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# Minimizing the uncertainty due to parton distribution and production mechanism in the determination of the $W$ mass

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DØ experiment

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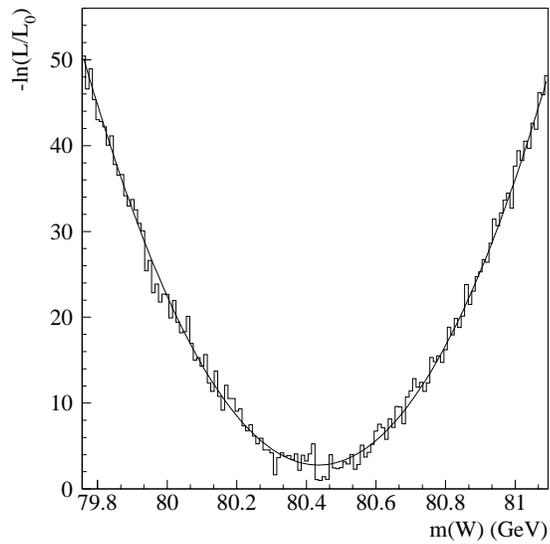
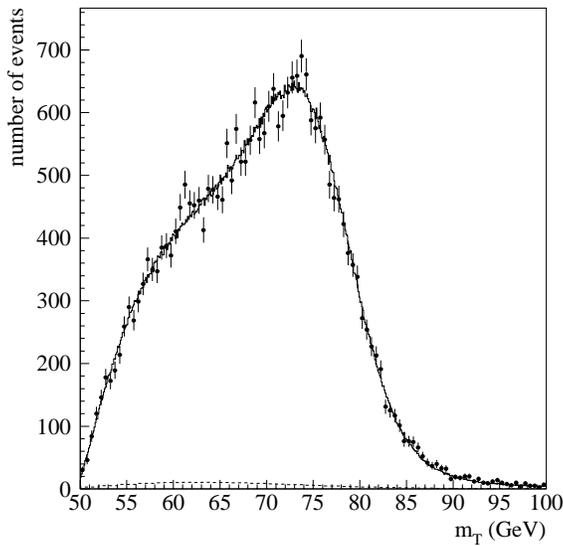
1. Theoretical limitations on the  $W$  mass determination at  $p\bar{p}$
2. The theoretical framework of the  $W$  production cross-section
  - parametrization of the  $W$ -mass resonance
  - resummed cross-section and  $d^2\sigma/dy dP_T$
  - non-perturbative contribution in  $d^2\sigma/dy dP_T$
3. The experimental constraints on the modelisation at DØ
  - error in  $W$ -mass distribution: the parton luminosity effects
  - constraint on  $d^2\sigma/dy dP_T$  spectrum: the choice of the pdf
  - constraint on  $d^2\sigma/dy dP_T$  spectrum: non-perturbative parameters
4. Conclusion

# Theoretical limitations on the W mass determination

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## Sources of errors in W-mass determination

- W-mass from maximum likelihood fit to  $M_T$  or  $E_T^e$



Sources of error (MeV)	$M_T^W$	$E_T^e$
Statistical	70	90
Energy Scale	75	75
Detector Syst.	91	55
Theoretical Syst.	33	61

⇒ Uncertainty on theoretical models becomes a limiting factor:

- For measurements with higher statistics (run II)
- For determination using  $E_T^e$  distributions

# The theoretical framework of the W-production cross-section

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Specify W cross-section in terms of:

- $m$  - Mass
- $y$  - Rapidity
- $P_T$  - Transverse momentum

$$\frac{\partial^3 \sigma}{\partial P_T \partial y \partial m}$$

A diagram showing the decomposition of the triple derivative of the cross-section. The expression  $\frac{\partial^3 \sigma}{\partial P_T \partial y \partial m}$  is at the top. Two arrows point downwards from it to the expression  $\left| \frac{\partial^2 \sigma}{\partial P_T \partial y} \right| \times \frac{d\sigma}{dm}$ . The double derivative term is enclosed in blue brackets, and the single derivative term is in red.

