Experiments on Physics Beyond the Standard Model I

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Content

1 The Standard Model of Particle Physics

1.1 Introduction

1.2 Quantum Electrodynamics

1.3 Quantum Chromodynamics

1.4 Quantum Flavordynamics

Literature

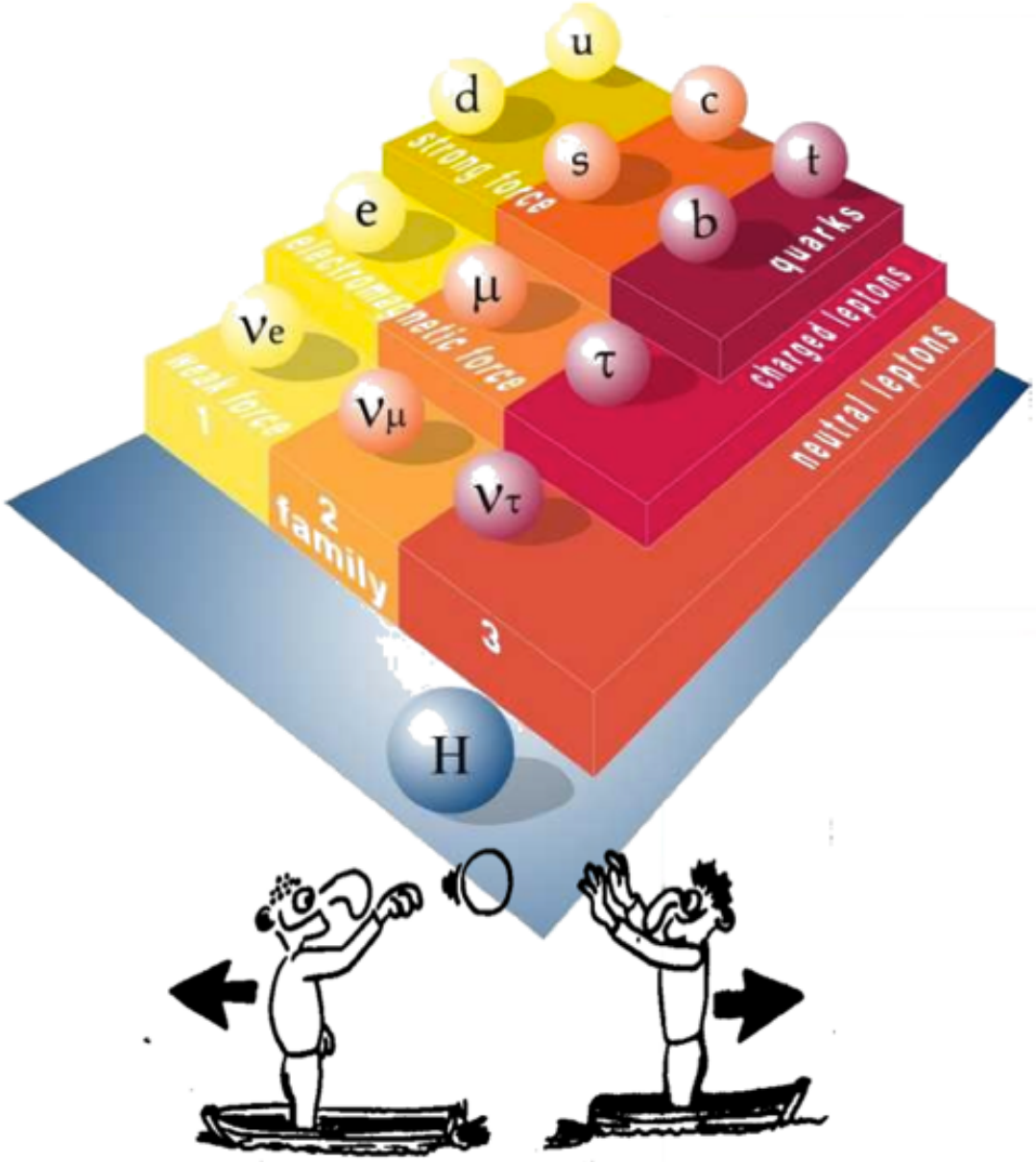
THEORY

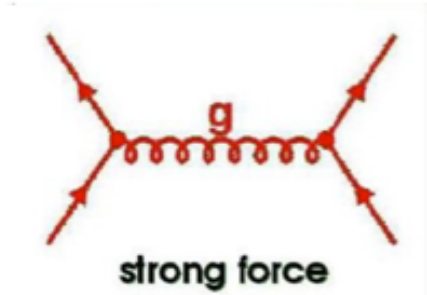
- An Introduction to Quantum Field Theory, Peskin and Schroeder
- Quarks and Leptons, Halzen & Martin
- Introduction to Gauge Field Theory, Bailin & Love
- Gauge Theory of Elementary Particle Physics, Cheng & Li
- Quantum Field Theory, Ryder
- Gauge Theories in Particle Physics, Aitchison & Hey (Vols. 1 & 2)
- Elementary Particle Physics: Concepts and Phenomena Nachtmann

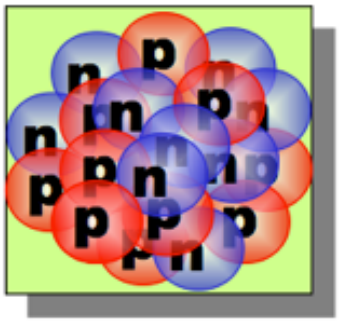
EXPERIMENT

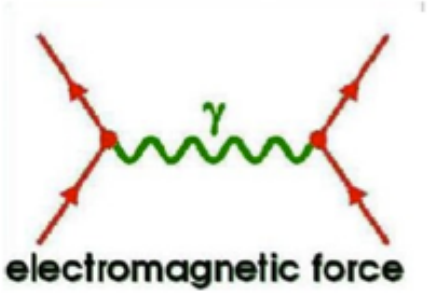
- Experimente zur Physik Jenseits des Standardmodells, Notizen zur Vorlesung SoSe 2014, Peter Schleper
  `http://www.desy.de/~schleper/lehre/BSM/SS_2014`
- Teilchen und Kerne: Eine Einführung in die physikalischen Konzepte (Springer-Lehrbuch), Povh, Rith, Scholz, Zetsche
- Introduction to Elementary Particles (Wiley), Griffiths

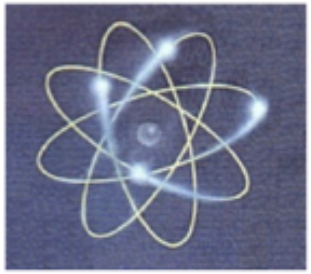
The Standard Model of Particle Physics

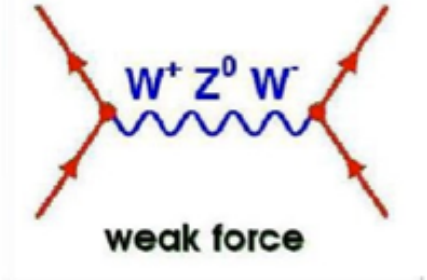


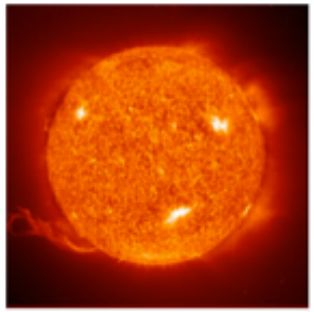


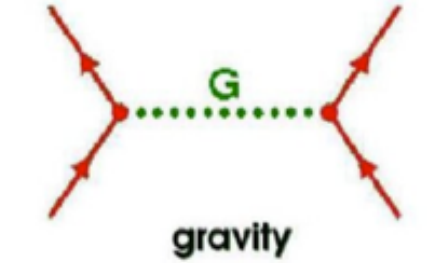


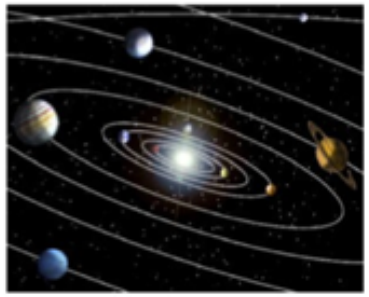




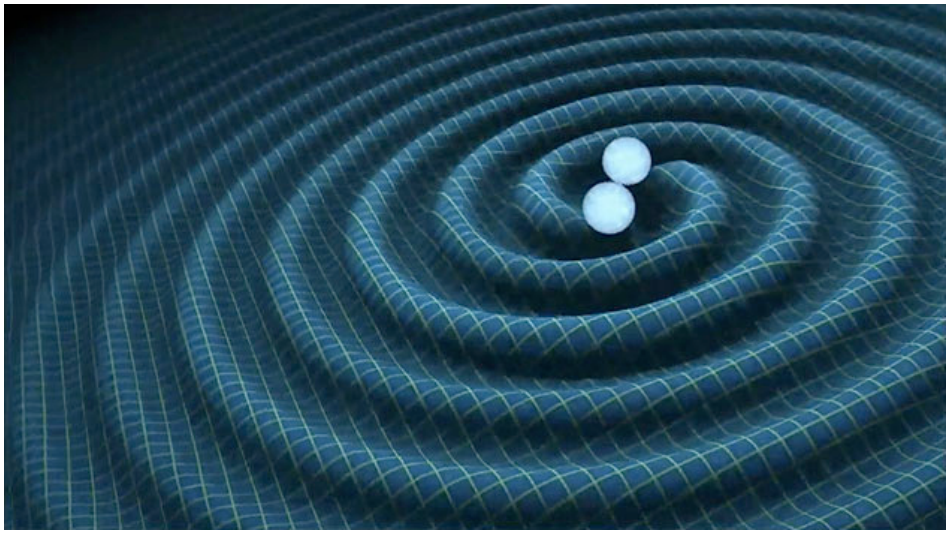




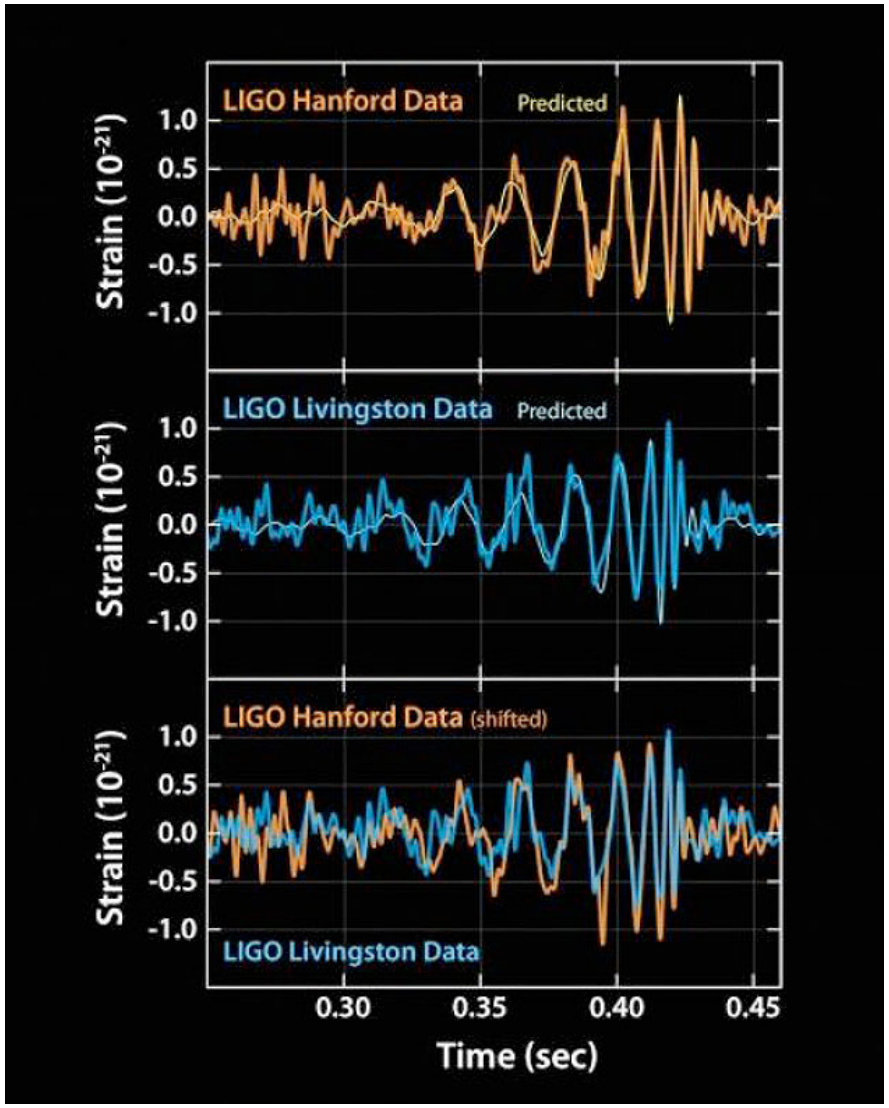


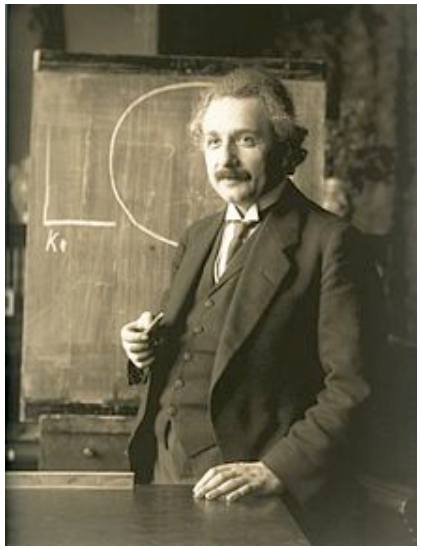


Gravitational Waves



computer simulation of gravitational waves in space-time, caused by two black holes rotating against each other



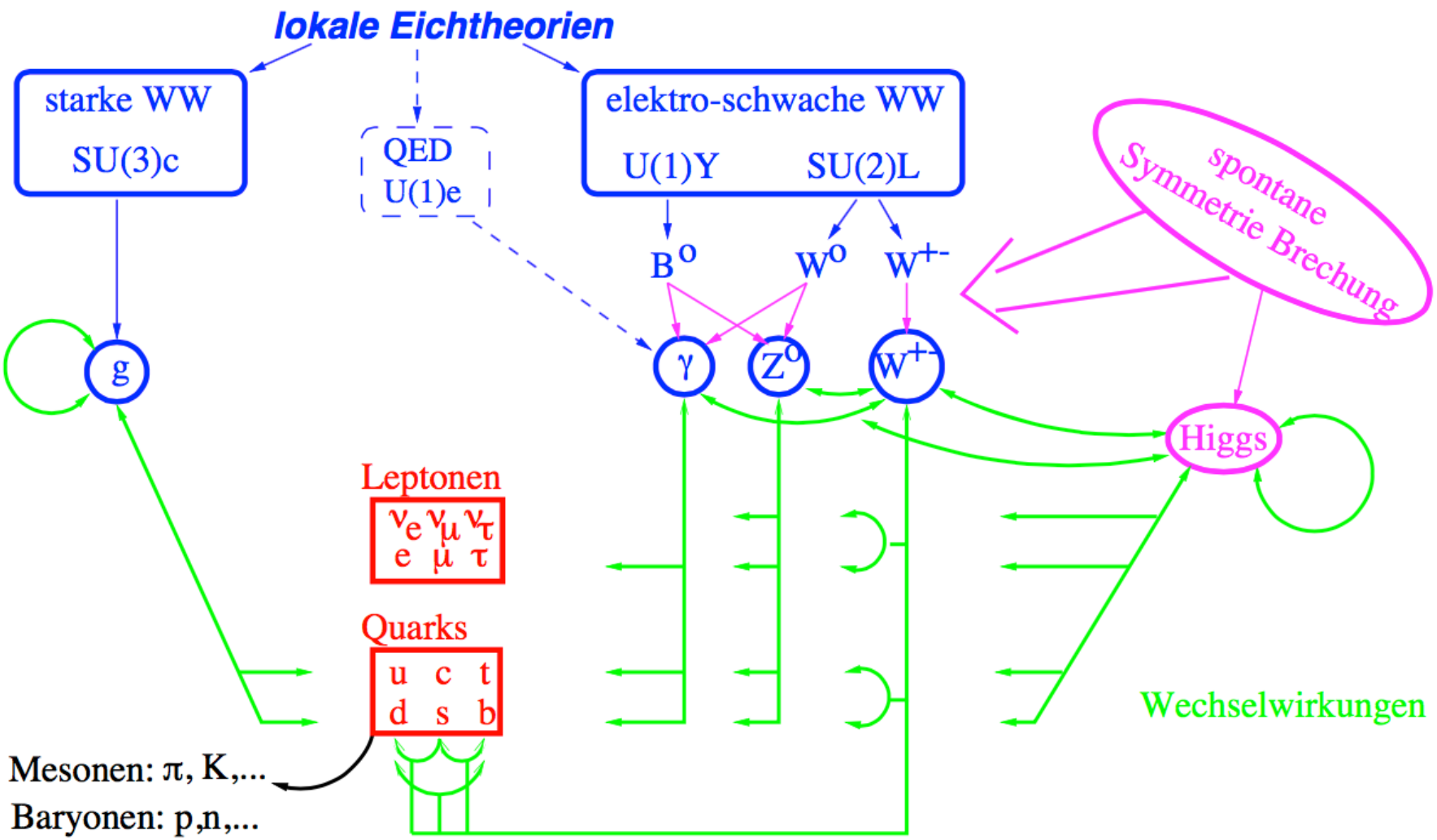


Albert Einstein
1879-1955

→ predicted in 1916 (Albert Einstein)
→ discovered in 2016 (LIGO-Collaboration)

The Standard Model

Das Standard Modell



Feynman Rules of QED

APPENDIX B

The Feynman Rules of QED

(From D. Nachtmann, Elementary Particle Physics, Springer, 1990)

We shall make use of the following correspondence between physical situations, the elements of diagrams, and analytical expressions. Spin indices are not written explicitly.

electron in initial state	incoming electron line
$u(p)$	$e^-(p)$
electron in final state	outgoing electron line
$\bar{u}(p)$	$e^-(p)$
positron in initial state	outgoing electron line
$\bar{v}(q)$	$e^+(q)$
positron in final state	incoming electron line
$v(q)$	$e^+(q)$
photon in initial state	incoming photon line
ε^μ	$\gamma(k)$
photon in final state	outgoing photon line
$\varepsilon^{\mu *}$	$\gamma(k)$
virtual photon	internal photon line
$\frac{-ig_{\mu\nu}}{k^2+i\varepsilon}$	k, μ, ν
virtual electron	internal electron line
$i\frac{\not{p}+m}{p^2-m^2+i\varepsilon}$	p
elementary process	vertex
$ie\gamma^\mu$	

The transition amplitude for a particular reaction is ~~given by~~ obtained from the sum of all the diagrams with the appropriate incoming and outgoing lines. Within the diagrams, any number of vertices are allowed. At each vertex, four-momentum must be conserved. This also guarantees overall four-momentum conservation, i.e., the sum of the incoming four-momenta is equal to the sum of the outgoing four-momenta. The four-momenta flow through the diagrams like electrical currents through a network of electrical conductors. Internal currents correspond to loop momenta. Diagrams with no loops are called *tree diagrams*. Examples are the diagrams of Fig. 10.2.

In Fig. B.1 we show an example of a diagram with one loop. The reaction is $\gamma(k_1)+\gamma(k_2)\to\gamma(k_3)+\gamma(k_4)$ and the loop momentum is l.

It is implicitly understood that loop momenta are to be integrated over with a measure given for a single loop by

$$\int \frac{dl}{(2\pi)^4}. \tag{B.1}$$

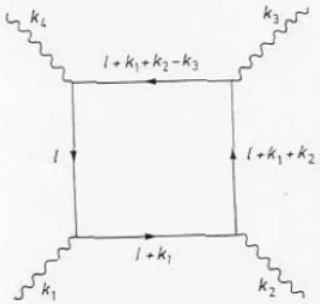


Figure B.1 A diagram for photon–photon scattering with one loop.

Feynman Rules of QED

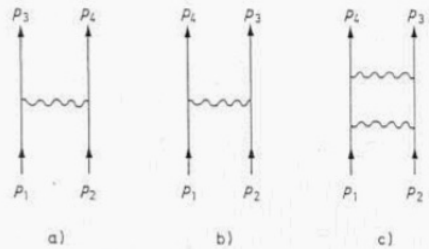


Figure B.2 Examples of diagrams for electron–electron scattering. The diagrams (b) and (c) receive a minus sign relative to (a).

these rules

The S-matrix element is obtained from ~~the transition amplitude~~ an additional by multiplication with the δ-function from the energy-momentum conservation:

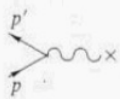
$$(2\pi)^4\delta\left(\sum_f p_f - \sum_i p_i\right), \tag{B.2}$$

where $p_i(p_f)$ are the momenta of the incoming (outgoing) particles.

A few other rules should be noted. For every closed fermion loop a factor of (-1) appears. Diagrams differing from one another only through a permutation of the labels of the external fermion lines receive relative to each other as factors the sign of the corresponding permutation. (An overall phase factor for the complete amplitude remains, of course, arbitrary.) Diagrams which are identical as far as the fermion lines and their labeling are concerned get no extra sign factor relative to each other. Some examples are shown in Fig. B.2.

To the interaction with an external electromagnetic field there corresponds the following analytical expression and vertex:

$$\mathrm{i}e\gamma^\mu \int \mathrm{d}x\, \mathrm{e}^{\mathrm{i}(p'-p)x} A_\mu^{\mathrm{ext}}(x) = -\mathrm{i}e\gamma^\mu \frac{1}{(p'-p)^2}\int \mathrm{d}x\, \mathrm{e}^{\mathrm{i}(p'-p)x} j_\mu^{\mathrm{ext}}(x).$$



If such a vertex appears in a diagram, then the δ-function of the energy-momentum conservation (B.2) is to be omitted in the S-matrix element.

For calculating scattering cross-sections for unpolarized particles in the initial (final) state one must average (sum) over the corresponding spin orientations. The spin sums for Dirac spinors are given in (A.22). The spin sums for photons require some further explanation.

Let us consider a reaction in which an unpolarized photon with four-momentum k is incident. The Feynman rules yield as a result the amplitude for a photon with polarization vector ε where the dependence on ε is of the general form

$$\langle \ldots|\mathbf{T}|\ldots\gamma(k,\varepsilon)\rangle = \mathcal{M}_\mu(k)\varepsilon^\mu. \tag{B.3}$$

As discussed in Chap. 7, we can replace the photon state by an equivalent

state (7.31). The T-matrix element remains unchanged, as do all physical quantities. If c is an arbitrary complex number and

$$\varepsilon' = \varepsilon + ck, \tag{B.4}$$

then it is easy to see that ε and ε' correspond to equivalent photon states:

$$\varepsilon^\mu \mathbf{a}_\mu^\dagger(k)|0\rangle \sim \varepsilon'^\mu \mathbf{a}_\mu^\dagger(k)|0\rangle. \tag{B.5}$$

Thus it must hold that

$$\mathcal{M}_\mu(k)\varepsilon^\mu = \mathcal{M}_\mu(k)\varepsilon'^\mu, \tag{B.6}$$

from which it follows that

$$\mathcal{M}_\mu(k)k^\mu = 0. \tag{B.7}$$

In calculating the cross-section σ for an unpolarized photon we must average over the physical polarization directions of the photon in the initial state:

$$\sigma \propto \sum_{\text{phys.}\,\varepsilon}{}' |\mathcal{M}_\mu(k)\varepsilon^\mu|^2. \tag{B.8}$$

To evaluate this sum we make use of the coordinate system defined in (7.24). As physical polarization vectors we can choose $\varepsilon_1 = (0, \boldsymbol{e}_1)$ and $\varepsilon_2 = (0, \boldsymbol{e}_2)$. Equation (B.7) yields

$$\begin{aligned} \mathcal{M}_\mu(k)k^\mu &= k^0(\mathcal{M}_0(k) - \vec{\mathcal{M}}(k)\boldsymbol{e}_3) = 0, \\ \mathcal{M}_0(k) &= \vec{\mathcal{M}}(k)\boldsymbol{e}_3. \end{aligned} \tag{B.9}$$

For the cross-section in (B.8) we then obtain

$$\begin{aligned} \sigma \propto \sum_{\text{phys.}\,\varepsilon}{}' |\mathcal{M}_\mu\varepsilon^\mu|^2 &= \tfrac{1}{2}\sum_{i=1}^{2}|\mathcal{M}_\mu\varepsilon_i^\mu|^2 = \tfrac{1}{2}\{|\vec{\mathcal{M}}\boldsymbol{e}_1|^2 + |\vec{\mathcal{M}}\boldsymbol{e}_2|^2\} \\ &= \tfrac{1}{2}\{|\vec{\mathcal{M}}\boldsymbol{e}_1|^2 + |\vec{\mathcal{M}}\boldsymbol{e}_2|^2 + |\vec{\mathcal{M}}\boldsymbol{e}_3|^2 - |\mathcal{M}_0|^2\} \\ &= \tfrac{1}{2}\mathcal{M}_\mu(-g^{\mu\nu})\mathcal{M}_\nu^*. \end{aligned} \tag{B.10}$$

An analogous argument holds for an unpolarized photon in the final state. Thus for calculating cross-sections in QED we can make the replacement

$$\sum_{\text{phys.}\,\varepsilon} \varepsilon^\mu\varepsilon^{\nu*} \to -g^{\mu\nu}. \tag{B.11}$$

A useful formula regarding a type of phase-space integral that frequently occurs in the calculation of cross-sections is

$$\begin{aligned} &\int \mathrm{d}q_1 \int \mathrm{d}q_2\, \delta_+(q_1^2 - m_1^2)\delta_+(q_2^2 - m_2^2)\delta(p - q_1 - q_2) \\ &\quad\cdot\{a + b^\mu(q_1 - q_2)_\mu + c^{\mu\nu}(q_1 - q_2)_\mu(q_1 - q_2)_\nu\} \\ &= \theta(p^0)\theta(p^2 - (m_1 + m_2)^2)\frac{\pi}{2p^2}w(p^2, m_1^2, m_2^2)\left\{a + b^\mu p_\mu \frac{m_1^2 - m_2^2}{p^2}\right. \\ &\quad\left. + c^{\mu\nu}\left[(4p_\mu p_\nu - g_{\mu\nu}p^2)\frac{1}{3(p^2)^2}w^2(p^2, m_1^2, m_2^2) - p_\mu p_\nu \frac{1}{p^2}(p^2 - 2m_1^2 - 2m_2^2)\right]\right\}. \end{aligned} \tag{B.12}$$

Feynman Rules of QED

Here p is an external momentum, $m_1 \geq 0$ and $m_2 \geq 0$ are masses, and a, b^μ, $c^{\mu\nu}$ are arbitrary constants. The function w is defined in (2.72) and

$$\delta_+(q^2 - m^2) \equiv \theta(q^0)\delta(q^2 - m^2) = \frac{1}{2q^0}\delta(q_0 - \sqrt{q^2 + m^2}). \qquad \text{(B.13)}$$

In order to prove (B.12) one must first realize that the integral is only nonzero for $p^2 \geq 0$ and $p^0 \geq 0$ and that the individual parts are invariant or covariant under Lorentz transformations. For $p^2 > 0$ the integral is easily calculated in a frame where the spatial part of p is zero. For $p^2 = 0$ the integral gives zero except when $m_1 = m_2 = 0$. In the latter case, we *define* it as the limiting value for $p^2 \to +0$.

