The math for the ADS hexapod system

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ABSTRACT

In this note we will calculate the forces and motions for the current ADS design of the DECam hexapod system. We will show that the maximum force in any hexapod actuator is about 2/3 of the total DECam weight and that an actuator resolution of 12xxx micrometers is enough to meet the current DES positioning requirements. The study of the effect of loads on the actuators will show that a position measurement feedback system close to the focal plane will be needed. The vibratonal modes will also be calculated.

1. INTRODUCTION

This note is an extension of a previous general note titled "The math of the hexapod system". Currently ADS is designing and bulding the hexapods for DECam, so we feel that it will be useful to update the general note mentioned above with the design parameters provided to us by ADS.

In Section 2 we will specify the ADS hexapod configurations and discuss the transformations that will be used to study the motion and forces in the hexapod system. In Section 3 we will calculate the forces on the actuators and study how these forces change as the camera and the cage are moved as a unit through space (keeping the actuator motors off). In Section 4 we will calculate how each actuator length has to be changed to move the motion plate relative to the fixed plate. The effect of moving one actuator at a time and the errors introduced by the fact that this can only be done in finite steps will also be studied in Section 4. In Section 5 we will calculate the effect of elastic deformations due to loads on the hexapod actuators. And finally in Section 6 we will study the hexapod vibrations.

2. THE ADS HEXAPOD SYSTEM

The hexapod system consists of six actuators and two plates which are referred to as the motion and fixed plates. The motion plate is rigidly attached to the camera while the fixed plate is rigidly attached to the cage. One end of the actuators inserts in the fixed plate, the other end inserts in the motion plate.

For the ADS Configuration, one end of the actuators insert in the motion plate as is shown in Figure 1 left, the other actuator end inserts in the fixed plate as is shown in the center figure. If each actuator is attached in each of the six plate insertion points using a ball joint then at each joint the actuator is allowed to freely rotate in all three dimensions. The camera can be held in space by applying the right set of forces at each of the six motion plate insertion points. At each of these points the actuators can independently push or pull by any amount. This means an arbitrary force components at each insertion point, for a total of six independent forces which is enough to satisfy the six static equations that come from the sum of all forces and all moments.

The six insertion points in the motion plate can be arbitrarily positioned with respect to the six insertion points in the fixed plate. Once this relative position is fixed the distances between the points can be calculated. This set of distances is unique, that is different positions of the motion plate relative to the fixed one will create a different set of six distances. This one-to-one correspondence means that the relation can be inverted, or that specifying a set of six distances will create a unique position of the motion plate relative to the fixed one. If one wishes to do so, only one of the six distances can be changed at a given time which guarantees that the hexapod

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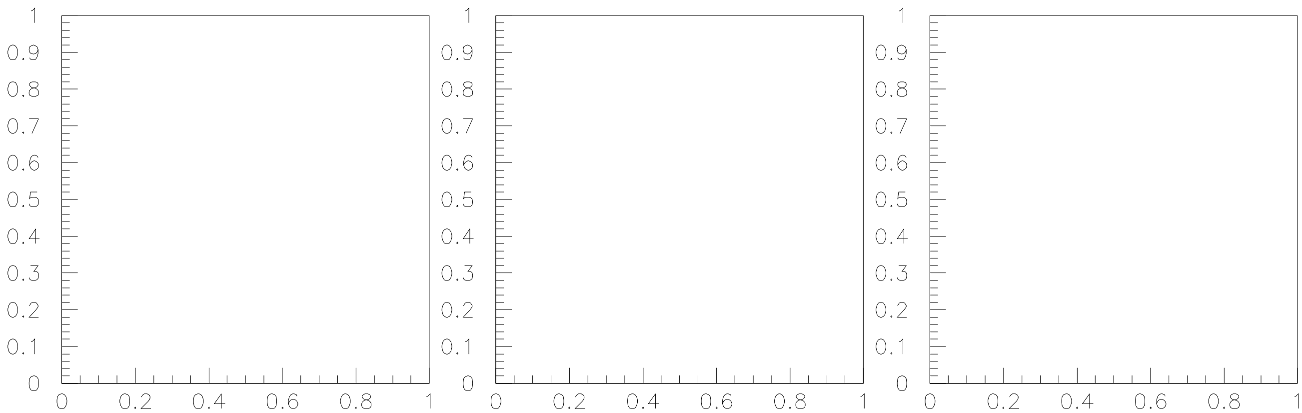

Figure 1. Motion plate (left), fixed plate (center), hexapod system (right).

system is not over-constrained. We will mathematically prove in Section 4 that this is the case, but before doing that we have to define the vectors for each of the plate insertion points and their rotations and translations.

The insertion points will be labeled as $\vec{r}_{0_i}$ for the motion plate and $\vec{R}_{0_i}$ for the fixed plate. They are calculated as

$$\vec{r}_{0_i} = [r\ cos(\alpha_i + \delta_0), r\ sin(\alpha_i + \delta_0), 0.5(A + A')] \tag{1}$$

$$\vec{R}_{0_i} = [R\, cos(\beta_i + \delta_0), R\, sin(\beta_i + \delta_0), 0.5(A - A')] \tag{2}$$

The angles α_i and β_i are given in Table 1. The motion plate radius r and the fixed plate radius R and the distances A and A' are given in Table 2. The angle δ_0 is arbitrary and allows us to rotate the insertion points around the z-axis. When the camera is pointing up the plane of the fixed plate forms the x-y plane of the coordinate system, the z-axis points up and the origin of the coordinate system is centered on the fixed plate.

Table 1. Angles, in degrees, corresponding to the points where the actuators insert into the motion and fixed plates.

		i	1	2	3	4	5	6
ADS configuration	motion plate	α_i	13	107	133	227	253	347
	fixed plate	β_i	47	73	167	193	287	313

Table 2. Motion and fixed plate insertion points radius r and R. The distances A, A', B and C are described in the text and shown in Figure 2. All numbers are in millimeters.

	r	R	A	A'	B	C
ADS configuration	660	680	850	553	100	578

Two planes will be selected to define rotations and translations. One will be the plane of the motion plate, and the other will be the focal plane. The translations on these planes will be defined as $(\Delta x, \Delta y, \Delta z)$. The rotations can be defined by the Euler angles $(\Delta\phi, \Delta\theta, \Delta\gamma)$ or by the rotations around the x, y and z axis (tip,tilt,twist)=$(\Delta\theta_x, \Delta\theta_y, \Delta\theta_z)$. The mathematics to perform these rotations is described in Appendix A.

Figure 2 shows distances between different parts of the DECam camera. The distance between the fixed and motion plates is labeled A. The distance between the motion plate and the camera's Center of Mass (CM) is B, and the distance between the CM and the focal plane is labeled C. Figure 2 also shows the planes where the actuators insert into the fixed and motion plates. The distance between these planes is labeled A'. The values of the previous distances are given in Table 2.

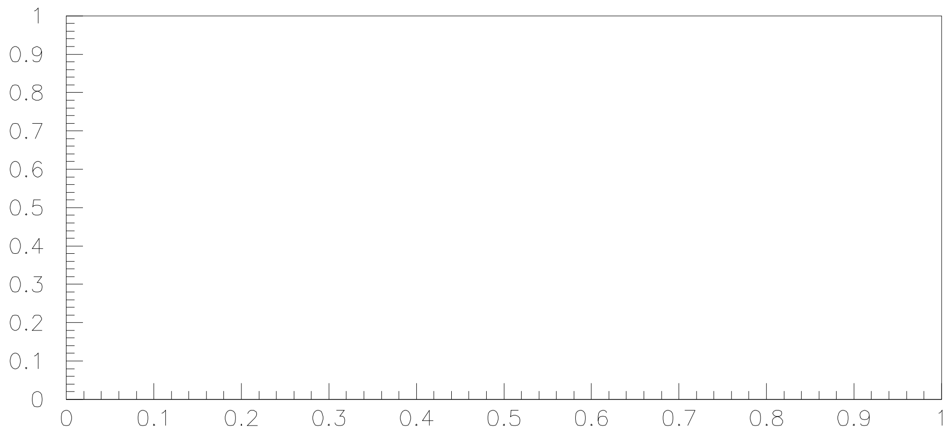

Figure 2. Side view of hexapod system and focal plane. Also shown is the camera Center of Mass (CM). The different distances are listed in Table 2 and described in the text.

To avoid coupling rotations with translations the rotation of either the motion or focal planes has to be done at the coordinate system origin. After the rotation is performed the plane can be translated to its designed position. This rotation affects the motion plate insertion points, so the final position of these insertion points is obtained as follows:

- Translate $\vec{r}_{0_i}$ until the motion plate or focal plane is at the coordinate systems origin. This translation will be given by the vector $\vec{d}$.
- Perform rotation *Rot*.
- Translate insertion points so the motion plate or focal plane is back at their design position. This translation is again given by $\vec{d}$.
- Performed translations $\vec{r}_0 = (\Delta x, \Delta y, \Delta z)$

Then the final position of the motion plate insertion points will be

$$\vec{r}_i = Rot\,(\vec{r}_{0_i} - \vec{d}) + \vec{d} + \vec{r}_0 \tag{3}$$

When defining the camera rotations and translations relative to the motion plate we have $\vec{d} = (0, 0, A)$. When the focal plane is used to define rotations and translations we have $\vec{d} = (0, 0, A + B + C)$. For motions relative to the Center of Mass $\vec{d} = (0, 0, A + B)$. For small angle rotations one can use tip, tilt and twist and *Rot* will be the matrix given in Appendix A Eq. 59. In the general case we can use the matrix given in Appendix A Eq. 51.

