$$\begin{pmatrix} \psi_{1L} \\ \psi_{2L} \end{pmatrix} \qquad SU(2)_L \qquad\qquad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\phi^\dagger \begin{pmatrix} \psi_{1L} \\ \psi_{2L} \end{pmatrix} = \phi_1^\dagger \psi_{1L} + \phi_2^\dagger \psi_{2L}$$

$$\phi^T \varepsilon \begin{pmatrix} \psi_{1L} \\ \psi_{2L} \end{pmatrix} = \phi_1 \psi_{2L} - \phi_2 \psi_{1L}$$

$$\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} = U_1 \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \qquad (U_1 \in U(3))$$

$$U_1^\dagger U_1 = \mathbb{1}$$

$$C_\ell \to U_1^\dagger C_\ell$$

$$\begin{pmatrix} \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \\ \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \\ \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix} \end{pmatrix} \to V_1 \begin{pmatrix} \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \\ \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \\ \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix} \end{pmatrix} \qquad V_1 \in U(3)$$

quark doublets:

$$V_2 \in U(3)$$

$$\begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} \rightarrow U_2 \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} \qquad U_2 \in U(3)$$

$$\begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix} \rightarrow U_3 \begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix} \qquad U_3 \in U(3)$$

$$C_\ell \to U_1^\dagger C_\ell V_1$$

$$C_q^1 \to U_2^\dagger C_q^1 V_2$$

$$C_q \to U_3^\dagger C_q V_2$$

$$C_\ell C_\ell^\dagger = U_1^\dagger C_\ell \underbrace{V_1 V_1^\dagger}_{= \mathbb{1}} C_\ell^\dagger U_1$$

$$= U_1^\dagger C_\ell C_\ell^\dagger U_1$$

$$= \begin{pmatrix} C_e^2 & 0 & 0 \\ 0 & C_\mu^2 & 0 \\ 0 & 0 & C_\tau^2 \end{pmatrix} \quad C_e, C_\mu, C_\tau \geq 0$$

$$C_l = \begin{pmatrix} c_e & 0 & 0 \\ 0 & c_\mu & 0 \\ 0 & 0 & c_\tau \end{pmatrix} W \qquad W \in U(3)$$

$$C_l C_l^\dagger = (\cdot) W W^\dagger (\cdot)$$

$$= \begin{pmatrix} c_e^2 & & \\ & \ddots & \\ & & \end{pmatrix}$$

$$V_l = W^\dagger$$

$$C_l \to \begin{pmatrix} c_e & 0 & \\ 0 & c_\mu & \\ & & c_\tau \end{pmatrix}$$

$$C_q^1 \to \begin{pmatrix} c_u & 0 & 0 \\ 0 & c_c & 0 \\ 0 & 0 & c_t \end{pmatrix} \qquad c_u, c_c, c_t \geq 0$$

$$C_q = \begin{pmatrix} c_d & 0 & 0 \\ 0 & c_s & 0 \\ 0 & 0 & c_b \end{pmatrix} V^\dagger \qquad V \in U(3)$$

choose $U_3 = V^\dagger$ $\qquad c_d, c_s, c_b \geq 0$

$$C_q \to V \begin{pmatrix} c_d & 0 & 0 \\ 0 & c_s & 0 \\ 0 & 0 & c_b \end{pmatrix} V^\dagger$$

$V \in U(3)$

CKM (Cabibbo-Kobayashi-Maskawa) matrix

$$V^\dagger V = V V^\dagger = \mathbb{1}_3 \qquad V \in U(3)$$

$$V^*_{ji} V_{jk} = \delta_{ik}$$

$$V_{ij} V^*_{kj} = \delta_{ik}$$

$$V_2 = U_2 = U_3 = U_\varphi = \begin{pmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 \\ 0 & 0 & e^{i\varphi_3} \end{pmatrix}.$$

$$V' = U_\varphi^+ V$$

$$U_\chi \in U(3) \qquad U_\chi = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix}$$

$$V' \longrightarrow U_\varphi^+ V U_\chi$$

$\varphi_1, \varphi_2, \varphi_3, \chi_1, \chi_2, \chi_3$ 6 free param.

$\Rightarrow$ only phase differences matter

$\Rightarrow$ 5 independent parameters

$V_{11} \geq 0 \quad V_{33} \geq 0$

$V_{12} \geq 0 \quad V_{11} \cdot V_{22} - V_{12} V_{21} \geq 0$

$V_{23} \geq 0$

Wolfenstein's parametrisation

$$\lambda = s_{12} \approx 0.22$$

$$A = \frac{s_{23}}{(s_{12})^2}$$

$$\rho - i\eta = \frac{s_{13} e^{-i\delta_{13}}}{s_{12} s_{23}}$$

$$c_{12} = \sqrt{1 - s_{12}^2}$$

$$= \sqrt{1 - \lambda^2} \quad O(\lambda^4)$$

$$= 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 + O(\lambda^6)$$

$$c_{23} = 1 + O(\lambda^4)$$

$$c_{13} = 1 + O(\lambda^6)$$

$$\phi(x) \quad \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H(x)) \end{pmatrix}$$

"mass eigenfields"

$$\begin{pmatrix} d(x) \\ s(x) \\ b(x) \end{pmatrix} = V^{\dagger} \begin{pmatrix} d'(x) \\ s'(x) \\ b'(x) \end{pmatrix}$$

$\rightarrow T_3 = -\frac{1}{2}$

$u, c, t : T_3 = +\frac{1}{2}$

$$J^{\lambda}_{em} = -\left(\bar{e}, \bar{\mu}, \bar{\tau}\right) \gamma^{\lambda} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$+\frac{2}{3}\left(\bar{u}, \bar{c}, \bar{t}\right) \gamma^{\lambda} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$$-\frac{1}{3}\left(\bar{d}, \bar{s}, \bar{b}\right) \gamma^{\lambda} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$
\begin{aligned}
&\left(\bar{d}', \bar{s}', \bar{b}'\right) \gamma^{\lambda} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} \\
&= \left(\bar{d}, \bar{s}, \bar{b}\right) V^{+} \gamma^{\lambda} V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \\
&= \left(\bar{d}, \bar{s}, \bar{b}\right) \gamma^{\lambda} \begin{pmatrix} d \\ s \\ b \end{pmatrix}
\end{aligned}
$$