. + \left(-6.637 - 1.200 n_q - 0.005 n_f^2 - 1.240 \frac{(\sum_q Q_q)^2}{\sum_q Q^2}\right) \left(\frac{\alpha_s(Q)}{\pi}\right)^3 + \ldots\right]
\end{aligned}$$

- **Experiment for $\sigma_{\mathrm{tot}}^{(\mathrm{e^+e^-})}$: QCD almost disappears . . .**



- **Can measure $\alpha_s(Q)$ this way, *but*, we'd like more**

- ***Are the Quarks and Gluons "Really There"? . . .***

ID. AFTER THE FACT: INFRARED SAFETY FOR JETS

QUARKS AND GLUONS

Unitarity and its Generalization:

- **The General Form:**

$$\sum_m A_m^* A_m = 2\mathrm{Im}\,(-iA)$$

- **Proof:**

$$\sum_m A_m^* A_m = \sum_{m=1}^{N} \prod_{j=m+1}^{N} \frac{1}{E_j - S_j - i\epsilon} (2\pi)\delta(E_m - S_m) \prod_{i=1}^{m-1} \frac{1}{E_i - S_i + i\epsilon}$$

$$2\mathrm{Im}\,(-iA) = -i\left[-\prod_{j=1}^{N} \frac{1}{E_j - S_j + i\epsilon} + \prod_{j=1}^{N} \frac{1}{E_j - S_j - i\epsilon} \right]$$

$$i\left(\frac{1}{x + i\epsilon} - \frac{1}{x - i\epsilon} \right) = 2\pi\delta(x)$$

- ***We Didn't Need to Integrate Over Momenta***

Infrared Safety of Jet Cross Sections

- **Example: Cone-Defined Two-Jet Cross Section**

$$\sigma_{2\mathrm{J}} = \sigma(E_{\mathrm{cones}} \geq (1-\epsilon)m_{\mathrm{Z}})$$

- **Use unitarity for any jet configuration in sum over states**

•This gives same cancellation as for the total cross section. Except at the sub-spaces of phase space that define the jets (more later).

- **Jet Cross Sections can be computed in PT in Z Decay or e^+e^- Annihilation**
- **Critereon: Any Cross Section that is Insensitive to Collinear Rearrangements and to Emission of Soft Gluons**

“Seeing” Quarks and Gluons With Jet Cross Sections

- **No unique jet definition. $\leftrightarrow$ Each event a sum of possible histories.**
- **Relation to quarks and gluons always approximate but corrections to the approximation computable.**

- **General Form**

$$\sigma_{\text{jet}} = \sigma_0 \sum_{n=0}^{\infty} c_n(y_i, N, C_F)\alpha_s^n(Q)$$

  - y_i **choices:** $\delta,\ \Omega_{\text{jet}},\ T,\ y_{\text{cut}}, \ldots$

- δ**, cone size;** Ω**, jet direction**

- **Shape Variable, e.g. thrust (**$T = 1$ **for "back-to-back" jets**

$$T = \frac{1}{s}\text{max}_{\hat{n}}\ \sum_i |\hat{n}\cdot\vec{p}_i|$$

- y_{cut} **Cluster Algorithm:** $y_{ij} > y_{\text{cut}}$**,**

$$y_{ij} = 2\text{min}\left(E_i^2, E_j^2\right)(1 - \cos\theta_{ij})$$

- **Quark-Gluon Dynamics: Perturbative Formation of the Final State**

- **For Large** Q **in** $\mathrm{e}^+\mathrm{e}^-$ **Annihilation, all times** $1/Q \leq t \leq 1/\Lambda_{\mathrm{QCD}}$ **are available for Perturbation Theory.**

- **Success of pQCD depends crucially on** $m_{u,d} \ll \Lambda_{\mathrm{QCD}}$**.**

- **Parton-Hadron Duality: Follow pQCD out to** $t \sim 1/\Lambda$**. Hadrons in Final State Follow Distribution of Partons**

Lect. 2. Factorization and jets

IIA. THE BASICS OF PQCD SUMMARIZED . . .

- **Infrared safety & asymptotic freedom:**

$$
\begin{aligned}
Q^2\, \hat{\sigma}_{\mathrm{SD}}(Q^2, \mu^2, \alpha_s(\mu)) &= \sum_n c_n(Q^2/\mu^2)\, {\alpha_s}^n(\mu) + \mathcal{O}\left(\frac{1}{Q^p}\right) \\
&= \sum_n c_n(1)\, {\alpha_s}^n(Q) + \mathcal{O}\left(\frac{1}{Q^p}\right)
\end{aligned}
$$

- $\mathrm{e}^+\mathrm{e}^-$ **total; jets: a sum over collinear rearrangements and soft emission organizes all long-time transitions, which must sum to** ≤ 1 **by unitarity. But not always as simple as it seems.**

- **Generalization: factorization**

$$Q^2\sigma_{\rm phys}(Q,m) = \omega_{\rm SD}(Q/\mu,\alpha_s(\mu)) \otimes f_{\rm LD}(\mu,m) + \mathcal{O}\left(1/Q^p\right)$$

μ = **factorization scale;** m= **IR scale (**m may **be perturbative)**

- **New physics in** $\omega_{\rm SD}$**;** $f_{\rm LD}$ **"universal"**

- **Deep-inelastic (**$p=2$**),** $\mathrm{p\bar{p}} \rightarrow Q\bar{Q}$ **. . .**

- **Decays:** $B \rightarrow \pi\pi$ **and "elastic" limits:** $\mathrm{e^+e^-} \rightarrow \mathrm{JJ}$ **as** $m_J \rightarrow 0$

⋆ When Can We Resum?

