The Standard Model of Particle Physics

Christian Schwanenberger

Tel.: +44 161 306-6466
+1 630 840-4242
Fax: +1 630 840-6650
schwanen@fnal.gov
http://www-d0.fnal.gov/~schwanen

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MANCHESTER 1824

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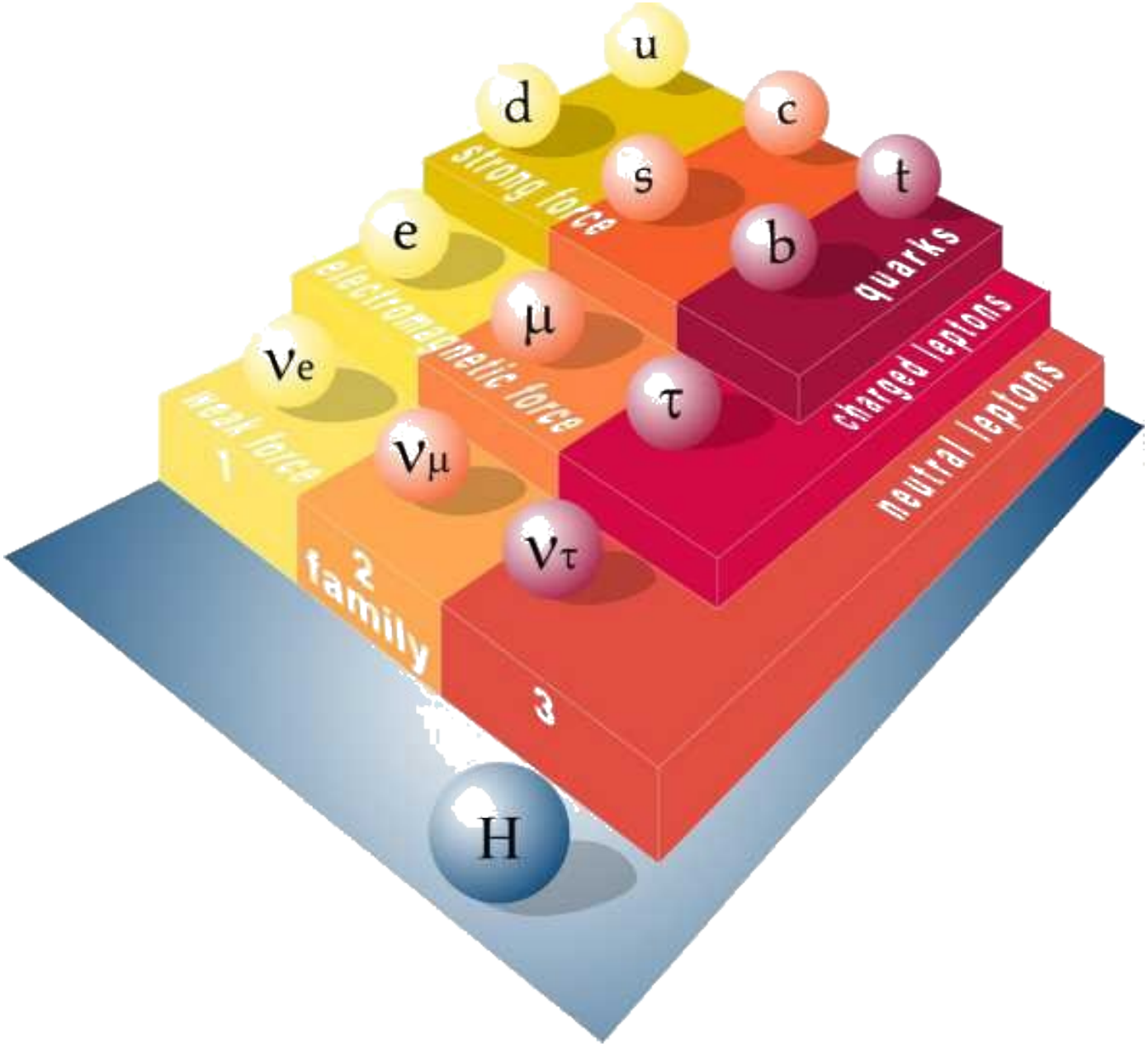
Literature

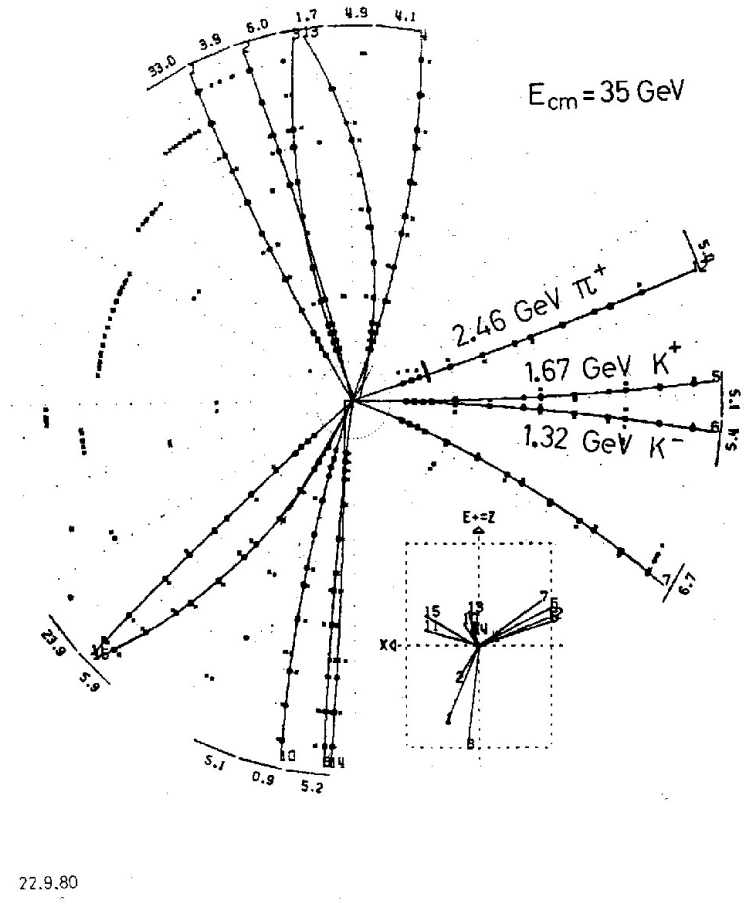
- Introduction to Quantum Field Theory,
  J. Forshaw webpage
- An Introduction to Quantum Field Theory,
  Peskin and Schroeder
- Quarks and Leptons,
  Halzen & Martin
- Introduction to Gauge Field Theory,
  Bailin & Love
- Gauge Theory of Elementary Particle Physics,
  Cheng & Li
- Quantum Field Theory,
  Ryder
- Gauge Theories in Particle Physics,
  Aitchison & Hey (Vols. 1 & 2)
- Elementary Particle Physics: Concepts and Phenomena
  Nachtmann

1. Introduction

The Standard Model

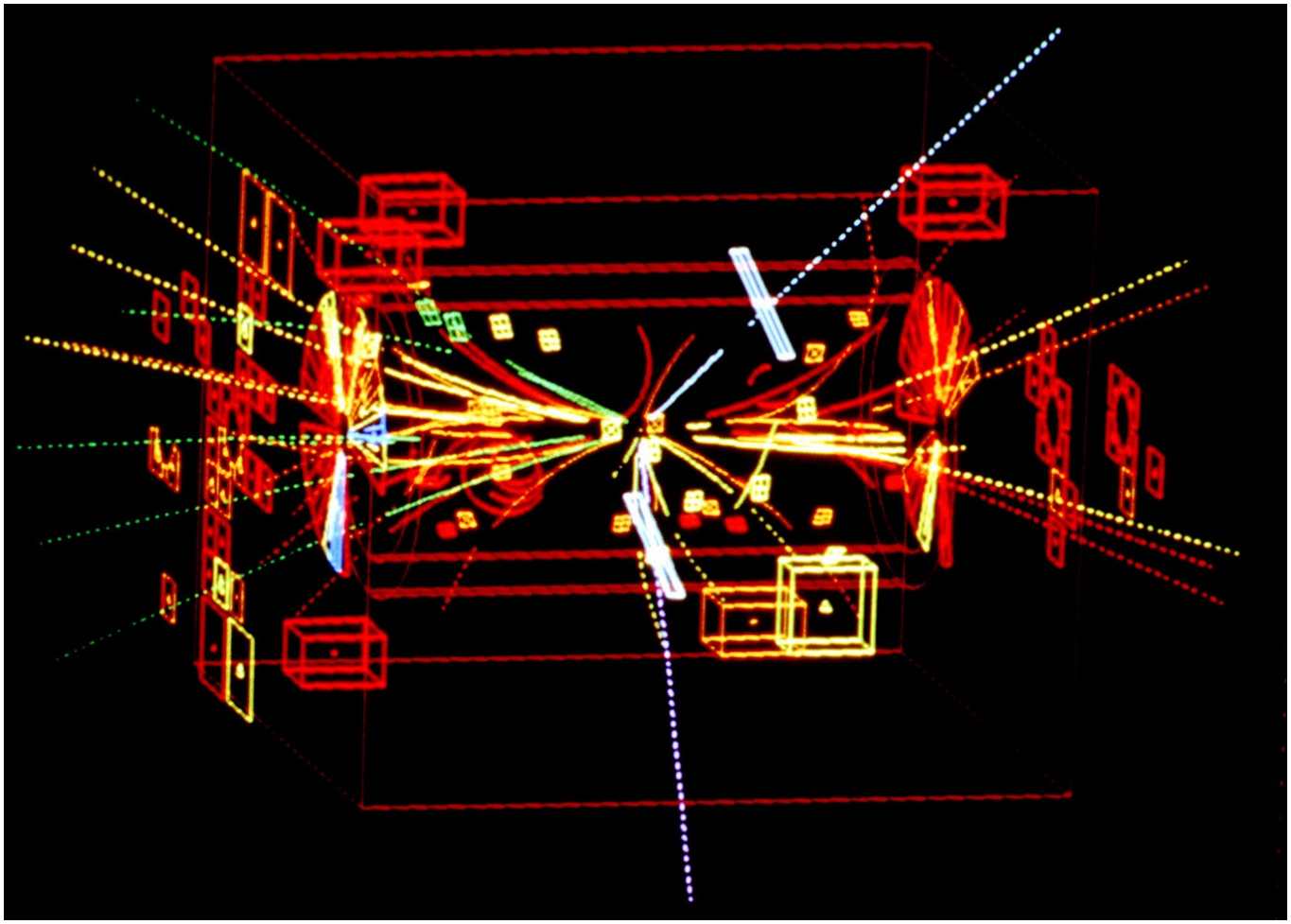


√s=35 GeV



gluon discovery at e^+e^- accelerator PETRA (DESY, Hamburg)