The fixed plate insertion points don't move, therefore

$$\vec{R}_i = \vec{R}_{0_i} \tag{4}$$

The actuators lie in the line that connects points $\vec{r}_i$ and $\vec{R}_i$. The vector difference between the points is

$$\vec{L}_i = \vec{r}_i - \vec{R}_i \tag{5}$$

Therefore the actuator's length is given by $L_i = |\vec{L}_i|$, and the unit vectors in the direction of the actuators are

$$\hat{f}_i = \frac{\vec{L}_i}{L_i} \tag{6}$$

The Center of Mass position $\vec{r}_{0_{CM}} = (0, 0, A + B)$ is also rotated and translated according to Eq. 3 to give $\vec{r}_{CM} = Rot\,(\vec{r}_{0_{CM}} - \vec{d}) + \vec{d} + \vec{r}_0$.

When the camera is rotated away from the vertical position we need to rotate all vectors associated with the camera or the cage (except gravity of course). This rotation is performed using the Eq. 51 matrix in Appendix A. That is

$$\vec{V'} = R(\phi, \theta, \gamma)\,\vec{V} \tag{7}$$

where $\vec{V}$ is any of the vectors calculated in this section.

3. FORCES ON THE HEXAPOD ACTUATORS

In this section we will study the forces exerted by the hexapod actuators. We define a force as positive when the actuator pushes on the motion plate which means that the actuator is under compression. When the force is negative the actuator is pulling on the motion plate and it is under tension.

To study the actuator forces both the fixed and motion plates (or cage and camera) will be rotated as a unit between 0 and 90 degrees around the y-axis and between 0 and 360 degrees around the camera axis. In terms of Euler angles (see Appendix A) this means $\phi = 0°$, $0° \leq \theta \leq 90°$ and $0° \leq \gamma \leq 360°$, which covers the full range of actuator forces.

The motion plate will be assumed to be parallel to the fixed plate and sharing the same axis. In the notation of the previous section this means $\Delta x = \Delta y = \Delta z = 0$ and $\Delta\phi = \Delta\theta = \Delta\gamma = 0$ (or $\Delta\theta_x = \Delta\theta_y = \Delta\theta_z = 0$). The cases where the motion plate is rotated or translated relative to the fixed plate can be easily studied too, but since these displacements are small and the motion plate will be oriented in every possible position, for simplicity we decided to just run the parallel and coaxial case.

As explained in the previous section once the positions of the fixed and motion plates (and the actuator insertion points) are given, the unit vectors $\hat{f}_i$ can be easily calculated (see Eqs 1 to 6). With the vectors $\hat{f}_i$, the relations given in Appendix B (Eqs 60 to 67) can be used to calculate the force per unit camera weight (F/w) exerted by the actuators.

When the camera is vertical all actuator forces will be equal and independent of γ. As the telescope rotates the force on the actuators will change. Perhaps the most interesting case to start the discussion with is the case in which the camera is horizontal ($\theta = 90°$) and rotates around its axis ($0° \leq \gamma \leq 360°$). Figure 3 shows F/w for $\theta = 90°$ as a function of γ for all six actuators. The actuators are paired according to the way they insert in the motion plate. When looking from the focal plane towards the hexapod, $\gamma = 0$ correspond to the case in which actuators 1 and 6 insert on both sides of 6 o'clock, with actuator 6 coming from the left and 1 from the right. Actuators 2 and 3 insert on both sides of 2 o'clock with actuator 2 going up and 3 coming down. Actuators 4 and 5 insert on both sides of 10 o'clock with actuator 4 coming down and 5 going up.

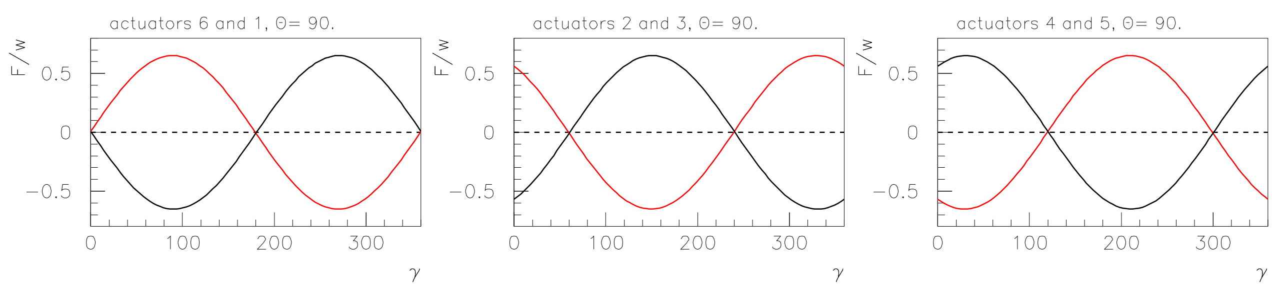

Figure 3. Actuators forces per unit camera weight (F/w) for $\theta = 90°$ as a function of γ. The actuators are paired according to the way they insert in the motion plate: 6 and 1 (left), 2 and 3 (center), 4 and 5 (right). Actuators 2, 4 and 6 are in red, 1, 3 and 5 in black.

As we can see from the left plot in Figure 3 the actuators on both sides of the 6 o'clock ($\gamma = 0$) insertion point experience an almost negligible force. The main purpose of these small forces is to cancel out the residual moments left from the push-pull of the forces around the 2 and 10 o'clock insertion points. At 6 o'clock there is also a small up or down component, but this force is small because in this position actuators 1 and 6 are almost horizontal, so the weight of the camera is being supported by the forces at the 2 and 10 o'clock insertion points.

The actuator forces for the 2 and 10 o'clock insertion points can be red out from the $\gamma = 0$ point at the center and right plots in Figure 3. We can see that the going up actuators (2 and 5) are pushing while the downward ones (3 and 4) are pulling. The pushing and pulling forces are almost equal. This produces a total force whose main component is on the vertical plane and a small horizontal component whose sign depends on the exact CM position. This is needed to support the camera weight and to cancel (together with the small horizontal forces at 6 o'clock) the moment produce by the fact that the CM is displaced from the motion plate.

As the angle γ increase the camera rotates counterclockwise. At $\gamma = 60°$ the 2 o'clock point has moved to 12 o'clock. At this point actuators 2 and 3 are almost horizontal (see center plot in Figure 3). This produces a force that only serves to cancel moments and the weight of the camera is supported by the now at 8 and 4 o'clock points. The maximum forces are close to the 3 (or 9) o'clock point because here there is one insertion point between 12 and 6 o'clock but 2 between 6 and 12 o'clock.

Figure 4 shows F/w for $\theta = 60°$ as a function of γ for all six actuators.

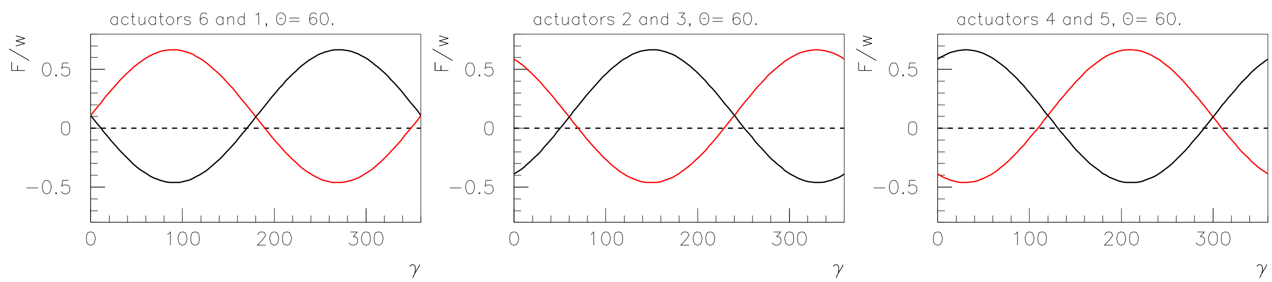

Figure 4. Actuators forces per unit camera weight (F/w) for $\theta = 60°$ as a function of γ. The actuators are paired according to the way they insert in the motion plate: 6 and 1 (left), 2 and 3 (center), 4 and 5 (right). Actuators 2, 4 and 6 are in red, 1, 3 and 5 in black.

So we can study all the actuator forces by just following one insertion point around the clock or if we only want the maximum force by following just one actuator. Figure 5 shows the forces on actuators 1 and 6 for different values of θ. The light blue, blue, green, red and black curves correspond to θ equals to 0, 30, 45, 60 and 90 degrees. We can see that the maximum force exerted by an actuator is about 2/3 of the total camera weight.