- **Whenever there is factorization, there is evolution**

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\mathrm{phys}}(Q, m)$$

$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}$$

- **Wherever there is evolution there is resummation**

$$\ln \sigma_{\mathrm{phys}}(Q, m) = \exp \left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

Factorization structure

$$\frac{d\sigma(Q, a+b \to N_{\rm jets})}{dQ} = H_{IJ} \otimes \prod_{c=a,b} \mathcal{P}_{c'/c} \times S_{JI} \times \prod_i J_i$$

- **A story with only these pieces:**

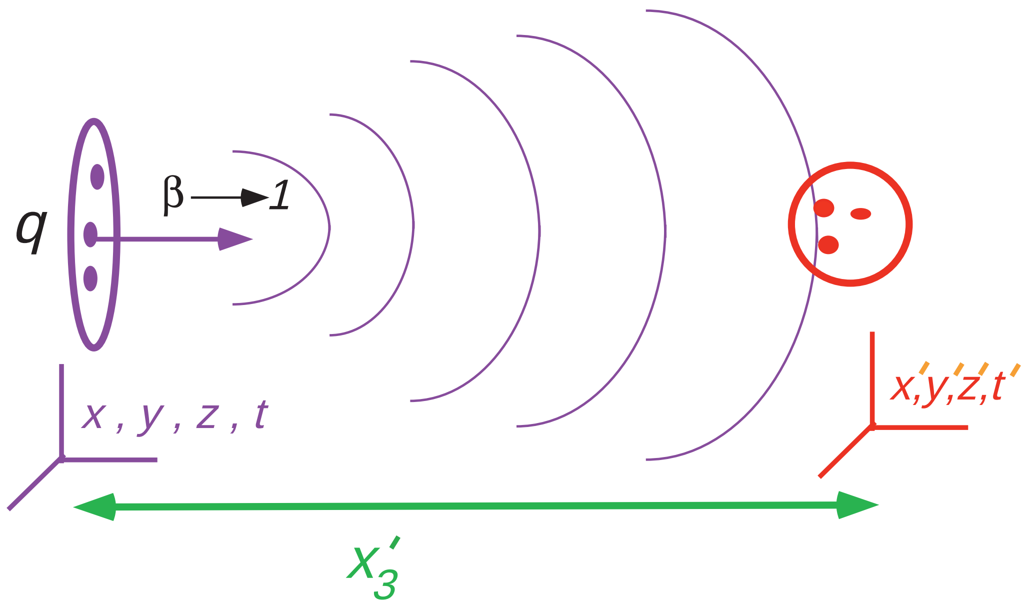
- **Evolved incoming partons $\mathcal{P}_{a'/a}$, $\mathcal{P}_{b'/b}$ collide at H_{IJ}, I, J label color exchange in amplitudes and their complex conjugates;**

- **Outgoing jets J_i and coherent soft emission S_{JI}.**

- **Holds to any fixed α_s^n, all $\ln^a \mu/Q$ to $\sim E_{\rm soft}/E_{\rm jet}$.**

IIB. THE PHYSICAL BASIS OF FACTORIZATION

Classical picture



$$\Delta \equiv \beta c t' - x'_3$$

- Why a classical picture isn't far-fetched . . .

  The correspondence principle is the key
  to the origin of IR divergences.

- Any accelerated charge must produce classical radiation,

  and infinite numbers of soft gluons are required
  to make a classical field.

Transformation of a scalar field:

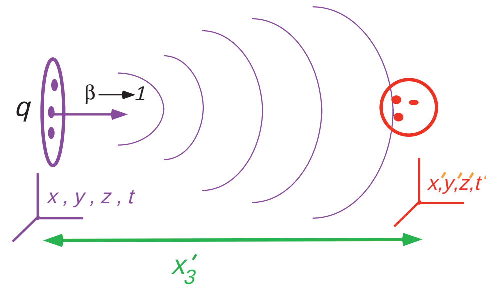
$$\phi(x) \;=\; \frac{q}{\sqrt{x_T^2 + x_3^2}} \;=\; \phi'(x') \;=\; \frac{q}{(x_T^2 + \gamma^2 \Delta^2)^{1/2}}$$

From the Lorentz transformation: $x_3 = \gamma(\beta c t' - x_3') \equiv \gamma \Delta'$.

Closest approach is at $\Delta' = 0$, **i.e.** $t' = \frac{1}{\beta c} x_3'$.

The scalar field transforms "like a ruler": At any fixed $\Delta' \neq 0$, the field decreases like $1/\gamma = \sqrt{1-\beta^2}$.

Why? Because when the source sees a distance x_3, the observer sees a much larger distance.