Dirac Matrices and Spinors

APPENDIX A

Dirac Matrices and Spinors

From O.Nachtmann, Elementary Particle Physics, Springer, 1990

Defining relation:

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}. \tag{A.1}$$

Definitions:

$$\not{a} = a^\mu\gamma_\mu, \tag{A.2}$$

$$\gamma_5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3, \tag{A.3}$$

$$\sigma^{\mu\nu} = \frac{\mathrm{i}}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu). \tag{A.4}$$

Relations:

$$\gamma^\mu\gamma_\mu = 4, \tag{A.5}$$

$$\gamma^\mu\not{a}\gamma_\mu = -2\not{a}, \tag{A.6}$$

$$\gamma^\mu\not{a}\not{b}\gamma_\mu = 4(a, b), \tag{A.7}$$

$$\gamma^\mu\not{a}\not{b}\not{c}\gamma_\mu = -2\not{c}\not{b}\not{a}, \tag{A.8}$$

$$\gamma^\mu\gamma_5 + \gamma_5\gamma^\mu = 0, \tag{A.9}$$

$$\gamma_5\gamma_5 = 1. \tag{A.10}$$

From (A.1) it immediately follows for the trace that

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}. \tag{A.11}$$

The trace of an odd number of γ-matrices vanishes:

$$\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n+1}) = 0. \tag{A.12}$$

To prove this we use the matrix γ_5, for which from (A.9), (A.10) we have

$$\begin{aligned}\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n+1}) &= \mathrm{Tr}(\gamma_5\not{a}_1\gamma_5\gamma_5\not{a}_2\gamma_5\ldots\gamma_5\not{a}_{2n+1}\gamma_5)\\ &= (-1)^{2n+1}\,\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n+1}).\end{aligned} \tag{A.13}$$

A further relationship is

$$\mathrm{Tr}(\not{a}_1\ldots\not{a}_n) = \mathrm{Tr}(\not{a}_n\ldots\not{a}_1) \tag{A.14}$$

for which the proof is similar to that of (A.13), but using the matrix $\mathbf{S}(C)$ (4.124) instead of γ_5.

Relations concerning γ_5:

$$\gamma_\mu\gamma_5 = \frac{\mathrm{i}}{6}\varepsilon_{\mu\nu\rho\sigma}\gamma^\nu\gamma^\rho\gamma^\sigma, \tag{A.15}$$

$$\mathrm{Tr}\,\gamma_5 = \mathrm{Tr}(\gamma^\mu\gamma_5) = \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) = \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma_5) = 0, \tag{A.16}$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = 4\mathrm{i}\varepsilon^{\mu\nu\rho\sigma}. \tag{A.17}$$

The trace of the product of an even number $(2n)$ of γ-matrices can be expressed in terms of traces over $2(n-1)$ γ-matrices:

$$\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n}) = \sum_{k=2}^{2n}(-1)^k(a_1, a_k)\,\mathrm{Tr}(\not{a}_2\ldots\not{a}_{k-1}\not{a}_{k+1}\ldots\not{a}_{2n}). \tag{A.18}$$

To prove this $\not{a}_1$ is moved to the right by repeated anticommutation:

$$\begin{aligned}\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n}) &= \mathrm{Tr}((\{\not{a}_1, \not{a}_2\} - \not{a}_2\not{a}_1)\not{a}_3\ldots\not{a}_{2n})\\ &= 2(a_1, a_2)\,\mathrm{Tr}(\not{a}_3\ldots\not{a}_{2n}) - \mathrm{Tr}(\not{a}_2\not{a}_1\not{a}_3\ldots\not{a}_{2n})\\ &= 2\sum_{k=2}^{2n}(-1)^k(a_1, a_k)\,\mathrm{Tr}(\not{a}_2\ldots\not{a}_{k-1}\not{a}_{k+1}\ldots\not{a}_{2n})\\ &\quad + (-1)^{2n-1}\,\mathrm{Tr}(\not{a}_2\not{a}_3\ldots\not{a}_{2n}\not{a}_1).\end{aligned} \tag{A.19}$$

Since the matrices in a trace can be cyclically permuted, (A.18) follows.

For the trace of four γ-matrices we obtain from (A.18)

$$\mathrm{Tr}(\gamma^\lambda\gamma^\mu\gamma^\nu\gamma^\rho) = 4\{g^{\lambda\mu}g^{\nu\rho} + g^{\lambda\rho}g^{\mu\nu} - g^{\lambda\nu}g^{\mu\rho}\}. \tag{A.20}$$

The following are some important relations satisfied by the solutions of the Dirac equation (4.38), (4.41), where $s, r = \pm\frac{1}{2}$ are the spin indices:

$$\begin{aligned}(\not{p} - m)u_s(p) &= 0,\\ (\not{p} + m)v_s(p) &= 0,\end{aligned} \tag{A.21}$$

$$\begin{aligned}\sum_{s=\pm 1/2} u_s(p)\bar{u}_s(p) &= \not{p} + m,\\ \sum_{s=\pm 1/2} v_s(p)\bar{v}_s(p) &= \not{p} - m,\end{aligned} \tag{A.22}$$

Dirac Matrices and Spinors

472 Appendix A Dirac Matrices and Spinors

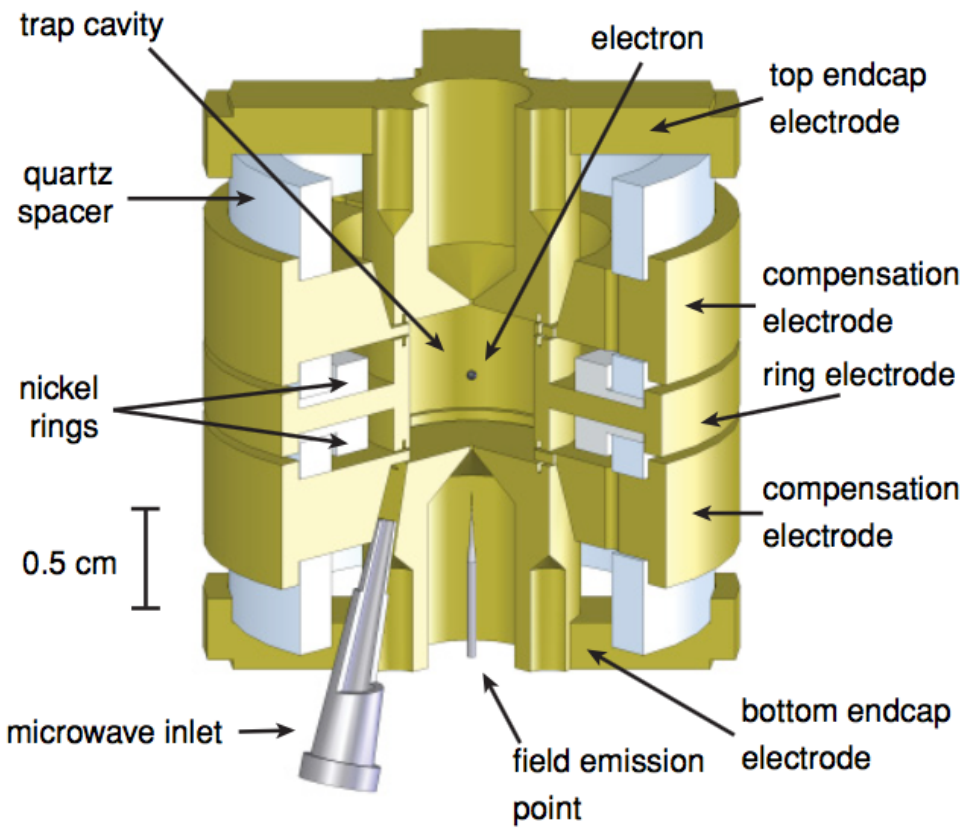
$$\bar{u}_r(p)u_s(p) = 2m\delta_{rs},$$
$$\bar{v}_r(p)v_s(p) = -2m\delta_{rs}, \tag{A.23}$$

$$\bar{u}_r(p)\gamma^\mu u_s(p) = 2p^\mu\delta_{rs},$$
$$\bar{v}_r(p)\gamma^\mu v_s(p) = 2p^\mu\delta_{rs}, \tag{A.24}$$

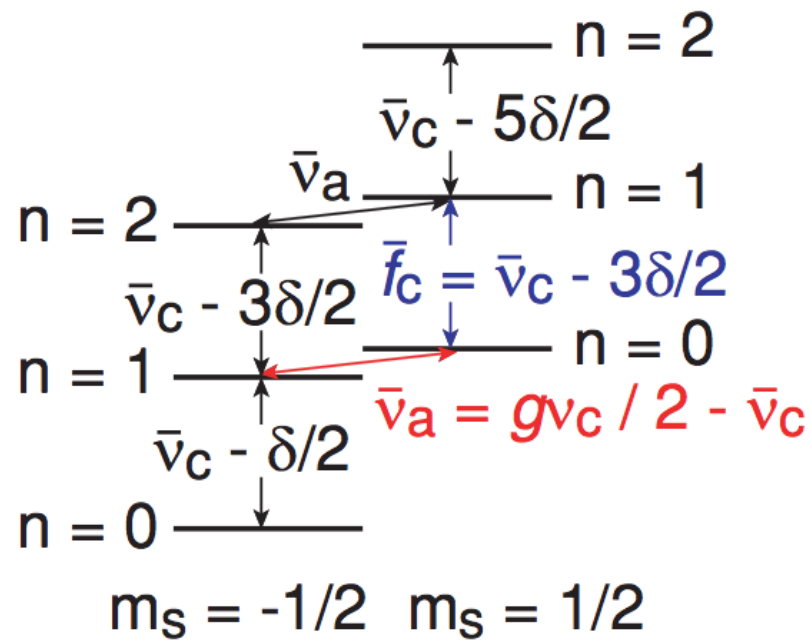
$$\bar{u}_r(p')\{2m\gamma^\mu - (p' + p)^\mu - i\sigma^{\mu\nu}(p' - p)_\nu\}u_s(p) = 0,$$
$$\bar{v}_r(p')\{2m\gamma^\mu + (p' + p)^\mu + i\sigma^{\mu\nu}(p' - p)_\nu\}v_s(p) = 0. \tag{A.25}$$

Measurement of Landé g_e Factor

cylindrical Penning trap cavity



lowest cyclotron and spin levels



experiment: $1/\alpha = 137.035\,999\,084\,(51)$ PRL 100, 120801 (2008)

theory: $1/\alpha = 137.035\,999\,070\,(98)$ PRL 99, 039902 (2007)

Quantum Electrodynamics

Summary QED Lagrangian

local phase invariance : $\psi(x) \to e^{i\alpha(x)}\psi(x)$

field strength tensor : $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$

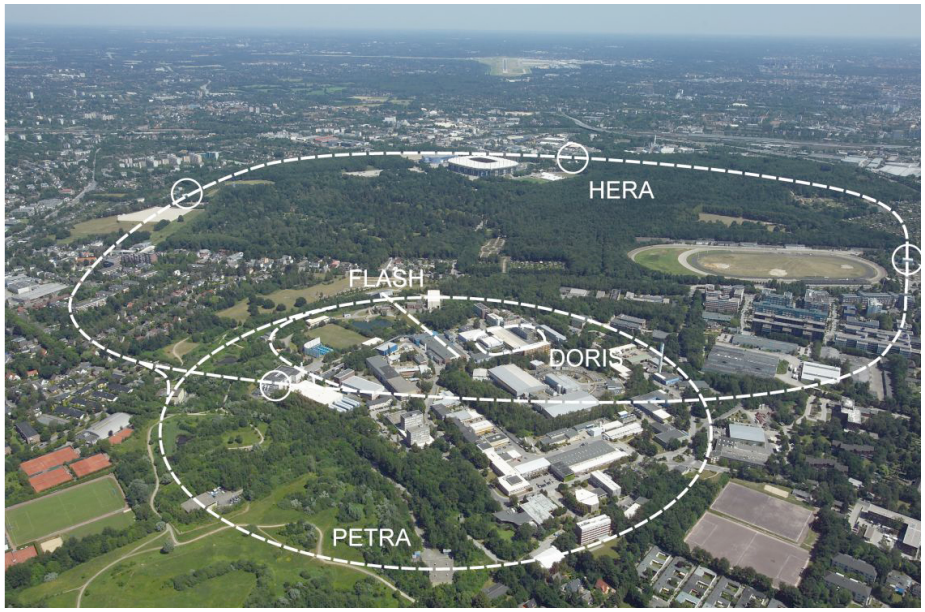
U(1) gauge symmetry

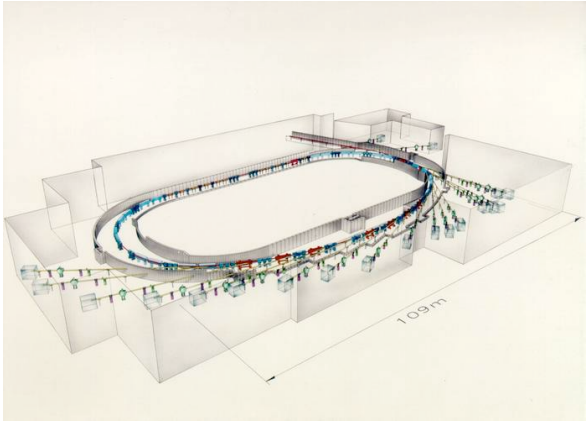
$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \bar{\psi}(x)\left(i\gamma_\mu \partial^\mu - m\right)\psi(x) + e A_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x)$$

free photon Dirac field interaction

PETRA and DORIS e^+e^- Colliders at DESY

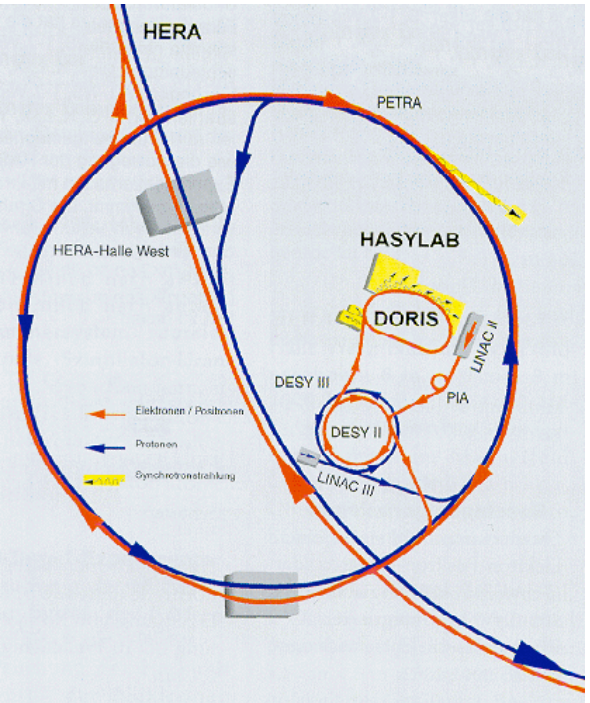
HERA
(Hadron-Elektron-Ring-Anlage)





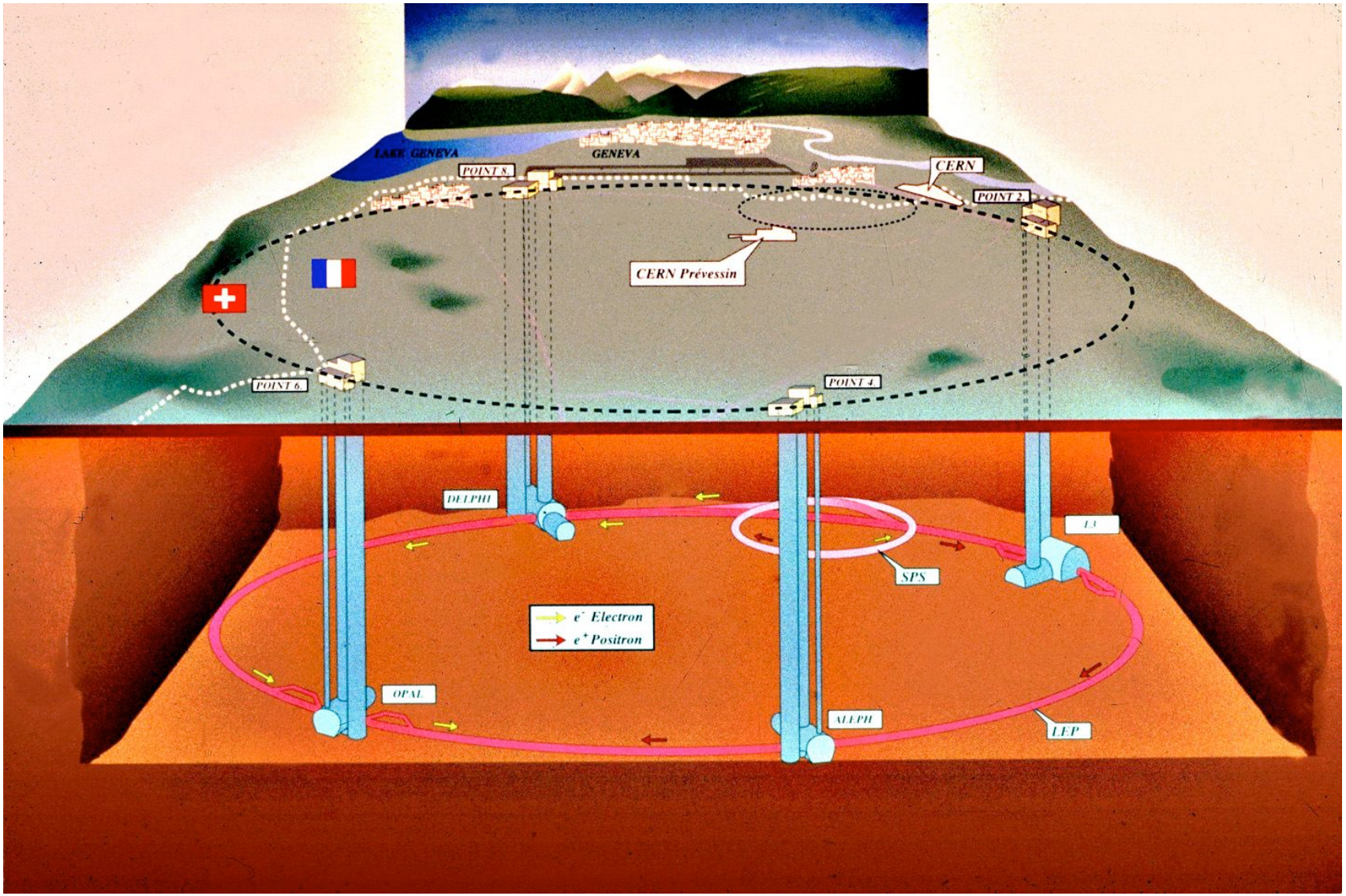
DORIS
(Doppel-Ring-Speicher)

PETRA
(Positron-Elektron-Tandem-Ring-Anlage)



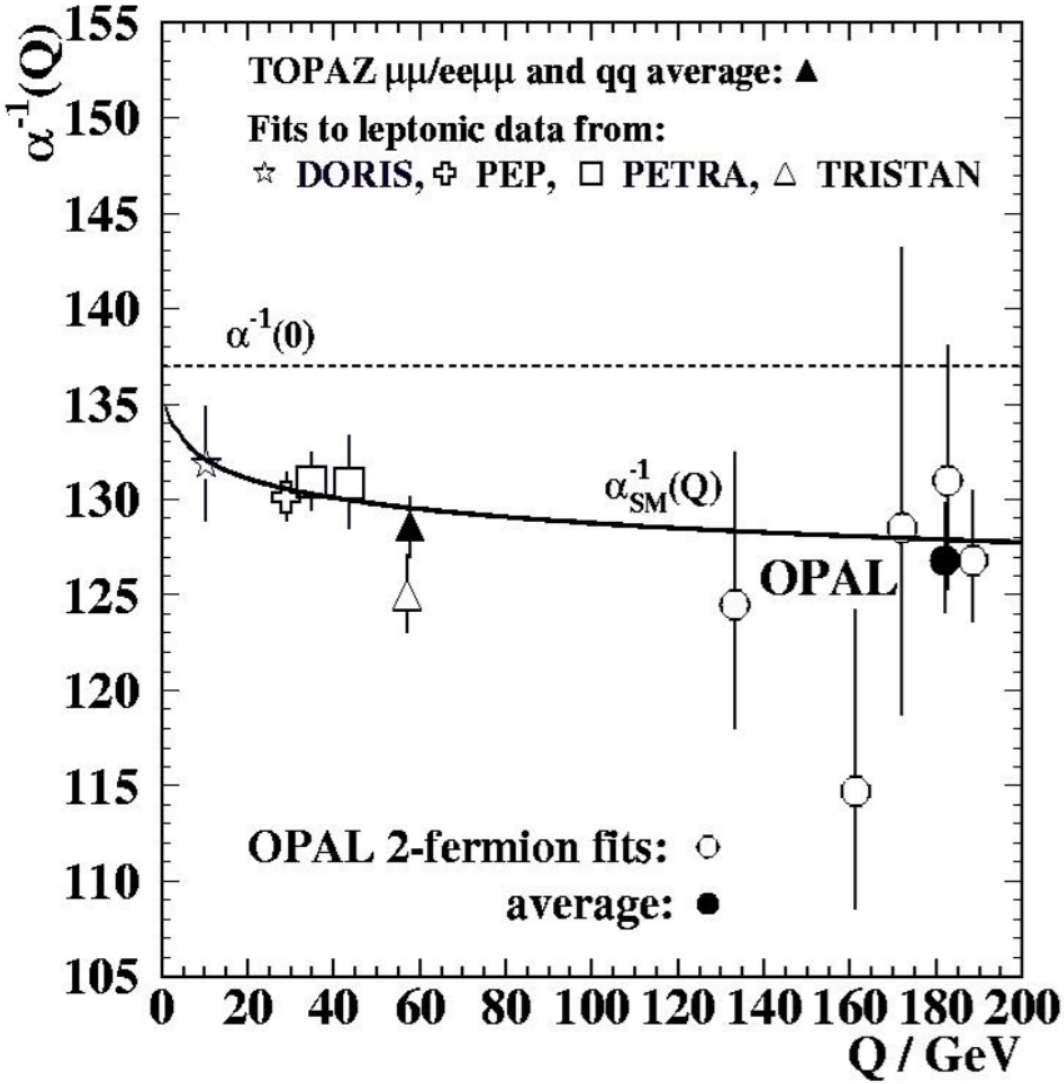
The LEP e^+e^- Collider at CERN

$\sqrt{s}$=90-209 GeV

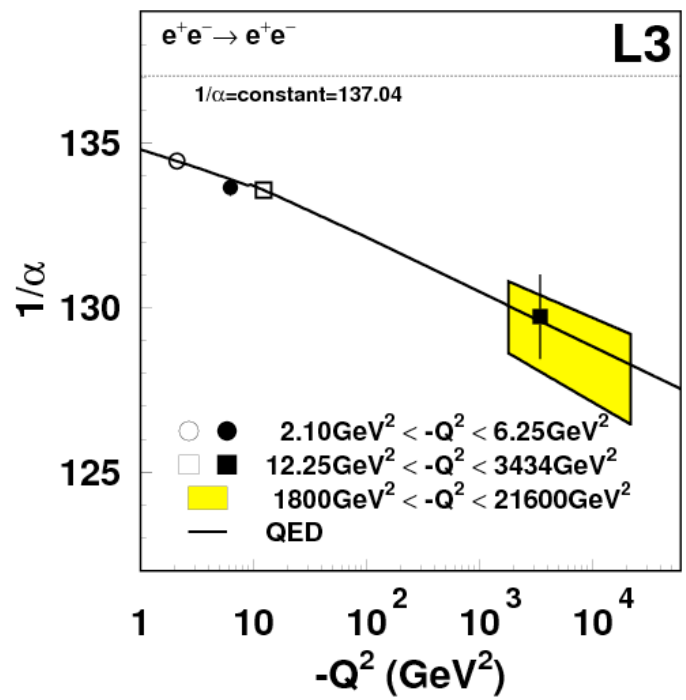


Running of fine structure constant

α^{-1}



$e^+e^- \to e^+e^-$



α^{-1}_{QED} as function of $\sqrt{s}$ in e^+e^- annihilation

Gell-Mann Matrices and Structure Constant

The λ_a $(a = 1, \ldots, 8)$ are the Gell-Mann matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{C.44}$$

Relations:

$$\mathrm{Tr}\, \lambda_a = 0, \tag{C.45}$$

$$\mathrm{Tr}(\lambda_a \lambda_b) = 2\delta_{ab}, \tag{C.46}$$

$$[\lambda_a, \lambda_b] = 2\mathrm{i} f_{abc} \lambda_c, \tag{C.47}$$

$$\{\lambda_a, \lambda_b\} = \tfrac{4}{3}\delta_{ab} + 2 d_{abc} \lambda_c, \tag{C.48}$$

where f_{abc} is totally antisymmetric, and d_{abc} is totally symmetric with values for the independent components as given in Table C.1.