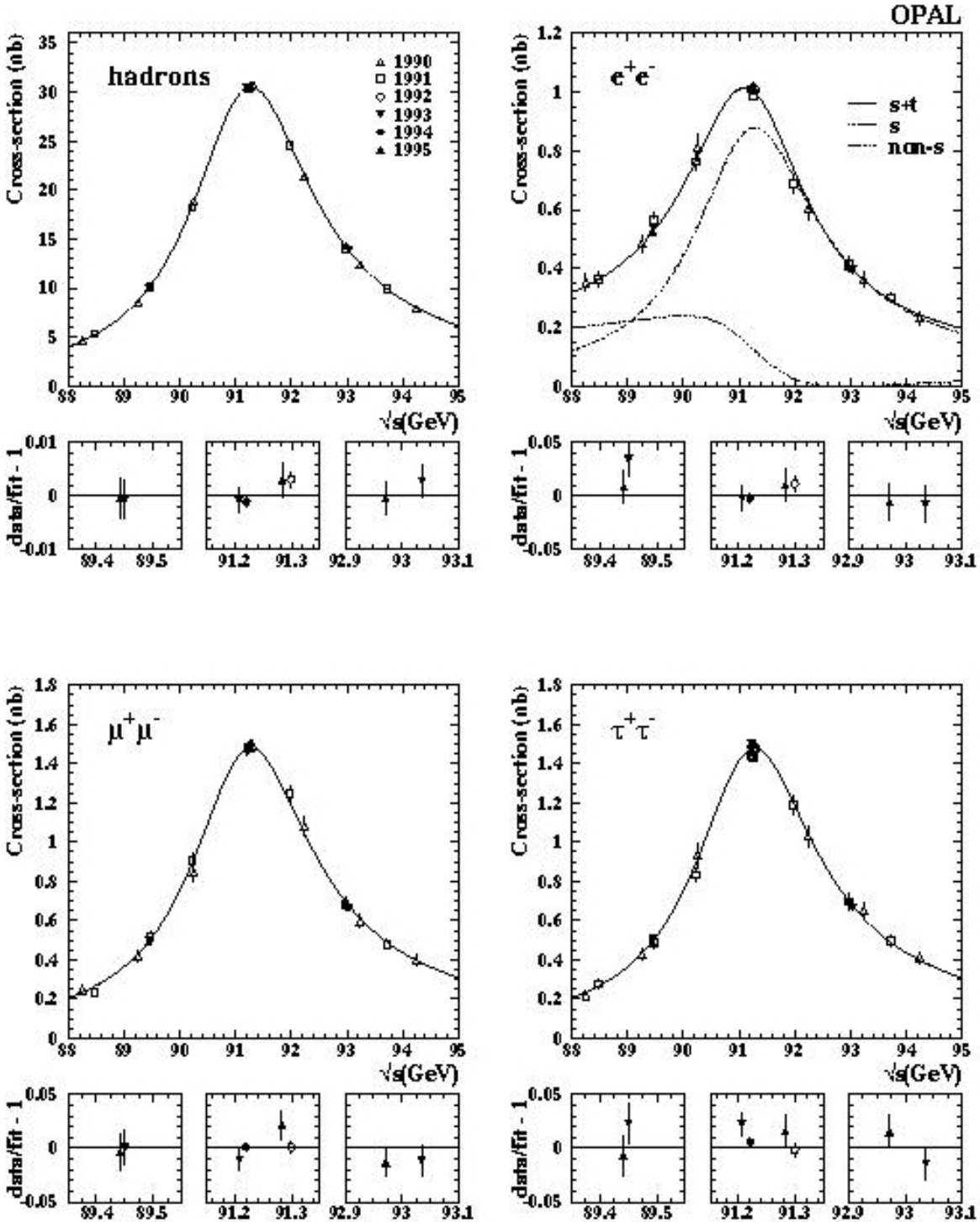
Z boson

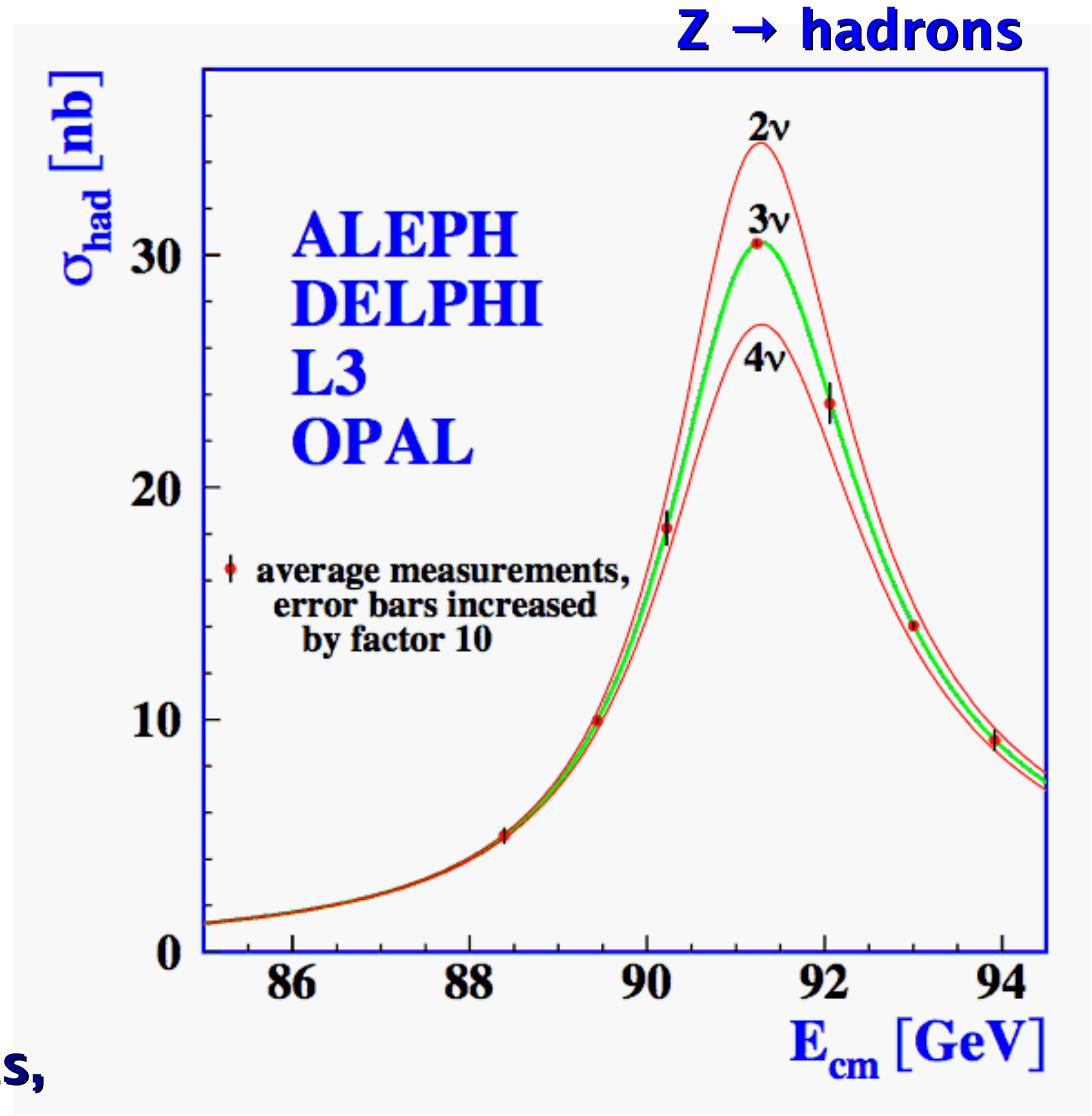


1983, UA1 experiment, $\sqrt{s}$=540 GeV

discovery of Z boson at $\bar{p}p$ accelerator SPS (CERN, Geneva)

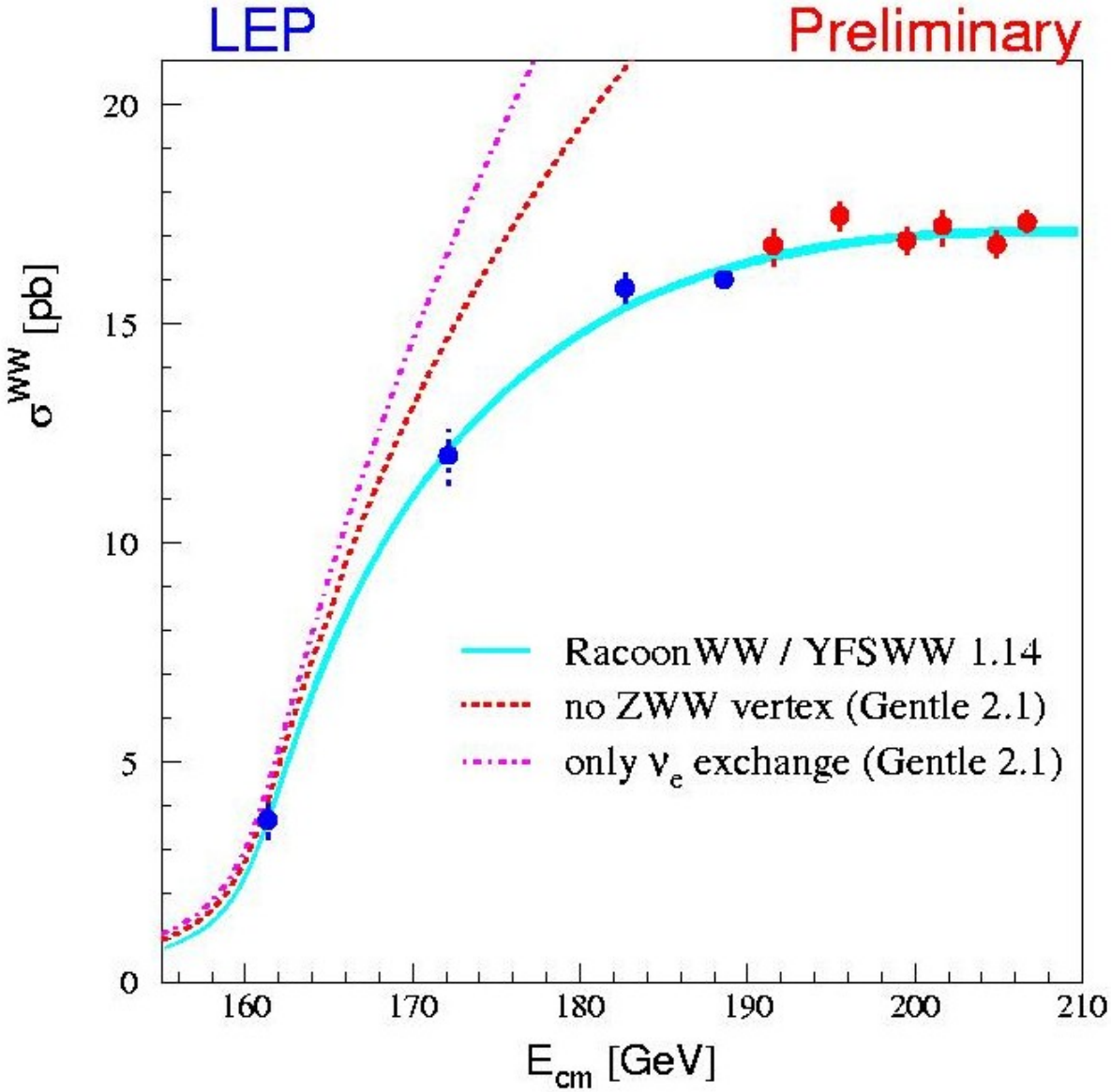
Cross section $\sigma(e^+e^- \rightarrow Z^0 \rightarrow f\bar{f})$ as function of center of mass energy by OPAL (LEP)



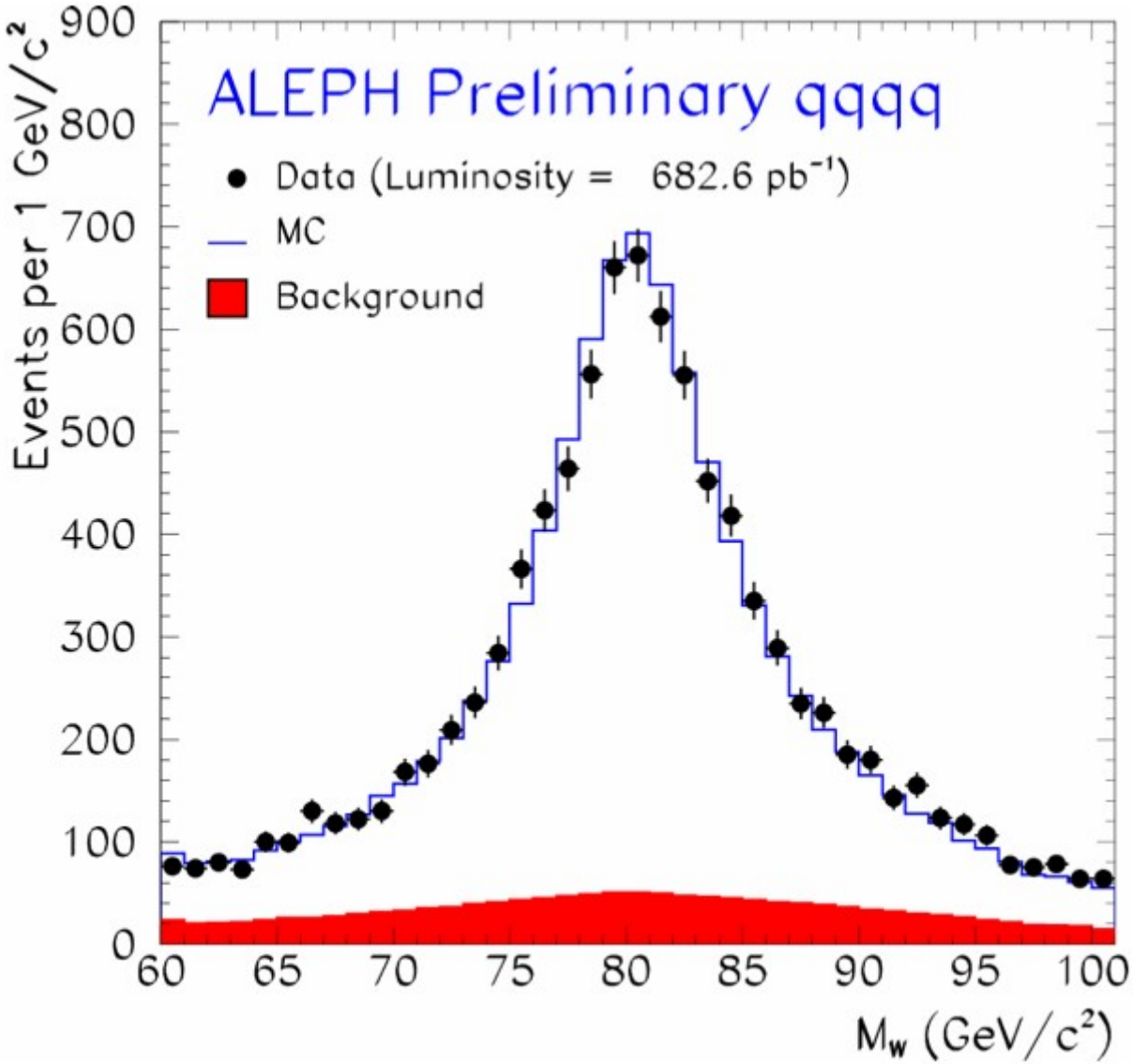


LEP experiments, CERN

Z mass distribution with calculations for 2, 3 and 4 neutrino types by all 4 experiments at LEP