<u>field</u>	<u>x frame</u>	<u>x' frame</u>
scalar	$\frac{q}{\lvert\vec{x}\rvert}$	$\frac{q}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
gauge $(-)$	$A^-(x) = \frac{q}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{-q\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
field strength	$E_3(x) = \frac{q}{\lvert\vec{x}\rvert^2}$	$E_3'(x') = \frac{-q\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$
Gauge fields :	$A_3 \sim \gamma^0,$	$E_3 \sim \gamma^{-2}$

- **The "gluon" $\vec{A}$ is enhanced, yet is a total derivative:**

$$A^{\mu} = q\frac{\partial}{\partial x_{\mu}}\ \ln\left(\beta c t' \ -\ x_3\right) + \mathcal{O}(1-\beta) \sim A^{-}$$

- **A^{-} is an unphysical polarization & can be removed by a gauge transformation!**

- **The "force" $\vec{\mathrm{E}}$ field of the incident particle does not overlap the "target" until the moment of the scattering.**

- **“Advanced” effects are corrections to the total derivative:**

$$1 - \beta \;\sim\; \frac{1}{2}\left[\sqrt{1-\beta^2}\right]^2 \;\sim\; \frac{m^2}{2E^2}$$

- **Power-suppressed! These are corrections to factorization.**

- Initial-state interactions decouple from hard scattering

- Summarized by multiplicative factors: the parton distributions

- Interactions after the scattering are too late to affect
  large momentum transfer, creation of heavy particle, etc.

- Fragmentation of partons to jets too late to know details of
  the hard scattering: factorization of fragmentation functions.

- $\Rightarrow$ Cross section for hard scattering is IR safe,
  with power-suppressed corrections.

- **The gauge-theory analog of our classical argument is the universal soft-parton factor:**

For soft gluon k emitted by fast quark p, Dirac eq. gives:

$$\bar{u}(p)\,(-ig_s\,\gamma^\mu\,)\,\frac{\not{p} + \not{k}\ +\ m}{(p+k)^2\ -\ m^2}\ =\ \bar{u}(p)\,(-ig_s\,)\,\frac{p^\mu}{p\cdot k}\ +\ (IR\ finite)$$

In a diagram p^μ will be contracted with a gluon propagator,

and in $p \cdot A = 0$ gauge, this term vanishes!

$$G^{\nu\mu}(k) = -\left(g^{\nu\mu}\ -\frac{p^\nu\,k^\mu\ +\ k^\nu\,p^\mu}{p\cdot k}\ +\ p^2\,\frac{k^\nu\,k^\nu}{(p\cdot k)^2}\right)$$

- **Notice this gauge depends on the momentum p.**

- **The origin of the "universality" of soft gluon interactions.**

- **But it is the same for every parton in a jet.**

IIC. FACTORIZATION FOR FRAGMENTATION

– **Hadron production at high p_T (e.g., from gluon fragmentation)**

– **Leading power result: factorization into fragmentation function**

$$
\begin{aligned}
d\sigma_{A+B\to H+X}(p_T) &= d\hat{\sigma}_{A+B\to g+X}(p_T/z\mu) \otimes D_{H/g}(z, m_c, \mu) \\
&\quad + \mathcal{O}(m_c^2/p_T^2)\,,
\end{aligned}
$$

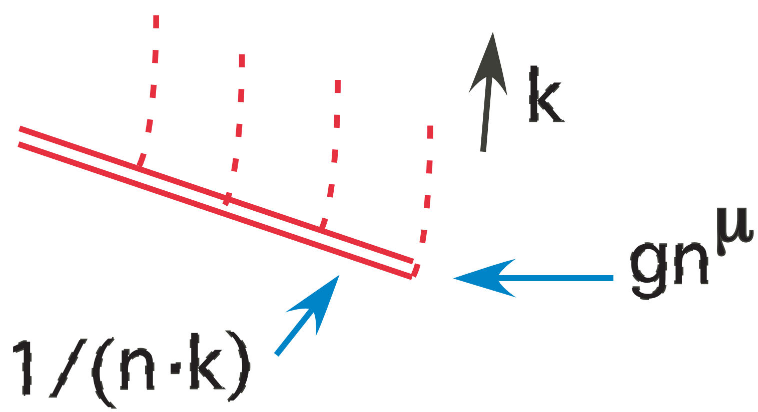
– **With $D_{H/g}$ defined as VeV:**

$$
\begin{aligned}
D_{H/g}(z, m_c, \mu) &\propto \frac{1}{P^+} \mathrm{Tr}_{color} \int dy^- \mathrm{e}^{-i(P^+/z)y^-} \\
&\quad \times \langle F^{+\lambda}(0)\, [\Phi_-^{(g)}(0)]^\dagger\, a_H^\dagger(P^+)\, a_H(P^+)\, \Phi_-^{(g)}(y^-)\, F_\lambda^+(y^-) \rangle_0
\end{aligned}
$$