(From O. Nachtmann, Elementary Particle Physics)

Table 4.1 The independent, nonvanishing components of f_{abc} and d_{abc}.

(a, b, c)	f_{abc}	(a, b, c)	d_{abc}
123	1	118	$1/\sqrt{3}$
147	1/2	146	1/2
156	−1/2	157	1/2
246	1/2	228	$1/\sqrt{3}$
257	1/2	247	−1/2
345	1/2	256	1/2
367	−1/2	338	$1/\sqrt{3}$
458	$\sqrt{3}/2$	344	1/2
678	$\sqrt{3}/2$	355	1/2
		366	−1/2
		377	−1/2
		448	$-1/(2\sqrt{3})$
		558	$-1/(2\sqrt{3})$
		668	$-1/(2\sqrt{3})$
		778	$-1/(2\sqrt{3})$
		888	$-1/\sqrt{3}$

Feynman rules of QCD

(O. Nachtmann, Elementary Particle Physics)

APPENDIX D

The Feynman Rules of QCD

In this Appendix we present the Feynman rules of QCD. The derivation of these rules from the Lagrange density (19.22) is best achieved using the path-integral formalism. Here we restrict ourselves to giving only the results.

The Feynman diagrams of QCD are constructed from three types of lines: quark lines, gluon lines, and so-called Faddev–Popov ghost lines. Within QCD it is not possible that lines corresponding to different quark flavors are joined together. The ghosts are to be treated like a complex scalar particle with zero mass. Thus ghost and antighost are distinct. The corresponding lines, like the quark lines, are given an arrow. The ghosts transform according to the adjoint representation of the color group SU(3) and thus carry a color index a ($a = 1, \ldots, 8$).

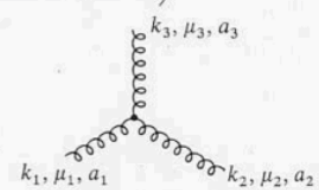
We shall make use of the following scheme to relate physical situations, parts of diagrams, and analytical expressions:

quark in initial state	incoming quark line
$u(p)$	$q(p)$
quark in final state	outgoing quark line
$\bar{u}(p)$	$q(p)$
antiquark in initial state	outgoing quark line
$\bar{v}(p)$	$\bar{q}(p)$
antiquark in final state	incoming quark line
$v(p)$	$\bar{q}(p)$
ghost in initial or antighost in final state	incoming ghost line
1	
ghost in final or antighost in initial state	outgoing ghost line
1	
virtual quark with flavour j	internal quark line
$\dfrac{\mathrm{i}}{\not{p} - m_j + \mathrm{i}\varepsilon}$	p
gluon in initial state	incoming gluon line
ε^{μ}	
gluon in final state	outgoing gluon line
$\varepsilon^{*\mu}$	
virtual gluon	internal gluon line
$\mathrm{i}\delta^{ab}\left\{\dfrac{-g_{\mu\nu}}{k^2 + \mathrm{i}\varepsilon} + \dfrac{(1-\xi)k_\mu k_\nu}{(k^2 + \mathrm{i}\varepsilon)^2}\right\}$	
virtual ghost	internal ghost line
$\mathrm{i}\dfrac{\delta^{ab}}{p^2 + \mathrm{i}\varepsilon}$	

The elementary processes of QCD correspond to the following vertices:

3-gluon vertex $\left(\sum_{j=1}^{3} k_j = 0, \text{ all momenta incoming}\right)$

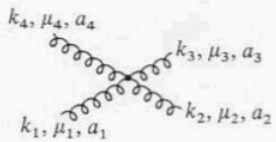
$$-g_s f_{a_1 a_2 a_3} \cdot \{(k_1 - k_2)_{\mu_3} g_{\mu_1\mu_2} + (k_2 - k_3)_{\mu_1} g_{\mu_2\mu_3} + (k_3 - k_1)_{\mu_2} g_{\mu_3\mu_1}\}$$



Feynman rules of QCD

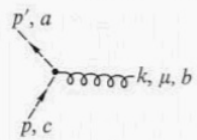
4-gluon vertex $\left(\sum_{j=1}^{4} k_j = 0, \text{ all momenta incoming}\right)$

$$ig_s^2 f_{a_1 a_2 b} f_{a_3 a_4 b} \cdot (g_{\mu_2\mu_3} g_{\mu_1\mu_4} - g_{\mu_1\mu_3} g_{\mu_2\mu_4}) + \text{cyclic perms. of } 1, 2, 3$$



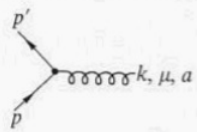
ghost–gluon vertex (p, k incoming; p' outgoing; $p + k = p'$)

$$g_s f_{abc} p'_\mu$$



quark–gluon vertex (p, k incoming; p' outgoing; $p + k = p'$)

$$-ig_s \gamma_\mu \tfrac{1}{2} \lambda_a$$



The parameters that appear in the above Feynman rules are the coupling constant g_s, the quark masses m_j ($j = 1, \ldots, f$), and the arbitrary gauge parameter ξ which must drop out in the expressions obtained for observable quantities. It is usually most convenient to set $\xi = 1$; this is the so-called Feynman gauge. We used this gauge throughout our discussion of QED where an analogous gauge parameter can be introduced.

The rules for loops and for the δ-function of energy-momentum conservation are the same as in QED (Appendix B). For every closed fermion loop and every ghost loop a factor of (-1) must be included. For closed gluon loops one must take the so-called statistical factor into account. This is obtained by counting up all possible contractions of field operators in the perturbation expansion, which, for QCD too, is as given in (9.14). The diagram in Fig. D.1(a), for example, has the statistical factor 1, and that of Fig. D.1(b) has the factor $\frac{1}{2}$. In order to see this we write the three-gluon coupling schematically as

$$\frac{1}{3!} :\mathbf{G}^3(x):,$$

where $\mathbf{G}(x)$ is the gluon field operator. The diagram of Fig. D.1(a) then corresponds to a certain Fourier transform of the vacuum expectation value

$$\langle 0| \frac{1}{3!} :\mathbf{G}^3(x): \mathbf{G}(x_1)\mathbf{G}(x_2)\mathbf{G}(x_3)|0\rangle \propto 1. \qquad \text{(D.1)}$$

The evaluation is carried out using Wick's theorem from Sect. 8.2. $\mathbf{G}(x_1)$ must be contracted in turn with each of the three operators from $:\mathbf{G}^3(x):$. For $\mathbf{G}(x_2)$

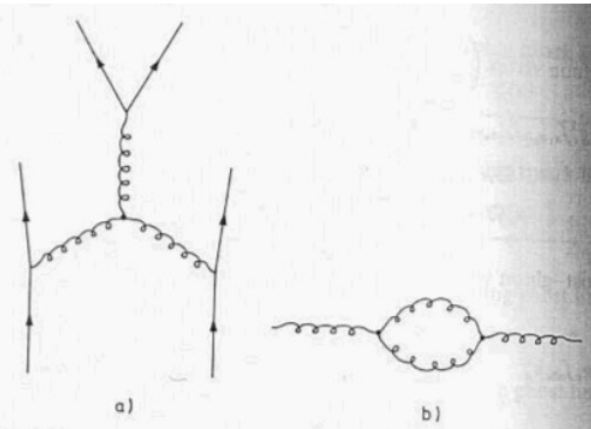


Figure D.1 A diagram with a three-gluon vertex in which no loop occurs, thus giving a statistical factor of 1(a). A self-energy diagram for the gluon propagator. In this a statistical factor of $\frac{1}{2}$ must be taken (b).

only two possibilities then remain, and for $\mathbf{G}(x_3)$ only one. The net result indicated in (D.1) is that the factor 1/3! is cancelled. If we now move on to the diagram of Fig. D.1(b) we find, with the factor 1/2! from the exponential series in (9.14), a corresponding vacuum expectation value of

$$\frac{1}{2!} \langle 0| \frac{1}{3!} :\mathbf{G}^3(x): \frac{1}{3!} :\mathbf{G}^3(y): \mathbf{G}(x_1)\mathbf{G}(x_2)|0\rangle \propto \tfrac{1}{2}. \qquad \text{(D.2)}$$

This statistical weight of $\frac{1}{2}$ can also be interpreted as phase-space factor for the two gluons in the intermediate state since these are indistinguishable.

When calculating the polarization sums for gluons in the initial or final states there are likewise a few points that should be noted. It is always possible to do the calculation noncovariantly, i.e., to consider only the transverse polarization degrees of freedom of the gluons (cf. Appendix E). Alternatively, one can work with covariant quantities, i.e., replace the polarization sum of a gluon in the initial or final state by

$$\sum_{\text{Spins}} \varepsilon_\mu \varepsilon_\nu^* = -g_{\mu\nu}. \qquad \text{(D.3)}$$

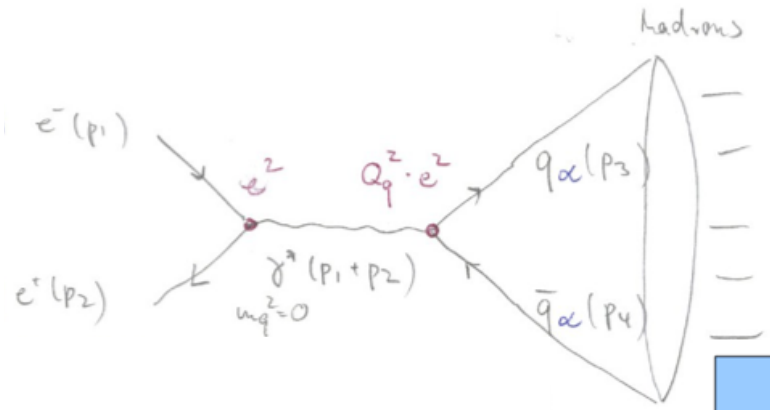
In this case, however, it is also necessary to take into account processes of the same order with incoming and outgoing ghost lines, treating ghosts and antighosts formally as if they were extra degrees of freedom for the gluons to be considered in their spin sums. (Of course only two external gluon lines can be replaced at a time by ghost lines since an odd number of external ghost lines is impossible, as is an odd number of external fermion lines.) Amplitudes corresponding to diagrams which are identical except for permutations of the labels of external ghost lines receive extra sign factors relative to each other

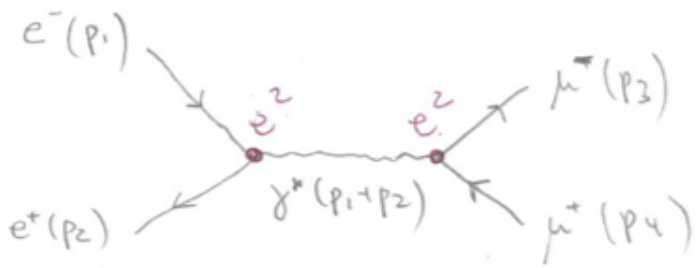
Feynman rules of QCD

in the same way as for fermions (cf. Appendix B). In calculating transition probabilities one has furthermore to include a factor $(-1)^{n/2}$ where n is the number of external ghost lines in the corresponding diagram. Thus some ghost processes, for instance, the production of a single ghost-antighost pair ($n = 2$), get assigned a negative probability (cf. Cutler 1978).

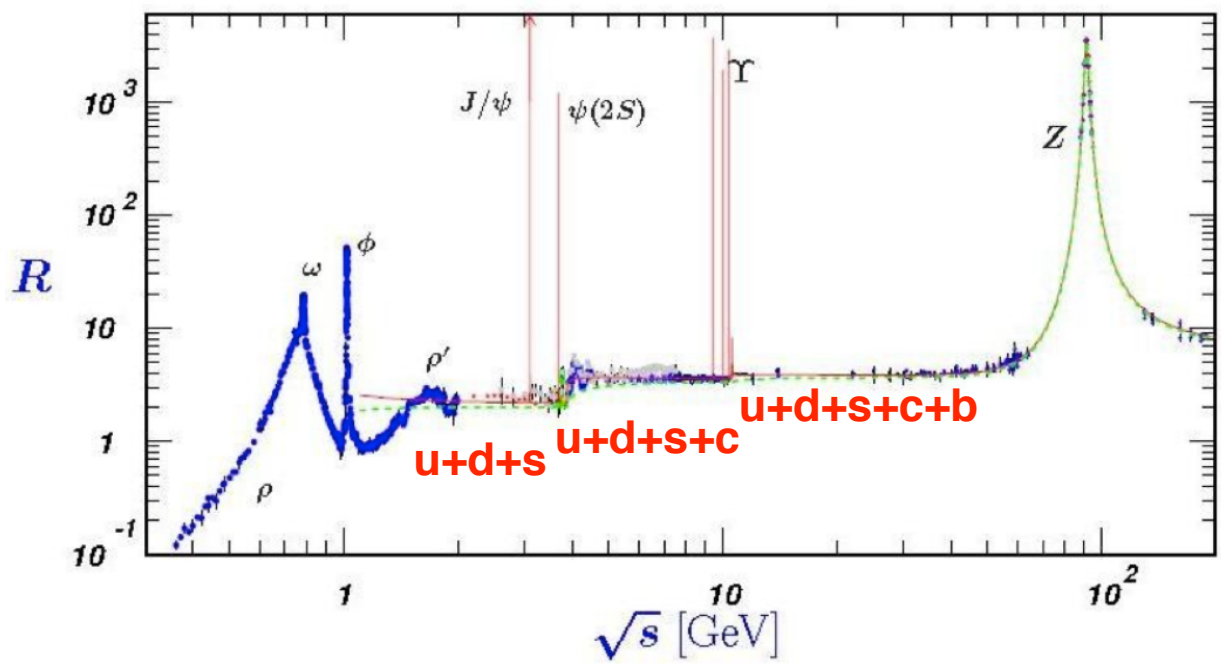
These rules are sufficient for the calculation of all tree diagrams, i.e., diagrams without loops. For the regularization and renormalization procedures to be applied beyond this level the reader is referred to the available textbooks, e.g., Itzykson (1980).

Search for Different Quark Flavors





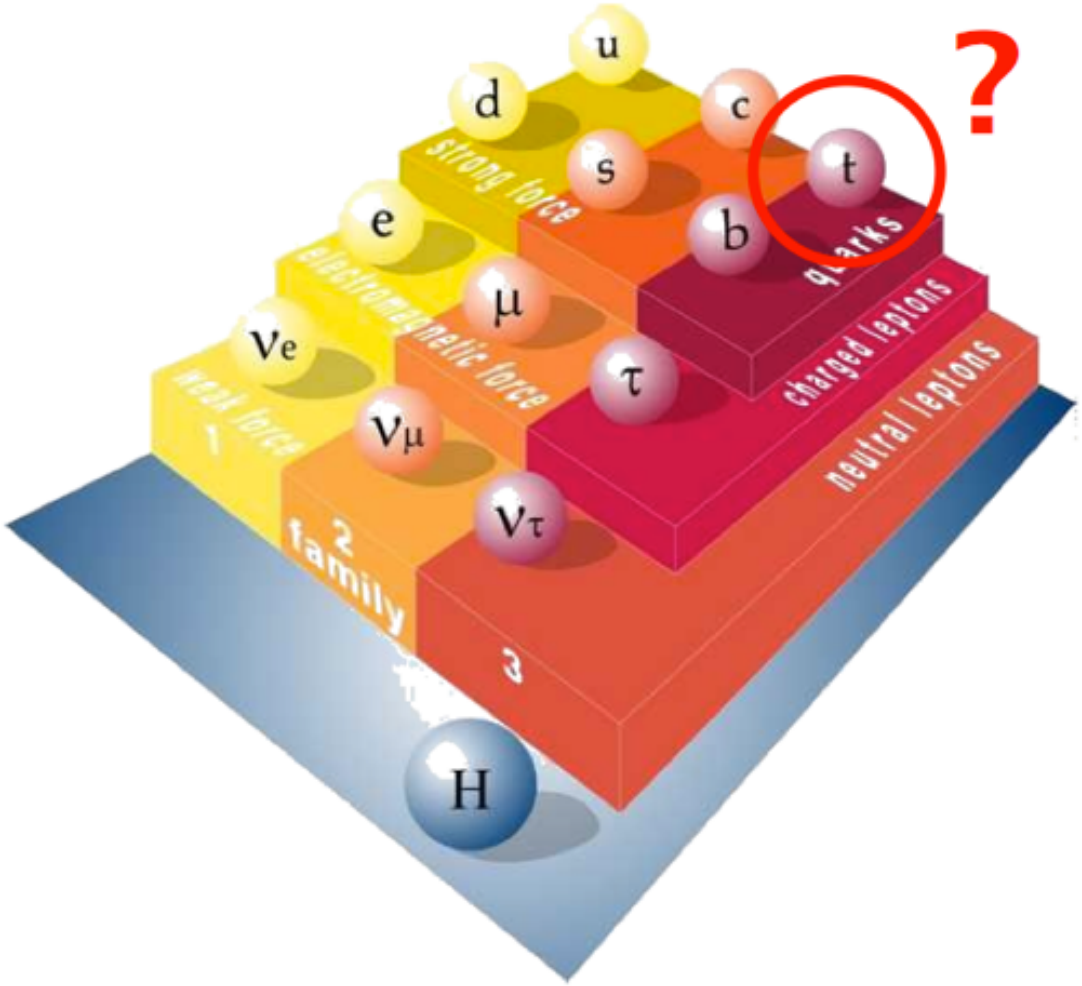
$$R = \frac{\sigma(e^+ + e^- \to hadrons)}{\sigma(e^+ + e^- \to \mu^+ + \mu^-)} = \frac{3\sum(quark\ charge)^2}{1^2}$$

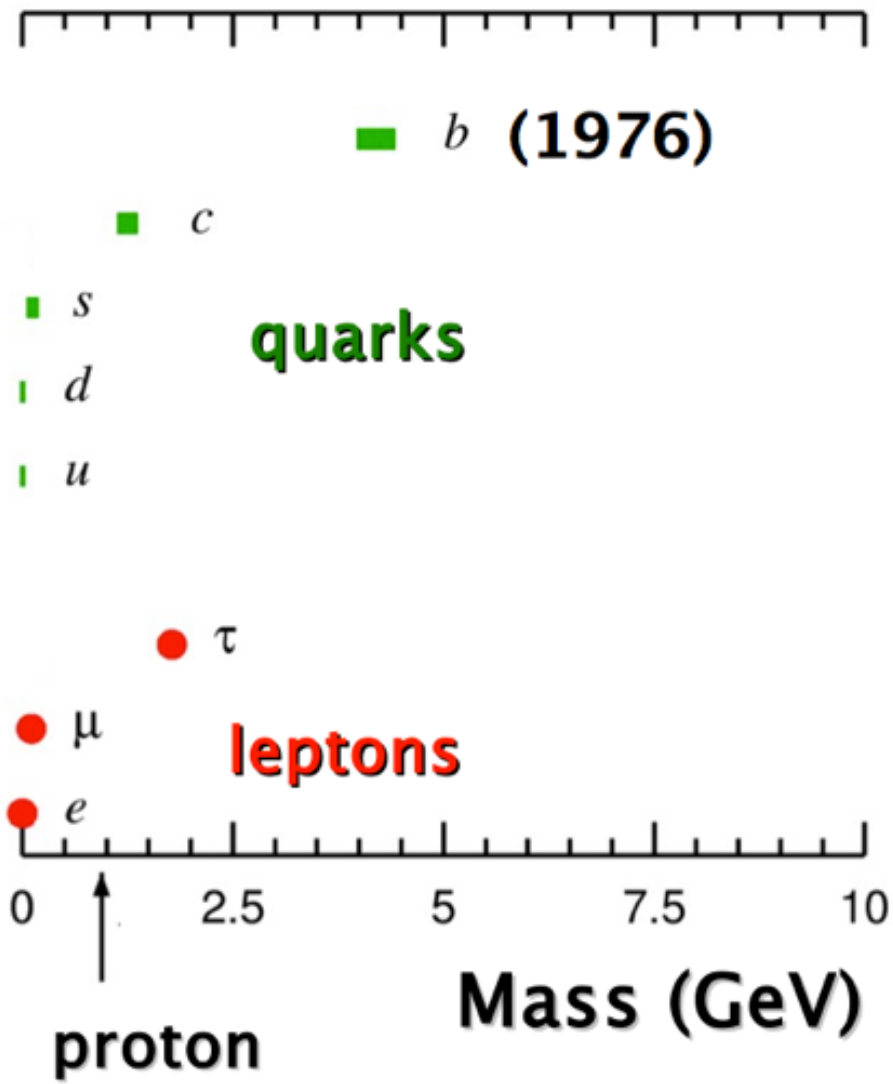


→ structure of quark families suggested existence of a 6th quark:

where is the top?

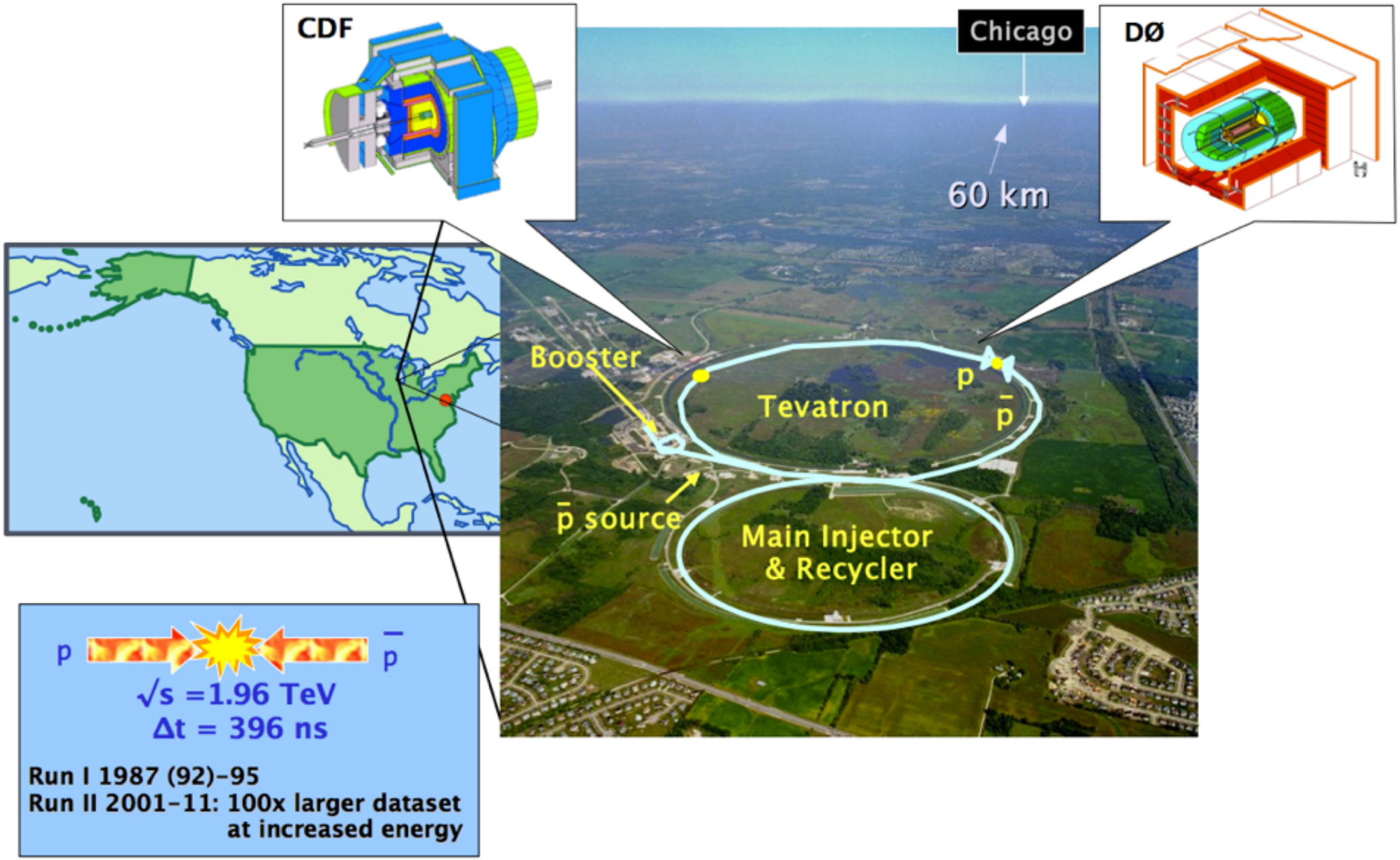
The Top Quark





neutrino masses ≪ 1 eV

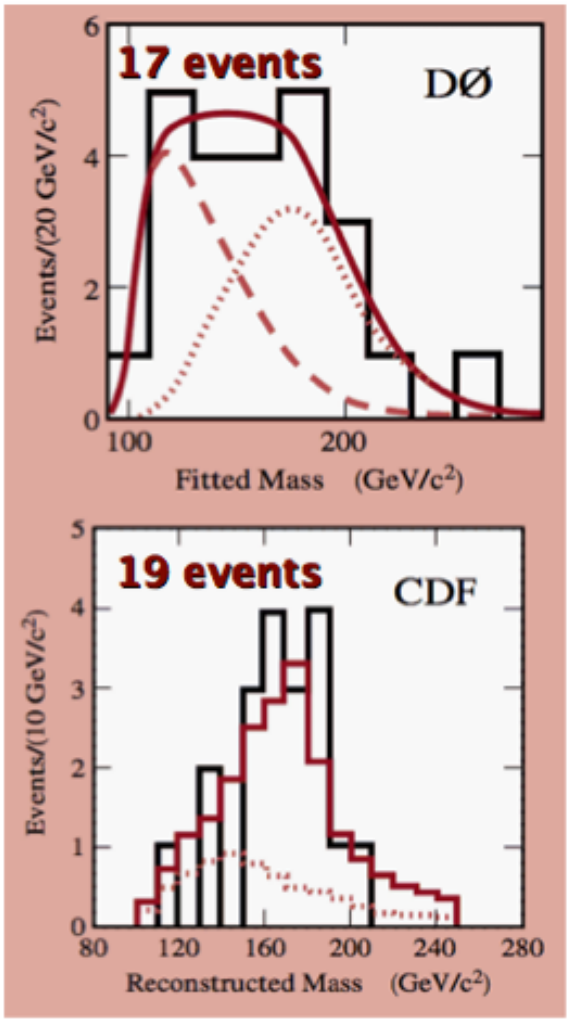
The Tevatron $p\bar{p}$ Collider at Fermilab