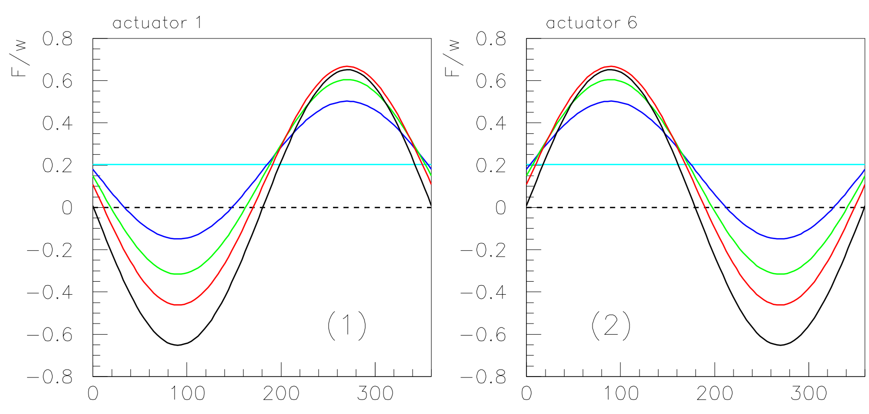

Figure 5. F/w as a function of γ for actuator 1 (plot 1), and actuator 6 (plot 2). The light blue, blue, green, red and black curves correspond to θ equals to 0, 30, 45, 60 and 90 degrees.

4. MOTION AND ACCURACY STUDIES

Now we will turn our attention to the study of how the hexapod system moves. Since we are trying to understand the accuracy with which the hexapod system can position the camera we will restrict ourselves to small motions (of the order of a millimeter at the focal plane) which, as explained in Appendix A.1, have the advantage that rotations commute and movements can be treated in a linear way.

The restriction to small motions will not change the generality of our conclusions for two reasons: 1) almost inevitably after large movements there will be small adjustments to reach the final camera position, so the final accuracy of the hexapod system relies on the ability to make small movements, and 2) given a change in linear translations and in angles the change in actuators length can be easily calculated in a very general way (see Section 2), so except for the inversion to go from actuator lengths to motion, every thing else calculated in this section can be easily generalized.

For small motions the relation between translations and rotations with respect to the three coordinate axis and the actuators length will be given by a 6x6 matrix M. Equation 59 in Appendix A.1 shows that for small rotations we can calculate each rotation independently and then add the effect of each one to obtain the final rotation. For translations this is also the case, so the columns of M can be calculated by performing one motion at a time. For example a translation along the x-axis by an amount Δx will induce a change in the length of the six actuators $(\Delta L_1^x, \Delta L_2^x, \Delta L_3^x, \Delta L_4^x, \Delta L_5^x, \Delta L_6^x)$ and this will constitute the first column of our matrix. A translation along the y-axis by an amount Δy will induce a change in the length of the six actuators $(\Delta L_1^y, \Delta L_2^y, \Delta L_3^y, \Delta L_4^y, \Delta L_5^y, \Delta L_6^y)$. And the effect of both translations will be $(\Delta L_1^x + \Delta L_1^y, \Delta L_2^x + \Delta L_2^y, \Delta L_3^x + \Delta L_3^y, \Delta L_4^x + \Delta L_4^y, \Delta L_5^x + \Delta L_5^y, \Delta L_6^x + \Delta L_6^y)$. So we calculate the 6x6 matrix M by performing small motions using the general equations given in Section 2. We define translations and rotations in two different places: 1) the motion plate, and 2) the focal plane. For the ADS hexapod configuration and translations and rotations defined at the motion plate we obtain

$$\begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} = \begin{pmatrix} 0.265 & -0.514 & 0.816 & 0.217 & -2.733 & -1.795 \\ -0.578 & -0.028 & 0.816 & 2.476 & 1.179 & 1.795 \\ 0.314 & 0.486 & 0.816 & 2.259 & 1.554 & -1.795 \\ 0.314 & -0.486 & 0.816 & -2.259 & 1.554 & 1.795 \\ -0.578 & 0.028 & 0.816 & -2.475 & 1.179 & -1.795 \\ 0.265 & 0.515 & 0.816 & -0.217 & -2.733 & 1.795 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} \tag{8}$$

where Δx, Δy and Δz are translations along the x, y and z axis in micrometers, $\Delta\theta_x$, $\Delta\theta_y$ and $\Delta\theta_z$ are rotations around the x, y and z axis in arcseconds and ΔL_1, ΔL_2, ΔL_3, ΔL_4, ΔL_5 and ΔL_6 are actuators length changes in micrometers.

Equation 8 can be inverted to give a relation between the change in actuators length and the camera translations and rotations:

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} = \begin{pmatrix} 0.049 & -0.564 & 0.514 & 0.514 & -0.564 & 0.049 \\ -0.622 & -0.269 & 0.354 & -0.354 & 0.269 & 0.622 \\ 0.204 & 0.204 & 0.204 & 0.204 & 0.204 & 0.204 \\ 0.060 & 0.132 & 0.071 & -0.071 & -0.132 & -0.060 \\ -0.117 & 0.006 & 0.111 & 0.111 & 0.006 & -0.117 \\ -0.093 & 0.093 & -0.093 & 0.093 & -0.093 & 0.093 \end{pmatrix} \begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} \tag{9}$$

The fact that Eq. 8 inverts without a problem proves that the hexapod system is not over-constraint. In other words we can move one actuator at a time and the entire system will move without problems. For example if the length of actuator 1 changes by $\Delta L_1 = 10\ \mu$m then the camera will move in x, y and z by 0.5, -6.2 and 2.0 micrometers and it will rotate around x, y and z by 0.6, -1.2 and -0.9 arcseconds. As expected there is a lot of symmetry in Eq. 9. For example actuators 1 and 6 insert in the motion plate at two points on oposite sides of the x-axis, so they will both push in x and z in the same direction but they will push in opposite directions in y, so the translation in z and rotations around the y-axis will have the same sign and translations in y and rotation around the x and z-axis will have opposite signs. And this is what we see in Eq. 9.

Equations 8 and 9 are essential to calculate motion but no particular number in the two matrices is very important, specially because these numbers depend on how we define the coordinate system with respect to the insertion points. So we need other numbers to understand how precisely the hexapod system can move the camera. We see for example from Eq. 8 that if we want to move by $\Delta x = 10\ \mu$m then the actuators lengths ΔL_1 to ΔL_6 will have to change by 2.65, -5.78, 3.14, 3.14, -5.78 and 2.65 micrometers respectively. If the minimum actuator step size is 1 micrometer then the changes will be 3, -6, 3, 3, -6 and 3 steps, so there will be errors due to the finite size of the actuators steps. Over many motions these errors will act randomly and generate a distribution of errors in all three translations and rotations. The rms of these distributions can be calculated by adding the numbers in Eq. 9 in quadratures. If we write Eq. 8 as $\overrightarrow{\Delta L} = M\,\overrightarrow{\Delta x}$ and Eq. 9 as $\overrightarrow{\Delta x} = M^{-1}\,\overrightarrow{\Delta L}$ and assume that the step size S_L is the same in all actuators then the rms of the distributions will be $\sigma_i = \sqrt{\sum_j (M^{-1}_{i,j})^2}\,\sigma_L$, with $\sigma_L = S_L/\sqrt{12} \approx 0.29\,S_L$ (the factor $1/\sqrt{12}$ is just the rms of a square distribution of width 1). For Eq. 9 these number are:

$$(\sigma_x, \sigma_y, \sigma_z, \sigma_{\theta_x}, \sigma_{\theta_y}, \sigma_{\theta_z}) = (1.08, 1.08, 0.50, 0.23, 0.23, 0.23)\,\sigma_L \tag{10}$$

As with every distribution we may wonder how far the tails of the above mentioned distributions go. So another useful number is the maximum error that can be introduced due to the finite actuator step size. If h is the minimum interval that contains all errors, and if we assume that all step size errors conspire to give the largest possible deviation then $h_i = (\sum_j |M^{-1}_{i,j}|)\,S_L$. For Eq. 9 these gives

$$(h_x, h_y, h_z, h_{\theta_x}, h_{\theta_y}, h_{\theta_z}) = (2.25, 2.25, 1.23, 0.53, 0.47, 0.56)\,S_L \tag{11}$$

We can now calculate the equivalent of Eqs 8 to 11 when translations and rotations are defined in the focal plane. They are

$$\begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} = \begin{pmatrix} 0.265 & -0.514 & 0.816 & -1.474 & -3.603 & -1.795 \\ -0.578 & -0.028 & 0.816 & 2.383 & 3.079 & 1.795 \\ 0.314 & 0.486 & 0.816 & 3.858 & 0.525 & -1.795 \\ 0.314 & -0.486 & 0.816 & -3.857 & 0.525 & 1.795 \\ -0.578 & 0.028 & 0.816 & -2.382 & 3.079 & -1.795 \\ 0.265 & 0.515 & 0.816 & 1.475 & -3.603 & 1.795 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} = \begin{pmatrix} -0.336 & -0.543 & 0.878 & 0.878 & -0.543 & -0.336 \\ -0.820 & -0.701 & 0.119 & -0.119 & 0.701 & 0.820 \\ 0.205 & 0.204 & 0.204 & 0.204 & 0.204 & 0.204 \\ 0.060 & 0.132 & 0.071 & -0.071 & -0.132 & -0.060 \\ -0.117 & 0.006 & 0.111 & 0.111 & 0.006 & -0.117 \\ -0.093 & 0.093 & -0.093 & 0.093 & -0.093 & 0.093 \end{pmatrix} \begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} \tag{13}$$

$$(\sigma_x, \sigma_y, \sigma_z, \sigma_{\theta_x}, \sigma_{\theta_y}, \sigma_{\theta_z}) = (1.53, 1.53, 0.50, 0.23, 0.23, 0.23)\,\sigma_L \tag{14}$$

$$(h_x, h_y, h_z, h_{\theta_x}, h_{\theta_y}, h_{\theta_z}) = (3.51, 3.28, 1.23, 0.53, 0.47, 0.56)\,S_L \tag{15}$$

We can see that the only differences between Eqs 8 and 12 are in the columns corresponding to $\Delta\theta_x$ and $\Delta\theta_y$. These are the only expected differences and they are due to the fact that rotations around the x and y axis at the motion plate produce translations at the focal plane.