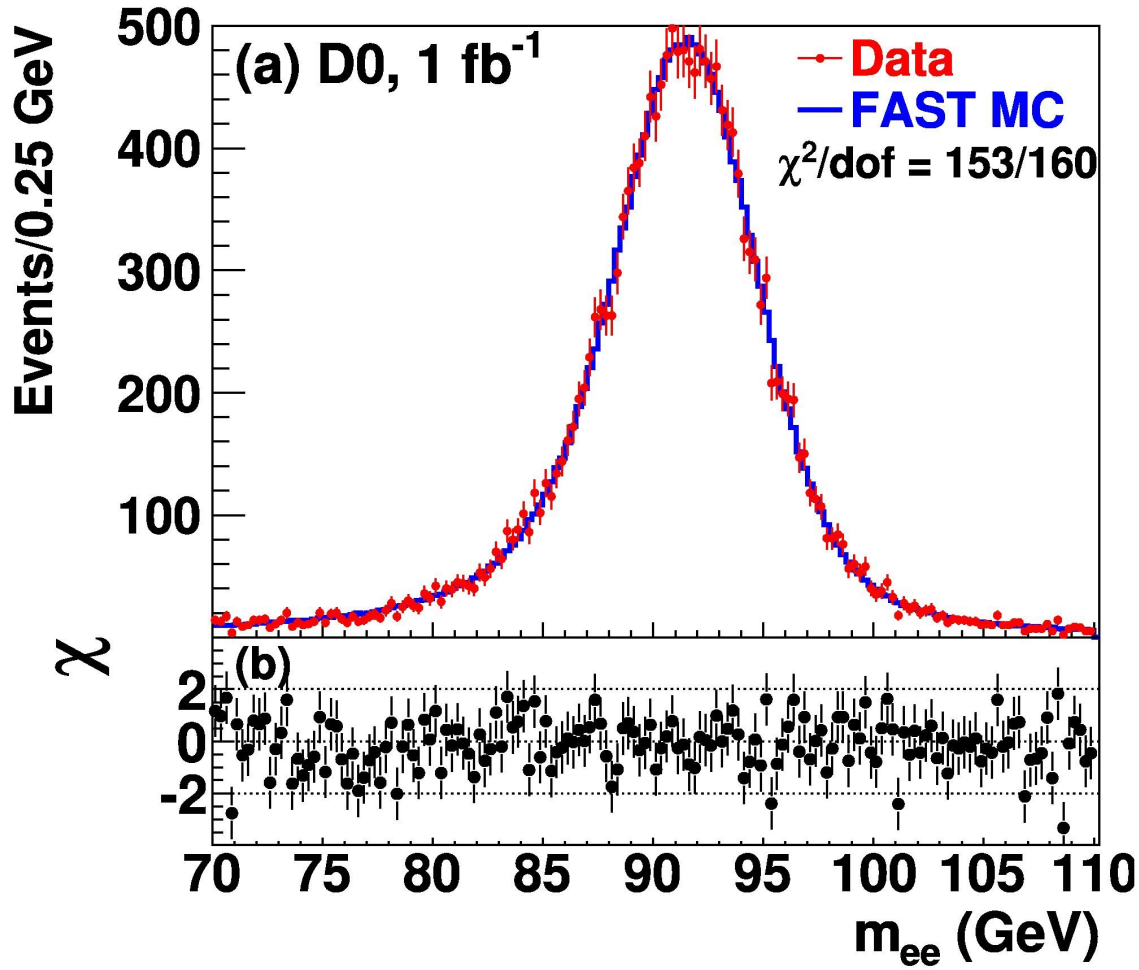


Measurement of $W^+ W^-$ at the threshold in $e^+ e^-$ interactions at LEP II



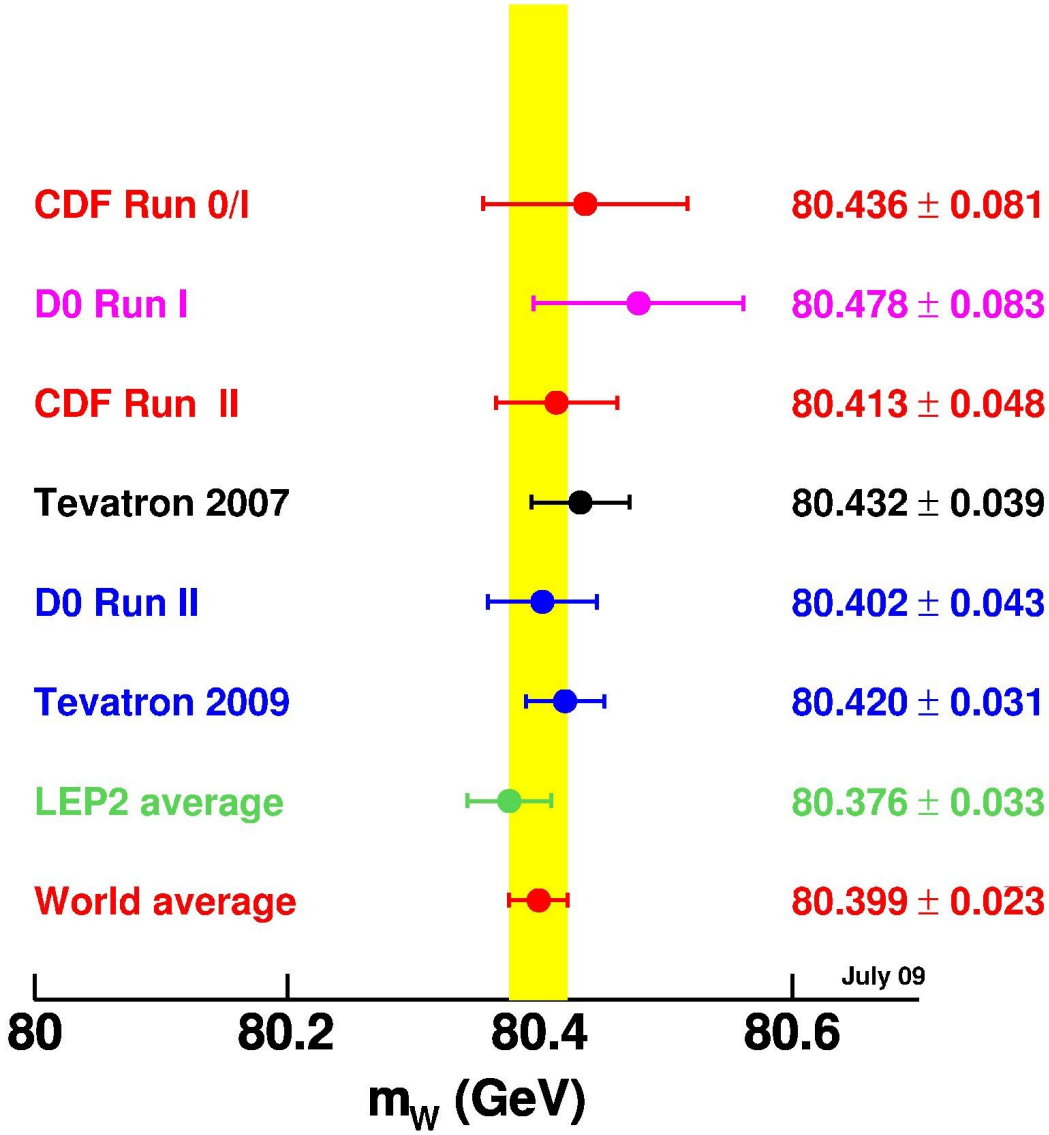
Reconstructed W mass in 4 jet events by ALEPH at LEP II

W mass measurements at the Tevatron



$m_W = 80.401 \pm 0.043$ (stat) GeV

±0.05%

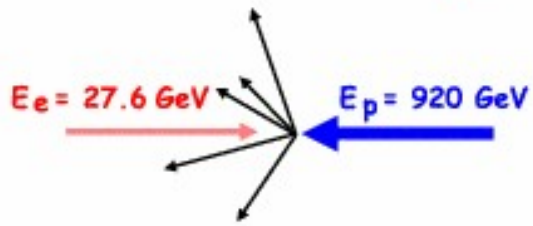


Summary: W mass measurements

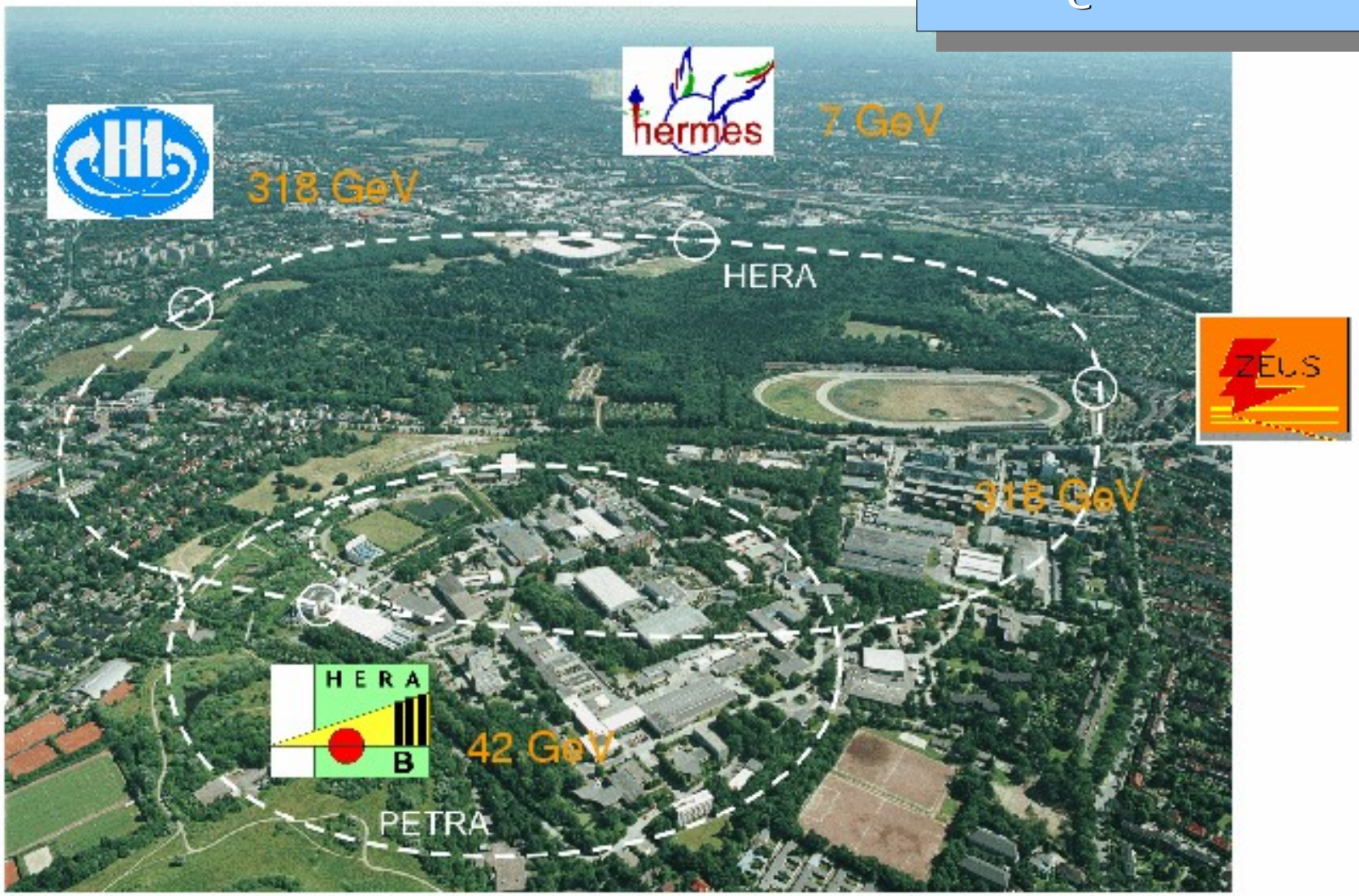
Speicherring	Standort	Typ	$\sqrt{s}$[GeV]
Tevatron	Fermilab, Chicago	$p\overline{p}$	1800/1960
LEP I	CERN, Genf	e^+e^-	M_Z
SLC	SLAC, Stanford	e^+e^-	M_Z
LEP II	CERN, Genf	e^+e^-	208