The Top Quark

discovery

PRL 74, 2632 (1995)
PRL 74, 2626 (1995)



1995, CDF and DØ experiments, Fermilab

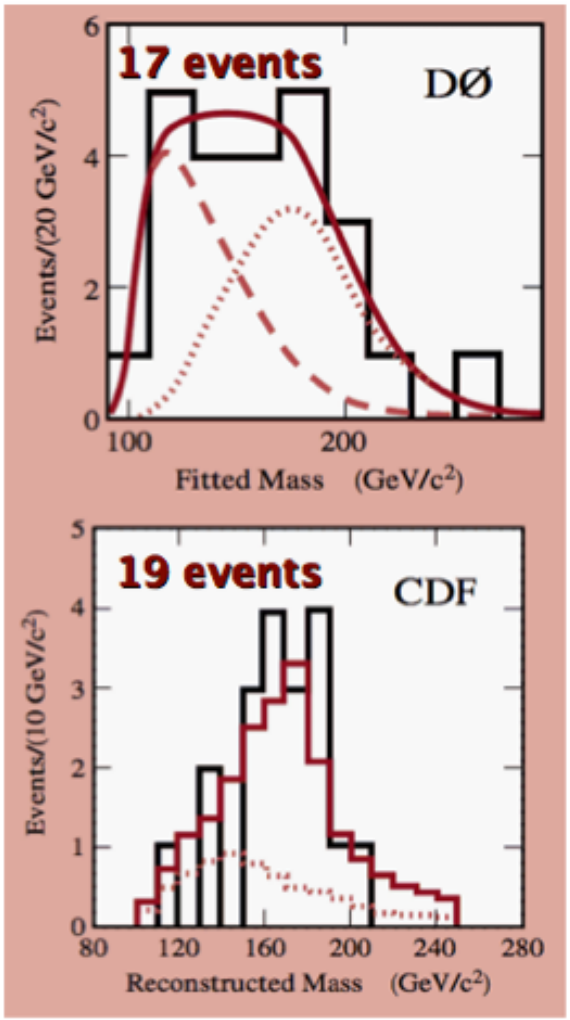
March 2nd, 1995:

First announcement of Top Discovery in public seminar at Fermilab

The Top Quark

discovery

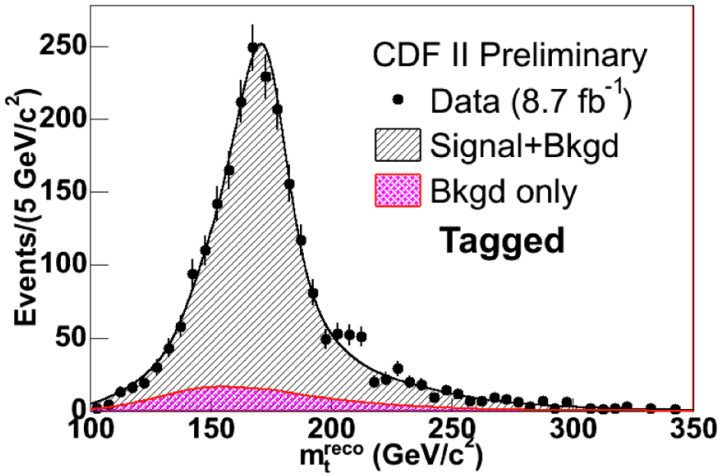
PRL 74, 2632 (1995)
PRL 74, 2626 (1995)



1995, CDF and DØ experiments, Fermilab

today

1000s of events

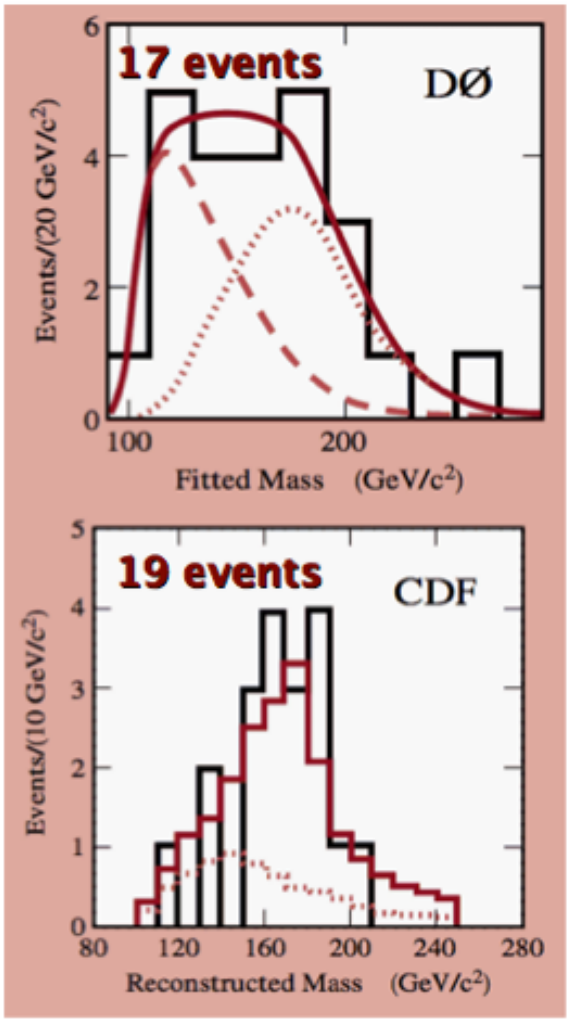


Tevatron

The Top Quark

discovery

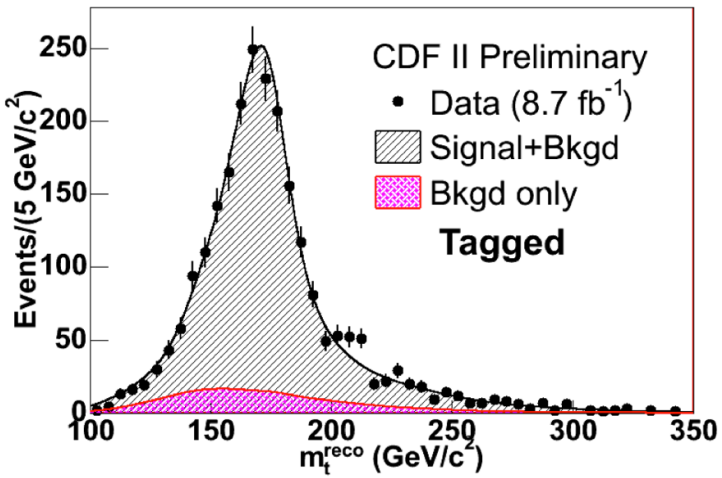
PRL 74, 2632 (1995)
PRL 74, 2626 (1995)



1995, CDF and DØ experiments, Fermilab

today

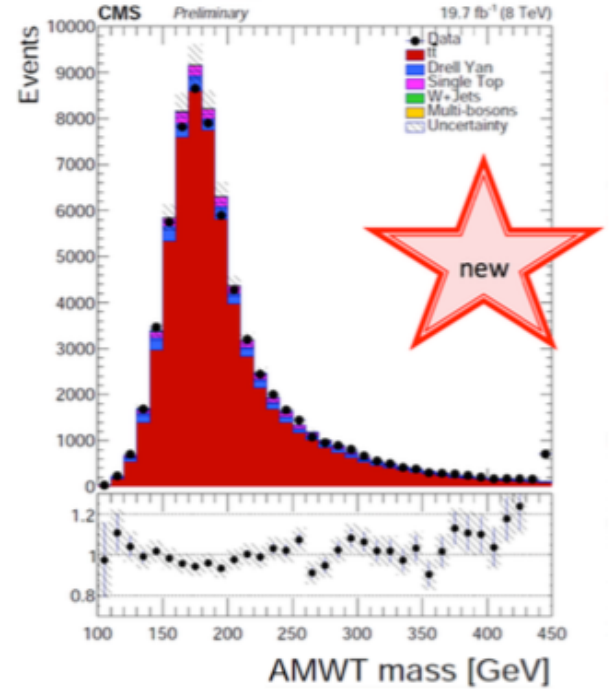
1000s of events



Tevatron

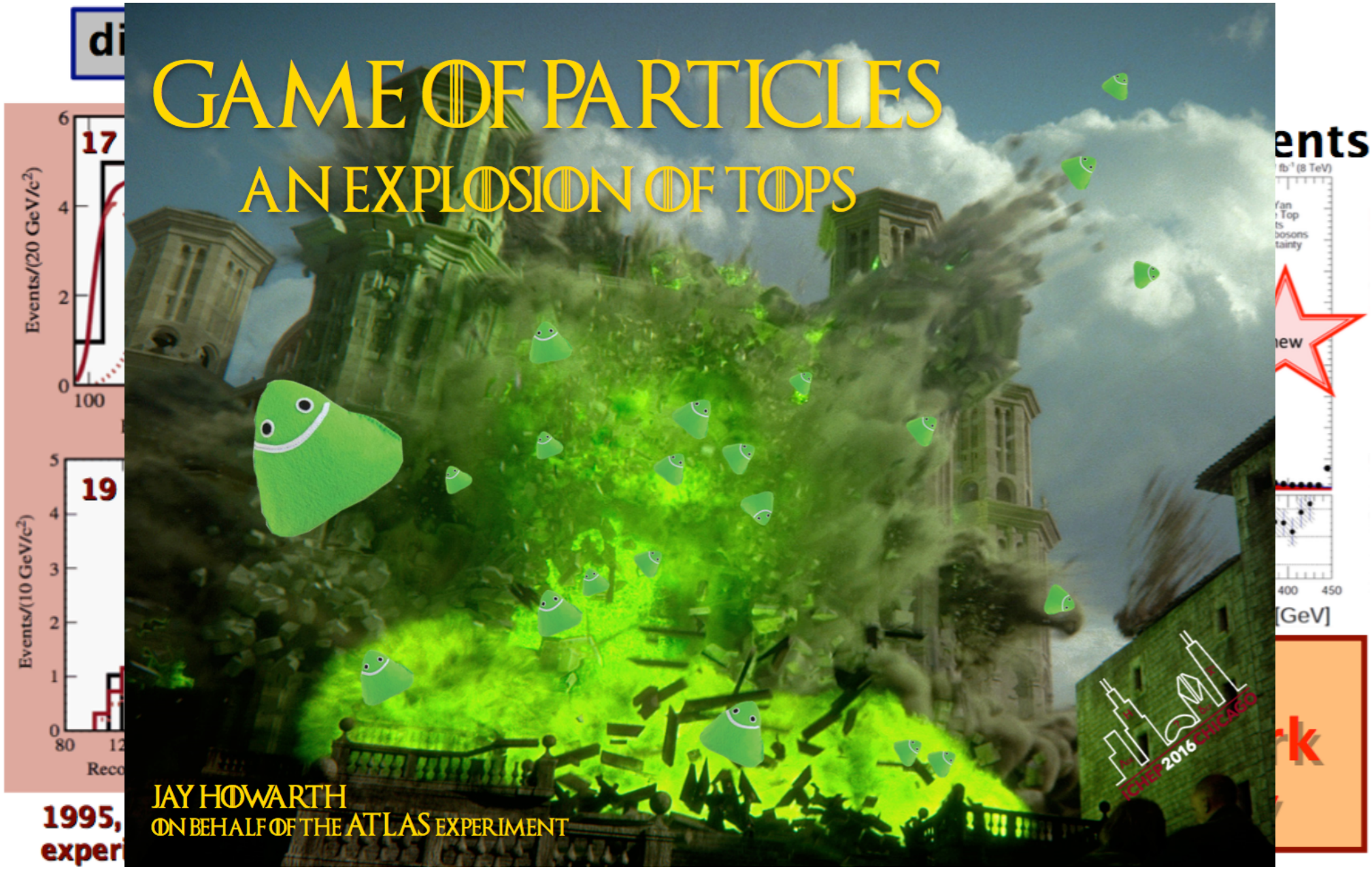
today

100000s of events

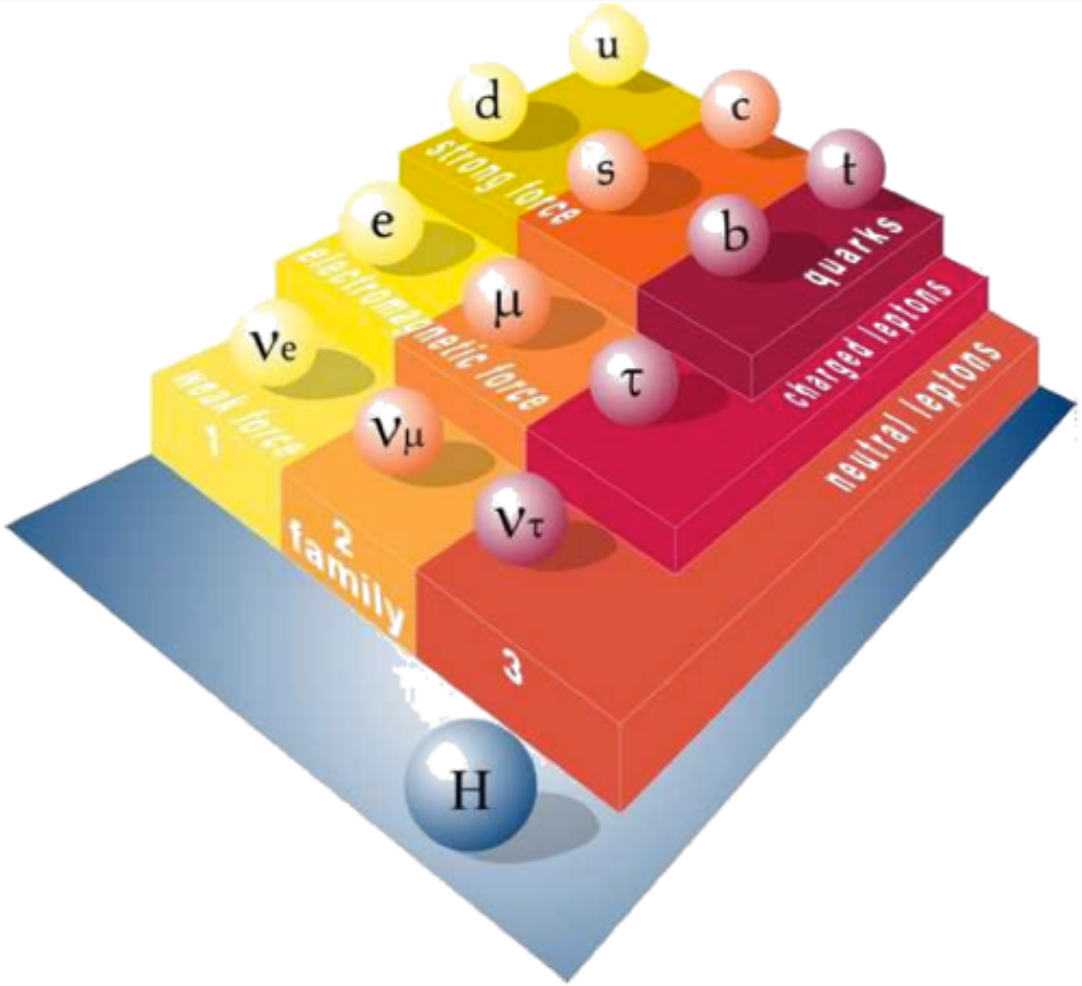


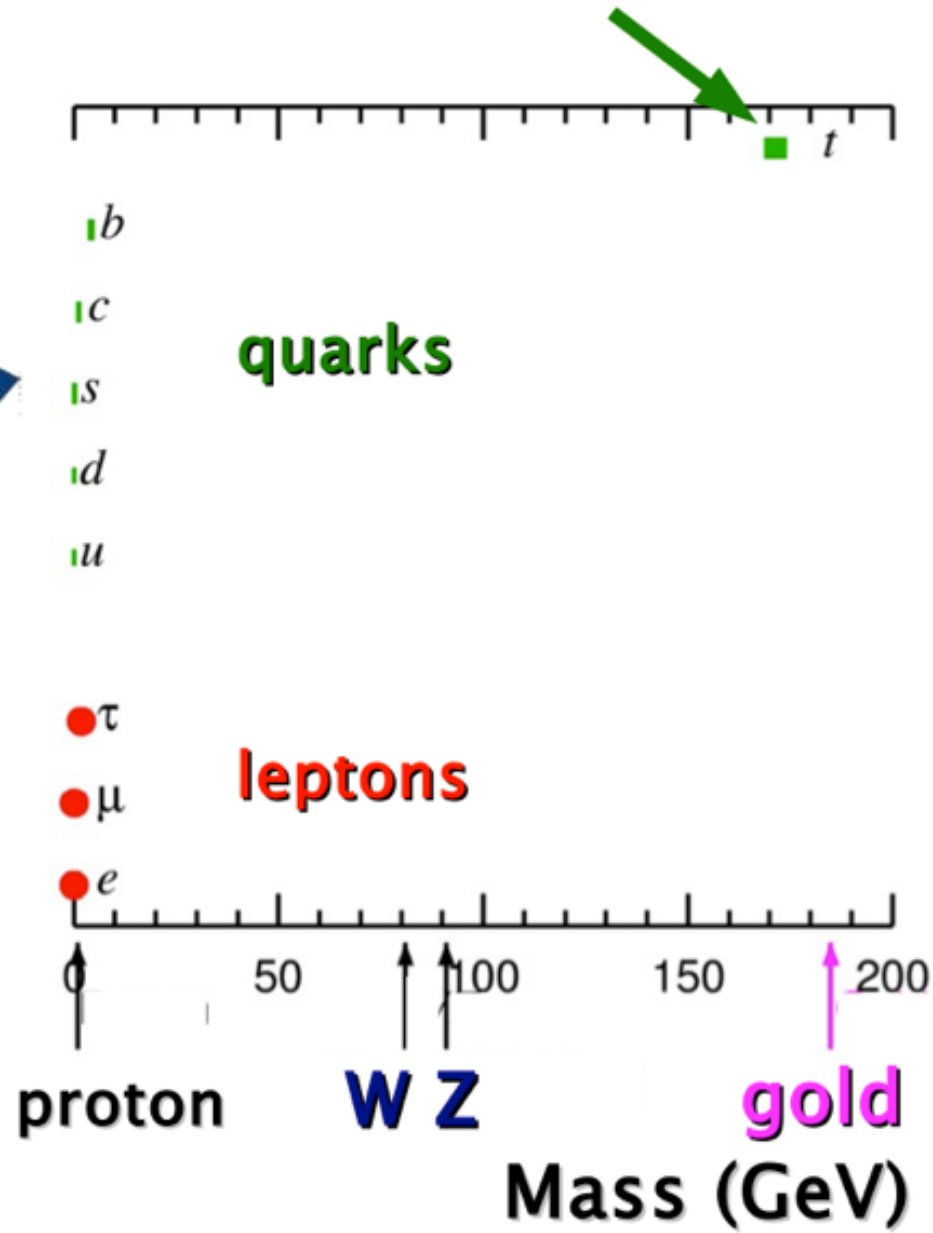
LHC: top quark factory

The Top Quark

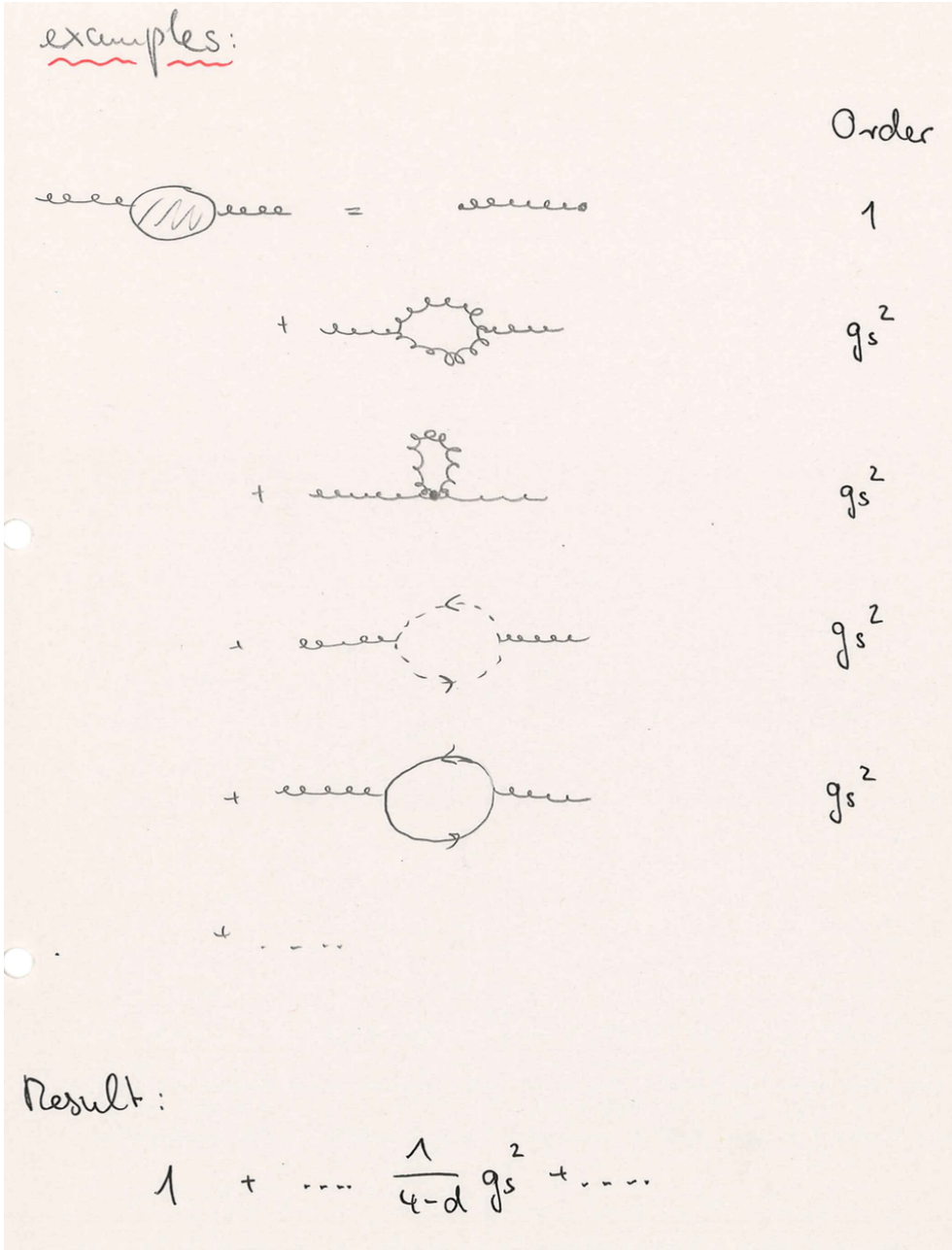


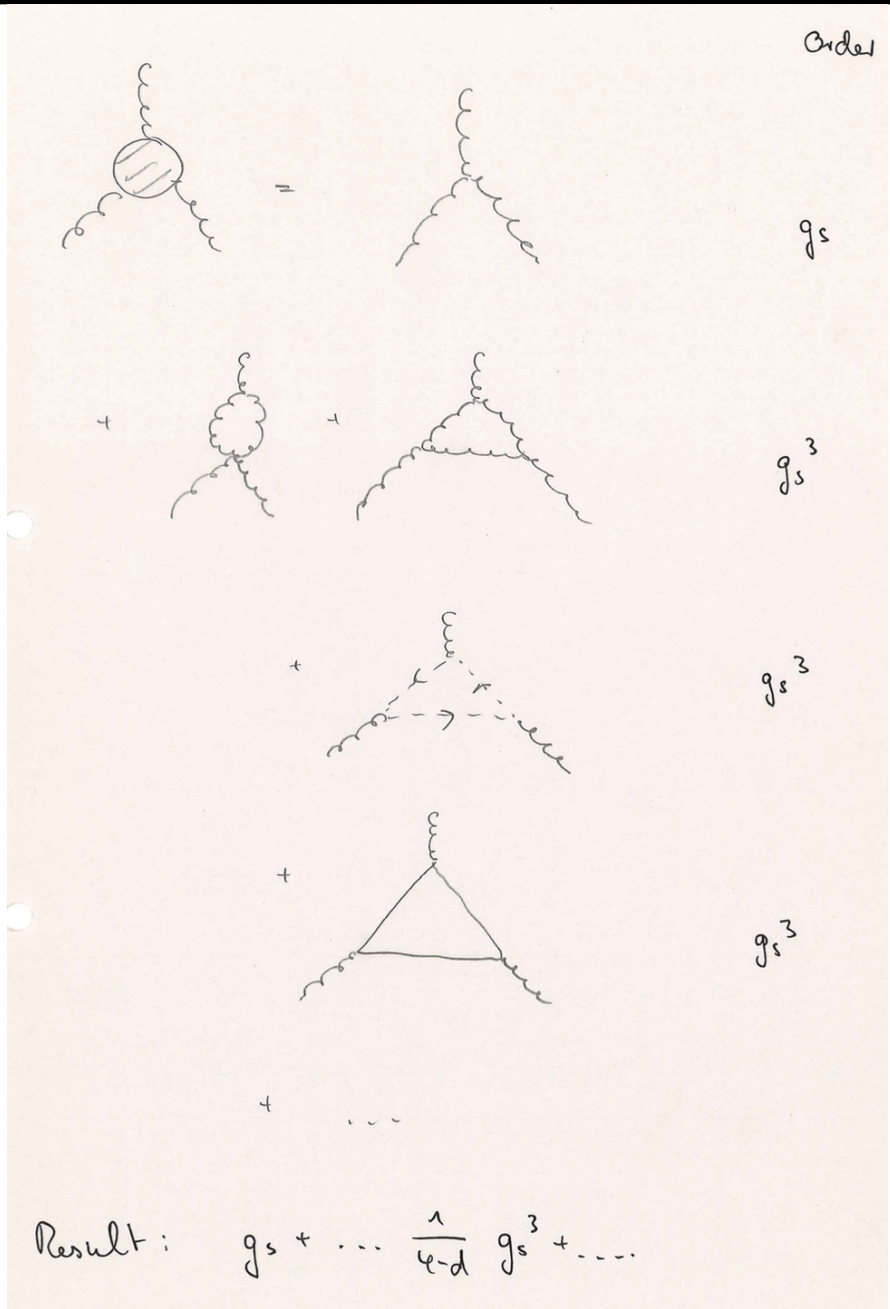
The Top Quark





Dimensional Regularisation





Asymptotic Freedom

$$\mu \frac{d}{d\mu} g_s(\mu) = \beta(g_s(\mu)) \qquad (x)$$

$$\beta(g) = -\frac{g}{2}\left(\frac{\beta_0}{2\pi}\alpha_s + \frac{\beta_1}{4\pi^2}\alpha_s^2 + \frac{\beta_2}{64\pi^3}\alpha_s^3 + \dots\right)$$