Eq. 12 inverts without any problem, which again proves that the hexapod system is not over-constraint over a large range of parameters. As briefly discussed in Section 2 this is a general property and it goes as follows. In general we have six actuator insertion points in the motion plate and six in the fixed plate. The six insertion points in the motion plate can be arbitrarily positioned with respect to the six insertion points in the fixed plate. Once this relative position is fixed the distances between the insertion points can be calculated. This set of distances is unique, that is different positions of the motion plate relative to the fixed one will create a different set of six distances. This is true as long as we don't line up two points in the motion plate with two points in the fixed plate identically. This will create degeneracies but also will make the camera unable to stand in space, because six non-degenerate parameters are needed to position a body in three dimensional space. So if the camera is to stand in space, for a given position of the motion and fixed plates the distances between the actuator insertion points is unique. This one-to-one correspondence means that the relation can be inverted, or that specifying a set of six distances will create a unique position of the motion plate relative to the fixed one. Therefore only one of the six distances can be changed at a given time which guarantees that the hexapod system is not over-constrained. Therefore as long as the actuators can rotate freely in three dimensions at the insertion points the hexapod system is not constrained at all. The only constraining will come from the binding at the actuator insertion points, so these insertions will have to be designed carefully to make sure that the binding is small enough so that the system will move when any of the actuators is moved by one step.

Table 3 summarizes the rms errors σ_i and the minimum intervals containing all possible errors h_i. As expected from the effect of x and y rotations just mentioned, the numbers for x and y translations increase when going from the motion plate to the focal plane. Other than that the numbers don't change much between configurations.

Table 3. Summary of rms errors σ_i and the minimum intervals containing all possible errors h_i.

		σ_x/σ_L	σ_y/σ_L	σ_z/σ_L	$\sigma_{\theta_x}/\sigma_L$	$\sigma_{\theta_y}/\sigma_L$	$\sigma_{\theta_z}/\sigma_L$
ADS configuration	motion plate	1.08	1.08	0.50	0.23	0.23	0.23
	focal plane	1.53	1.53	0.50	0.23	0.23	0.23
		h_x/S_L	h_y/S_L	h_z/S_L	h_{θ_x}/S_L	h_{θ_y}/S_L	h_{θ_z}/S_L
ADS configuration	motion plate	2.25	2.25	1.23	0.53	0.47	0.56
	focal plane	3.51	3.28	1.23	0.53	0.47	0.56

The only point that remains to be studied is the stability with respect to the selection of the coordinate system. The angle δ_0 in Eqs 1 and 2 rotates the insertion points around the z-axis. We studied the dependence of the numbers in Table 3 as a function of δ_0. All six rms's σ_i are completely independent of δ_0, and the values of h_i only change slightly. Figure 6 shows the values of h_x/S_L to h_{θ_z}/S_L as a function of δ_0. We can see that the numbers are very stable.

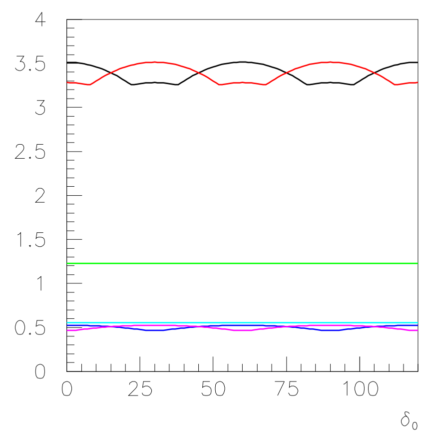

Figure 6. This figure shows $(h_x, h_y, h_z, h_{\theta_x}, h_{\theta_y}, h_{\theta_z})/S_L$ (black, red, green, blue, magenta and light blue) as a function of δ_0 with displacements and rotations defined at the focal plane. The angle δ_0 is defined in Eqs 1-2.

The tolerances specified in the hexapod RFP * are: $\Delta_x = \Delta_y = \pm\ 25\ \mu$m, $\Delta_z = \pm\ 7.50\ \mu$m and $\Delta\theta_x = \Delta\theta_y = \pm\ 3$ arcseconds (1 arcsec = 4.8 μrad). Using Table 3 and the focal plane as a reference for rotations and translations the previous specifications translate into the following step sizes S_L: 14 μm (50/3.5) for lateral

*See "Request for proposals (RFP). DECam Hexapod Position Adjustment System Specification". R. French Leger, Dave McGinnis, Andy Stefanik, Darren DePoy, Gaston Gutierrez, Brenna Flaugher. February 4, 2008 - Revision 3.

motion, 12 μm (15/1.23) for focusing and 12 μm (6/0.5) for tip and tilt. So we see that an actuator resolution of 12 μm is enough to satisfy our specifications.

5. EFFECT OF LOADS ON THE ACTUATORS

The design stiffnes of the hexapod actuators is 240 N/μm. Then a displacement ΔL produces a force F given by

$$F[N] = 240\ \Delta L[\mu m] \tag{16}$$

with ΔL in micrometers and F in Newtons. For a camera weight of 35000 Newtons the maximum actuator load will be 23000 Newtons, which will produce an actuator deformation of 96 microns. The effect of these deformations can be easily calculated as follow: 1) calculate forces as in Section 3, 2) with Eq. 16 calculate the actuator deformations and 3) with Eq. 13 calculate the focal plane motion due to these deformations. The result of these calculations is shown in Figure 7.

Plots 7.1 to 7.6 show the focal plane displacements $(\Delta x, \Delta y, \Delta z, \Delta\theta_x, \Delta\theta_y, \Delta\theta_z)$ as a function of γ due to the elastic deformation of the actuators. The light blue, blue, green, red and black curves in each plot correspond to θ values of 0, 30, 45, 60 and 90 degrees. This range of θ and γ covers the entire motion of the telescope. The displacements shown in the plots exceed our specifications. This of course means that as the telescope moves we will have to step the actuators to correct for the actuators elastic deformations. One can do this by monitoring the length of the actuators, or by installing devices to measure the position of the camera relative to the cage. In our minds the problem of just controlling the length of the actuators is that most likely there will be other deformations (like at the actuator insertion points) that can not be compensated that way. So it appears to us that installing devices to measure the position of the camera with respect to the cage is the right thing to do, specially if these devices are installed near the focal plane.