Collins-Soper-Sterman [CSS]

Davies-Stirling-Weber [DSW]

Arnold & Kauffman [AK]

Ladinsky-Yuan [LY]

$$\times \frac{e^{-\beta m}}{m} \frac{m^2}{(m^2 - M_W^2)^2 + m^4 \Gamma_W^2 / M_W^2}$$

# the parametrization of the W-mass resonance

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## Dependance of the W-mass distribution

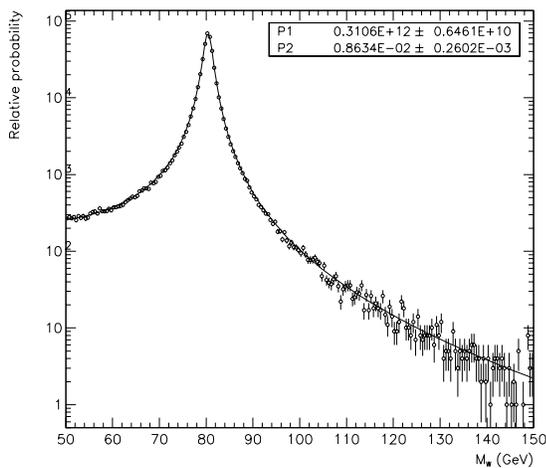
$$\frac{d\sigma}{dm} = \frac{e^{-\beta m}}{m} \times \frac{m^4 \Gamma_W^2 / M_W^2}{(m^2 - M_W^2)^2 + m^4 \Gamma_W^2 / M_W^2}$$

→ Relativistic Breit-Wigner: Dependence in  $\Gamma_W$

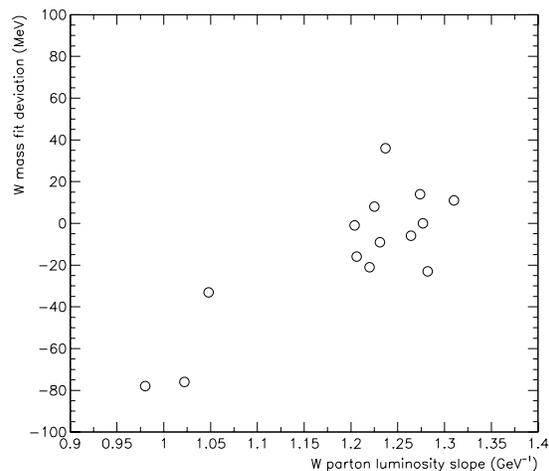
→ Parton Luminosity factor:

$$\frac{e^{-\beta m}}{m} \equiv \sum_{i,j} f_i \left( \frac{m}{\sqrt{s}} e^y, m^2 \right) f_j \left( \frac{m}{\sqrt{s}} e^{-y}, m^2 \right)$$

- $f_i$ : is the pdf for the parton i/proton.  $\beta$  is the slope
- parton with momentum fraction  $me^y/\sqrt{s}$  at  $m^2$



$M_W$  distribution (MRSA)



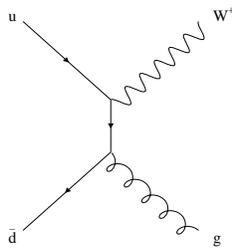
$M_W$  vs  $\beta_W$

# The resummed cross-section and $d\sigma/dy dP_T$

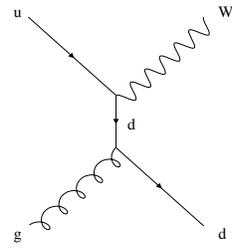
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## W-Production cross-section

- **Drell-Yan** processus :  $P_T^V \approx 0$  wrt  $p\bar{p}$  axis  
 $\Rightarrow$  Higher order QCD-corrections  $\Rightarrow P_T^V \neq 0$