$$\alpha_s = \frac{g^2}{4\pi} \qquad (*)$$

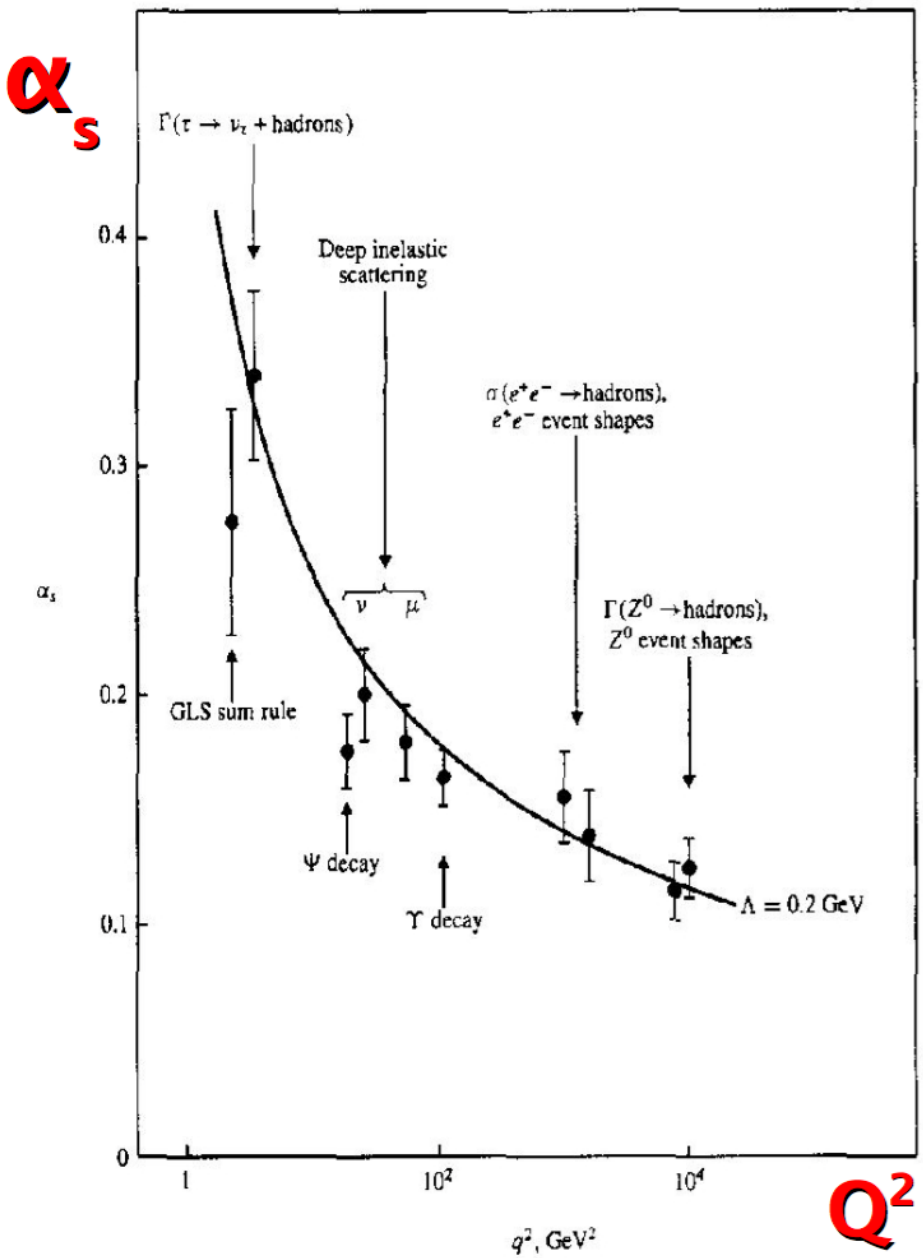
$$\beta_1 = 51 - \frac{19}{3} n_f$$

$$\beta_2 = 2857 - \frac{5033}{9} n_f + \frac{325}{27} n_f^2,$$

etc.

Also the term β_3 is known by rather lengthy...

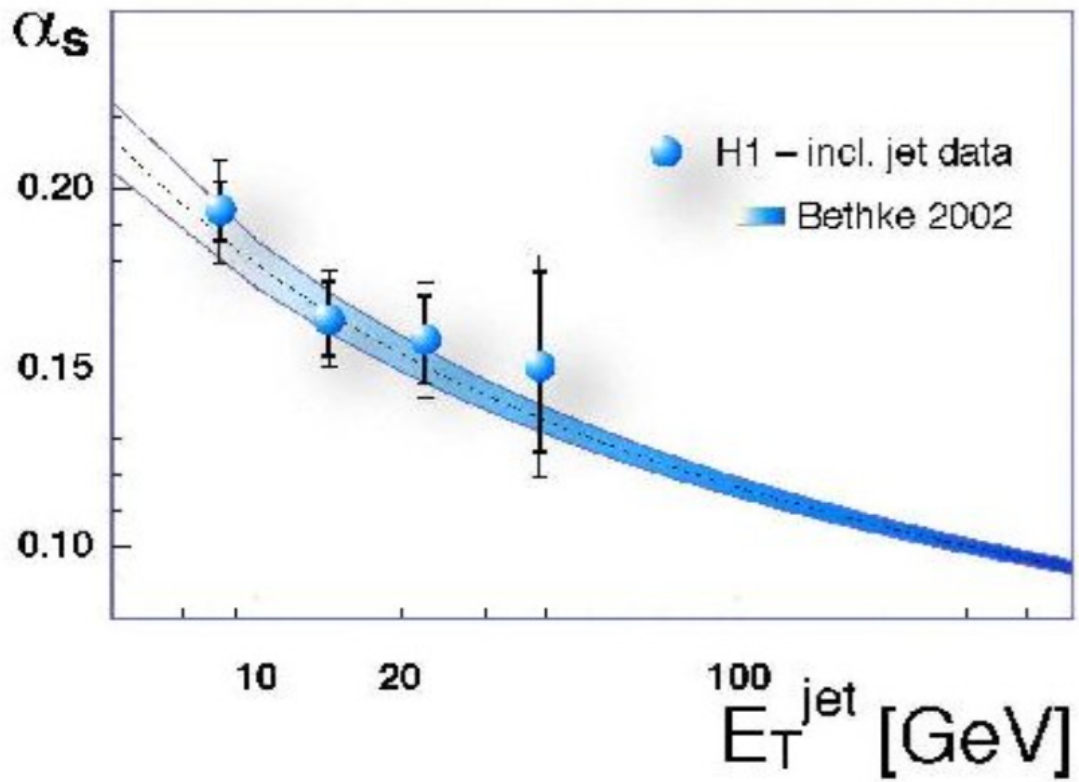
Asymptotic Freedom



strong coupling $\alpha_s(Q^2)$

Asymptotic Freedom

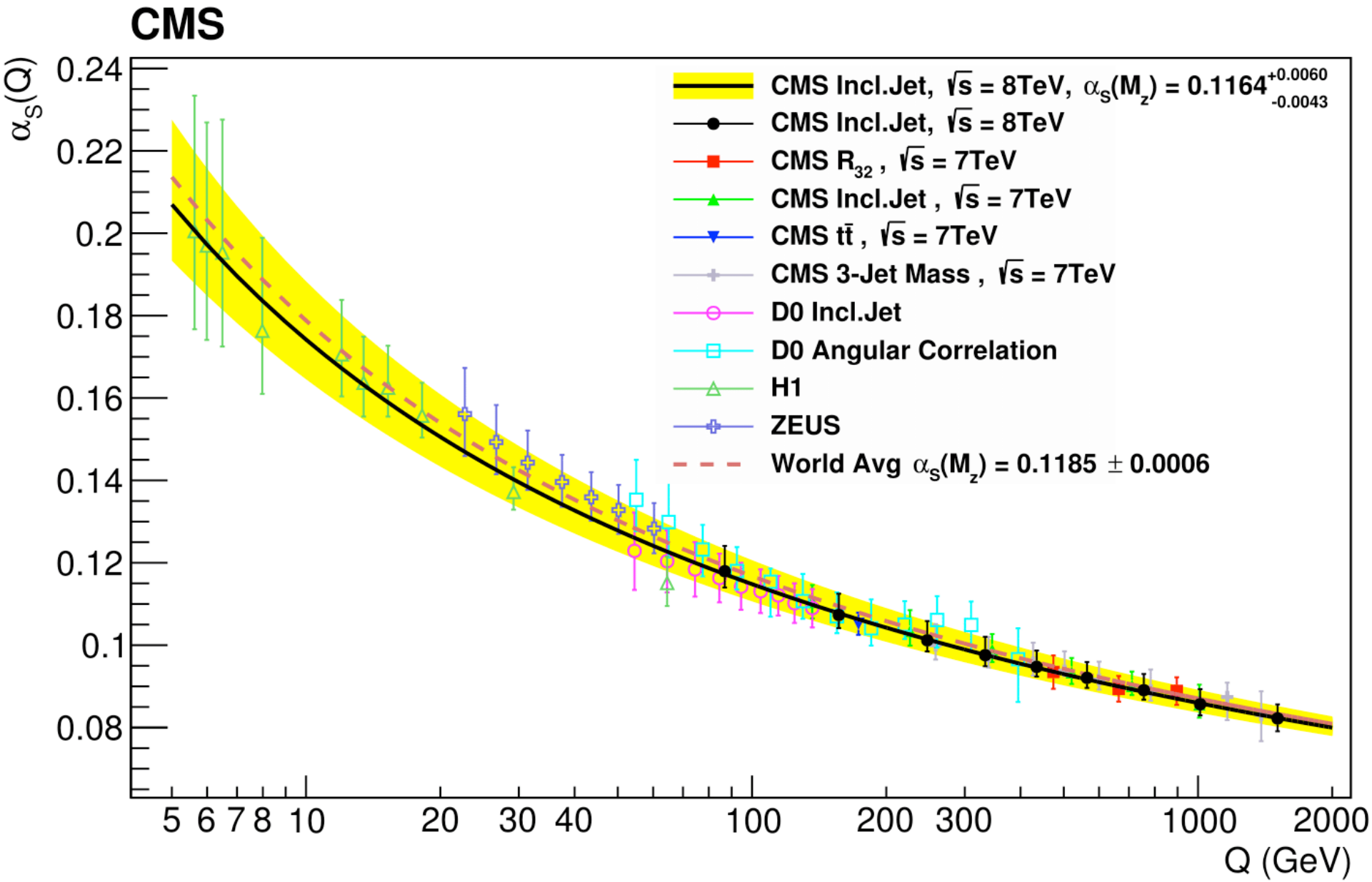
David Gross, David Politzer and Frank Wilczek awarded the 2004 Nobel Prize in Physics for the discovery of asymptotic freedom



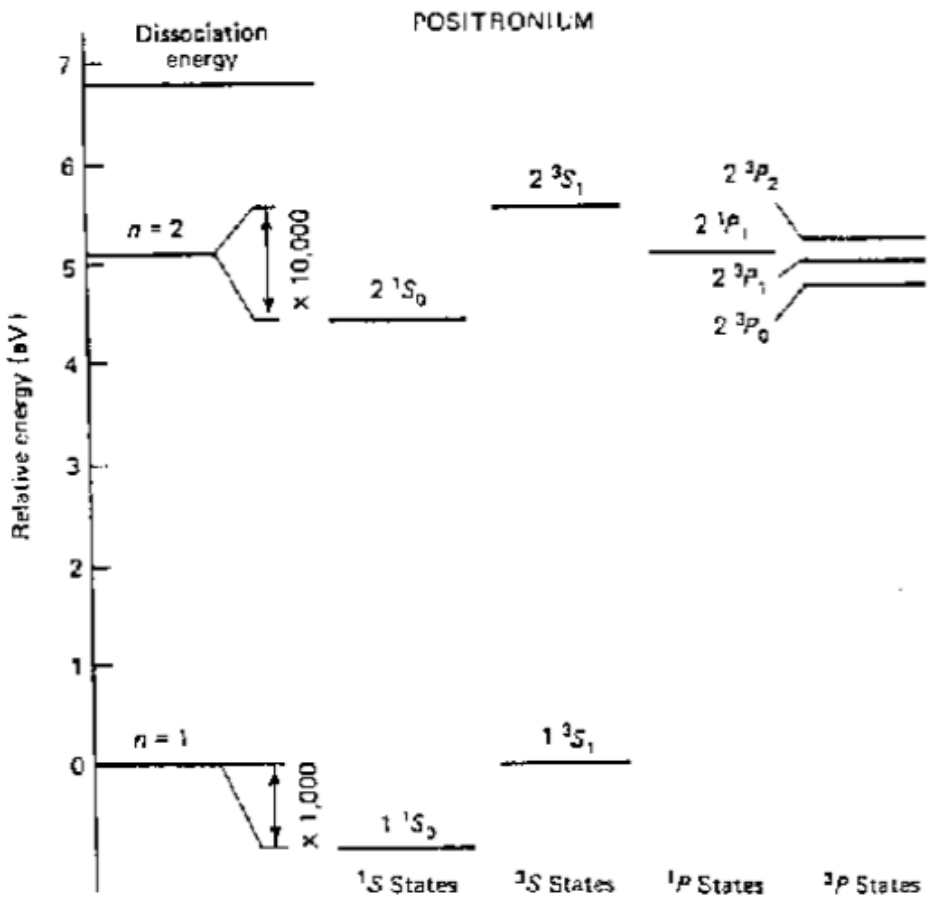
Twenty years ago, David Gross, David Politzer and Frank Wilczek discovered **asymptotic freedom** in the theory of the strong interactions. Measurements published by **H1** in the year 2000 beautifully illustrate this effect: the strong coupling a_s is seen to decrease as the energy at which it is measured, E_T^{jet}, increases.

Noble Prize 2004 for running of strong coupling α_s

Asymptotic Freedom

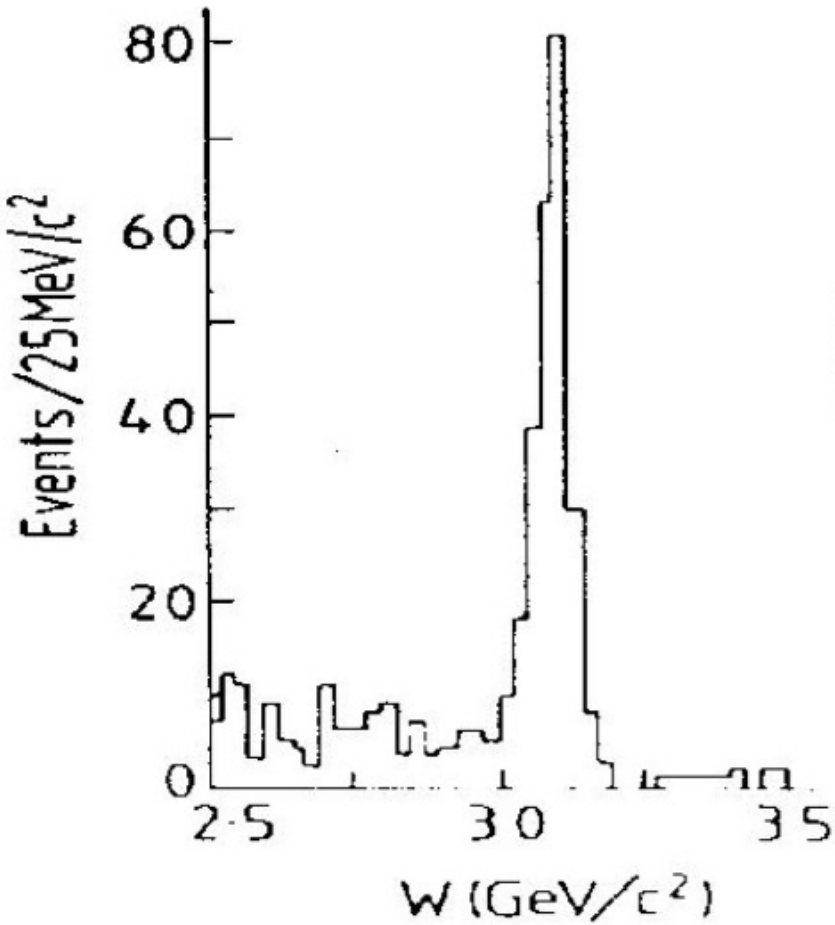


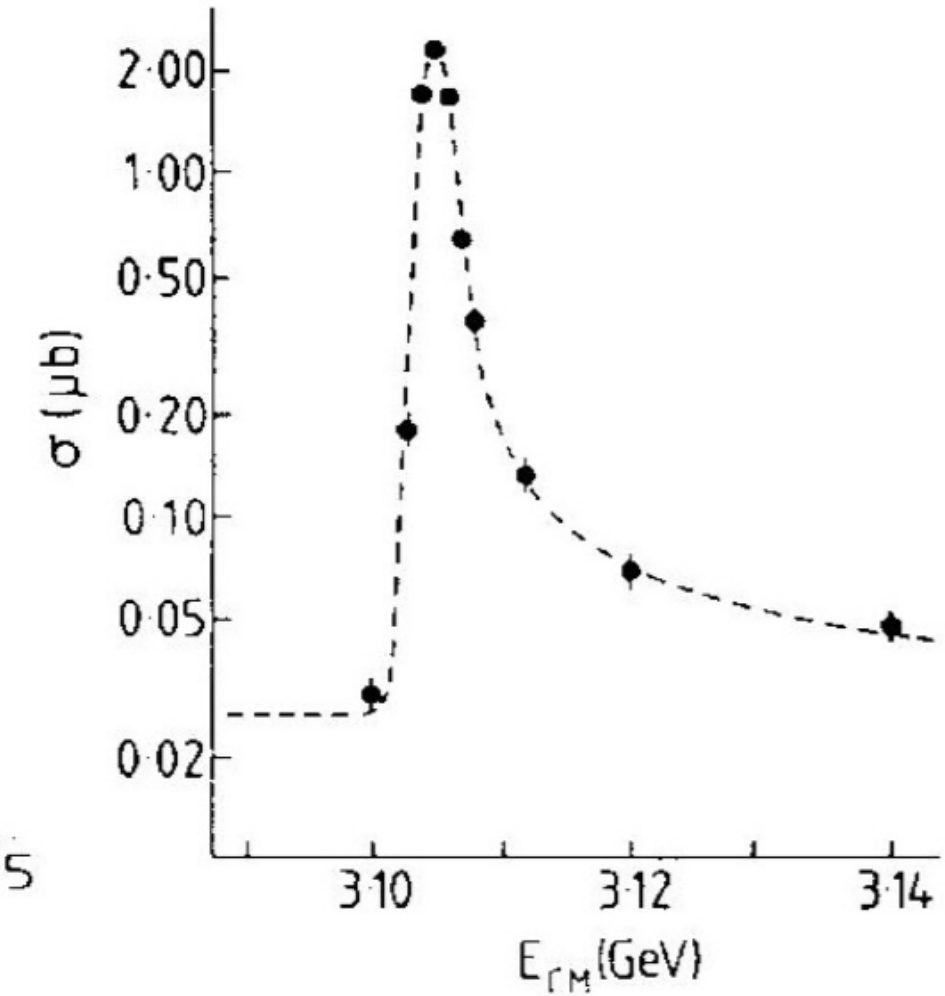
Discovery of J/ψ



energy levels for positronium

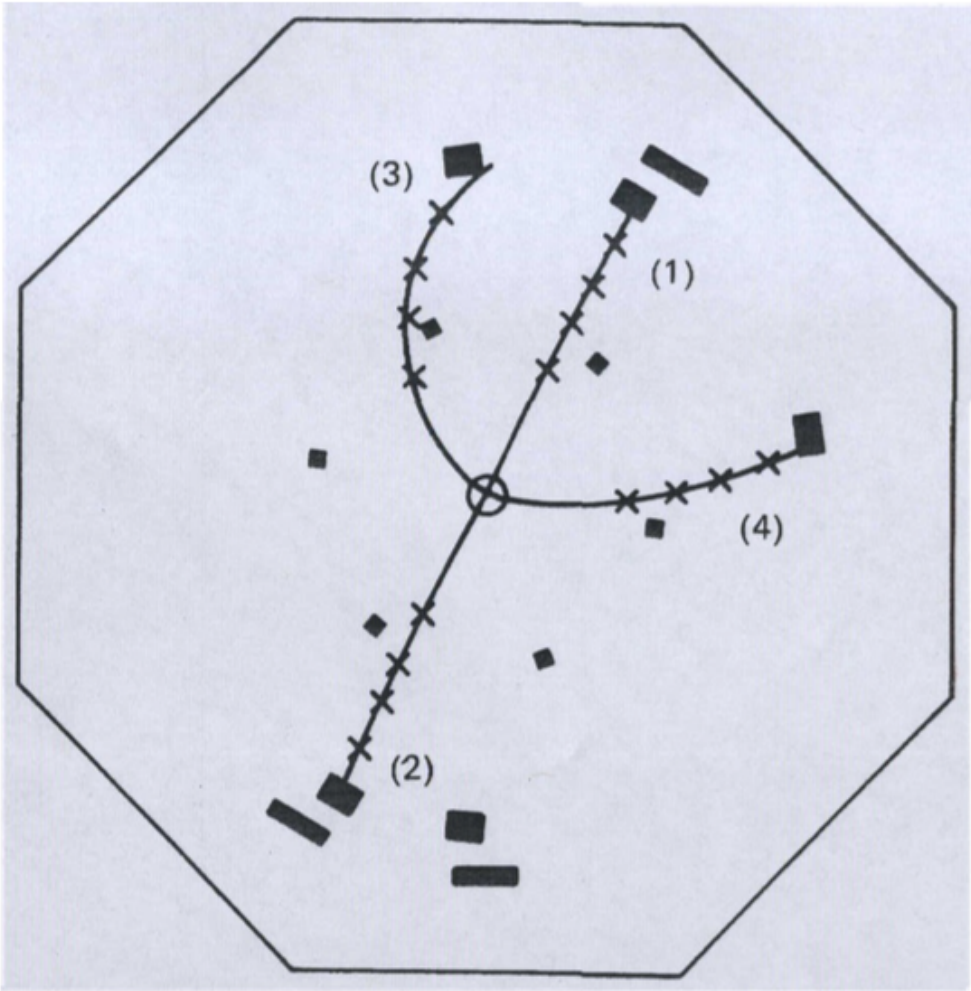
Discovery of J/ψ





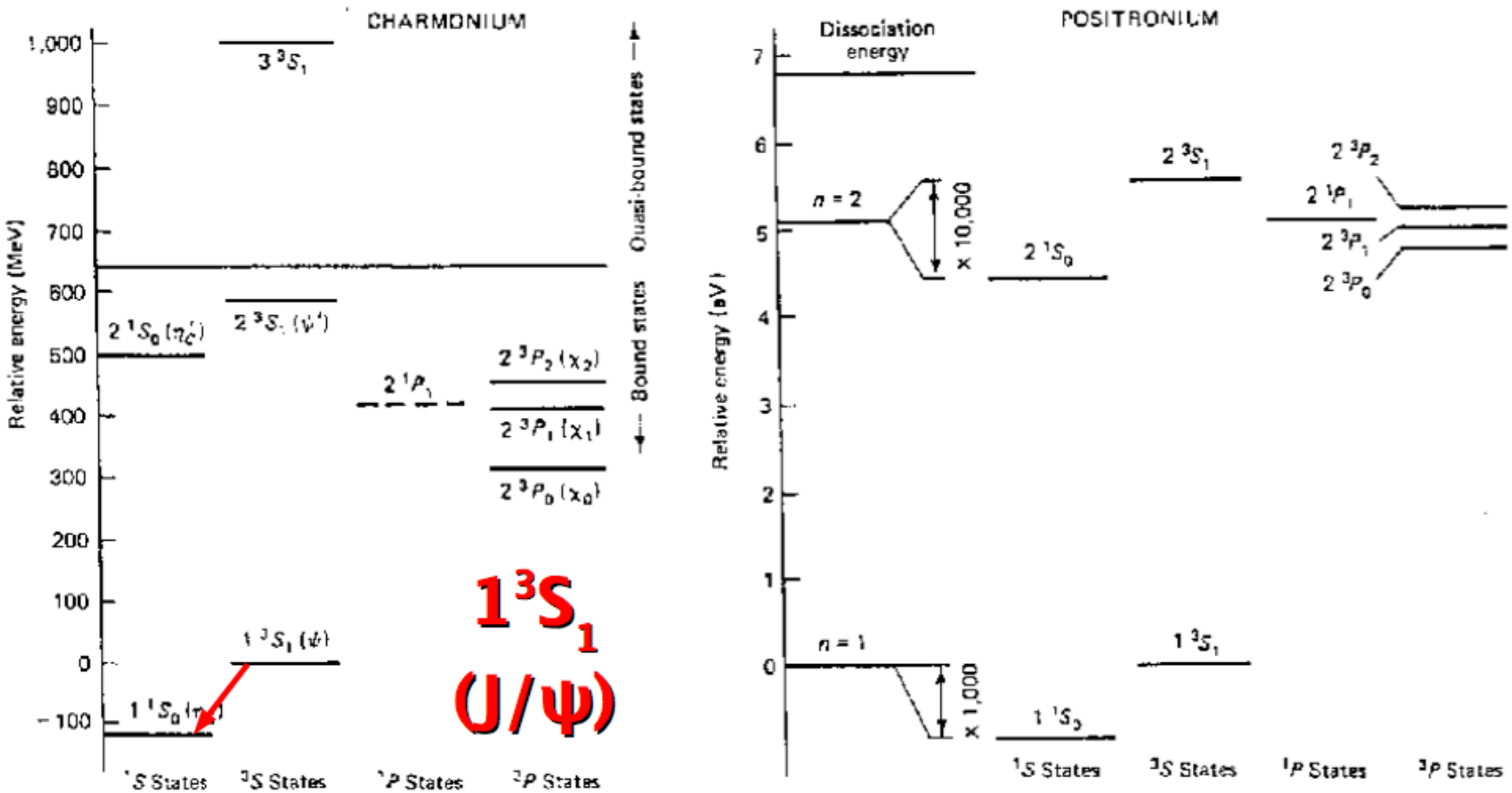
Discovery of J/ψ:
pp- (Brookhaven) and e^+e^--scattering (SLAC)