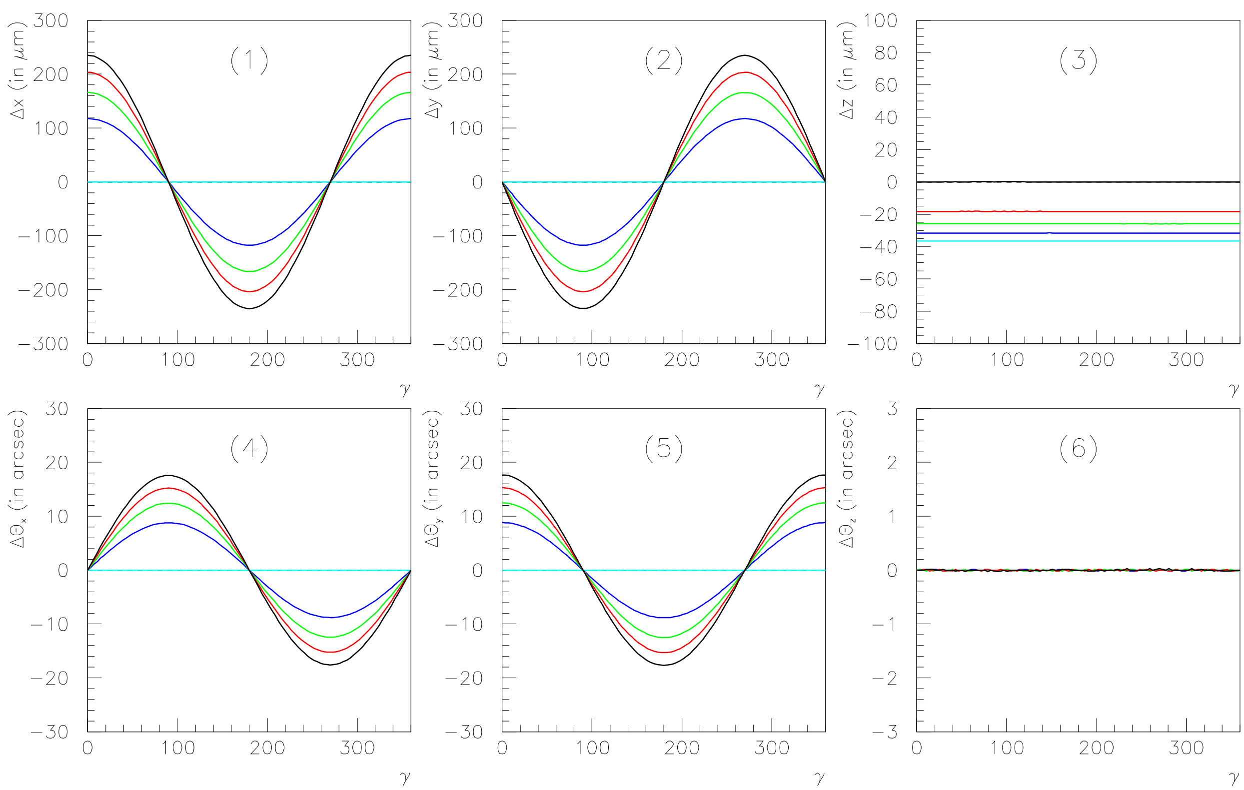

Figure 7. Plots 1-6 show the focal plane motions $(\Delta x, \Delta y, \Delta z, \Delta\theta_x, \Delta\theta_y, \Delta\theta_z)$ produced by the elastic deformations of the actuators as the telescope moves. The displacements are plotted as a function of γ and the light blue, blue, green, red and black curves correspond to θ values of 0, 30, 45, 60 and 90 degrees. Lengths are in micrometers, angles in arcseconds.

We want now to turn our attention to the problem of trying to focus the camera during exposures. As we can see from Eq. 13 if we step all actuators by the same amount ($\Delta L_1 = \Delta L_2 = \Delta L_3 = \Delta L_4 = \Delta L_5 = \Delta L_6 = \Delta L$) then the focal plane moves in the z direction by the amount $\Delta z = 1.22\,\Delta L$, and all the other motions are zero. Then assuming that all actuators can act equally the motion of focusing the camera should be smooth and should only affect the z-motion. This is true except for elastic deformations. As the actuators move theirs lengths will change and since the loads on the actuators is usually very different they will deform by different amounts. We studied this problem in the following way: 1) for a given camera position we calculated all the actuator loads, 2) we moved the camera axially by 10 millimeters and recalculated all the loads, 3) we calculated the elastic deformation on the actuators due to the change in loads and 4) we used Eq. 13 to calculate all the displacements. The results of this calculation are shown in Figure 8. The plots are defined as in Figure 7. We can see that all motions are well within specifications. So we should be able to focus the camera smoothly if all actuators can be made to move in sync by the same amounts.

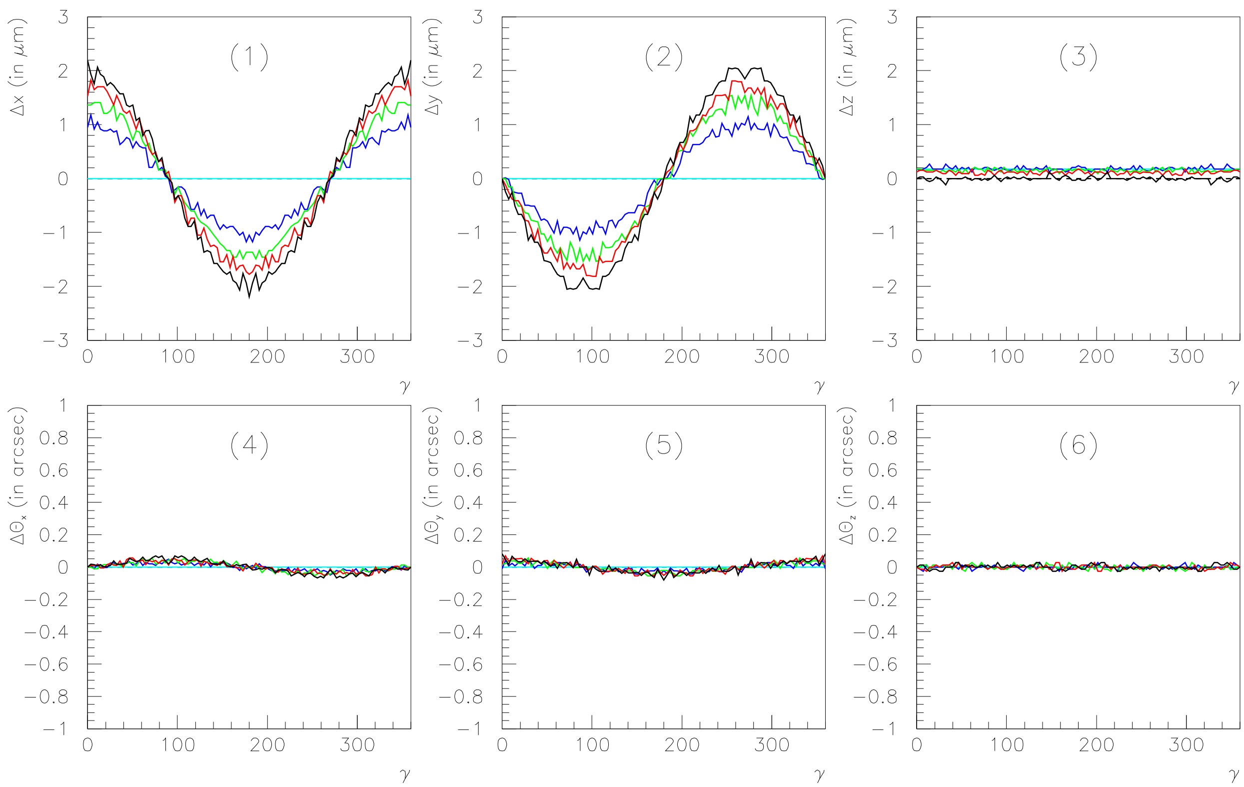

Figure 8. Plots 1-6 show the focal plane motions ($\Delta x, \Delta y, \Delta z, \Delta\theta_x, \Delta\theta_y, \Delta\theta_z$) produced by the elastic deformations of the actuators as the camera is moved by 10 millimeters along its axis. The displacements are plotted as a function of γ and the light blue, blue, green, red and black curves correspond to θ values of 0, 30, 45, 60 and 90 degrees. Lengths are in micrometers, angles in arcseconds.

6. CAMERA VIBRATIONS

In this section we will calculate eigenvalues and eigenvectors for the hexapod vibrations. We will make two (in our minds) very reasonable simplifying assumption. The first one is that the principal axis of the moment of inertia tensor coincide with the camera axis. The second one is that vibrations are dominated by the elongation of the hexapod legs.

Due to the filters and the electronic crates the camera does not have rotational symmetry around the telescope's axis. But we will assume that this deviation from symmetry will have a small effect in the vibration modes. In this case the moment of inertia tensor will be diagonal in the coordinate system of the camera.

The fixed and motion plates can be designed such that they deform very little in comparison to the deformations of the haxapod legs, so it is reasonable to ignore these deformations. Also we believe that the transverse vibrations of the hexapod legs can be ignored. The reason is that the change in length of the hexapos legs due to tranverse vibrations is negligible, and therefore the camera motion should not be affected by these vibrations. We will further assume that the elongation of the hexapos legs is elastic and can be described by a single constant. This constant will be dominated by the weakest point in the leg, and could be either the actuator or the joint.

In the presence of motion equations 61 and 62 have to be modified to read

$$\sum_{i=1}^{6} F_i \, \hat{f}_i \; + \; \vec{w} = \frac{d(m\vec{v})}{dt} \tag{17}$$

$$\sum_{i=1}^{6} F_i \, \vec{\Delta}_i \times \hat{f}_i = \frac{d(I\,\vec{w})}{dt} \tag{18}$$

where (as before) the translations and rotations are defined with respect to the center of mass. For small deformations the motion of the camera due to the elastic deformation of the hexapod legs can be considered linear. Therefore vibrations will be linearly superimposed to the camera motion due to gravity, then for calculating vibrations we can set $\vec{w} = 0$ in Eq. 17. Following the notation of Eq. 64 we can write

$$\begin{pmatrix} f_{x_1} & f_{x_2} & f_{x_3} & f_{x_4} & f_{x_5} & f_{x_6} \\ f_{y_1} & f_{y_2} & f_{y_3} & f_{y_4} & f_{y_5} & f_{y_6} \\ f_{z_1} & f_{z_2} & f_{z_3} & f_{z_4} & f_{z_5} & f_{z_6} \\ n_{x_1} & n_{x_2} & n_{x_3} & n_{x_4} & n_{x_5} & n_{x_6} \\ n_{y_1} & n_{y_2} & n_{y_3} & n_{y_4} & n_{y_5} & n_{y_6} \\ n_{z_1} & n_{z_2} & n_{z_3} & n_{z_4} & n_{z_5} & n_{z_6} \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{pmatrix} = \frac{d^2}{dt^2} \begin{pmatrix} m\,\Delta x \\ m\,\Delta y \\ m\,\Delta z \\ I_x\,\Delta\theta_x \\ I_y\,\Delta\theta_y \\ I_z\,\Delta\theta_z \end{pmatrix} \tag{19}$$