Particle accelerators used for precision measurements for W and Z bosons

HERA: ep Collider and Experiments

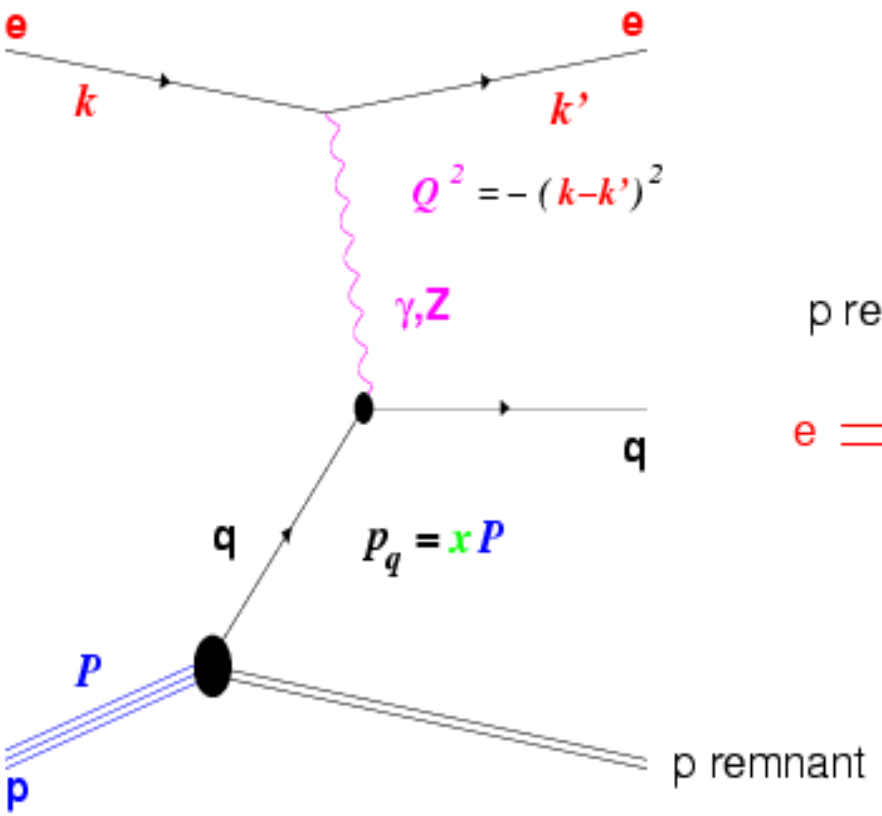


$\sqrt{s}$ = 320 GeV
$Q^2 \approx 100000$ GeV^2
$\lambda_c \approx 10^{-3}$ fm

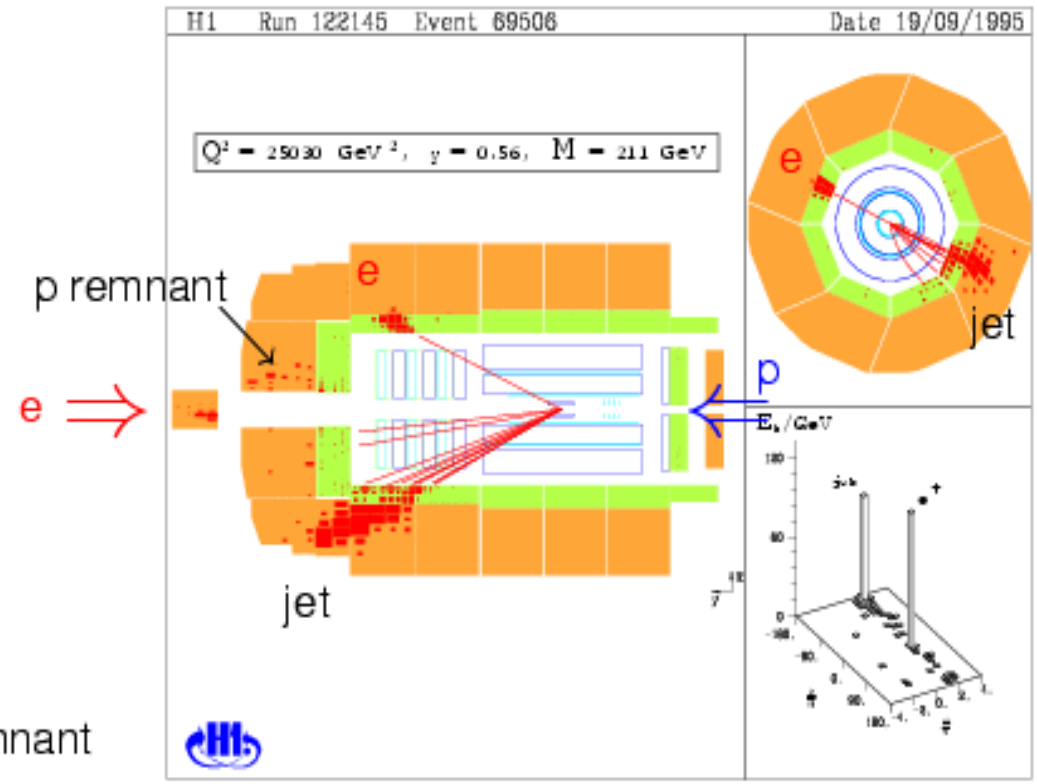


DIS – Neutral Current (NC)

deep inelastic scattering (DIS):



H1 detector



Q^2: four-momentum transfer

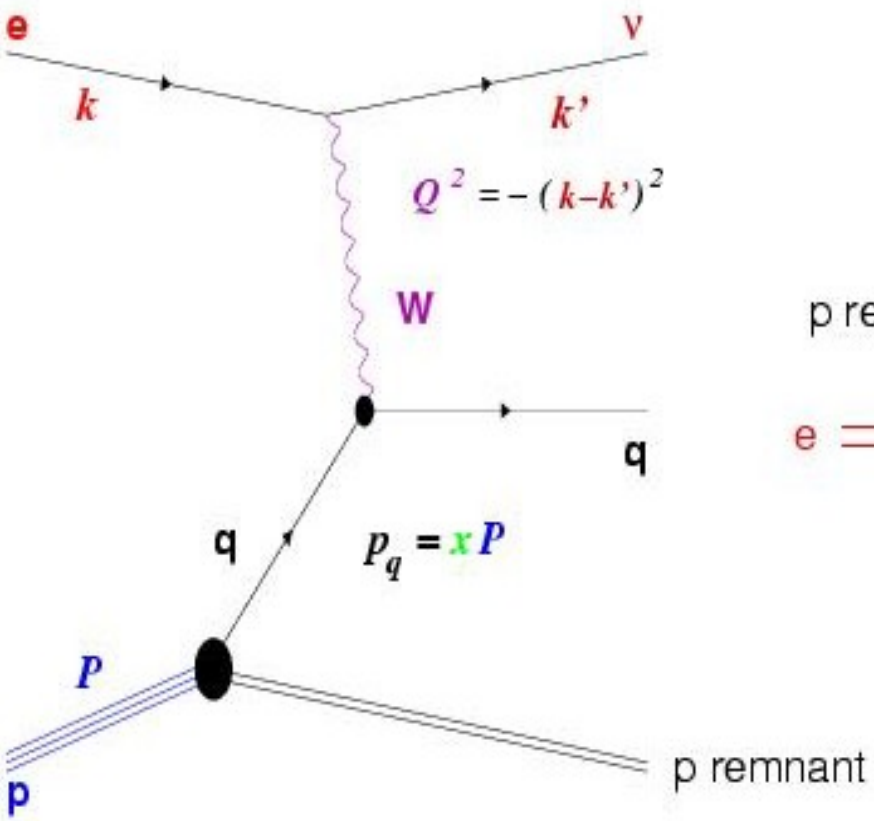
spatial resolution $\sim 1/Q$

$\Rightarrow 10^{-16}$ cm

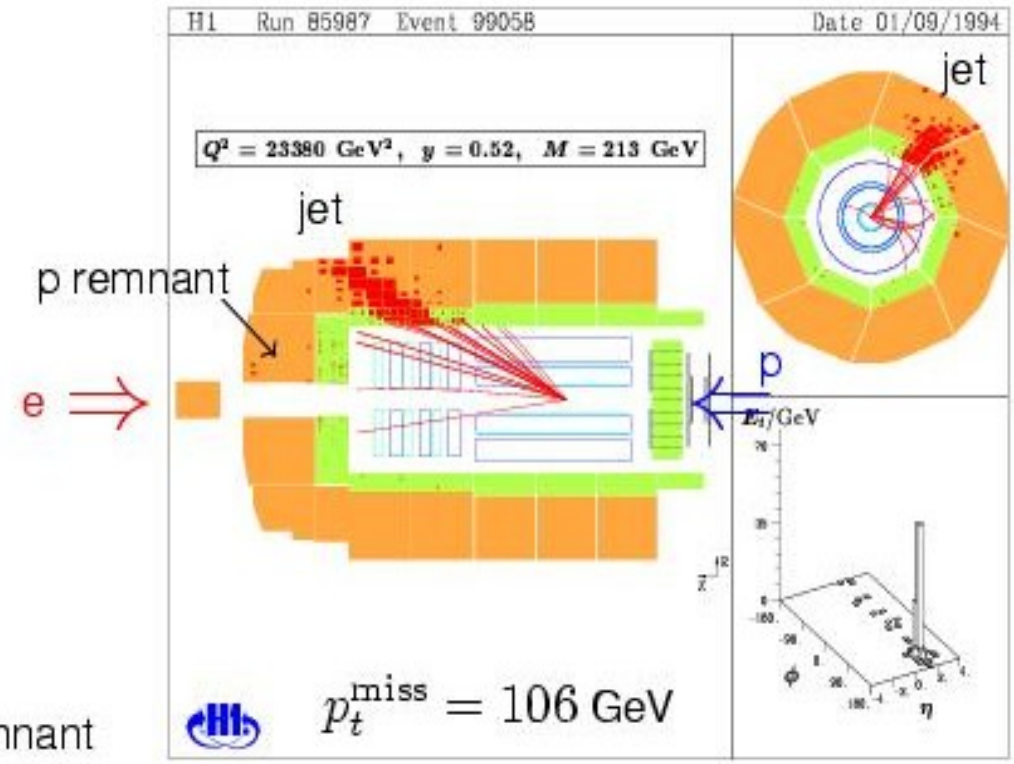
x: fractional momentum of the struck quark

DIS – Charged Current (CC)

deep inelastic (DIS) scattering:

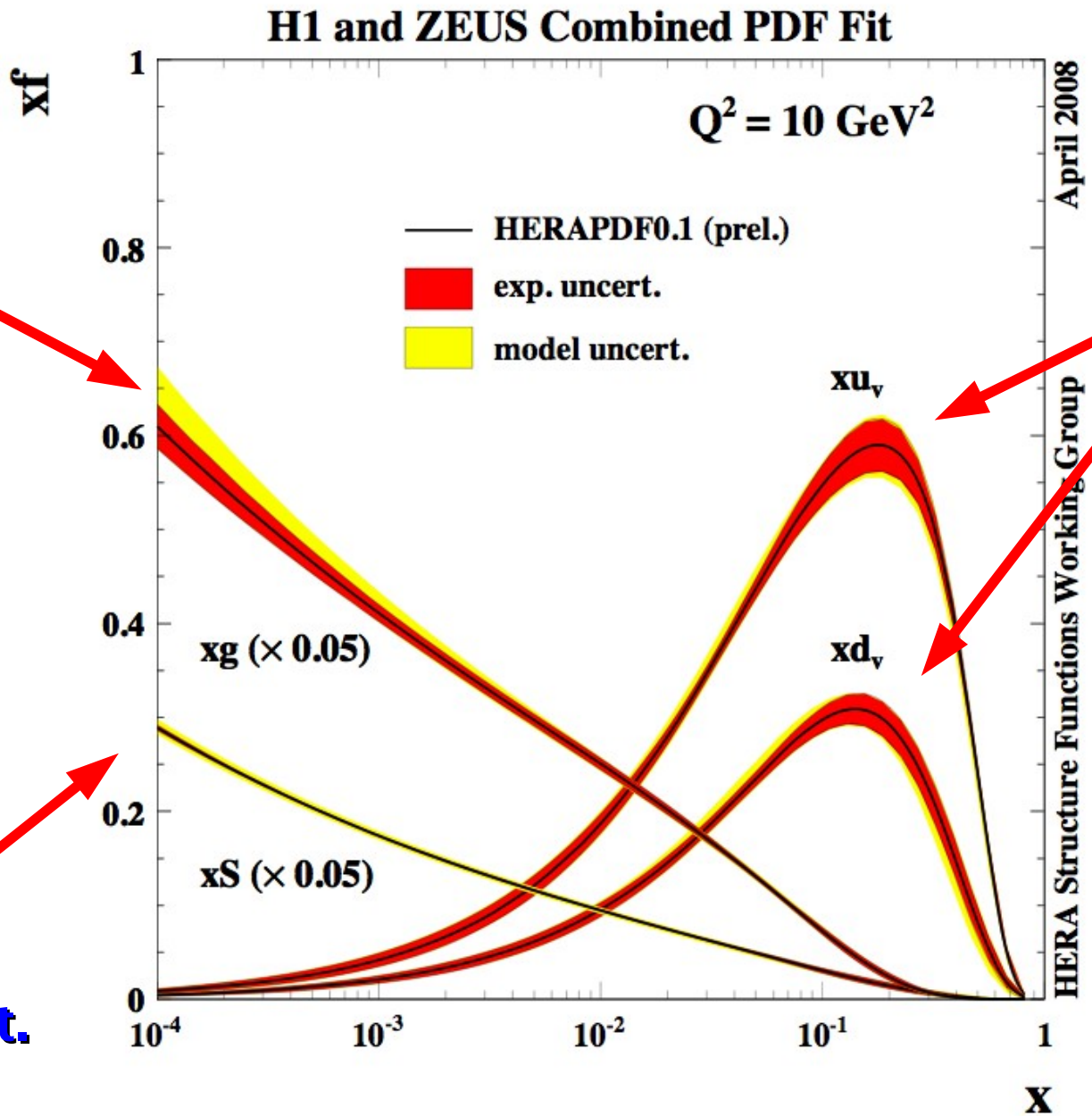


H1 detector



many **gluons** with small momentum fraction

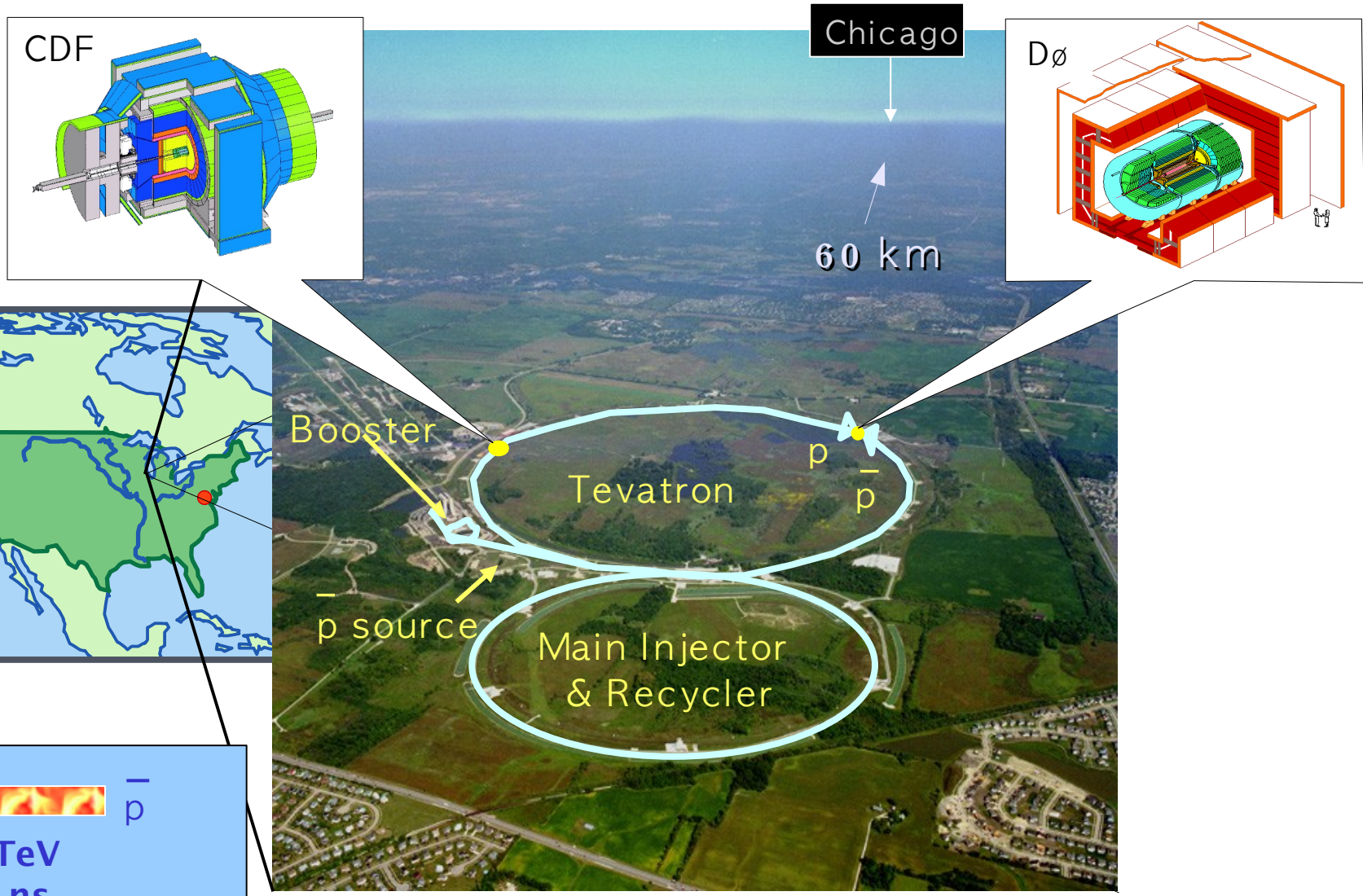
sea quarks: lead to scaling violation, $F_2(Q^2) \neq$ const.

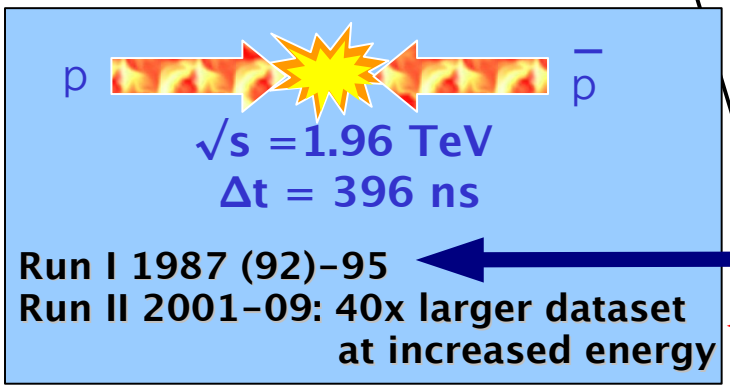


valence quarks: half of proton momentum

QCD fits to F_2 data using DGLAP equations

The Tevatron at FERMILAB: $p\bar{p}$ Collisions

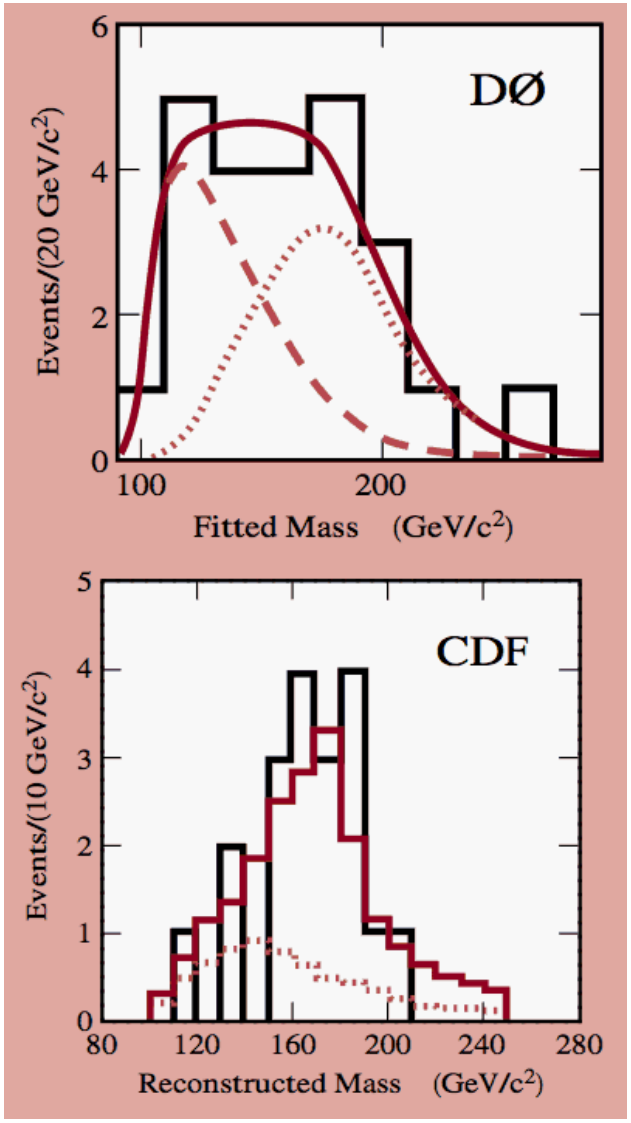




top quark discovery

measure properties with high precisions: is it really the particle expected in SM?