Gluon ISR



Compton q-g

$\Rightarrow$  Divergence at low  $P_T$  of  $\alpha_s \log(Q^2/P_T^2)$  terms at  $Q$  scale

## The Collins-Sopner-Sterman framework

- Re-summation of **LO** and **NLO** terms:

$$\frac{d\sigma_{ij}}{dP_T dy} \propto \int d^2b \times e^{i\vec{b}\vec{P}_T} W(b, Q, \dots)$$

$W(b, Q, \dots)$  depends on [AK]:

- parameters:  $\mathbf{b}$  (impact par.),  $\mathbf{s}$ ,  $\mathbf{m}$  (mass),  $\frac{Q}{\sqrt{s}}e^y$  ( $x_1$ ),  $\frac{Q}{\sqrt{s}}e^{-y}$  ( $x_2$ )
- pdfs :  $f_i(\dots)$  and  $f_j(\dots)$
- the **re-summed** terms in exponential form
- a **Non-Perturbative** Term, important for  $P_T \leq 10$  GeV

# The non-perturbative contribution in $d\sigma/dy dP_T$

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## Non-perturbative correction to the resummed cross-section

- $S_{NP}$  controls the shape of  $W$  for low  $P_T < P_T^{\min}$  ( $b > b_{max}$ )

$$W(b, Q\dots) \rightarrow W(b^*, Q\dots)e^{-S_{NP}(b^*)}, \quad b^* = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$

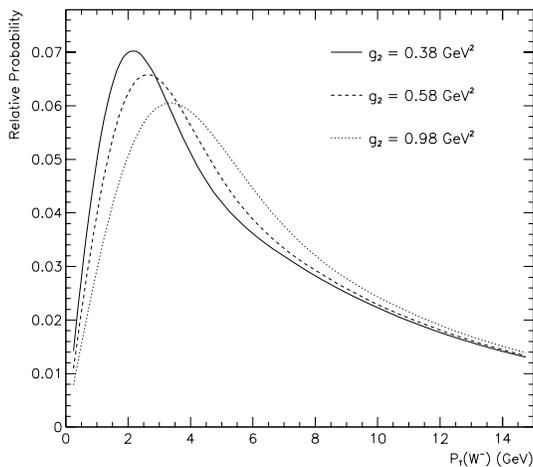
## Parametrization of the non-perturbative function $S_{NP}$

- constrained by low  $P_T$  Drell-Yan data fit

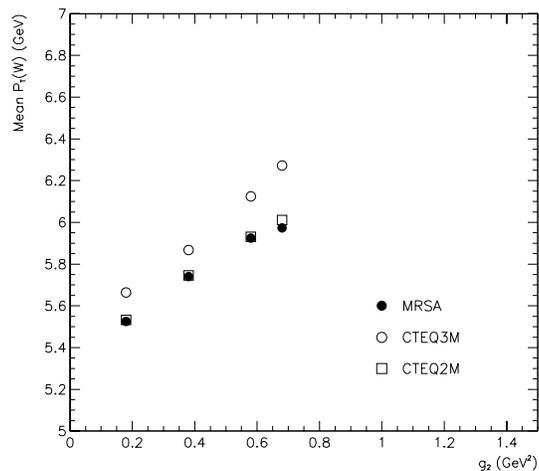
⇒ Modelisation of [DSW] improved by [LY]:

$$S_{NP}(b) = b^2 \left( g_1 + g_2 \ln \left( \frac{Q}{2Q_0} \right) \right) + b g_1 g_3 \ln(100x_1x_2)$$