Discovery of J/ψ



$\psi'(3.7) \rightarrow \psi(3.1)\ \pi^+\pi^- \rightarrow e^+e^-\ \pi^+\pi^-$
in a spark chamber detector

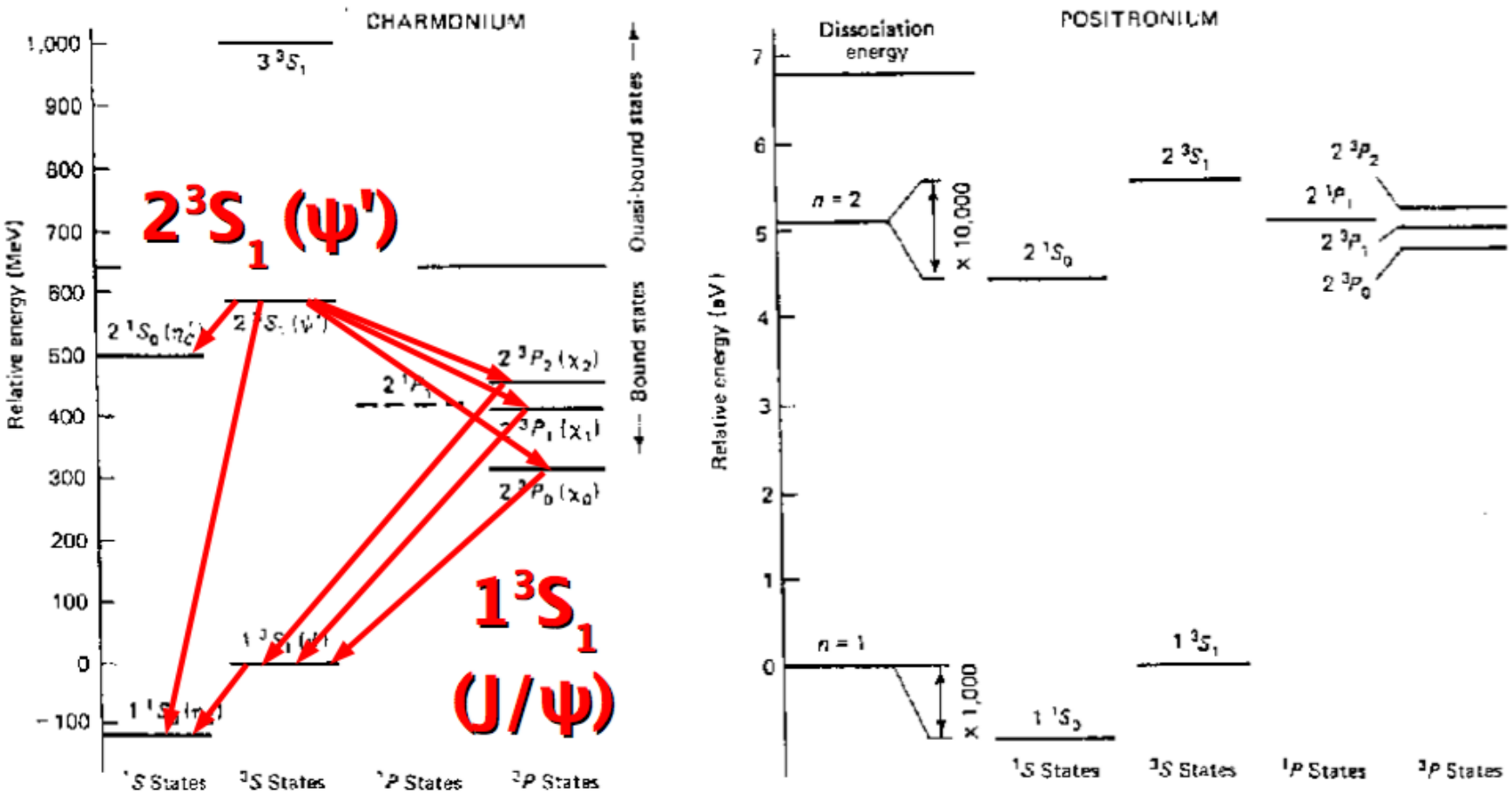
Discovery of J/ψ



$c\bar{c}$

energy levels for charmonium and positronium

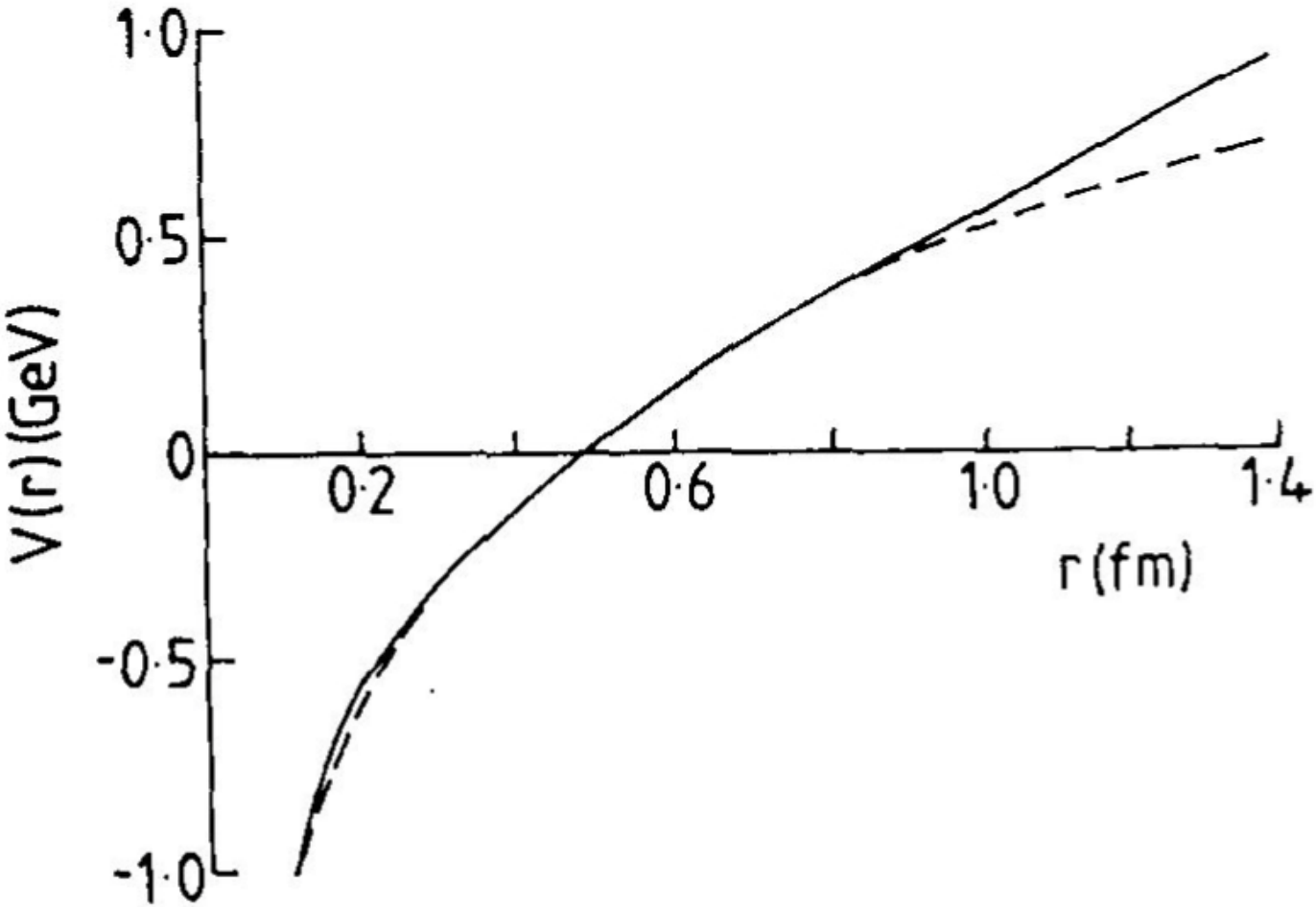
Discovery of J/ψ



$c\bar{c}$

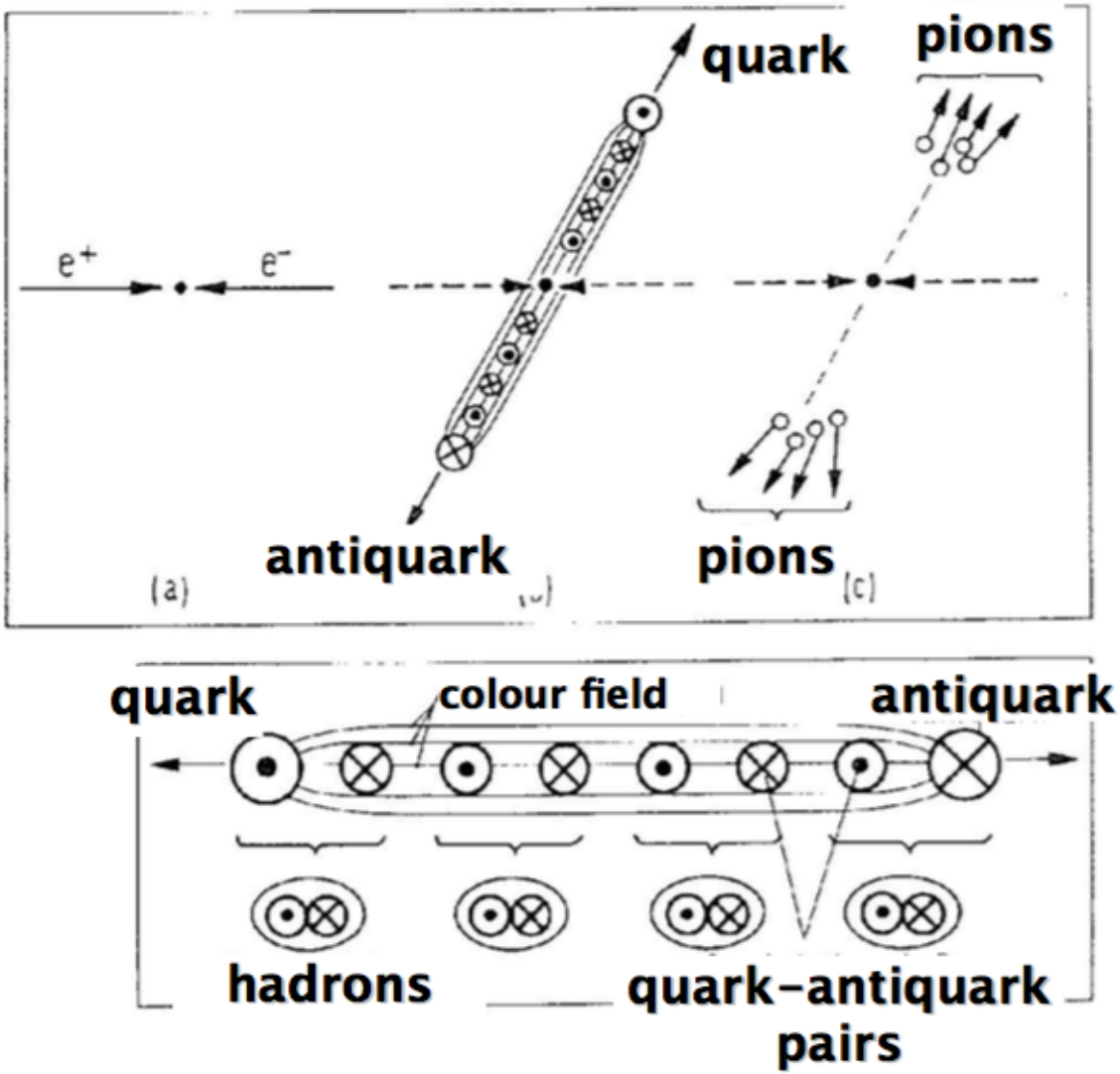
energy niveaux for charmonium and positronium

Confinement



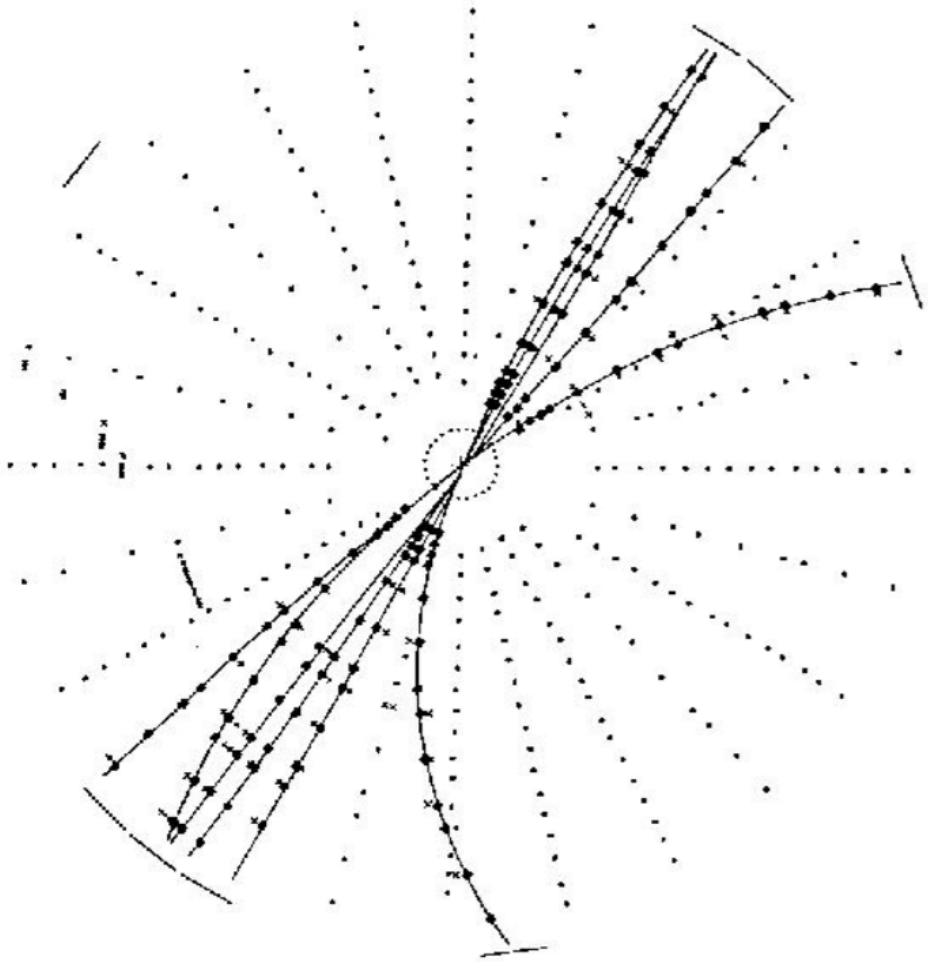
Potential of strong quark interaction from charmonium and bottomium data

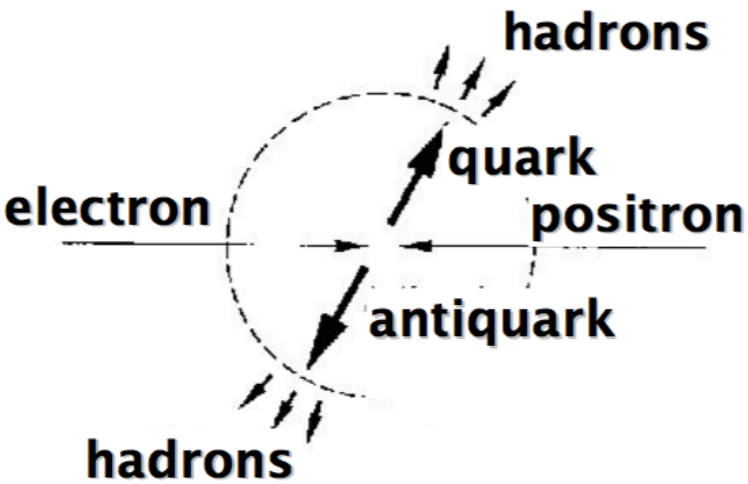
Confinement



Schematic Hadronisation

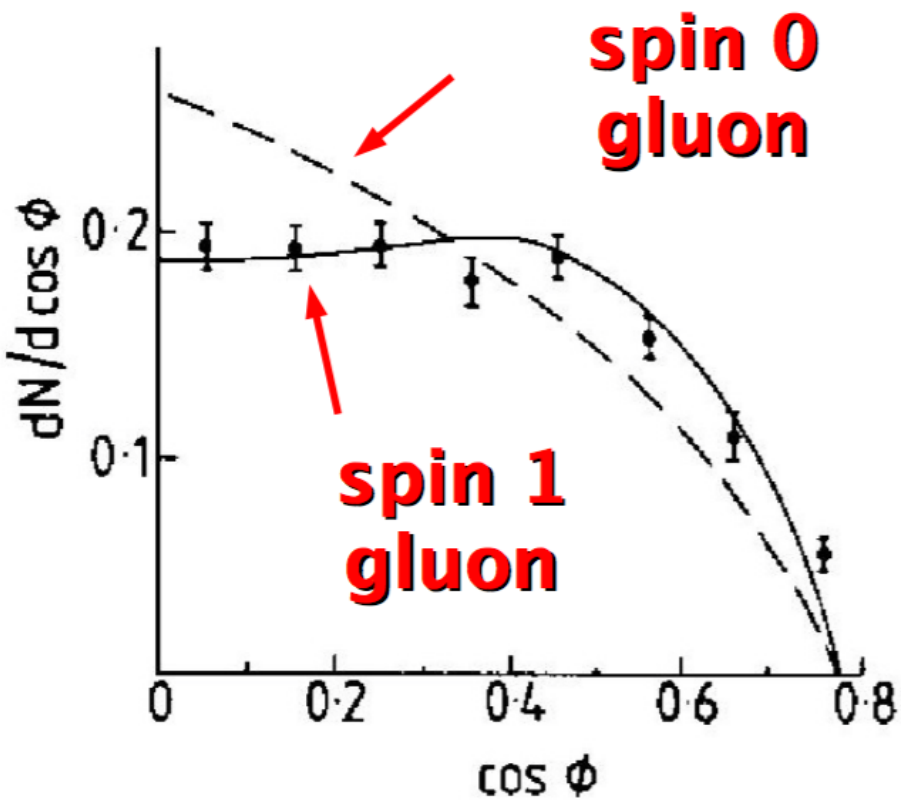
Quark Pair Production





2 jet event at e^+e^- accelerator PETRA

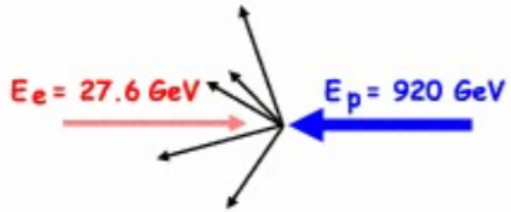
Gluon Discovery



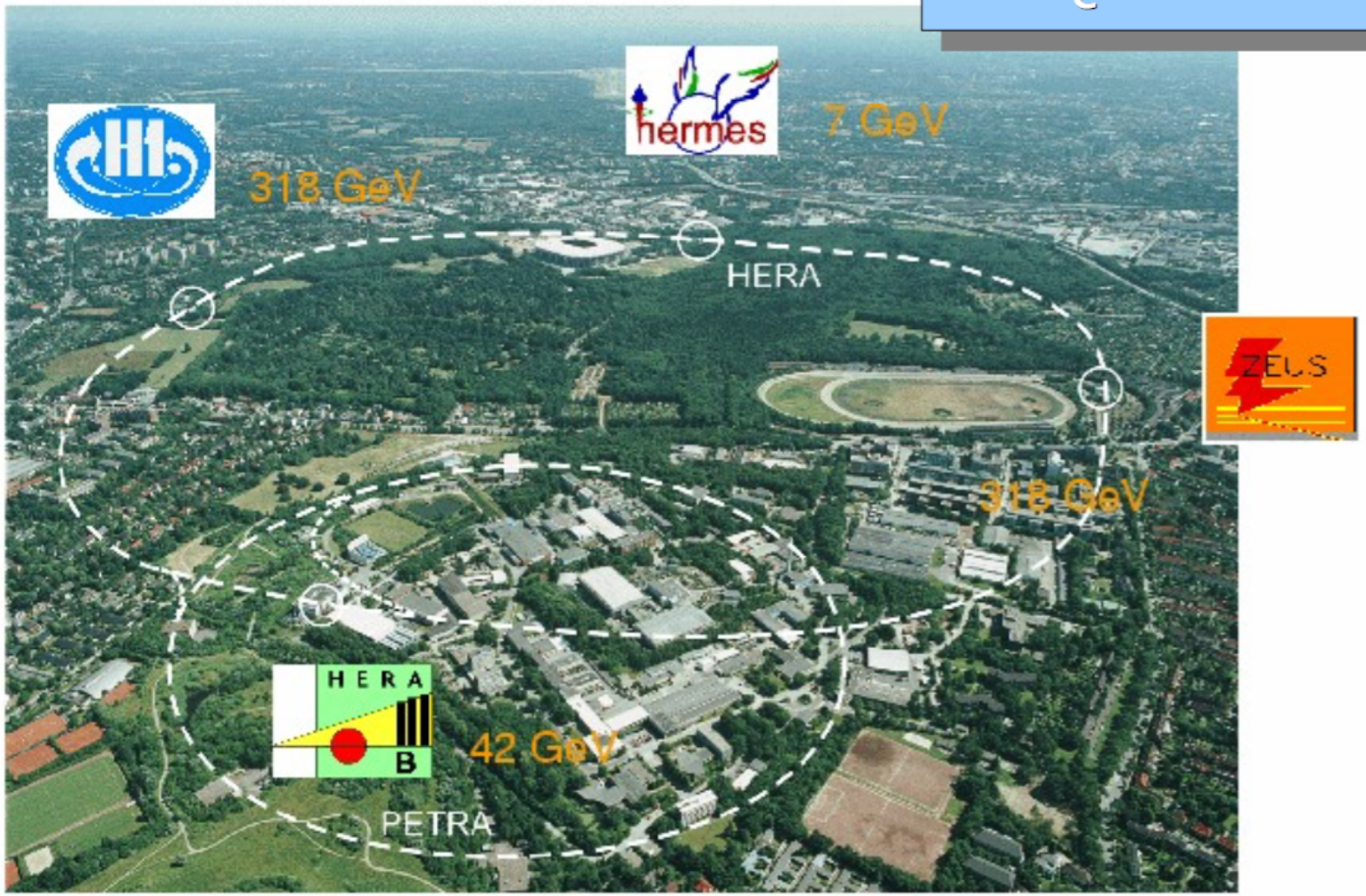
3 jet event and angular distribution at e^+e^- accelerator PETRA

The HERA ep Collider at DESY

HERA
(Hadron-Elektron-Ring-Anlage)