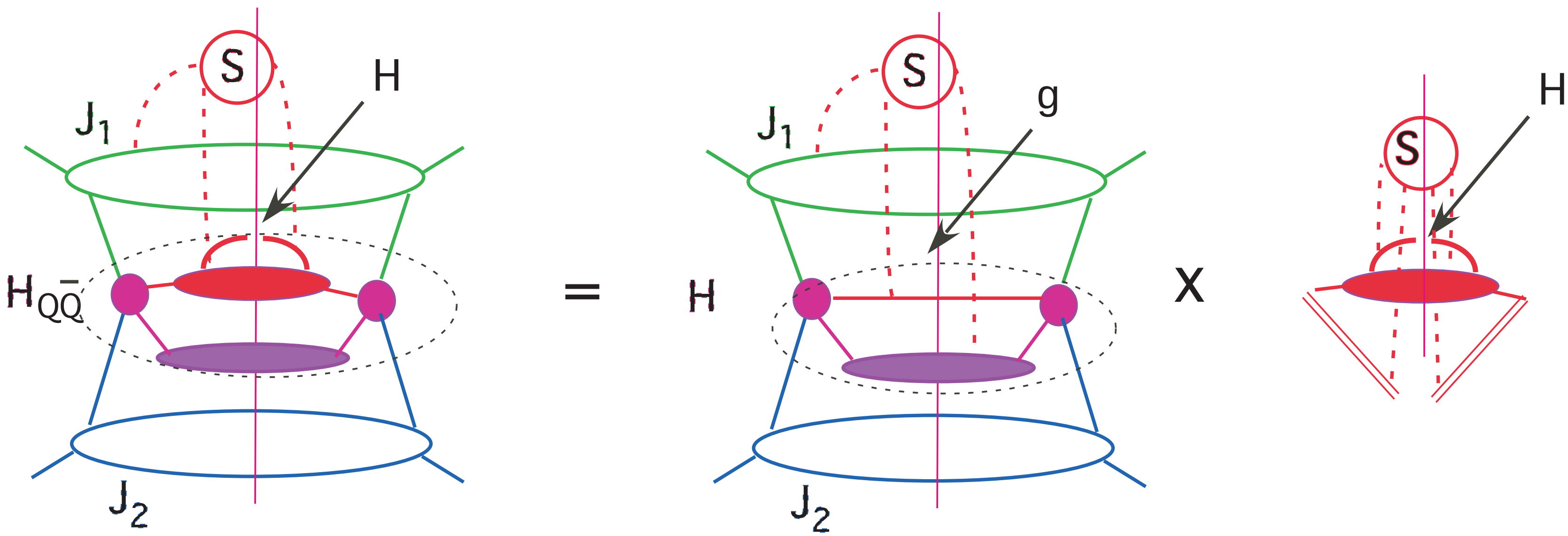
– **The Wilson line [a.k.a. path ordered exp, nonabelian phase, eikonal line] in x^- direction $(n^\mu = \delta_{\mu -})$:**

$$\Phi_{-}^{(g)}(x^-) = P \exp\left[-ig\int_0^\infty n\cdot A^{(adj)}\left((x^- + \lambda)n\right)\right]$$

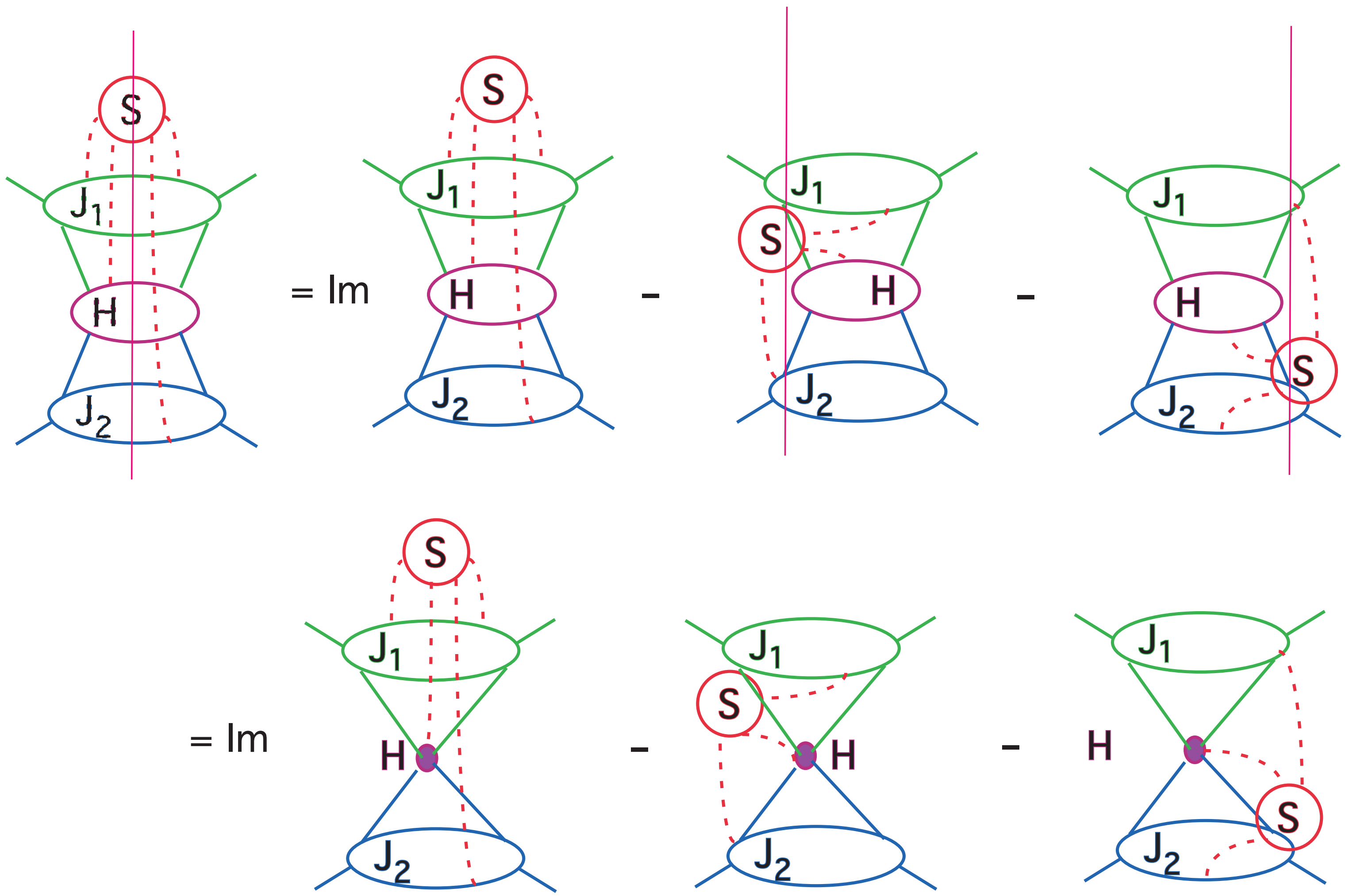
– **To the jet, all that's left of the rest of the world is gluon source!   Fragmentation analog of "current quark" in DIS.**