→ need strong constraint on  $g_2$



$P_T^W$  vs  $g_2$

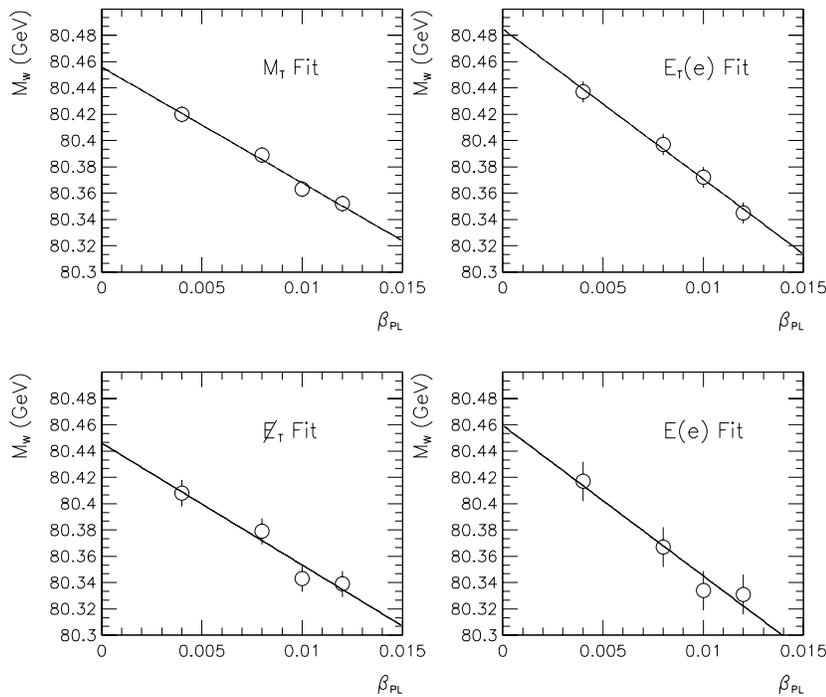


$\langle P_T^W \rangle$  vs  $g_2$

# Errors in the W-Mass distribution: the parton luminosity

## Uncertainty from the parton luminosity

- compute  $\beta$  for pdfs [MRS A/CTEQ2M/CTEQ3M/MRSD-](#)
- for each  $\beta$  generate MC and perform fit to data



Translated error on fitted W-mass for  $\Delta\beta = 0.001 \text{ GeV}^{-1}$

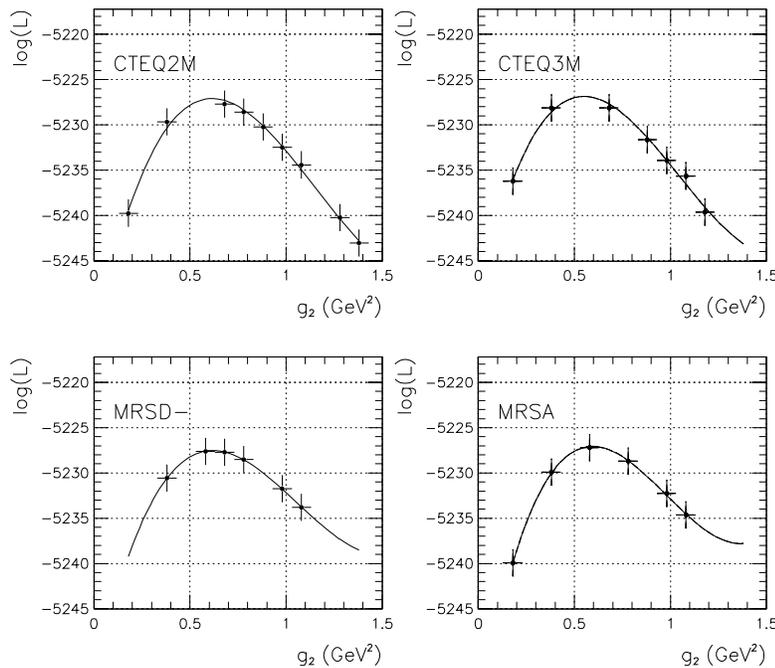
Observables	$M_T^W$ Fit	$E_T^e$ Fit
$\Delta M_W$ (MeV)	$-8.8 \pm 1.0$	$-11.4 \pm 1.3$