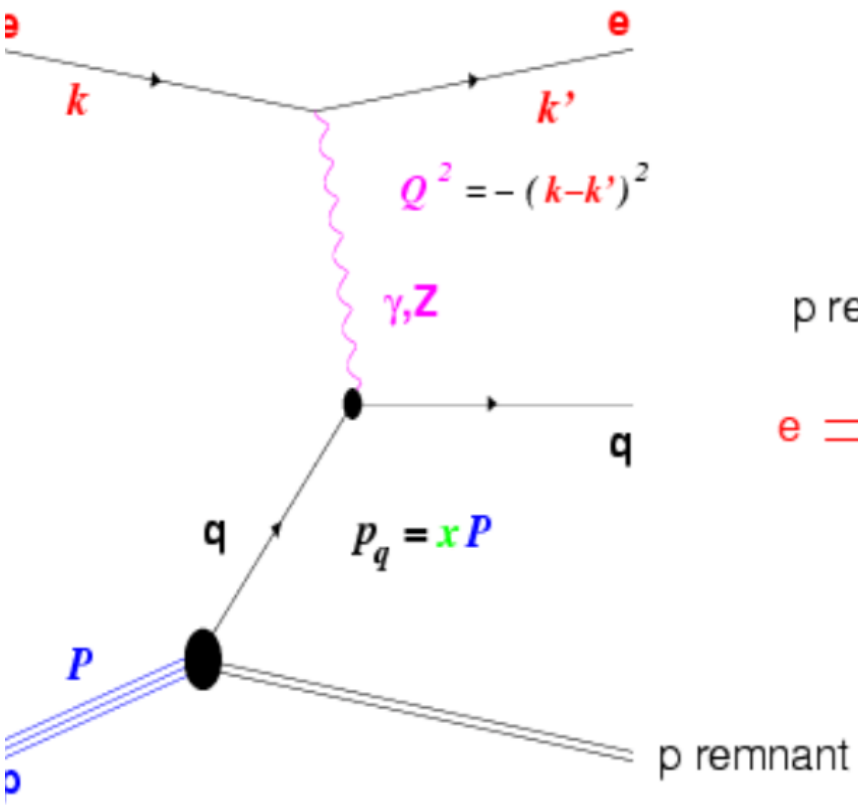


$\sqrt{s}$ = 320 GeV
$Q^2 \approx 100000$ GeV^2
$\lambda_c \approx 10^{-3}$ fm

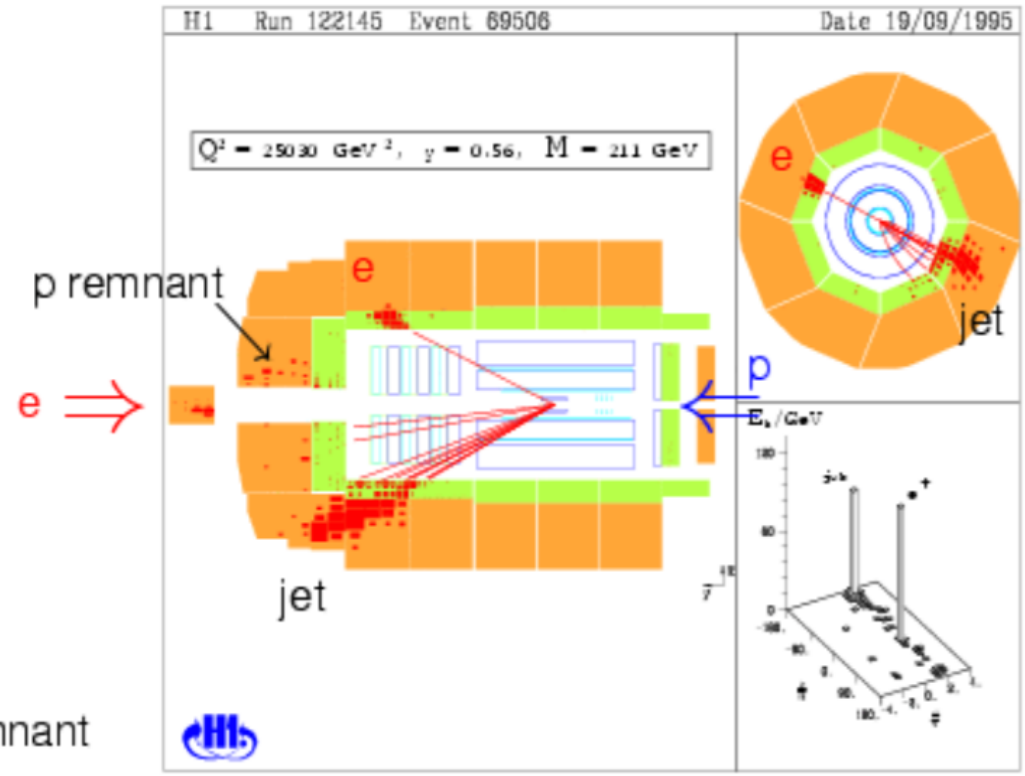


Deep Inelastic Scattering: Neutral Current

deep inelastic scattering (DIS):



H1 detector



Q^2: four-momentum transfer

spatial resolution $\sim 1/Q$

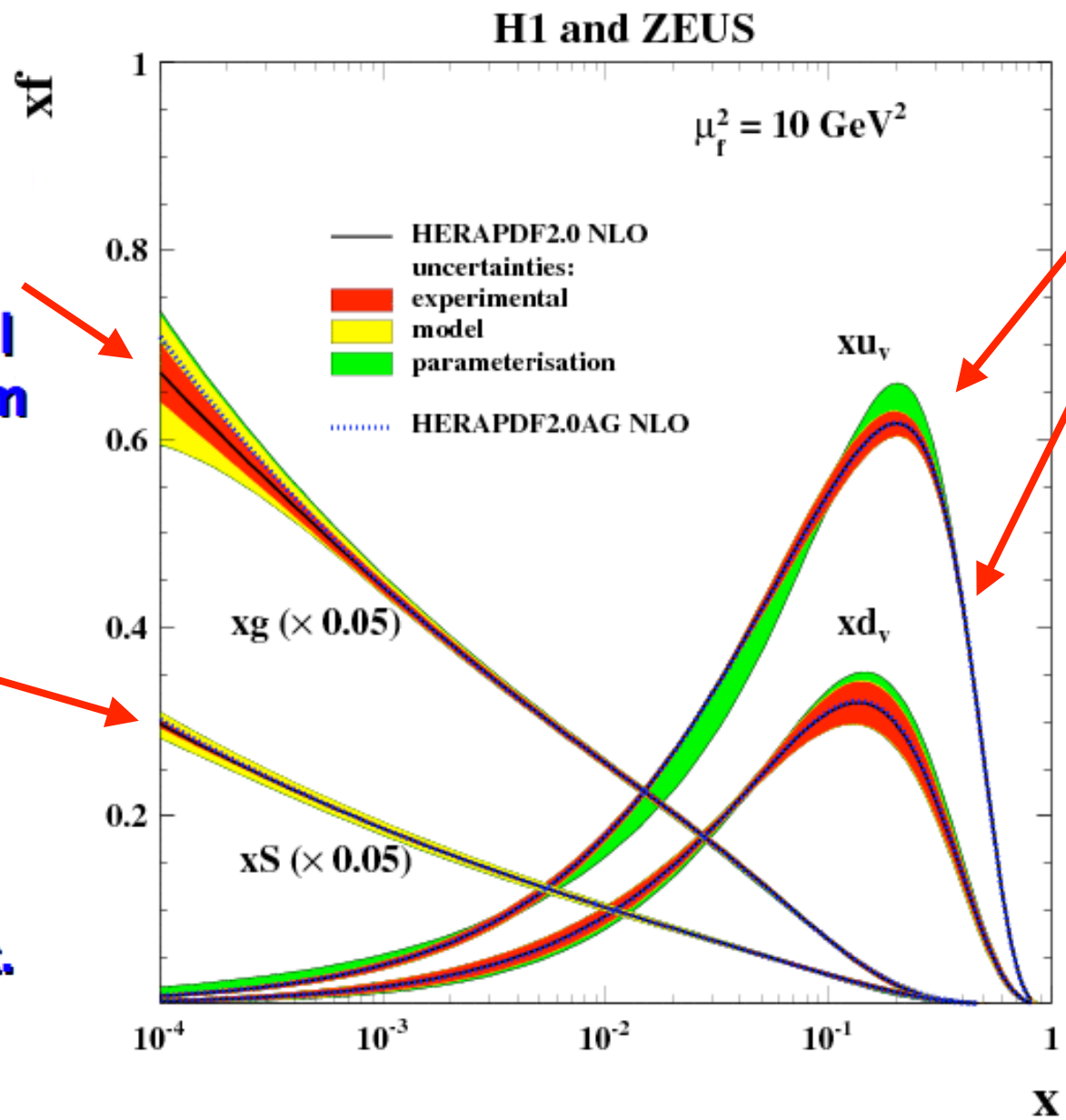
$\Rightarrow 10^{-16}$ cm

x: fractional momentum of the struck quark

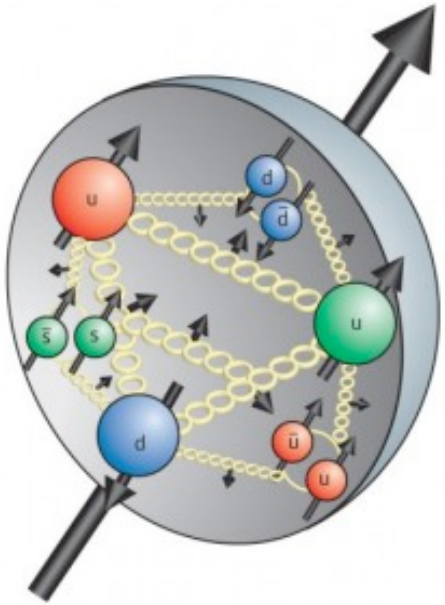
Parton Density Functions

many **gluons** with small momentum fraction

sea quarks: lead to scaling violation, $F_2(Q^2) \neq$ const.



valence quarks: half of proton momentum



Quantum Chromodynamics

Summary QCD Lagrangian

invariance under lokal gauge transformation in color space : $q^j(x) \to U(x)\, q^j(x) \qquad j = 1, \dots f$

field strength tensor : $G_{\lambda\rho}(x) = \partial_\lambda G_\rho(x) - \partial_\rho G_\lambda(x) + i g_s [G_\lambda(x), G_\rho(x)]$

SU(3) gauge symmetry

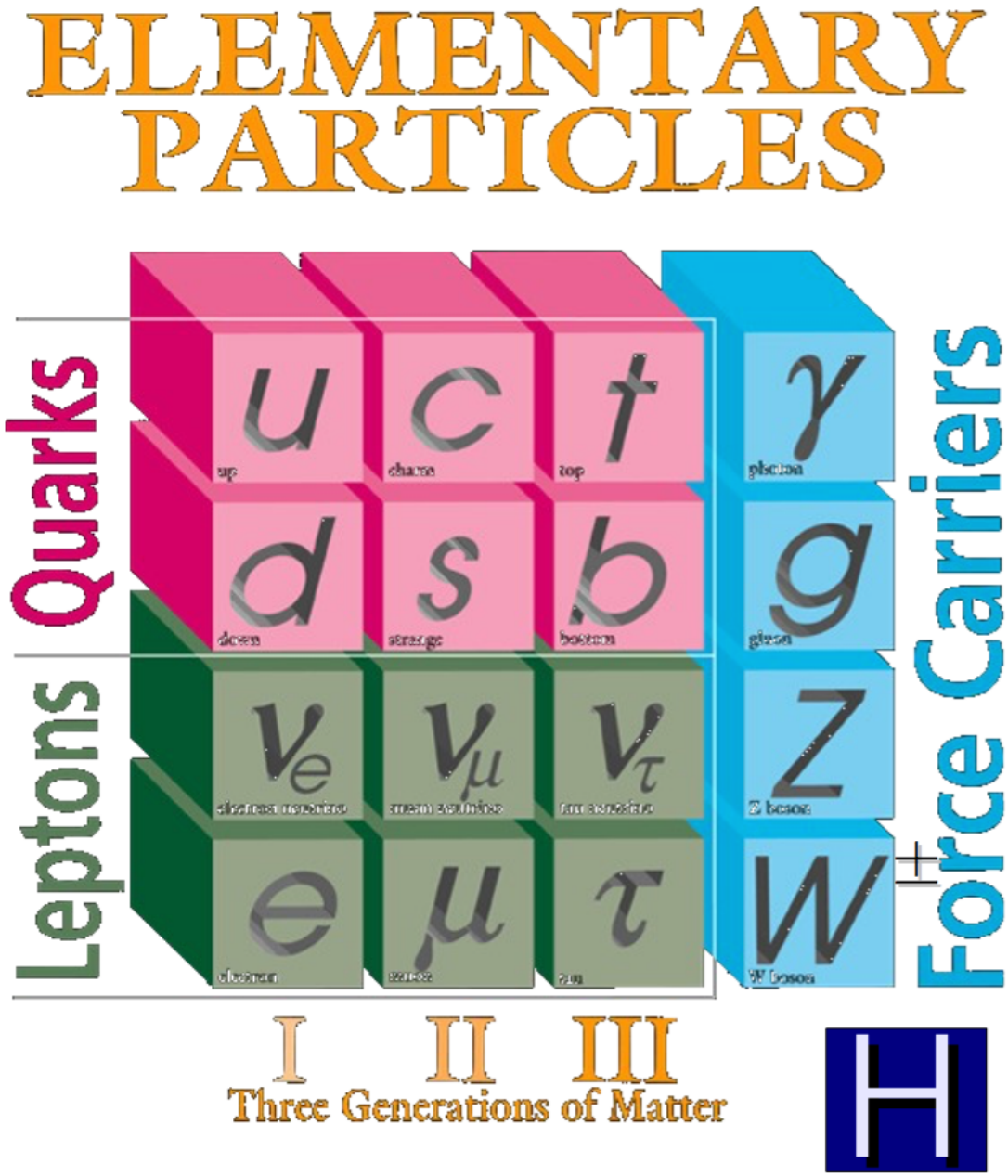
$$\mathcal{L}_{QCD}(x) = \underbrace{-\frac{1}{2} \mathrm{Tr}(G_{\lambda\rho}(x)\, G^{\lambda\rho}(x))}_{\text{free gluon}} + \underbrace{\sum_{j=1}^{f} \bar{q}^j(x) [i\gamma^\lambda \partial_\lambda - m_j]\, q^j(x)}_{\text{Dirac field}}$$

$$\underbrace{- g_s \sum_{j=1}^{f} \bar{q}^j(x)\, \gamma^\lambda\, G_\lambda(x)\, q^j(x)}_{\text{interaction}}$$

Comparison between QED and QCD

	QED	QCD
quantum number	electric charge	colour
fermions	electrons	quarks (colour triplets)
vector bosons	photon (uncharged)	gluon (colour octett)
gauge group	U(1) abelian	SU(3) (non-abelian)
coupling constant	e $\alpha = \frac{e^2}{4\pi}$	g_s $\alpha_s = \frac{g_s^2}{4\pi}$
bound states	positronium	hadrons
free particles (asymptotic states)	electrons, positrons, photons	only hadrons (quarks, gluons are confined)

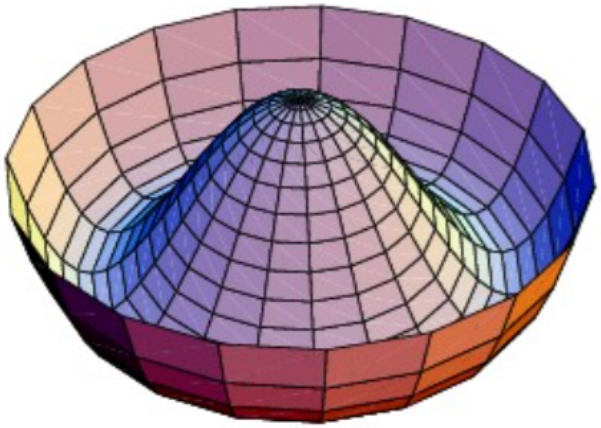
Standard Model Particles



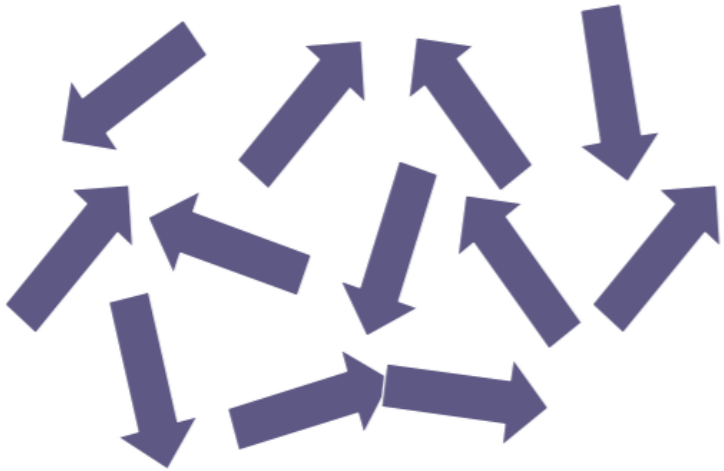
Higgs-boson:
explains masses

Spontaneous Symmetry Breaking (SSB)

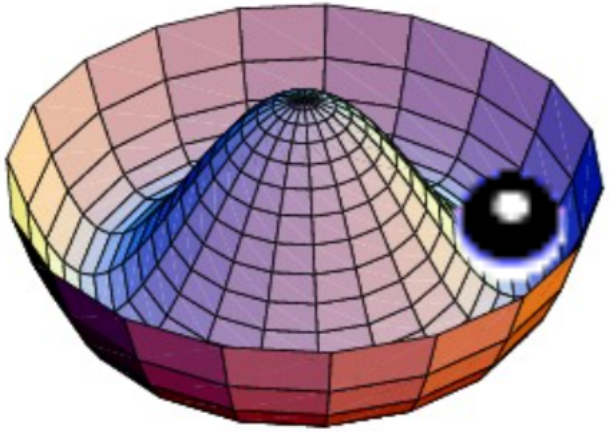
Higgs potential:



unmagnetised material:

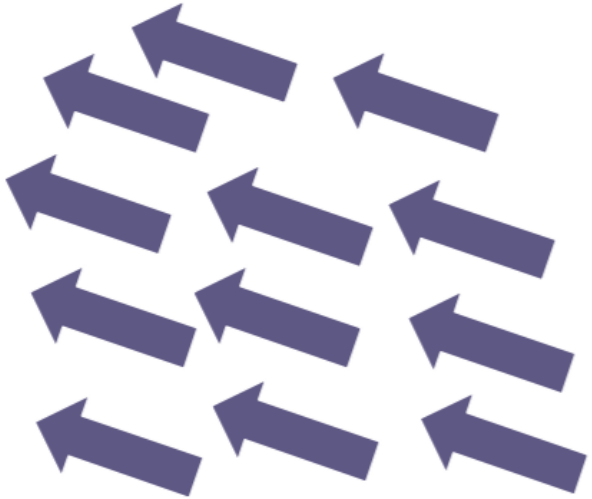


Higgs ground state:



SSB

ferromagnetic material:



Summary QFD Lagrangian

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\left(W_{\lambda\rho}W^{\lambda\rho}\right) - \frac{1}{4}B_{\lambda\rho}B^{\lambda\rho}$$

$$+\,\bar{\nu}_{eL}\, i\gamma^\lambda \partial_\lambda \nu_{eL} + \bar{e}\, i\gamma^\lambda \partial_\lambda e$$

$$+\,W^+_\lambda W^{-\lambda} m_W^2 \left(1+\frac{H}{v}\right)^2$$

$$+\,\frac{1}{2} Z_\lambda Z^\lambda m_Z^2 \left(1+\frac{H}{v}\right)^2$$

$$-\,m_e \bar{e} e \left(1+\frac{H}{v}\right)$$

$$+\,\frac{1}{2}(\partial_\lambda H)(\partial^\lambda H)$$

$$-\,\frac{1}{2} m_H^2 H^2 \left[1+\frac{H}{v}+\frac{1}{4}\left(\frac{H}{v}\right)^2\right]$$

$$+\,\mathcal{L}_{int}$$

$$Z_\lambda = \cos\vartheta_W W_\lambda^3 - \sin\vartheta_W B_\lambda$$

$$A_\lambda = \sin\vartheta_W W_\lambda^3 + \cos\vartheta_W B_\lambda$$

weak mixing angle

$$\sin\vartheta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$\cos\vartheta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$$SU(2)_L \times U(1)_Y \xrightarrow[SSB]{} U_{em}(1)$$

$$\phi(x) = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\left(v + H(x)\right) \end{pmatrix}$$

Summary QFD Lagrangian

$$\mathcal{L}_{int} = -e \Big\{ A_\lambda j^\lambda_{em} + \frac{1}{\sqrt{2}\sin\vartheta_W} \left(W^+_\lambda \bar{\nu}_{eL} \gamma^\lambda e_L + W^-_\lambda \bar{e}_L \gamma^\lambda \nu_{eL} \right) + \frac{1}{\sin\vartheta_W \cos\vartheta_W} Z_\lambda j^\lambda_{NC} \Big\}$$

charged weak current

V-A structure

$$\bar{\nu}_{eL} \gamma^\lambda e_L = \bar{\nu}_e \gamma^\lambda \tfrac{1}{2}(1-\gamma_5) e$$

em current

$$j^\lambda_{em} = -\bar{e}_L \gamma^\lambda e_L - \bar{e}_R \gamma^\lambda e_R = -\bar{e}\gamma^\lambda e$$

neutral current

$$j^\lambda_{NC} = \tfrac{1}{2} \bar{\nu}_{eL} \gamma^\lambda \nu_{eL} - \tfrac{1}{2} \bar{e}_L \gamma^\lambda e_L - \sin^2\vartheta_W j^\lambda_{em}$$

Flavor quantum numbers of quarks & leptons

			T	T_3	Y	Q
$\begin{pmatrix}\nu_{e_L}\\ e_L\end{pmatrix}$	$\begin{pmatrix}\nu_{\mu_L}\\ \mu_L\end{pmatrix}$	$\begin{pmatrix}\nu_{\tau_L}\\ \tau_L\end{pmatrix}$	1/2	1/2	− 1/2	0
			1/2	− 1/2	− 1/2	− 1
e_R	μ_R	τ_R	0	0	− 1	− 1
$\begin{pmatrix}u_L\\ d'_L\end{pmatrix}$	$\begin{pmatrix}c_L\\ s'_L\end{pmatrix}$	$\begin{pmatrix}t_L\\ b'_L\end{pmatrix}$	1/2	1/2	1/6	2/3
			1/2	− 1/2	1/6	− 1/3
u_R	c_R	t_R	0	0	2/3	2/3
d_R	s_R	b_R	0	0	− 1/3	− 1/3

T : weak isospin, T_3 third component,

Y : hypercharge, Q el. charge

Mass eigenfields and CKM Matrix

$$\begin{pmatrix} d(x) \\ s(x) \\ b(x) \end{pmatrix} = V^{\dagger} \begin{pmatrix} d'(x) \\ s'(x) \\ b'(x) \end{pmatrix}$$

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix}$$

$$V^{\dagger} V = V V^{\dagger} = \mathbb{1}_3$$

$$V_{11} \geq 0 ,$$

$$V_{12} \geq 0 ,$$

$$V_{23} \geq 0 ,$$

$$V_{33} \geq 0 ,$$

$$V_{11} V_{22} - V_{12} V_{21} \geq 0$$

$$\equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

SM Lagrangian

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{2}\,\mathrm{Tr}\left(W_{\lambda\rho}\,W^{\lambda\rho}\right) \\
& -\frac{1}{4}\,B_{\lambda\rho}\,B^{\lambda\rho} \\
& + W^{+}_{\lambda}\,W^{-\lambda}\,m_W^2\left(1+\frac{H}{v}\right)^2 \\
& + \frac{1}{2}\,Z_{\lambda}\,Z^{\lambda}\,m_Z^2\left(1+\frac{H}{v}\right)^2 \\
& + \left(\bar{\psi}\,\frac{i}{2}\,\gamma^{\lambda}\partial_{\lambda}\,\psi + h.c.\right) \\
& - \bar{\psi}\,M\,\psi\left(1+\frac{H}{v}\right) \\
& + \frac{1}{2}\,\partial_{\lambda}H\,\partial^{\lambda}H - \frac{1}{2}\,m_H^2\,H^2\left[1 + \frac{H}{v} + \frac{1}{4}\left(\frac{H}{v}\right)^2\right] \\
& + \mathcal{L}_{Int}
\end{aligned}
$$

SM Lagrangian: Yukawa term

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\left(W_{\lambda\rho}\,W^{\lambda\rho}\right)$$

$$-\frac{1}{4}\,B_{\lambda\rho}\,B^{\lambda\rho}$$

$$+\,W^{+}_{\lambda}W^{-\lambda}\,m_W^2\left(1+\frac{H}{v}\right)^2$$

$$+\frac{1}{2}\,Z_\lambda Z^\lambda\,m_Z^2\left(1+\frac{H}{v}\right)^2$$

$$+\left(\bar{\psi}\,\frac{i}{2}\,\gamma^\lambda\partial_\lambda\psi + h.c.\right)$$

$$-\,\bar{\psi}\,M\psi\left(1+\frac{H}{v}\right)$$

$$+\frac{1}{2}\,\partial_\lambda H\,\partial^\lambda H - \frac{1}{2}\,m_H^2\,H^2\left[1+\frac{H}{v}+\frac{1}{4}\left(\frac{H}{v}\right)^2\right]$$

$$+\,\mathcal{L}_{Int}$$

$$\mathcal{L}_{Yuk} = -\Big\{(\bar{e},\bar{\mu},\bar{\tau})\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$+\,(\bar{u},\bar{c},\bar{t})\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}\begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$$+\,(\bar{d}',\bar{s}',\bar{b}')\,V\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}V^{+}\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}\Big\}$$

$$\left(1+\frac{H}{v}\right)$$

SM Lagrangian: Yukawa term

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{2} \mathrm{Tr}\left(W_{\lambda\rho} W^{\lambda\rho}\right) \\
& -\frac{1}{4} B_{\lambda\rho} B^{\lambda\rho} \\
& + W^{+}_{\lambda} W^{-\lambda} m_W^2 \left(1 + \frac{H}{v}\right)^2 \\
& + \frac{1}{2} Z_\lambda Z^\lambda m_Z^2 \left(1 + \frac{H}{v}\right)^2 \\
& + \left(\bar{\psi} \frac{i}{2} \gamma^\lambda \partial_\lambda \psi + h.c.\right) \\
& - \bar{\psi} M \psi \left(1 + \frac{H}{v}\right) \\
& + \frac{1}{2} \partial_\lambda H \partial^\lambda H - \frac{1}{2} m_H^2 H^2 \left[1 + \frac{H}{v} + \frac{1}{4}\left(\frac{H}{v}\right)^2\right] \\
& + \mathcal{L}_{Int}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{Yuk} = \Bigg\{ & -(\bar{e}, \bar{\mu}, \bar{\tau}) \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \\
& - (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix} \\
& - (\bar{d}, \bar{s}, \bar{b}) \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \Bigg\} \\
& \cdot \left(1 + \frac{H}{v}\right)
\end{aligned}
$$