As shown in Eqs 8, 12 and **??** for small hexapod motions we can establish a linear relation between the camera movement and the change in length of the hexapod legs. The relationship will be given by a matrix E defined as:

$$\begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} = E \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} \tag{20}$$

The force F_i along a hexapod leg and the leg's deformation ΔL_i are related by $F_i = -k\,\Delta L_i$. The minus sign is due to the fact that the forces F_i are defined as positive when they push on the camera. For vibrations when ΔL_i is positive the forces are pulling on the camera and therefore the need for the minus sign. Then Eq. 19 reduces to

$$(M \cdot E) \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} = -\frac{m}{k}\,\frac{d^2}{dt^2} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \tilde{I}_x\,\Delta\theta_x \\ \tilde{I}_y\,\Delta\theta_y \\ \tilde{I}_z\,\Delta\theta_z \end{pmatrix} \tag{21}$$

where M and E are the matrices in Eqs 19 and 20, and $\tilde{I}_i = I_i/m$. For the Configuration 1 parameters given in Tables 1 and 2 we have

$$
(a) = (M \cdot E) = \begin{pmatrix} 1.440 & 0.000 & 0.000 & 0.000 & -0.494 & 0.000 \\ 0.000 & 1.440 & 0.000 & 0.494 & 0.000 & 0.000 \\ 0.000 & 0.000 & 3.120 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.494 & 0.000 & 0.670 & 0.000 & 0.000 \\ -0.494 & 0.000 & 0.000 & 0.000 & 0.670 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.924 \end{pmatrix} \tag{22}
$$

which clearly decouples Eq. 21 into the following equations

$$
\begin{pmatrix} a_{11} & a_{15} \\ a_{51} & a_{55} \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta\theta_y \end{pmatrix} = -\frac{1}{\omega_0^2} \frac{d^2}{dt^2} \begin{pmatrix} \Delta x \\ \tilde{I}_y \, \Delta\theta_y \end{pmatrix} \tag{23}
$$

$$
\begin{pmatrix} a_{22} & a_{24} \\ a_{42} & a_{44} \end{pmatrix} \begin{pmatrix} \Delta y \\ \Delta\theta_x \end{pmatrix} = -\frac{1}{\omega_0^2} \frac{d^2}{dt^2} \begin{pmatrix} \Delta y \\ \tilde{I}_x \, \Delta\theta_x \end{pmatrix} \tag{24}
$$

$$
a_{33} \, \Delta z = -\frac{1}{\omega_0^2} \frac{d^2}{dt^2} (\Delta z) \tag{25}
$$

$$
a_{66} \, \Delta\theta_z = -\frac{1}{\omega_0^2} \frac{d^2}{dt^2} \left(\tilde{I}_z \, \Delta\theta_z \right) \tag{26}
$$

with $a_{11} = a_{22} = 1.440$, $a_{15} = a_{51} = -a_{24} = -a_{42} = -0.494$, $a_{55} = a_{44} = 0.670$, $a_{33} = 3.120$ and $a_{66} = 0.924$. For convinience we have written $\omega_0 = \sqrt{k/m}$ which is the angular frequency of one haxapod arm when loaded with the entire camera mass. The solutions of the harmonic oscillator equations 25 and 26 are $sin(\omega t)$ and $cos(\omega t)$ with $\omega_z = \omega_0 \sqrt{a_{33}}$ for Eq. 25 and $\omega_{\theta_z} = \omega_0 \sqrt{a_{66}/\tilde{I}_z}$ for Eq. 26.

To solve Eq. 23 (or Eq. 24) we make a linear transformation to a new set of variable (α, β)

$$
\begin{pmatrix} \Delta x \\ \Delta\theta_y \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} = U \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} \tag{27}
$$

with this transformation Eq. 23 becomes

$$
\left[U^1 \begin{pmatrix} a_{11} & a_{15} \\ a_{51}/\tilde{I}_y & a_{55}/\tilde{I}_y \end{pmatrix} U \right] \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} = -\frac{1}{\omega_0^2} \frac{d^2}{dt^2} \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} \tag{28}
$$

and now we construct U such that the matrix in square brackets becomes diagonal. That is

$$
U^1 \begin{pmatrix} a_{11} & a_{15} \\ a_{51}/\tilde{I}_y & a_{55}/\tilde{I}_y \end{pmatrix} U = \begin{pmatrix} \lambda_\alpha & 0 \\ 0 & \lambda_\beta \end{pmatrix} \tag{29}
$$

so we arrive at having to diagonalize the 2x2 matrix

$$
\begin{pmatrix} A - \lambda_j & B \\ C & D - \lambda_j \end{pmatrix} \begin{pmatrix} u_{1j} \\ u_{2j} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{30}
$$

with $(A, B, C, D) = (a_{11}, a_{15}, a_{51}/\tilde{I}_y, a_{55}/\tilde{I}_y)$ for Eq. 23, and $(A, B, C, D) = (a_{22}, a_{24}, a_{42}/\tilde{I}_x, a_{44}/\tilde{I}_x)$ for Eq. 24. Once the diagonalization problem is solved Eq. 28 reduces to

$$
\frac{d^2 \alpha(t)}{dt^2} + \omega_0^2 \, \lambda_\alpha \, \alpha(t) = 0 \tag{31}
$$

$$
\frac{d^2 \beta(t)}{dt^2} + \omega_0^2 \, \lambda_\beta \, \beta(t) = 0 \tag{32}
$$

which are just the equations of the harmonic oscillator with frequencies $\omega_i = \omega_0 \sqrt{\lambda_i}$. The solution of Eq. 30 gives the eigenvalues

$$\lambda_{\pm} = \frac{(A+D) \pm \sqrt{(A-D)^2 + 4CB}}{2} \tag{33}$$

and the eigenvectors

$$U = \begin{bmatrix} 1 & 1 \\ -(A-\lambda_-)/B & -(A-\lambda_+)/B \end{bmatrix} \tag{34}$$

Since we are assuming rotational symmetry we have that $I_x = I_y$, then the eigenvalues of Eqs 23 and 24 are the same. Then there will be six vibration resonances with frequecies $f_i = f_0\sqrt{\lambda_i}$ with

$$\begin{aligned}
\lambda_{1,2} &= 0.5 \left[(a_{22} + a_{44}/\tilde{I}) - \sqrt{(a_{22} - a_{44}/\tilde{I})^2 + 4a_{24}^2/\tilde{I}} \right] && (35) \\
\lambda_{3,4} &= 0.5 \left[(a_{22} + a_{44}/\tilde{I}) + \sqrt{(a_{22} - a_{44}/\tilde{I})^2 + 4a_{24}^2/\tilde{I}} \right] && (36) \\
\lambda_5 &= a_{33} && (37) \\
\lambda_6 &= a_{66}/\tilde{I}_z && (38)
\end{aligned}$$

Writing $c1 = -(a_{22} - \lambda_1)/a_{24}$ and $c2 = -(a_{22} - \lambda_3)/a_{24}$ the corresponding matrix of column eigenvectors will be

$$U = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & 0 \\ -c_1 & 0 & -c_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{39}$$

The moments of inertia per unit mass of a solid cylinder are $\tilde{I}_x = \tilde{I}_y = (3R^2 + H^2)/12$ and $\tilde{I}_z = R^2/2$, where R and H are the radius and hight of the cylinder. For a cylinder with all the mass concentrated in the radial skin we have $\tilde{I}_x = \tilde{I}_y = (6R^2 + H^2)/12$ and $\tilde{I}_z = R^2$.

Assuming that DECam is a solid cylinder of R=0.6 meters and H=2 meters ($\tilde{I} = 0.423$ and $\tilde{I}_z = 0.180$) and Configuration 1 for the hexapods we have for the eigenvalues and eigenvectors

$$f = f_0 \begin{pmatrix} 0.865 & 0.865 & 1.508 & 1.508 & 1.766 & 2.265 \end{pmatrix} \tag{40}$$

$$U = \begin{pmatrix} 1.000 & 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & -1.400 & 0.000 & 1.687 & 0.000 & 0.000 \\ 1.400 & 0.000 & -1.687 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 1.000 \end{pmatrix} \tag{41}$$

Then the picture of the resonance modes is clear. For the two lowest frequencies we have (A_0 is just an arbitrary constant)

$$\begin{array}{c} \Delta x = 1.000\, A_0\, cos(2\pi f_1\, t) \\ \Delta\theta_y = 1.400\, A_0\, cos(2\pi f_1\, t) \end{array} \text{, and} \quad \begin{array}{c} \Delta y = 1.000\, A_0\, cos(2\pi f_1\, t) \\ \Delta\theta_x = -1.400\, A_0\, cos(2\pi f_1\, t) \end{array} \tag{42}$$

we see that $(\Delta x, \Delta\theta_y)$ oscillate with the same sign, which means that when the camera moves towards positive x it rotates clockwise around the y-axis. The same is true for $(\Delta y, \Delta\theta_x)$ because as the camera moves towards positive y it rotates counter clockwise around the x-axis. So this is like a person rocking standing on his feet.