- **How it works, in two steps:**
- **Step 1: Fragmentation factorizes from the rest**



– **Step 2: Cancellation of remaining IR final state: Uncut loops a short distance. (This is why jet cross sections are calculable.)**



A good example is (we'll come back to this in resummation) pions at measured transverse momentum.
PDFs $\otimes$ hard scattering $\otimes$ fragmentation functions:

$$
\begin{aligned}
\frac{p_T^3\, d\sigma(x_T)}{dp_T} \;\; &= \;\; \sum_{a,b,c} \int_0^1 dx_1\, f_{a/H_1}\left(x_1, \mu_F^2\right) \int_0^1 dx_2\, f_{b/H_2}\left(x_2, \mu_F^2\right) \\
&\times \int_0^1 dz\, z^2\, D_{h/c}\left(z, \mu_F^2\right) \\
&\times \int_0^1 d\hat{x}_T\; \delta\left(\hat{x}_T - \frac{x_T}{z\sqrt{x_1 x_2}}\right) \int_{\hat{\eta}_-}^{\hat{\eta}_+} d\hat{\eta}\, \frac{\hat{x}_T^4\; \hat{s}}{2}\, \frac{d\hat{\sigma}_{ab\to cX}(\hat{x}_T^2, \hat{\eta})}{d\hat{x}_T^2 d\hat{\eta}}
\end{aligned}
$$

with

$$
\begin{aligned}
x_T^2 \;\; &= \;\; \frac{4p_T^2}{S} \\
\hat{\eta}_+ \;\; &= \;\; -\hat{\eta}_- = \ln\left[(1+\sqrt{1-\hat{x}_T^2})/\hat{x}_T\right]
\end{aligned}
$$

$\star$ **Inclusive Jets**

– **Factorized Cross Sections (e.g.** $A + B \to J(p_J) + X$)

$$p_J^4 \frac{d\sigma_{\rm phys}(p_J, m)}{dp_J^2} \;=\; f_{\rm LD,A}(\mu, m) \otimes \omega_{\rm SD}\left(\frac{p_J^2}{\hat{s}}, \frac{\hat{s}}{\mu^2}, \alpha_s(p_J)\right) \otimes f_{\rm LD,B}(\mu, m)$$

– **But what's a jet?** $\leftrightarrow$ **define "**X**" and calculate** ω

– **Need to construct jets from final states: algorithms**
G. Blazey et al., *Run II Jet Physics* **hep-ph/0005012**

IID. JET ALGORITHMS

* **Cone algorithms**: towers $\rightarrow$ protojets $\rightarrow$ jets
  - Calorimeter tower mta. (directions y_i, ϕ_i)
  - Cluster within cones

$$i \subset C \quad : \quad \sqrt{\left(y^i - y^C\right)^2 + \left(\phi^i - \phi^C\right)^2} \leq R.$$

  - Task I: to identify "centers" y_C, ϕ_C
    (high-p_T towers as "seeds" (but IR safety problematic))
  - Result: "protojets"
* Task II: interpret overlapping protojets: "merge/split"
* Naive interpretation is to find jets that "really" come from a single parton, but this is not a well-defined concept.
* For single jet inclusive, a cleaner method would be to scan all possible protojets, identify largest p_T

* **The k_T algorithm: preclusters $\longrightarrow$ jets**
* **Starts with measurements in calorimeter "towers" p_i**
* **"For each precluster i in the list, define**