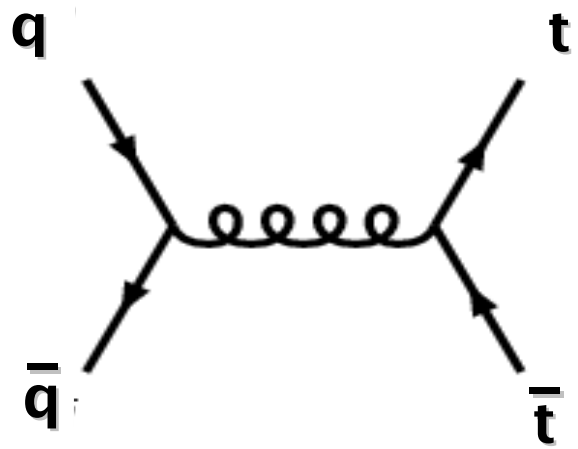
top quark



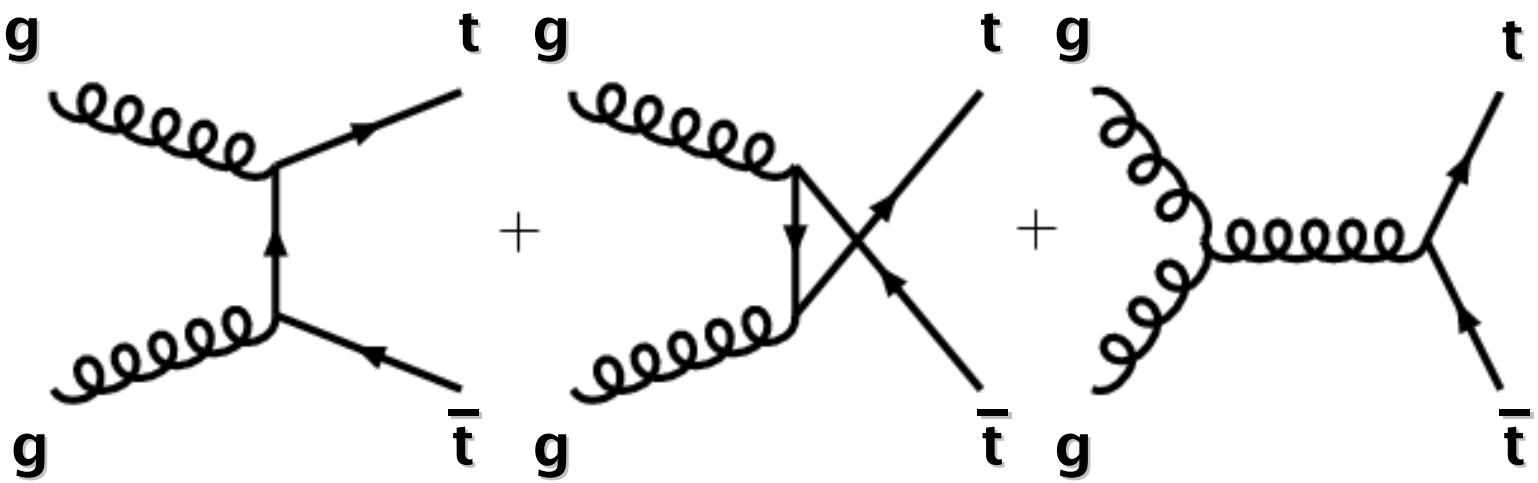
1995, CDF and DØ experiments, √s=1.8 TeV

discovery of top quark at $p\bar{p}$ accelerator Tevatron (Fermilab, Batavia)

Top Quark Pair Production



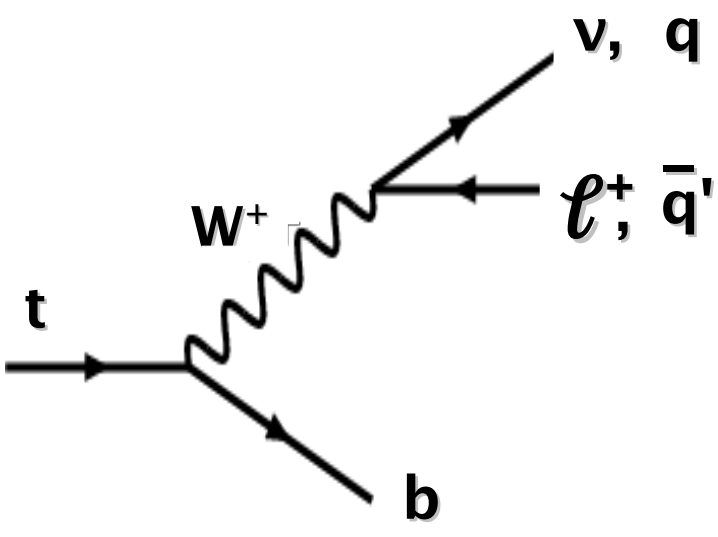
85%



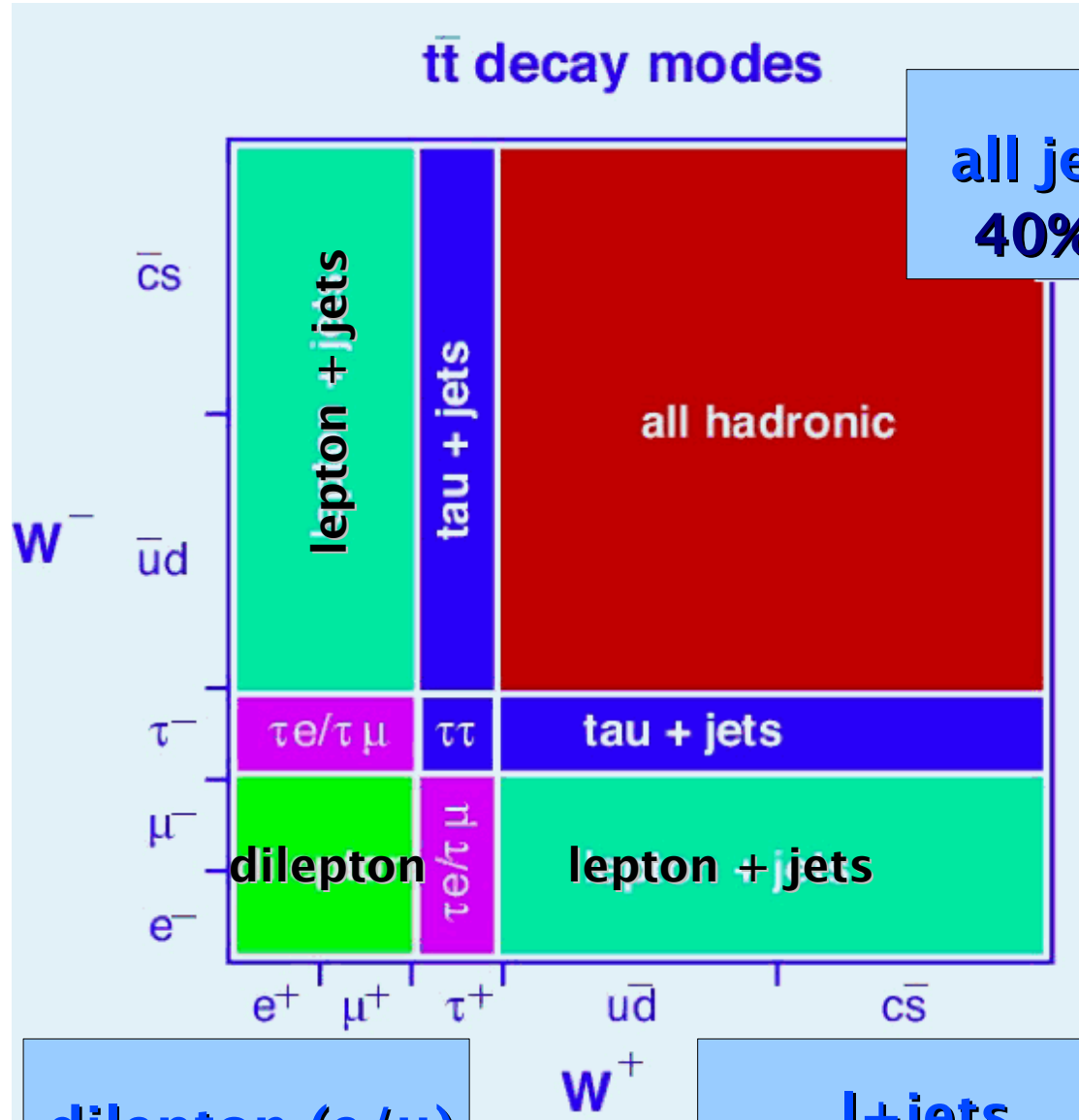
15%

Top Antitop Signatures

- **top decay:** ~100%



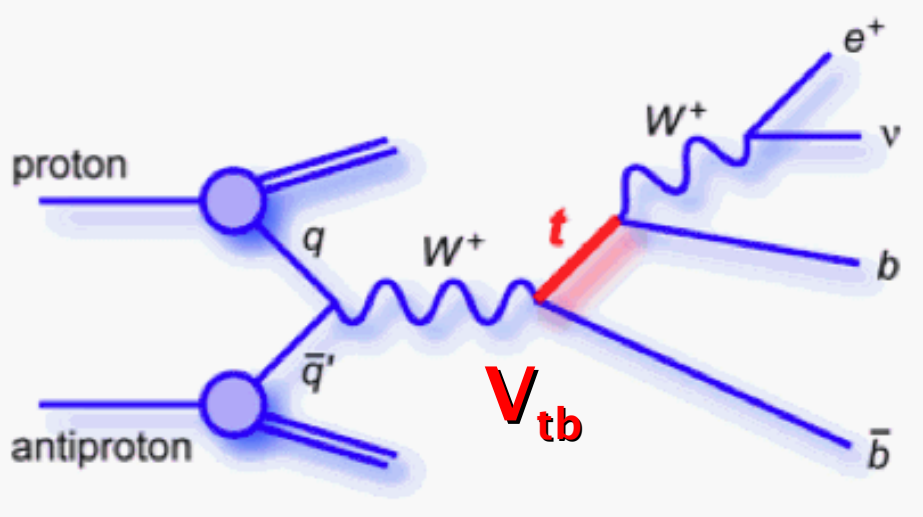
- reconstruct and identify: electrons, muons, jets, b-jets and missing transverse energy



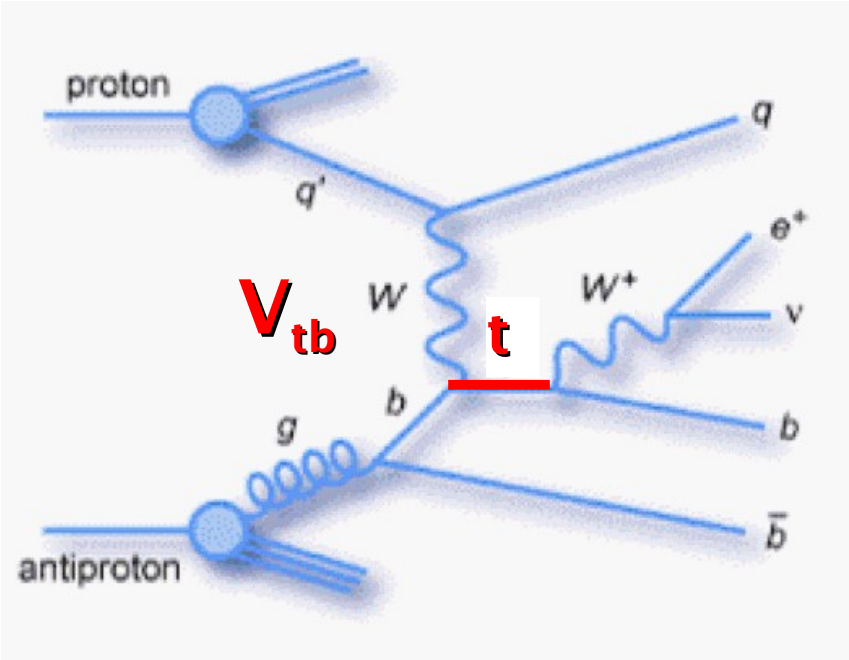
Single Top Quark Production

- **first direct measurement of $|V_{tb}|$**

s-channel:

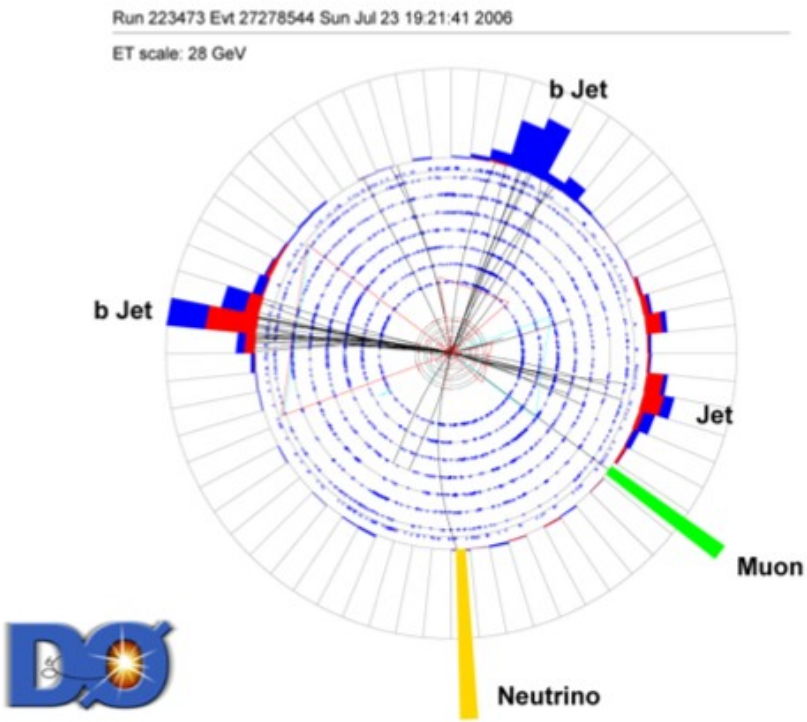


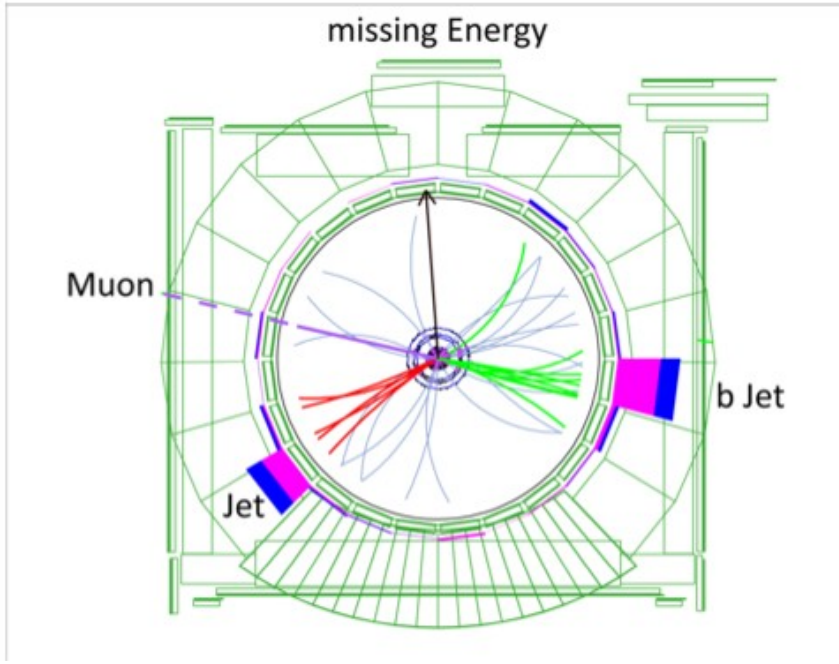
t-channel:

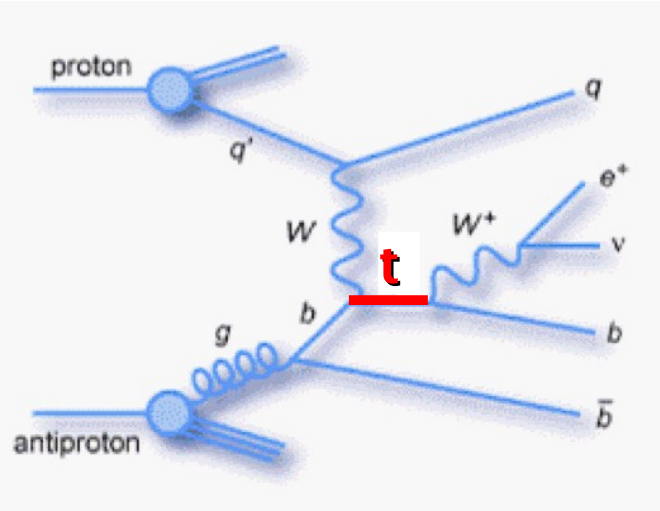


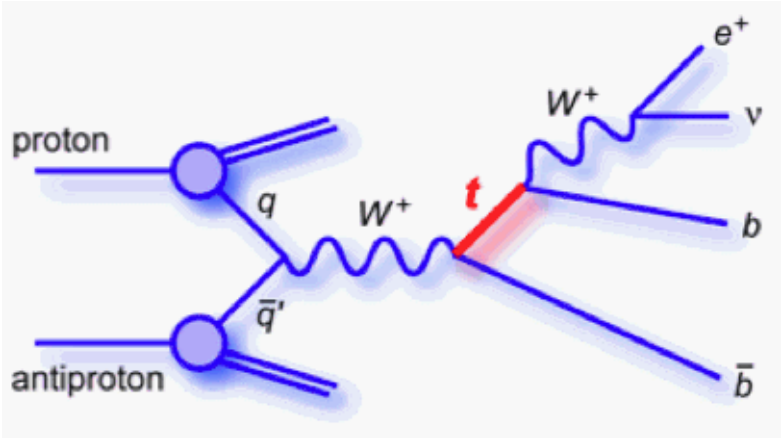
- **jets**
- **isolated lepton**
- **missing E_T**
- **b-jets**