For the next two modes we have

$$\begin{aligned} \Delta x &= 1.000\,A_0\,cos(2\pi f_1\,t) \\ \Delta\theta_y &= -1.687\,A_0\,cos(2\pi f_3\,t) \end{aligned} \quad , \text{ and } \quad \begin{aligned} \Delta y &= 1.000\,A_0\,cos(2\pi f_1\,t) \\ \Delta\theta_x &= 1.687\,A_0\,cos(2\pi f_3\,t) \end{aligned} \tag{43}$$

and this oscillation is like a person hanging from his head. The other two modes correspond to oscillations along and around the z-axis.

Figure 9.1 shows how the normalize frequencies vary with the moment of inertia per unit mass ($f_{1,2}/f_0$ are in red, $f_{3,4}/f_0$ in blue, f_5/f_0 in light blue and f_6/f_0 in green). The left dots correspond to the assumption that the camera is a solid uniform cylinder, the dots on the right to the assumtion that the weight of the camera is concentrated on the walls of the cylinder.

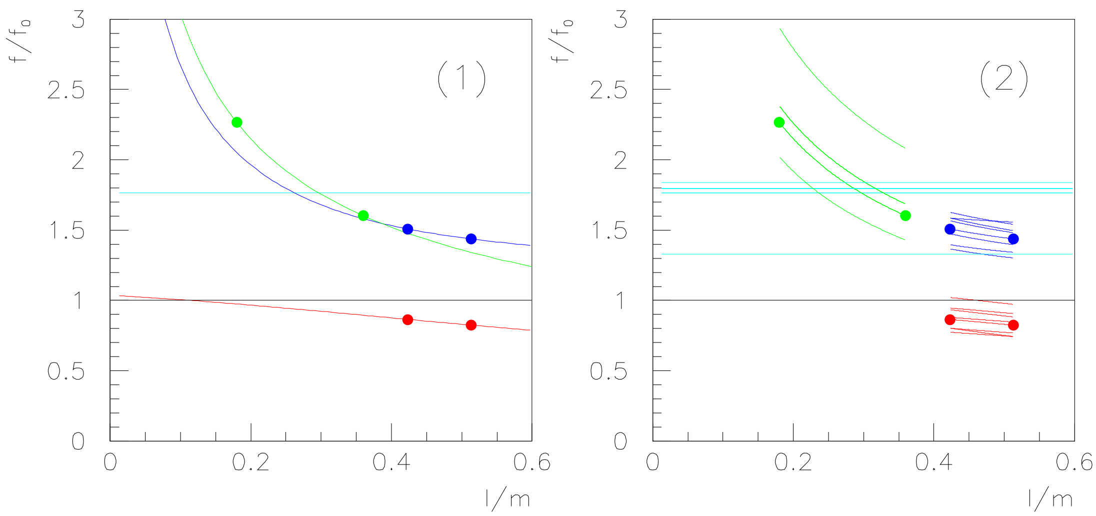

Figure 9. Left plot shows the normalize frequencies as a function of the moment of inertia per unit mass. The difference colors correspond to: red = $f_{1,2}/f_0$, blue = $f_{3,4}/f_0$, light blue = f_5/f_0 and green = f_6/f_0. The left (right) dots correspond to I/m for a solid (hollow) cylinder. The plot on the right shows the variation of f/f_0 for the different hexapod parameters listed in Table 4.

Table 4. List of the hexapod parameters used to calculate resonance frequencies. The parameters r to B are in millimeters, the resonant frequencies $f_{i/0} = f_i/f_0$ are calculated using the moment of inertia corresponding to a hollow cylinder.

	r	R	A	A'	B	a_{22}	a_{24}	a_{44}	a_{33}	a_{66}	$f_{1/0}$	$f_{3/0}$	$f_{5/0}$	$f_{6/0}$
Cnf. 1	612.5	715.0	697	697	100	1.440	0.494	0.670	3.120	0.924	0.82	1.44	1.77	1.60
	612.5	715.0	697	697	0	1.440	0.350	0.585	3.120	0.924	0.88	1.34	1.77	1.60
	612.5	715.0	697	697	200	1.440	0.638	0.783	3.120	0.924	0.77	1.54	1.77	1.60
Cnf. 2	612.5	715.0	697	500	100	1.389	0.404	0.710	3.223	1.022	0.91	1.40	1.80	1.68
	612.5	715.0	697	500	0	1.389	0.265	0.643	3.223	1.022	0.97	1.30	1.80	1.68
	612.5	715.0	697	500	200	1.389	0.542	0.805	3.223	1.022	0.85	1.50	1.80	1.68
	612.5	715.0	460	300	100	2.116	0.498	0.442	1.769	1.557	0.74	1.56	1.33	2.08
	612.5	612.5	697	697	100	1.307	0.586	0.739	3.386	0.736	0.74	1.48	1.84	1.43

We also studied the stability of the resonant frequencies with respect to different hexapod parameters. Table 4 shows a list of the parameters that were varied: r and R are the radius of the motion and fixed plates, A is the distance between the motion and fixed plates, A' is the distance between the two planes formed by the hexapod joint insertion points, and B is the distance between the motion plate and the Center of Mass. The matrix elements a_{22} to a_{66} are used to calculate the resonant frequencies in Eqs 35 to 38. The resonant frequencies $f_{i/0} = f_i/f_0$ are calculated using the moment of inertia corresponding to a hollow cylinder. Figure 9.2 shows the normalize resonance frequencies for all the cases listed in Table 4.

Looking at Figure 9.2 and Table 4 it is fair to say that the lowest resonant frequency will most likely be larger than $0.75\,f_0$. To calculate $f_0 = \sqrt{k/m}/2\pi$ we will assume a camera mass of $m = 3500$ Kg, and $k = (75, 100, 150, 200)$ Newtons/μm, then $f_0 = (23.3,\ 26.9,\ 32.9,\ 38.0)$ Hz. Then we believe that it is save to say that the lowest resonance mode will have a frequency larger than $0.75 \times 23.3 = 17.5$ Hz.

7. CONCLUSIONS

In this note we have shown that

- Six actuators arranged in a hexapod configuration form a system that is not over-constrained and that is stable with respect to changes in the design parameters.
- Given the space we plan to utilize for our hexapod system and the range of CM positions, the maximum force in any of the actuators is about 2/3 of the camera weight.
- The actuator to plate joints should be designed such that the actuators can rotate in all three dimensions and that the binding in the joints is small enough to allow for a single actuator step motion.
- The actuators should be made as stiff and short as possible in order to minimize camera motion due to elastic deformation in the actuators. Also the stiffer the actuators the larger the one step force exerted on the plates, which will facilitate the design of the actuator to plate joints.
- We have calculated the rms and the minimum interval that covers all possible motion errors due to the finite actuator step size. We find that an actuator positioning errors of 3 μm is enough to satisfy all the current positioning requirements. We have also shown that the requirement on tip and tilt determine the 3 μm positioning error in the actuators.

Perhaps one last comment on plate design versus actuators length is in order. Using long actuators allows to group the insertion points in pairs, which is equivalent to having three insertion points in each plate. This minimizes the moments on the plates but increases the elastic deformations in the actuators. If the plates can be designed in a very stiff way the insertion points can be opened up which will shorten the actuators length and minimize the camera motion due to elastic deformations.

APPENDIX A. ACTIVE ROTATIONS

This section describes the calculation of the matrix needed to perform active rotations in three dimensions. In an active rotation the objects are rotated while the coordinate system remains fixed. In Figure 10 the vector labeled $\overrightarrow{x}_1$ has been rotated by an angle θ to a new position $\overrightarrow{x}_2$. In terms of the coordinates (x, y) this rotation is written as

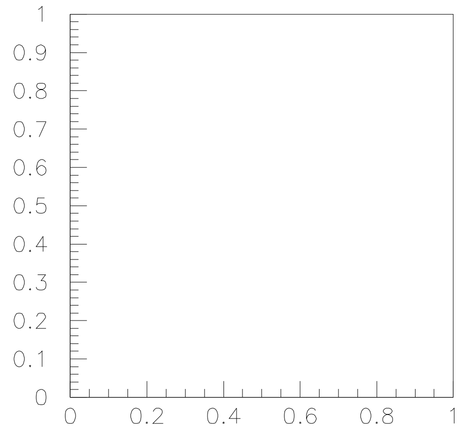

Figure 10. Two dimensional rotation.