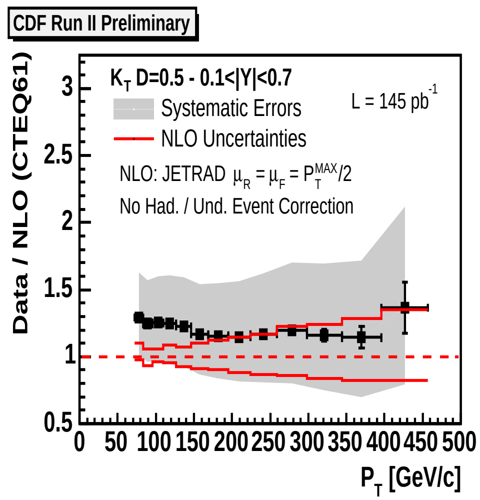
$$d_i = p^2_{T,i}$$

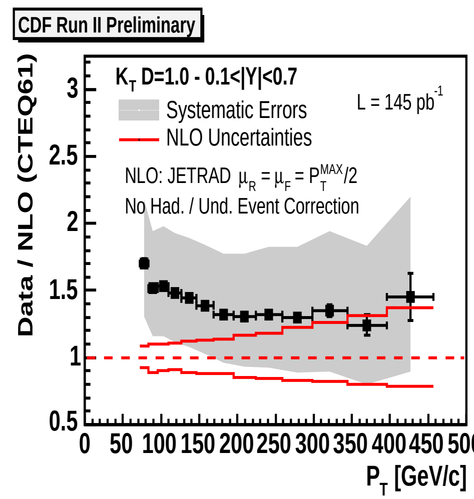
* **For each pair (i, j) of preclusters ($i \neq j$), define**

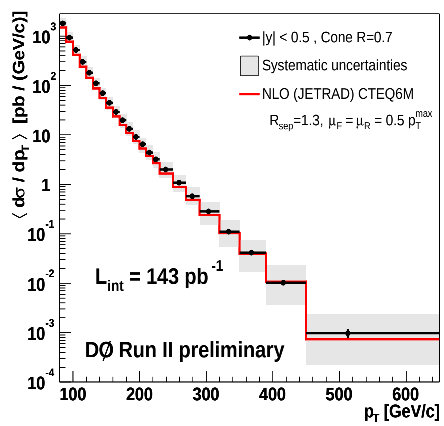
$$d_{ij} = \min\left(p^2_{T,i}, p^2_{T,j}\right) \frac{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}{D^2} \text{ "}$$

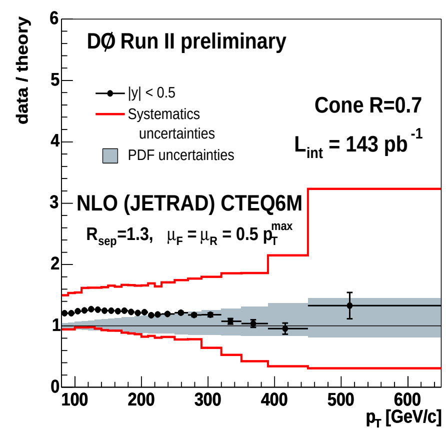
* **Find $d_{\min}$ among all d_i, d_{ij}**
* **if $d_{\min}$ is a d_i: identify as "jet"**
* **if $d_{\min}$ is a d_{ij}: combine into new precluster $p_{ij} = p_i + p_j$**
* **Repeat (leaving out "jets")**
* **End result: list of "jets" (most with small d_i)**

⋆ Tevatron Run II Jets









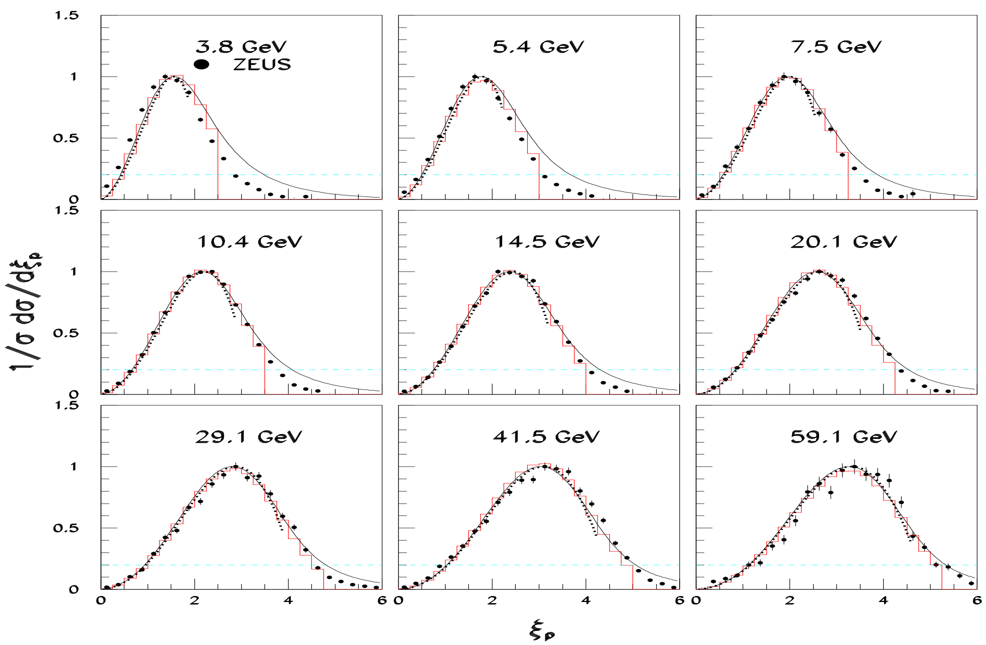
– **What have we learned so far?**
  * **Extraordinary tracking of predicted shape to highest energies**
  * **Energy uncertainty remains large but will decrease with more statistics**
  * **Poorly-understood excess towards lower p_T**
  * **CDF k_T algorithm shows excess at largest p_T**
  * **But algorithms may evolve**
  * **Remaining discrepancies probably due to still incomplete understanding of particle and energy flow**