SM Lagrangian: interaction

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\left(W_{\lambda\rho}\,W^{\lambda\rho}\right)$$

$$-\frac{1}{4}\,B_{\lambda\rho}\,B^{\lambda\rho}$$

$$+\,W^{+}_{\lambda}\,W^{-\lambda}\,m_W^2\left(1+\frac{H}{v}\right)^2$$

$$+\frac{1}{2}\,Z_\lambda\,Z^\lambda\,m_Z^2\left(1+\frac{H}{v}\right)^2$$

$$+\left(\bar{\psi}\,\frac{i}{2}\,\gamma^\lambda\partial_\lambda\,\psi + h.c.\right)$$

$$-\,\bar{\psi}\,M\,\psi\left(1+\frac{H}{v}\right)$$

$$+\frac{1}{2}\,\partial_\lambda H\,\partial^\lambda H - \frac{1}{2}\,m_H^2\,H^2\left[1 + \frac{H}{v} + \frac{1}{4}\left(\frac{H}{v}\right)^2\right]$$

$$+\,\mathcal{L}_{Int}$$

$$\mathcal{L}_{Int} = -e\left\{A_\lambda\,J^\lambda_{em} + \frac{1}{\sin\vartheta_W\cos\vartheta_W}\,Z_\lambda\,J^\lambda_{NC} + \frac{1}{\sqrt{2}\,\sin\vartheta_W}\left(W^{+}_{\lambda}\,J^\lambda_{CC} + W^{-}_{\lambda}\,J^{\lambda\,+}_{CC}\right)\right\}$$

$$J^\lambda_{em} = \bar{\psi}\,\gamma^\lambda\,(T_3 + Y)\,\psi = \bar{\psi}\,\gamma^\lambda\,Q\,\psi$$

$$(Q = T_3 + Y)$$

$$J^\lambda_{NC} = \bar{\psi}\,\gamma^\lambda\,T_3\,\psi - \sin^2\vartheta_W\,J^\lambda_{em}$$

$$J^\lambda_{CC} = \bar{\psi}\,\gamma^\lambda\,(T_1 + i\,T_2)\,\psi$$

SM Lagrangian: em interaction

$$J^\lambda_{em} = -(\bar{e}, \bar{\mu}, \bar{\tau})\gamma^\lambda \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} + \frac{2}{3}(\bar{u}, \bar{c}, \bar{t})\gamma^\lambda \begin{pmatrix} u \\ c \\ t \end{pmatrix} - \frac{1}{3}(\bar{d}, \bar{s}, \bar{b})\gamma^\lambda \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\mathcal{L}_{Int} = -e\left\{ A_\lambda J^\lambda_{em} + \frac{1}{\sin\vartheta_W \cos\vartheta_W} Z_\lambda J^\lambda_{NC} + \frac{1}{\sqrt{2}\sin\vartheta_W}\left(W^+_\lambda J^\lambda_{CC} + W^-_\lambda J^{\lambda\dagger}_{CC}\right)\right\}$$

$$J^\lambda_{em} = \bar{\Psi}\gamma^\lambda(T_3 + Y)\psi = \bar{\Psi}\gamma^\lambda Q \psi$$

$$(Q = T_3 + Y)$$

SM Lagrangian: em interaction

$$J^\lambda_{em} = -(\bar{e}, \bar{\mu}, \bar{\tau})\gamma^\lambda \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} + \frac{2}{3}(\bar{u}, \bar{c}, \bar{t})\gamma^\lambda \begin{pmatrix} u \\ c \\ t \end{pmatrix} - \frac{1}{3}\underbrace{(\bar{d}, \bar{s}, \bar{b})\gamma^\lambda \begin{pmatrix} d \\ s \\ b \end{pmatrix}}$$

$$\mathcal{L}_{Int} = -e\Big\{A_\lambda J^\lambda_{em} + \frac{1}{\sin\vartheta_W \cos\vartheta_W} Z_\lambda J^\lambda_{NC} + \frac{1}{\sqrt{2}\sin\vartheta_W}\left(W^+_\lambda J^\lambda_{CC} + W^-_\lambda J^{\lambda\dagger}_{CC}\right)\Big\}$$

$$J^\lambda_{em} = \bar{\Psi}\gamma^\lambda(T_3 + Y)\psi = \bar{\Psi}\gamma^\lambda Q \psi$$

$$(Q = T_3 + Y)$$

$$= (\bar{d}, \bar{s}, \bar{b})\, V^\dagger \gamma^\lambda V \begin{pmatrix} d \\ s \\ b \end{pmatrix} = (\bar{d}, \bar{s}, \bar{b})\gamma^\lambda \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

SM Lagrangian: NC interaction

$$\mathcal{L}_{Int} = -e \Big\{ A_\lambda J^\lambda_{em} + \frac{1}{\sin\vartheta_W \cos\vartheta_W} Z_\lambda J^\lambda_{NC} + \frac{1}{\sqrt{2}\sin\vartheta_W} \left(W^+_\lambda J^\lambda_{CC} + W^-_\lambda J^{\lambda +}_{CC} \right) \Big\}$$

$$J^\lambda_{NC} = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\, \gamma^\lambda \frac{1}{2} \frac{1-\gamma_5}{2} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$+ (\bar{e}, \bar{\mu}, \bar{\tau})\, \gamma^\lambda \left(-\frac{1}{2} \frac{1-\gamma_5}{2} + \sin^2\vartheta_W \right) \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$+ (\bar{u}, \bar{c}, \bar{t})\, \gamma^\lambda \left(\frac{1}{2} \frac{1-\gamma_5}{2} - \frac{2}{3} \sin^2\vartheta_W \right) \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$$+ (\bar{d}, \bar{s}, \bar{b})\, \gamma^\lambda \left(-\frac{1}{2} \frac{1-\gamma_5}{2} + \frac{1}{3} \sin^2\vartheta_W \right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$J^\lambda_{NC} = \bar{\Psi} \gamma^\lambda T_3 \Psi - \sin^2\vartheta_W J^\lambda_{em}$$

SM Lagrangian: CC interaction

$$\mathcal{L}_{Int} = -e \Big\{ A_\lambda \, J^\lambda_{em} + \frac{1}{\sin\vartheta_W \cos\vartheta_W} Z_\lambda \, J^\lambda_{NC} + \frac{1}{\sqrt{2}\sin\vartheta_W} \left(W^+_\lambda J^\lambda_{CC} + W^-_\lambda J^{\lambda\,+}_{CC} \right) \Big\}$$

$$J^\lambda_{CC} = (\bar{\nu}_{eL}, \bar{\nu}_{\mu L}, \bar{\nu}_{\tau L}) \gamma^\lambda \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} + \underbrace{(\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^\lambda \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}}$$

$$= (\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^\lambda \mathbf{V} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

$$J^\lambda_{CC} = \bar{\Psi} \gamma^\lambda (T_1 + i T_2) \Psi$$

General Form of CKM matrix

$$V = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

$$c_{ij} = \cos \vartheta_{ij} \quad , \quad s_{ij} = \sin \vartheta_{ij}$$

$$0 \leq \vartheta_{ij} \leq \pi/2 \quad , \quad 0 \leq \delta_{13} < 2\pi$$

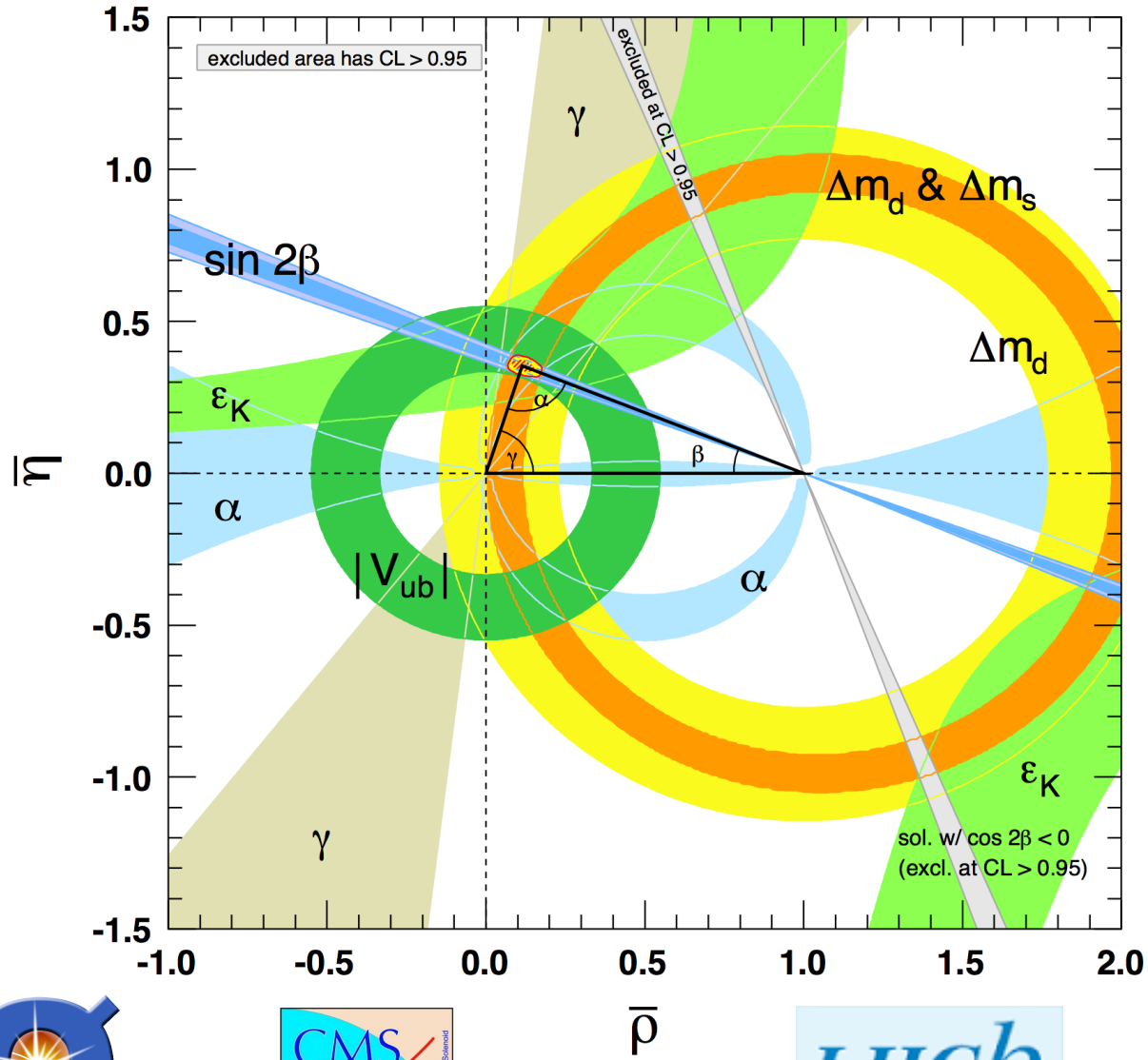
Wolfenstein parameterisation of CKM matrix

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix}$$

$$+ O(\lambda^4)$$

$$\lambda = s_{12}\,, \quad A = s_{23}/(s_{12})^2\,, \quad \rho - i\eta = \frac{s_{13}\, e^{-i\delta_{13}}}{s_{12}\, s_{23}}$$

Measurements of CKM matrix



QFD Feynman rules

The following rules—specific to QFD—represent the additions.

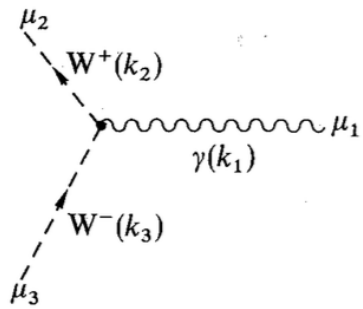
W^- initial state	incoming W line
$\varepsilon(k)$	$W^-(k)$
W^- in final state	outgoing W line
$\varepsilon^*(k)$	$W^-(k)$
W^+ in initial state	outgoing W line
$\varepsilon(k)$	$W^+(k)$
W^+ in final state	incoming W line
$\varepsilon^*(k)$	$W^+(k)$

Z in initial (final) state	external Z line
$\varepsilon(k)(\varepsilon^*(k))$	$Z(k)$
Higgs particle in initial (final) state	external H line
1	$H(k)$
virtual W boson	internal W line
$\dfrac{i\left(-g^{\mu\nu}+\dfrac{k^\mu k^\nu}{m_W^2}\right)}{k^2-m_W^2+i\varepsilon}$	$W(k)$ μ ν
virtual Z boson	internal Z line
$\dfrac{i\left(-g^{\mu\nu}+\dfrac{k^\mu k^\nu}{m_Z^2}\right)}{k^2-m_Z^2+i\varepsilon}$	$Z(k)$ μ ν
virtual Higgs particle	internal H line
$\dfrac{i}{k^2-m^2+i\varepsilon}$	$H(k)$

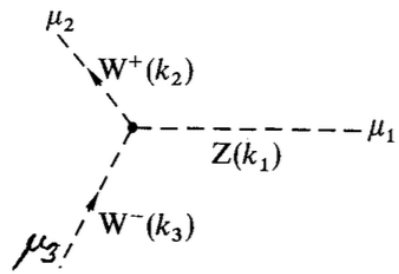
QFD Feynman rules

Vertices with 3 vector bosons $\left(\sum_{j=1}^{3} k_j = 0, \text{ all momenta incoming}\right)$:

$$ie\{(k_1 - k_2)_{\mu_3} g_{\mu_1\mu_2} + (k_2 - k_3)_{\mu_1} g_{\mu_2\mu_3} + (k_3 - k_1)_{\mu_2} g_{\mu_3\mu_1}\}$$

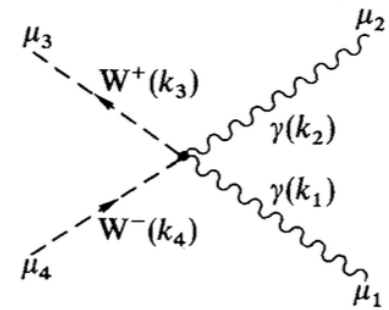


$$ie\frac{\cos\vartheta_W}{\sin\vartheta_W}\{\text{as above}\}$$

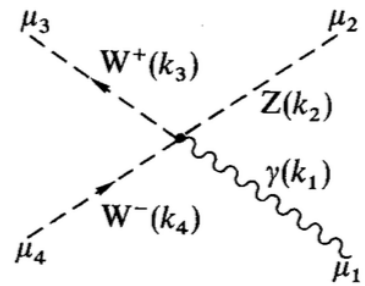


Vertices with 4 vector bosons $\left(\sum_{j=1} k_j = 0, \text{ all momenta incoming}\right)$:

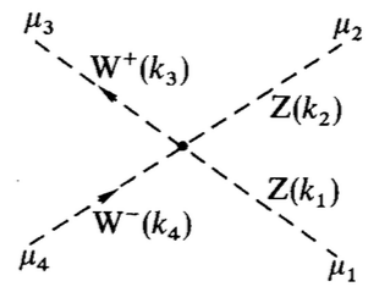
$$ie^2\{g_{\mu_1\mu_3} g_{\mu_2\mu_4} + g_{\mu_1\mu_4} g_{\mu_2\mu_3} - 2g_{\mu_1\mu_2} g_{\mu_3\mu_4}\}$$



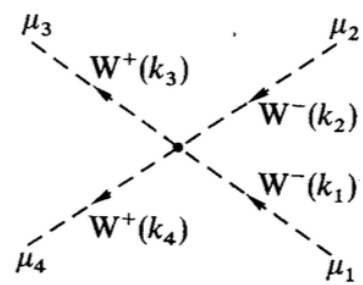
$$ie^2\frac{\cos\vartheta_W}{\sin\vartheta_W}\{\text{as above}\}$$



$$ie^2\frac{\cos^2\vartheta_W}{\sin^2\vartheta_W}\{\text{as above}\}$$



$$-ie^2\frac{1}{\sin^2\vartheta_W}\{\text{as above}\}$$

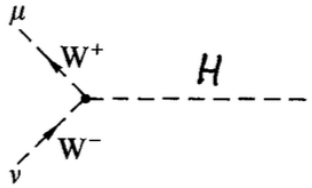


Vertices with Higgs bosons:

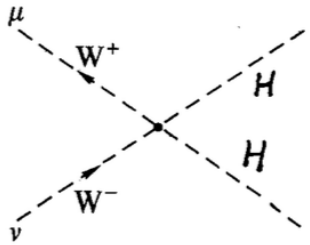
QFD Feynman rules

Vertices with Higgs bosons:

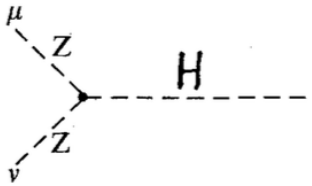
$$ig_{\mu\nu}\frac{2m_W^2}{v}$$



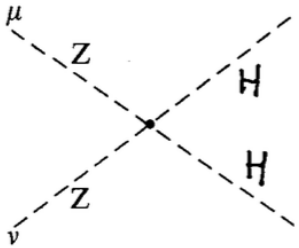
$$ig_{\mu\nu}\frac{2m_W^2}{v^2}$$



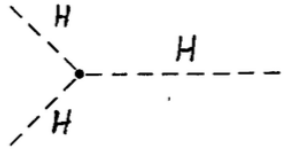
$$ig_{\mu\nu}\frac{2m_Z^2}{v}$$



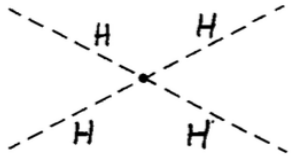
$$ig_{\mu\nu}\frac{2m_Z^2}{v^2}$$



$$-3i\frac{m_H^2}{v}$$

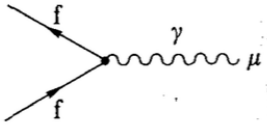


$$-3i\frac{m_H^2}{v^2}$$

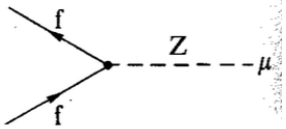


Fermion–Boson vertices:

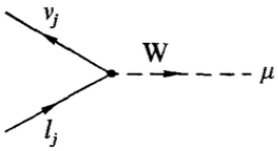
$$-ieQ_f\gamma^\mu$$



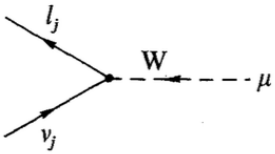
$$-i\frac{e}{\sin\vartheta_W\cos\vartheta_W}\left\{T_3^f\gamma^\mu\frac{1-\gamma_5}{2}-\sin^2\vartheta_W Q_f\gamma^\mu\right\}$$



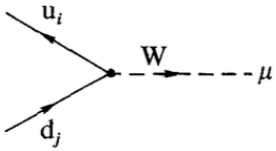
$$-i\frac{e}{\sqrt{2}\sin\vartheta_W}\gamma^\mu\frac{1-\gamma_5}{2}$$



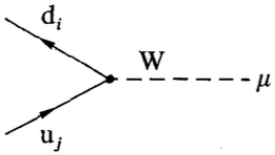
$$-i\frac{e}{\sqrt{2}\sin\vartheta_W}\gamma^\mu\frac{1-\gamma_5}{2}$$



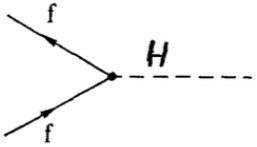
$$-i\frac{e}{\sqrt{2}\sin\vartheta_W}V_{ij}\gamma^\mu\frac{1-\gamma_5}{2}$$



$$-i\frac{e}{\sqrt{2}\sin\vartheta_W}V_{ji}^*\gamma^\mu\frac{1-\gamma_5}{2}$$



$$-i\frac{m_f}{v}$$



BACKUP

Maxwell Equations

MG1 $\nabla \vec{E} = \frac{\varrho}{\epsilon_0}$ $\oiint \vec{E}\, d\vec{A} = \frac{q}{\epsilon_0}$

MG2 $\nabla \vec{B} = 0$ $\oiint \vec{B}\, d\vec{A} = 0$

MG3 $\nabla \times \vec{E} = -\partial_t \vec{B}$ $\oint \vec{E}\, d\vec{s} = -\partial_t \iint \vec{B}\, d\vec{A}$

MG4 $\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \partial_t \vec{E}$ $\oint \vec{B}\, d\vec{s} = \mu_0 I + \mu_0 \epsilon_0 \partial_t \iint \vec{E} d\vec{A}$

Lorentz force: $\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$

Quantum Electrodynamics

Summary QED Lagrangian

local phase invariance : $\psi(x) \to e^{i\alpha(x)}\psi(x)$ (i)

minimal substitution : $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu(x)$ (ii)

gauge invariance : $A_\mu(x) \longrightarrow A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x)$ (iii)

field strength tensor : $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$

U(1) gauge symmetry

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \bar{\psi}(x)(i\gamma_\mu\partial^\mu - m)\psi(x) + eA_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x)$$

free photon Dirac field interaction

Quantum Chromodynamics

Summary QCD Lagrangian

invariance under lokal gauge transformation in color space : $q^j(x) \to U(x)\, q^j(x) \qquad j = 1, \dots f$ (i)

minimal substitution : $\partial_\lambda \to D_\lambda = \partial_\lambda + i g_s G_\lambda(x)$ (ii)

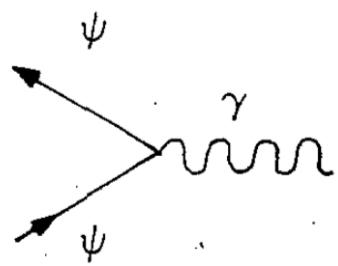
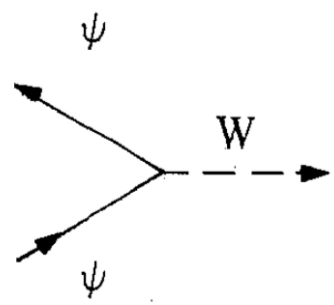
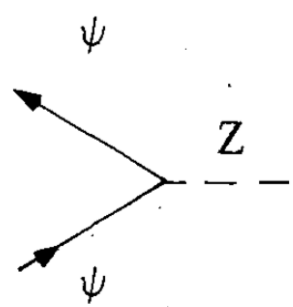
gauge invariance : $G_\lambda(x) \to G'_\lambda(x) = U(x)\, G_\lambda(x)\, U^+(x) - \frac{i}{g_s} U(x) \partial_\lambda U^+(x)$ (iii)

field strength tensor : $G_{\lambda\rho}(x) = \partial_\lambda G_\rho(x) - \partial_\rho G_\lambda(x) + i g_s [G_\lambda(x), G_\rho(x)]$

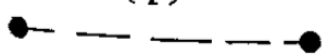
SU(3) gauge symmetry

$$\mathcal{L}_{QCD}(x) = \underbrace{-\frac{1}{2} \mathrm{Tr}(G_{\lambda\rho}(x)\, G^{\lambda\rho}(x))}_{\text{free gluon}} + \underbrace{\sum_{j=1}^{f} \bar{q}^j(x) [i\gamma^\lambda \partial_\lambda - m_j]\, q^j(x)}_{\text{Dirac field}} \underbrace{- g_s \sum_{j=1}^{f} \bar{q}^j(x)\, \gamma^\lambda G_\lambda(x)\, q^j(x)}_{\text{interaction}}$$

Fermion-gauge boson vertices

Diagrammteil	analytischer Ausdruck
	$-i\,e\,\mathbf{Q}\,\gamma^{\lambda}$
	$-i\,\frac{e}{\sqrt{2}\,\sin\vartheta_W}\,(\mathbf{T}_1 + i\,\mathbf{T}_2)\,\gamma^{\lambda}$
	$-i\,\frac{e}{\sin\vartheta_W\,\cos\vartheta_W}\,(\mathbf{T}_3 - \sin^2\vartheta_W\,\mathbf{Q})\,\gamma^{\lambda}$

Gauge boson propagators

Diagrammteil	analytischer Ausdruck
$\gamma(q)$	$\dfrac{-\mathrm{i}\, g_{\rho\lambda}}{q^2}$
$\mathrm{W}(q)$	$\dfrac{-\mathrm{i}\left(g_{\rho\lambda} - \dfrac{q_\rho q_\lambda}{m_\mathrm{W}^2}\right)}{q^2 - m_\mathrm{W}^2} \xrightarrow[q^2 \ll m_\mathrm{W}^2]{} \dfrac{\mathrm{i}\, g_{\rho\lambda}}{m_\mathrm{W}^2}$
$\mathrm{Z}(q)$ 	$\dfrac{-\mathrm{i}\left(g_{\rho\lambda} - \dfrac{q_\rho q_\lambda}{m_\mathrm{Z}^2}\right)}{q^2 - m_\mathrm{Z}^2} \xrightarrow[q^2 \ll m_\mathrm{Z}^2]{} \dfrac{\mathrm{i}\, g_{\rho\lambda}}{m_\mathrm{Z}^2}$