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} cos\,\theta & -sin\,\theta \\ sin\,\theta & cos\,\theta \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \tag{44}$$

It is easy to understand the structure of the above matrix as follows. To preserved lengths the 2x2 matrix responsible for the rotation has to be unitary, which means that the matrix can be written in terms of sines and cosines. For a zero angle rotation the matrix has to be the identity matrix, which means that the cosines have to be along the diagonal and the sines off the diagonal. In order for the scalar product of rows 1 and 2 to be zero one of the sines need to have a negative sign. It is easy to see where to put the negative sign by looking a which coordinate gets smaller. In the rotation shown in Figure 10 the x coordinate gets smaller after the rotation therefore the minus sign is in row 1. The extension to three dimensions is obviously given by:

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} cos\,\theta & -sin\,\theta & 0 \\ sin\,\theta & cos\,\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \tag{45}$$

The above rotation is said to be around the z-axis, and the rotation is defined as positive when the objects rotate around the z-axis as a right handed cork screw. To simplify the notation we can write

$$\overrightarrow{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{46}$$

and

$$\overrightarrow{x}_2 = R_z(\theta)\overrightarrow{x}_1 \tag{47}$$

Writing $c_\theta = cos\,\theta$ and $s_\theta = sin\,\theta$, the rotation $R_z(\theta)$ can simply be written as

$$R_z(\theta) = \begin{pmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{48}$$

In the same way we can define 3-dimensional active rotations around the x-axis and y-axis as

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\theta & -s_\theta \\ 0 & s_\theta & c_\theta \end{pmatrix} \text{ , and } \; R_y(\theta) = \begin{pmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{pmatrix} \tag{49}$$

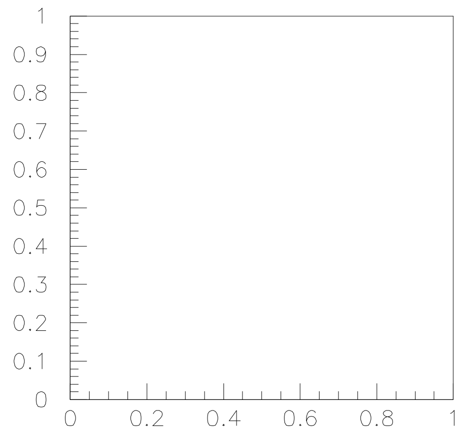

Figure 11. Three dimensional rotation using Euler angles.

As we can see in Figure 11 if a body is rotated around the x-axis the y-coordinate is the one that gets smaller and the minus sign has to be in the second row. For a rotation around the y-axis the z-coordinate gets smaller and the minus sign is in the third row.

Using Euler angles as defined in Figure 11 a general 3-dimensional rotation can be written as

$$R(\phi, \theta, \gamma) = R_z(\phi)\; R_y(\theta)\; R_z(\gamma) \tag{50}$$

Or in matrix form as

$$\begin{aligned} R(\phi,\theta,\gamma) &= \begin{pmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{pmatrix} \begin{pmatrix} c_\gamma & -s_\gamma & 0 \\ s_\gamma & c_\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_\theta\, c_\gamma & -\,c_\theta\, s_\gamma & s_\theta \\ s_\gamma & c_\gamma & 0 \\ -\,s_\theta\, c_\gamma & s_\theta\, s_\gamma & c_\theta \end{pmatrix} \\ &= \begin{pmatrix} c_\phi\, c_\theta\, c_\gamma - \, s_\phi\, s_\gamma & -\,c_\phi\, c_\theta\, s_\gamma - \, s_\phi\, c_\gamma & c_\phi\, s_\theta \\ s_\phi\, c_\theta\, c_\gamma + \, c_\phi\, s_\gamma & -\,s_\phi\, c_\theta\, s_\gamma + \, c_\phi\, c_\gamma & s_\phi\, s_\theta \\ -\,s_\theta\, c_\gamma & s_\theta\, s_\gamma & c_\theta \end{pmatrix} \end{aligned} \tag{51}$$

Rotations around the x-axis using Euler angles can be produced by

$$R_x(\theta) = R(3\pi/2, \theta, -3\pi/2) \tag{52}$$

This is easy to see by noting that $sin(3\pi/2) = -1$ and $cos(3\pi/2) = 0$, and therefore $c_\phi = c_\gamma = 0$ and $s_\gamma = -s_\phi = 1$. Replacing these values in Eq. 51 we obtain the rotation around the x-axis shown in Eq. 49. It is also easy to see that the rotations around the y-axis and z-axis are given by

$$R_y(\theta) = R(0, \theta, 0) \text{ , and } \; R_z(\theta) = R(\theta, 0, 0) = R(0, 0, \theta) \tag{53}$$

A.1 Small rotations

Small rotations commute, so it is convenient in this case to use rotations around the x, y and z axis defined as (tip,tilt,twist)=(θ_x, θ_y, θ_z). For small angles we can write Eqs 48 and 49 as

$$R_i(\theta) = I(1 - \frac{\theta^2}{2}) + M_i\,\theta + O(\theta^3) \tag{54}$$

where I is the identity matrix and

$$M_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\ ,\quad M_y = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}\ ,\ \text{and}\quad M_z = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{55}$$

Neglecting terms in θ^3 or higher, the result of a tip, tilt and twist rotation will be given by

$$\begin{aligned} R_{xyz}(\theta_x, \theta_y, \theta_z) &= R_x(\theta_x)\, R_y(\theta_y)\, R_z(\theta_z) & (56)\\ &= \left[I(1 - \frac{\theta_x^2}{2}) + M_x\,\theta_x\right]\left[I(1 - \frac{\theta_y^2}{2}) + M_y\,\theta_y\right]\left[I(1 - \frac{\theta_z^2}{2}) + M_z\,\theta_z\right] & (57)\\ &= I + (M_x\,\theta_x + M_y\,\theta_y + M_z\,\theta_z) + (M_xM_y\,\theta_x\theta_y + M_xM_z\,\theta_x\theta_z + M_yM_z\,\theta_y\theta_z) & \\ &\quad -I(\frac{\theta_x^2}{2} + \frac{\theta_y^2}{2} + \frac{\theta_z^2}{2}) & (58) \end{aligned}$$

The required resolution for tip and tilt is 1 arcsec = 4.85 μrad. So we want the non-linear terms in Eq. 58 to be smaller than 1 arcsec. That means $\theta^2 < 4.85$ μrad or $\theta < 2.20$ mrad = 454 arcsec. So as long as the tip, tilt and twist rotations are smaller than 400 arcsec (which translates into about 1.5 mm at the focal plane) we can safely use the approximation

$$R_{xyz}(\theta_x, \theta_y, \theta_z) = I + (M_x\,\theta_x + M_y\,\theta_y + M_z\,\theta_z) \tag{59}$$

APPENDIX B. CALCULATING ACTUATOR FORCES

B.1 Static forces when the actuator's weight is neglected

In this appendix we will calculate the static forces on the hexapod actuators when the weight of the actuators can be negelcted. In Section B.2 the forces will be calculated including the actuators weight. The static problem we want to solve requires the forces on the DECam camera, so we will calculate the forces exerted by the actuators on the motion plate. The forces on the actuators themselves are just the negative of the forces we will calculate. Since these forces are in the direction of the actuators we will write them as

$$\vec{F}_i = F_i\,\hat{f}_i \tag{60}$$

where $\hat{f}_i$ is a unit vector in the direction of actuator i, and points from the fixed to the motion plate. With this definition of $\hat{f}_i$ the force F_i will be positive when the actuator is pushing on the motion plate, therefore $F_i > 0$ means that the hexapod is under compression.

The static problem that we have to solve is:

$$\sum_{i=1}^{6} F_i\ \hat{f}_i\ +\ \vec{w} = 0 \tag{61}$$

$$\sum_{i=1}^{6} F_i\ \vec{\Delta}_i \times \hat{f}_i = 0 \tag{62}$$

where $\overrightarrow{w}$ is the DECam weight applied in the DECam Center of Mass (CM) and the moments in Eq. 62 are calculated relative to the CM, that is

$$\overrightarrow{\Delta}_i = \overrightarrow{r}_i - \overrightarrow{r}_{CM} \tag{63}$$

The previous equations form a 6x6 linear system with the following structure

$$\begin{pmatrix} f_{x_1} & f_{x_2} & f_{x_3} & f_{x_4} & f_{x_5} & f_{x_6} \\ f_{y_1} & f_{y_2} & f_{y_3} & f_{y_4} & f_{y_5} & f_{y_6} \\ f_{z_1} & f_{z_2} & f_{z_3} & f_{z_4} & f_{z_5} & f_{z_6} \\ n_{x_1} & n_{x_2} & n_{x_3} & n_{x_4} & n_{x_5} & n_{x_6} \\ n_{y_1} & n_{y_2} & n_{y_3} & n_{y_4} & n_{y_5} & n_{y_6} \\ n_{z_1} & n_{z_2} & n_{z_3} & n_{z_4} & n_{z_5} & n_{z_6} \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{pmatrix} = \begin{pmatrix} -w_x \\ -w_y \\ -w_z \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{64}$$

with $\overrightarrow{n}_i = \overrightarrow{\Delta}_i \times \hat{f}_i$. If we write Eq. 64 as $M\,\overrightarrow{F} = \overrightarrow{U}$ then the solution is

$$\overrightarrow{F} = M^{-1}\,\overrightarrow{U} \tag{65}$$

The weight is always in the -z direction, $\overrightarrow{w} = (0, 0, -w)$, therefore the force actuator i exerts on the motion plate is given by

$$F_i = M^{-1}_{i3}\,w \tag{66}$$

and the forces per unit weight are

$$F_i/w = M^{-1}_{i3} \tag{67}$$

B.2 Static forces when the actuator's weight is included