# Errors in $d^2\sigma/dy dP_T$ spectrum: the choice of the pdf

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## Constraints from the $P_T^Z$ measurement

- fixe  $g_1, g_3$  from [LY] fit
- fit  $g_2$  from  $P_T(ee)$  distribution in  $Z \rightarrow e^-e^+$
- enter  $g_2$  as input in  $P_T^W$



→ compute  $\text{Max}\{M_W(\text{pdf}) - M_W(\text{MRSA})\}$  as systematics

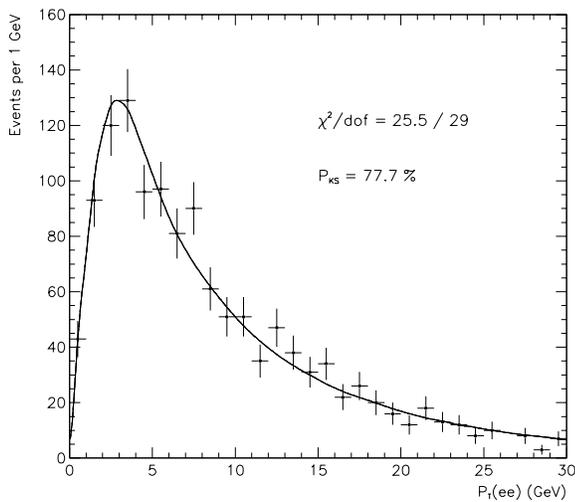
## Translated errors on fitted W-mass

Observables	$M_T^W$ Fit	$E_T^e$ Fit
$\Delta M_W$ (MeV)	21	48

# Errors in $d^2\sigma/dy dP_T$ from the non-perturbative contribution

## The $g_2$ determination from $P_T^Z$ measurements

- fixe  $g_1$  and  $g_3$  from [LY] fit
- for MRSA pdf , fit  $g_2$  from  $P_T(ee)$  in  $Z \rightarrow e^-e^+$
- enter  $g_2$  as input to  $P_T^W$  , compute systematics and changes in pdf



Results for MRSA :

$$g_2 = 0.587 \pm 0.095 \pm 0.052 \pm 0.043 \text{ GeV}^2$$

stat.    syst.    pdfs

Theoretical Predictions:

$$g_2 = 0.59 \pm 0.12 \text{ GeV}^2 \text{ [LY]}$$

## Translated error in the fitted W-mass

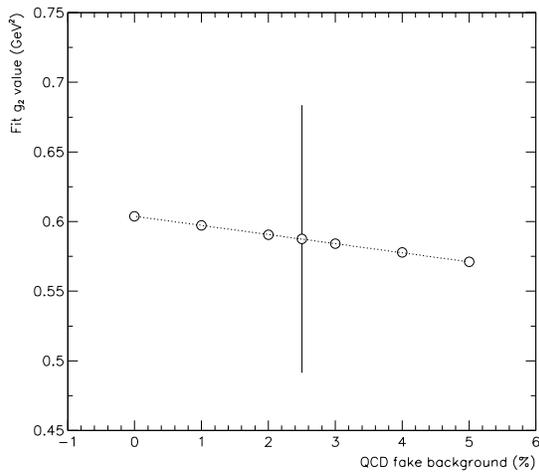
- Main limitation: statistics at run I ( systematics at run II)

