QFD: $SU_L(2) \times U_Y(1) \xrightarrow[SSB]{} U_{em}(1)$

$\underbrace{V_1, W_2, V_3 \quad B}$

$W^{\pm} \quad Z \quad A$

5.3 The extension of QFD to more fermions

$$\psi = \begin{pmatrix} \nu_{e_L} \\ e_L \\ \vdots \\ \vdots \\ b_R \end{pmatrix}$$

$T_a \quad (a = 1, 2, 3)$

Y

generators of

$SU(2)_L \times U(1)_Y$

$$D_\lambda \psi = \left(\partial_\lambda + i g W_\lambda^a \bar{T}_a + i g' B_\lambda Y \right) \psi$$

(ii)

$$\mathcal{L}_{Yuk} = \bar{\psi}(x) \left(\phi_1(x) C_1 + \phi_2(x) C_2 \right) \psi(x)$$

$$+ h.c.$$

$C_{1,2}$ complex matrices

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\left(W_{\lambda\rho}W^{\lambda\rho}\right) - \frac{1}{4}B_{\lambda\rho}B^{\lambda\rho}$$
$$+ \left\{\bar{\psi}\,\frac{i}{2}\,D_{\lambda_1}\psi + \text{h.c.}\right\}$$
$$+ \mathcal{L}_{\text{Yuk}}$$
$$+ \left(D_\lambda \phi\right)^\dagger \left(D^\lambda \phi\right) - V(\phi)$$

SSB: $\phi(x) = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H(x)) \end{pmatrix}$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(W_{\mu\nu}W^{\mu\nu}\right) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$+ W_\mu^+ W^{-\mu} m_W^2 \left(1 + \frac{H}{v}\right)^2$$

$$+ \frac{1}{2} Z_\mu Z^\mu m_Z^2 \left(1 + \frac{H}{v}\right)^2$$

$$+ \left\{ \bar{\psi} \frac{i}{2} \gamma^\mu D_\mu \psi + \text{h.c.} \right\} \quad (*)$$

$$- \bar{\psi} M \psi \left(1 + \frac{H}{v}\right)$$

$$+ \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2} m_H^2 H^2 \left[1 + \frac{H}{v} + \frac{1}{4}\left(\frac{H}{v}\right)^2\right]$$

$$M = -\left(C_2 + C_2^{+} \right) \frac{v}{\sqrt{2}}$$

mass matrix of fermions

$$(*) = \left(\bar{\psi} \frac{i}{2} \gamma^{\lambda} \partial_{\lambda} \psi + h.c. \right) + \mathcal{L}_{int}$$

$$\mathcal{L}_{int} = -e \left\{ A_{\lambda} J^{\lambda}_{em} + \frac{1}{\sin\vartheta_W \cos\vartheta_W} Z_{\lambda} J^{\lambda}_{NC} \right.$$

$$\left. + \frac{1}{\sqrt{2}\sin\vartheta_W} \left(W_{\lambda}^{+} J^{\lambda}_{cc} + W_{\lambda}^{-} J^{\lambda}_{cc}{}^{+} \right) \right\}$$

$$J^{\mu}_{em} = \bar{\psi}\,\gamma^{\mu}\left(T_3 + Y\right)\psi = \bar{\psi}\,\gamma^{\mu} Q\,\psi$$

$$Q = T_3 + Y$$

$$\psi = \begin{pmatrix} \nu_{eL} \\ e_L \\ e_R \end{pmatrix}$$

$$T_3 + Y = \begin{pmatrix} \tfrac{1}{2} & 0 & 0 \\ 0 & -\tfrac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} -\tfrac{1}{2} & 0 & 0 \\ 0 & -\tfrac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$= Q$$

$$J^{\lambda}_{NC} = \bar{\psi} \gamma^{\lambda} T_3 \psi - \sin^2 \theta_W J^{\lambda}_{em}$$

$$J^{\lambda}_{CC} = \bar{\psi} \gamma^{\lambda} \left(T_1 + i T_2 \right) \psi$$