⋆ Jet Particle Flow

∗ **Low-z spectrum at Zeus; from Khoze/Ochs hep-ph/0110295**

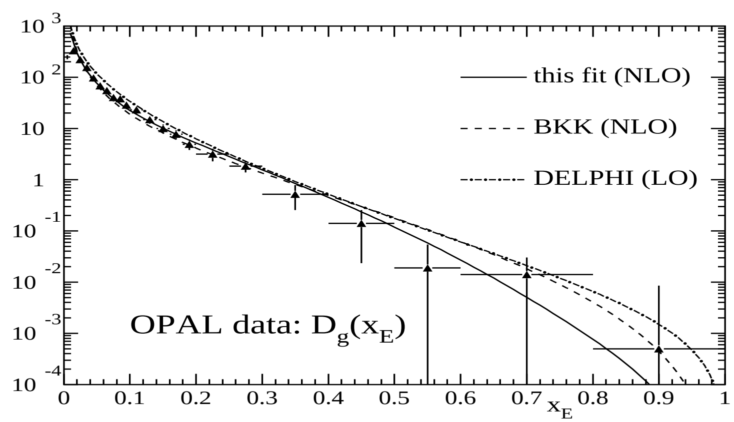
$$\xi = \ln\left(\frac{E_J}{E_{\text{particle}}}\right)$$

Angular ordering at branching $\to$ suppn. at large ξ; Gaussian-like shape.



$*$ **Large-z fragmentation function fit; from Kretzer hep-ph/0003177**

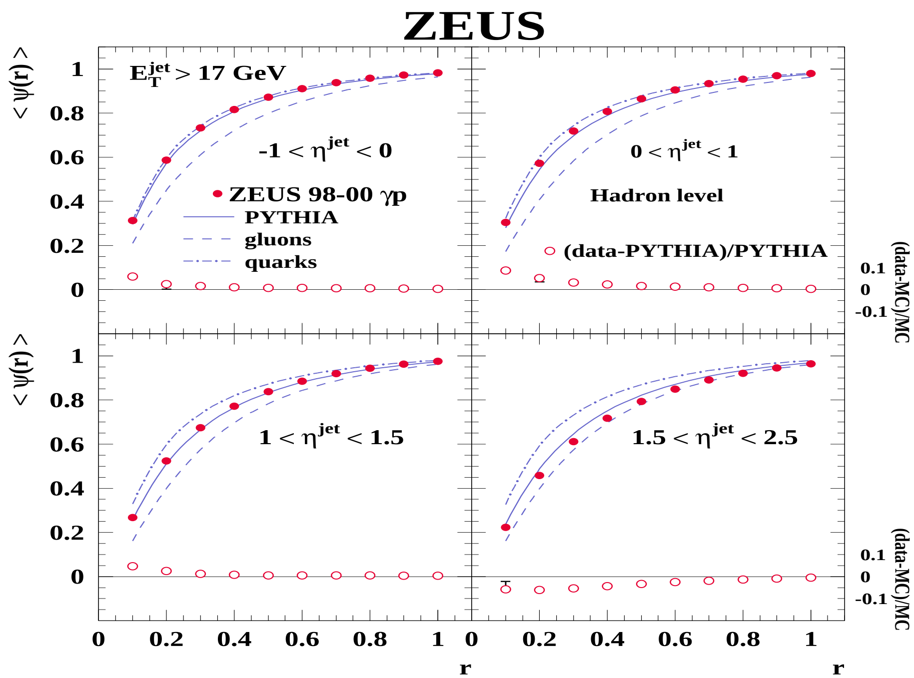
$$\frac{d\sigma^h_{P=T,L}}{dz} = \sum_{i=q,\bar{q},g} \int_z^1 \frac{d\zeta}{\zeta} C^i_P\left(\zeta, Q^2, \mu^2_{F,R}\right) D^h_i\left(\frac{z}{\zeta}, \mu^2_F\right)$$



⋆ Jet Energy Flow

– The "Jet Shape"

$$\psi(r) = \frac{E_T(r)}{E_{T,\mathrm{jet}}},$$



Special point: what makes a jet-finding algorithm infrared safe?

– A set of collinear particles and soft particles define a subspace of loop/phase space momenta