Observables	$M_T^W$ Fit	$E_T^e$ Fit
$\Delta M_W$ (MeV)	5	28

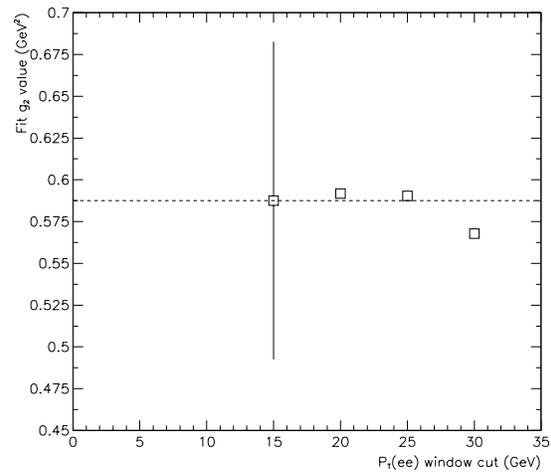
# Errors in $d^2\sigma/dy dP_T$ from the non-perturbative contribution

## Systematic errors in $g_2$ ( $\text{GeV}^2$ ) determination

Background	0.018
$e^-$ energy resolution	0.02
$e^-$ energy scale	0.02
$u_{//}$ efficiency	0.02
$\Phi$ acceptance	0.01
Radiative decays	0.03
Systematical	<b>0.052</b>
TOTAL	0.108



$g_2$  vs background



$g_2$  vs  $P_T$ -window

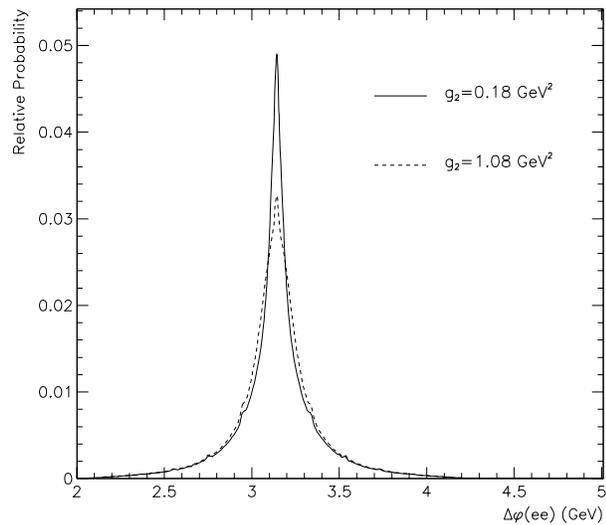
## Errors in $d^2\sigma/dy dP_T$ from the non-perturbative contribution

The determination of  $g_2$  from  $\Delta\phi(ee)$

- use of the correpondance  $\Delta\Phi(ee)$  versus  $P_T^Z$  for electrons in  $Z \rightarrow e^+e^-$

Higher  $\langle P_T^Z \rangle$  values

$\Rightarrow$  wider  $\Delta\Phi(ee)$



$\Delta\Phi(ee)$  vs  $g_2$

The determination of  $g_2$  from  $\Delta\phi(ee)$  :

– carries less informations than  $P_T(ee)$  :  $\Rightarrow$  statistically limited

+ is less sensitive to reconstruction biases in:  
 $e^-$  energy scale and resolution

$\Rightarrow$  important at run II

## Conclusions

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Theoretical errors for production models become a limiting factor:

1. for a **high statistics** run II at  $p\bar{p}$  collider
2. for all measurements based on a fit to  $E_T^e$  spectrum  
(lower systematics wrt  $M_T^W$  fit)

Theoretical errors in production model originate from:

Sources	$M_T^W$ Fit	$E_T^e$ Fit	$E_T^e$ Fit
$\Delta\Gamma_W$	9	7	6
Parton luminosity	10	10	10
$d^2\sigma/dP_T dy$ : pdf	21	48	30
$d^2\sigma/dP_T dy$ : $g_2$	5	28	9
TOTAL	25	57	33

→ dominated by errors in the non-resonant part to  $d^2\sigma/dP_T dy$

Theoretical error can be experimentally constrained by **both** :

1. a precise determination of  $g_2$  from  $P_T(ee)$  (run I)  
or from  $\Delta\Phi(ee)$  (run II)
2. stringent constraints on the pdf (**asymmetry** measurement )