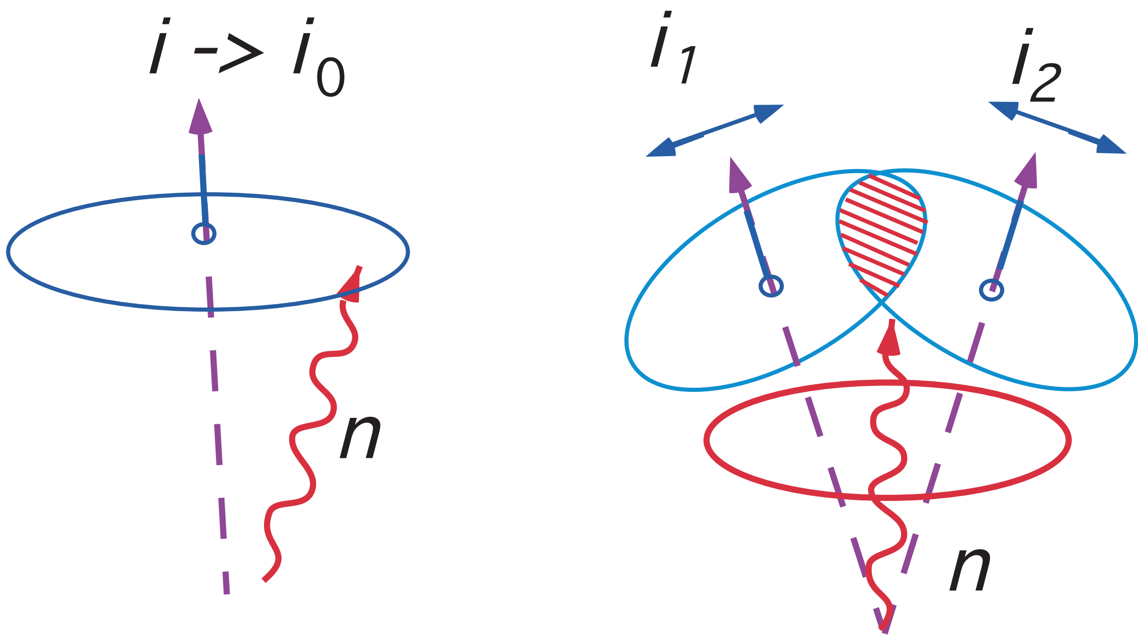
– "Normal variables" (n = soft gluon energies, relative k_T for collinear particles, etc.) determine where the subspace is.

– "Instrinsic variables" (i = hard particle energy, direction, etc.) parameterize the subspace. Number of jets changes at $i = i_0$.

– Generic cancellation between virtual and real states:

$$\int \frac{di}{Q} \int_0 \frac{dn}{n} \, (\, F(n) - F(0) \,)$$

– **Example on the left.**

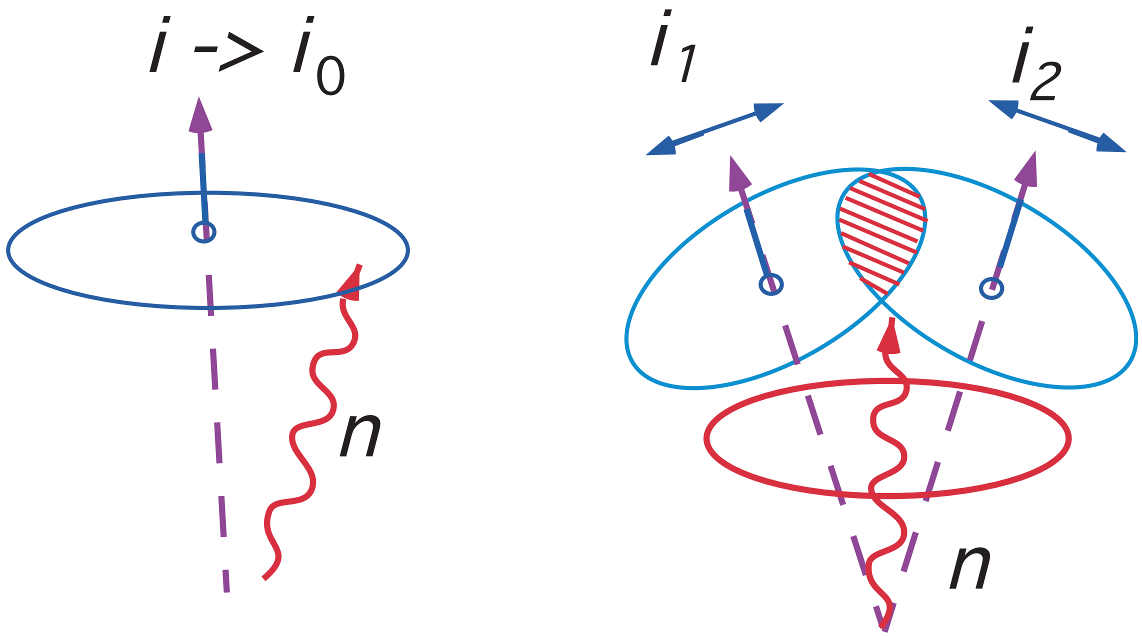


– **For energy i below minimum value i_0, soft gluon emission required to make up the difference near the subsurface $i = i_0$.**

$$\int^{i_0} \frac{di}{Q} \left(\int_{i-i_0} \frac{dn}{n} F(0) \right) = - \int_{i_0} \frac{di}{Q} \ln(i - i_0)\, F(0)$$

– **OK if singularities aren't worse than logarithmic.**

– **Example on the right:**



– **Soft gluon on the right reduces number of jets any energy** n **on a subspace of the same dimension as the singular surface.**

$$\int \frac{di}{Q} \int \frac{dn}{n} F(0) \to \infty$$

- IR safety requires that jet number change only on subspace of lower dimension than the relevant singular surface.

- Problem if trial cones are centered only on particles (seeds).
  Seymour (1998)

- "Midpoint algorithm" deals with the example above
  but the problem is more general unlesss all cones are sampled.

- Very recently, "Practical seedless cone algortithm": G. Salam & G. Soyez (2007) identify cones with hard particles "at the edges"