Single Top Quark Observation

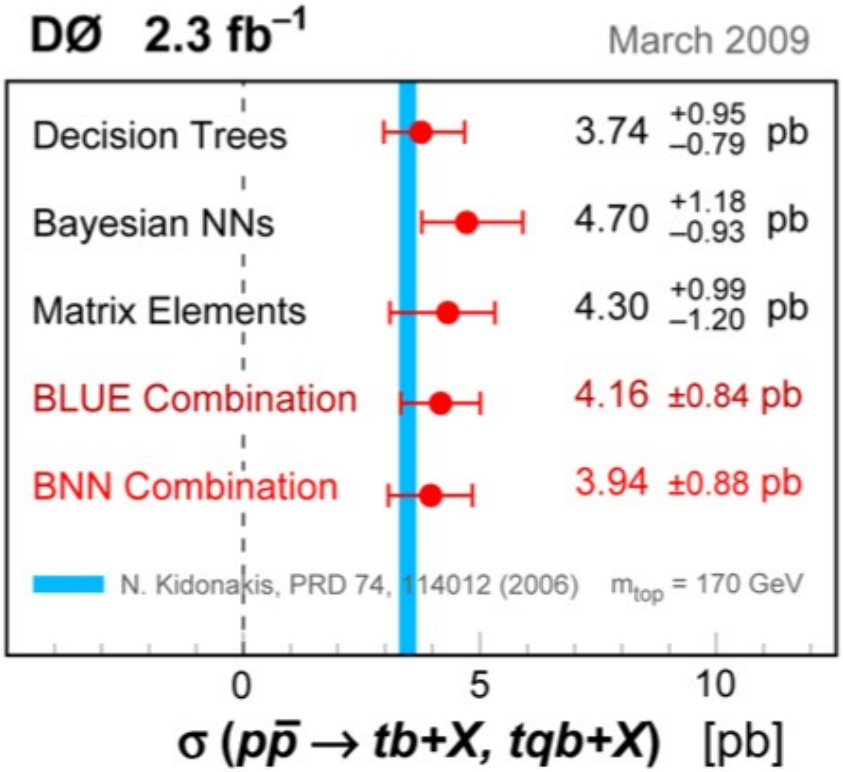




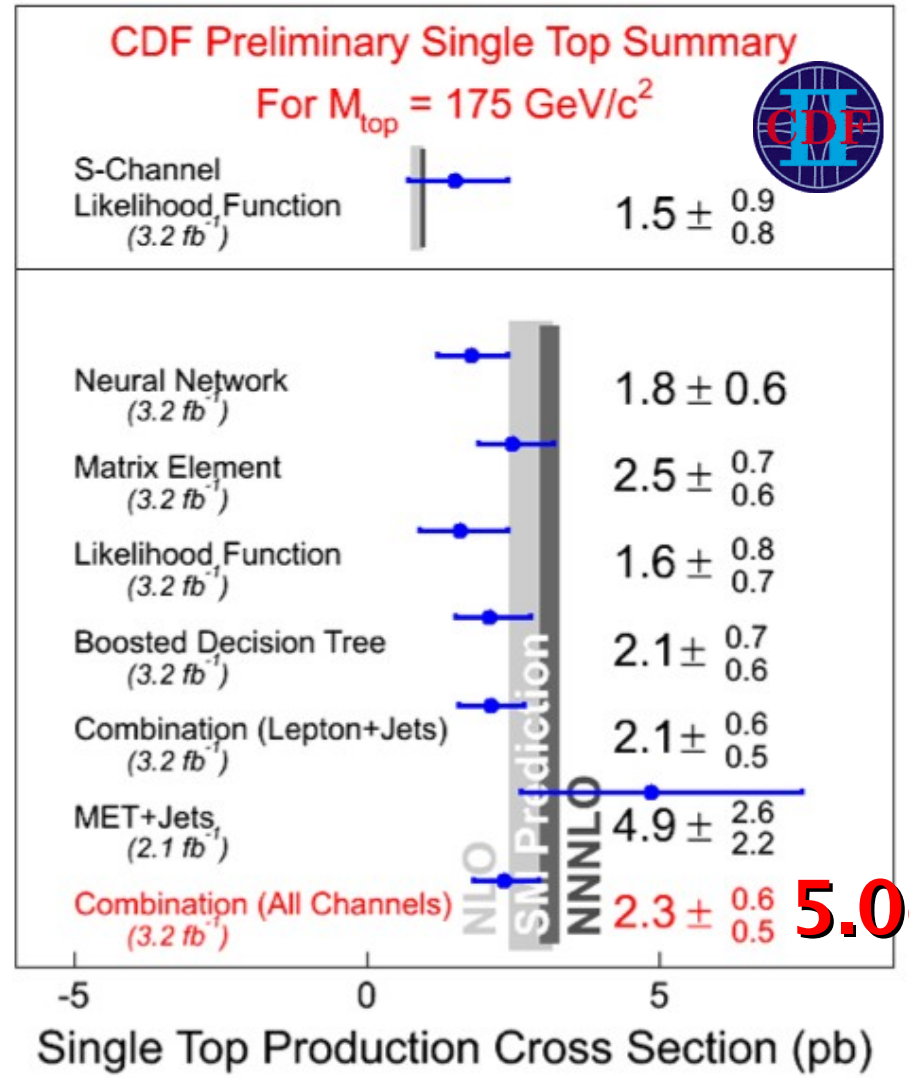




Single Top Quark Observation



5.0σ



5.0σ

$$|V_{tb}| = 1.07 \pm 0.12$$

$$|V_{tb}| = 0.91 \pm 0.11$$

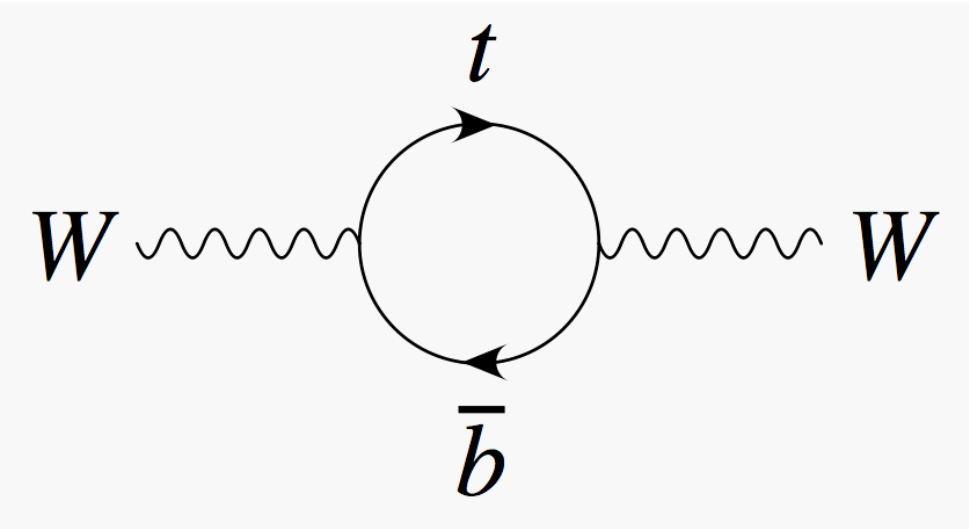
Single Top Quark Observation

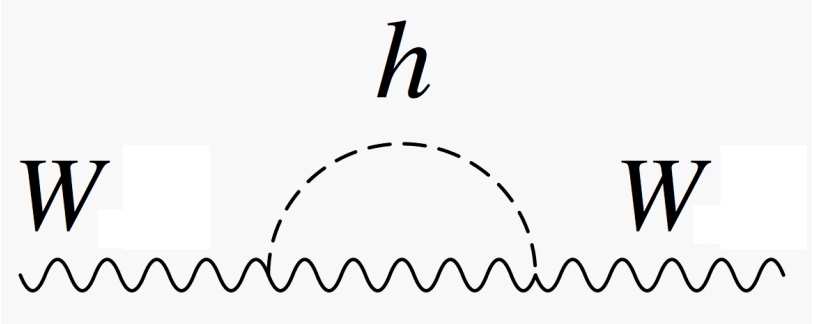
1 year ago

The Top Quark Mass

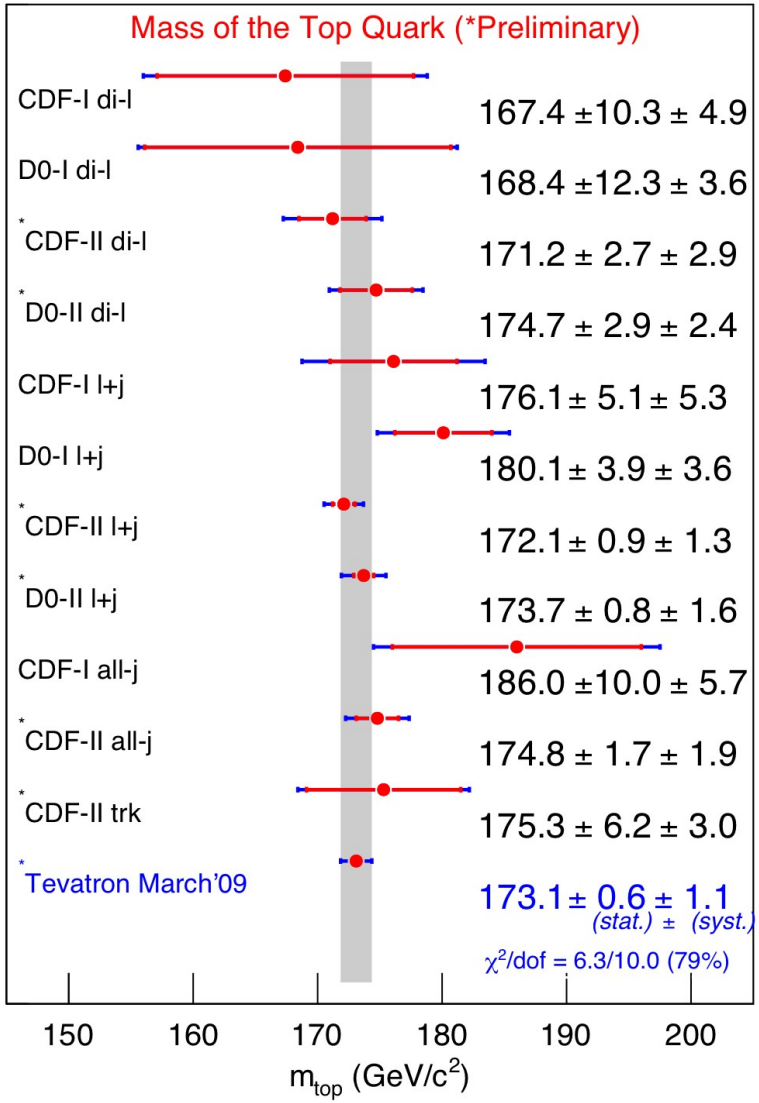
- free parameter in the Standard Model

- check the self-consistency of the Standard Model in combination with W mass measurement





Tevatron Combination: Spring 2009



world average:

$$m_{top} = 173.1 \pm 0.6\ (stat) \pm 1.1\ (syst)\ \ GeV$$

±0.75%

Top, W and Higgs Mass in the SM

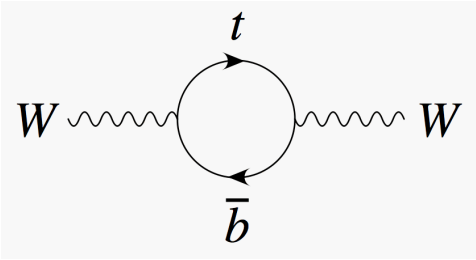
Heinemeyer, Hollik, Stockinger, Weber, Weiglein 2009

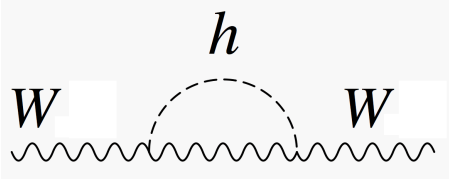
$$m_{top} = 173.1 \pm 1.3 \text{ GeV}$$

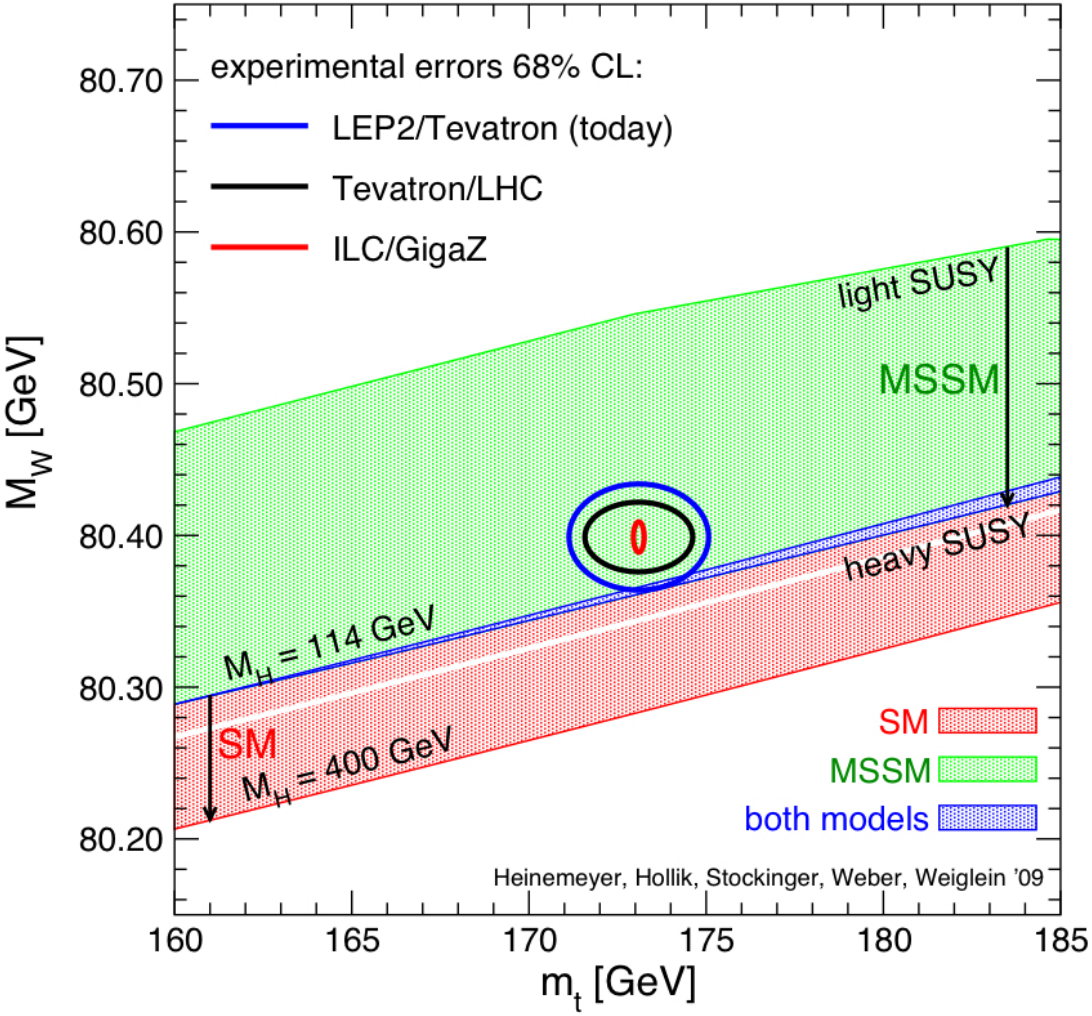
world average (March 2009)

$$m_W = 80399 \pm 23 \text{ MeV}$$

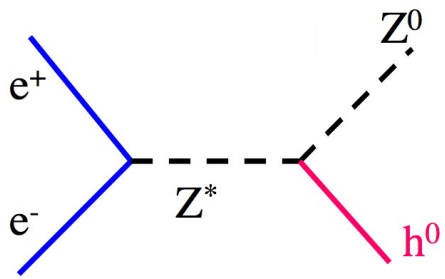
world average (March 2009)

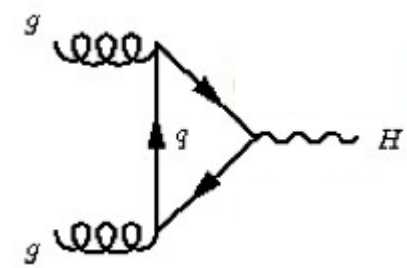


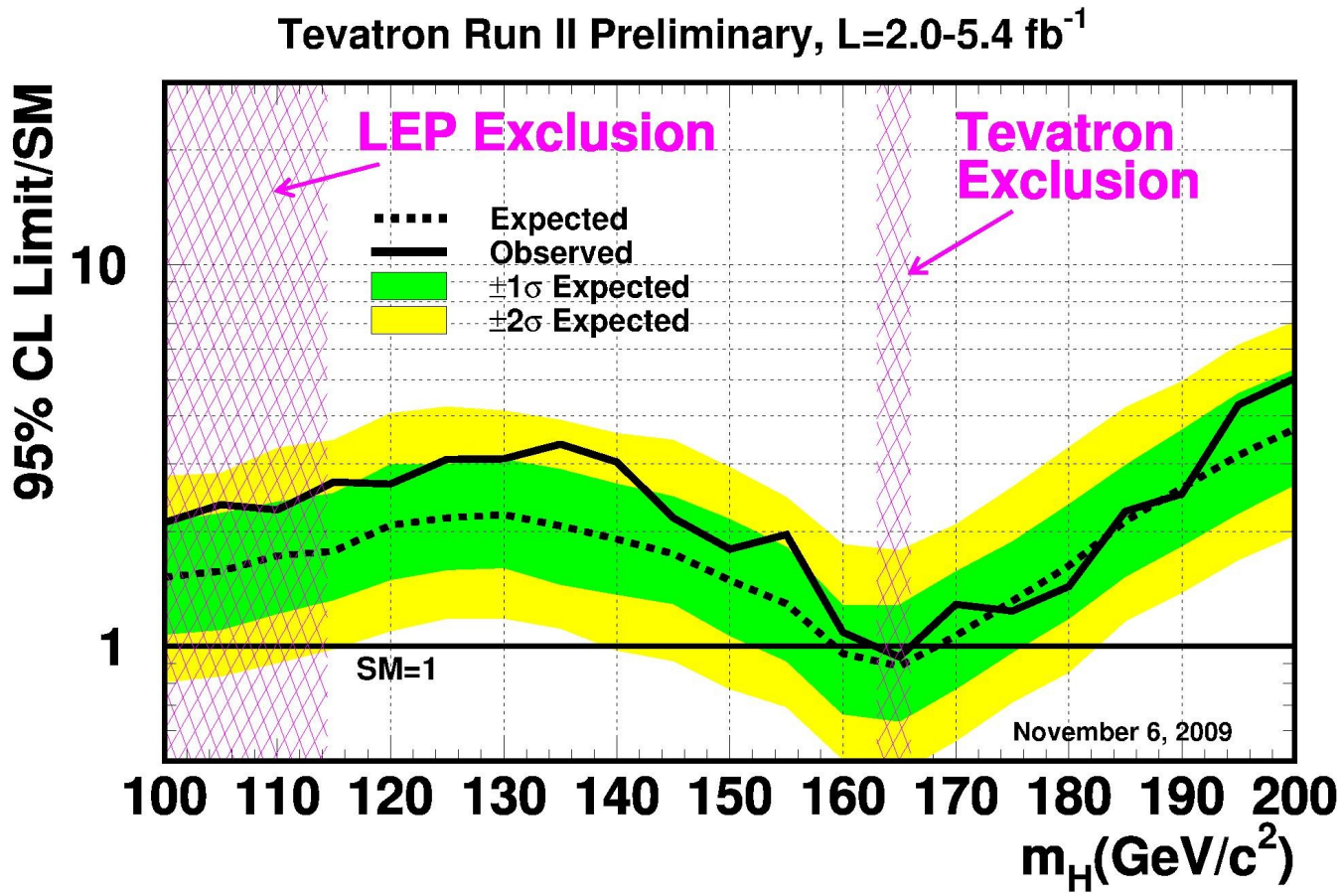




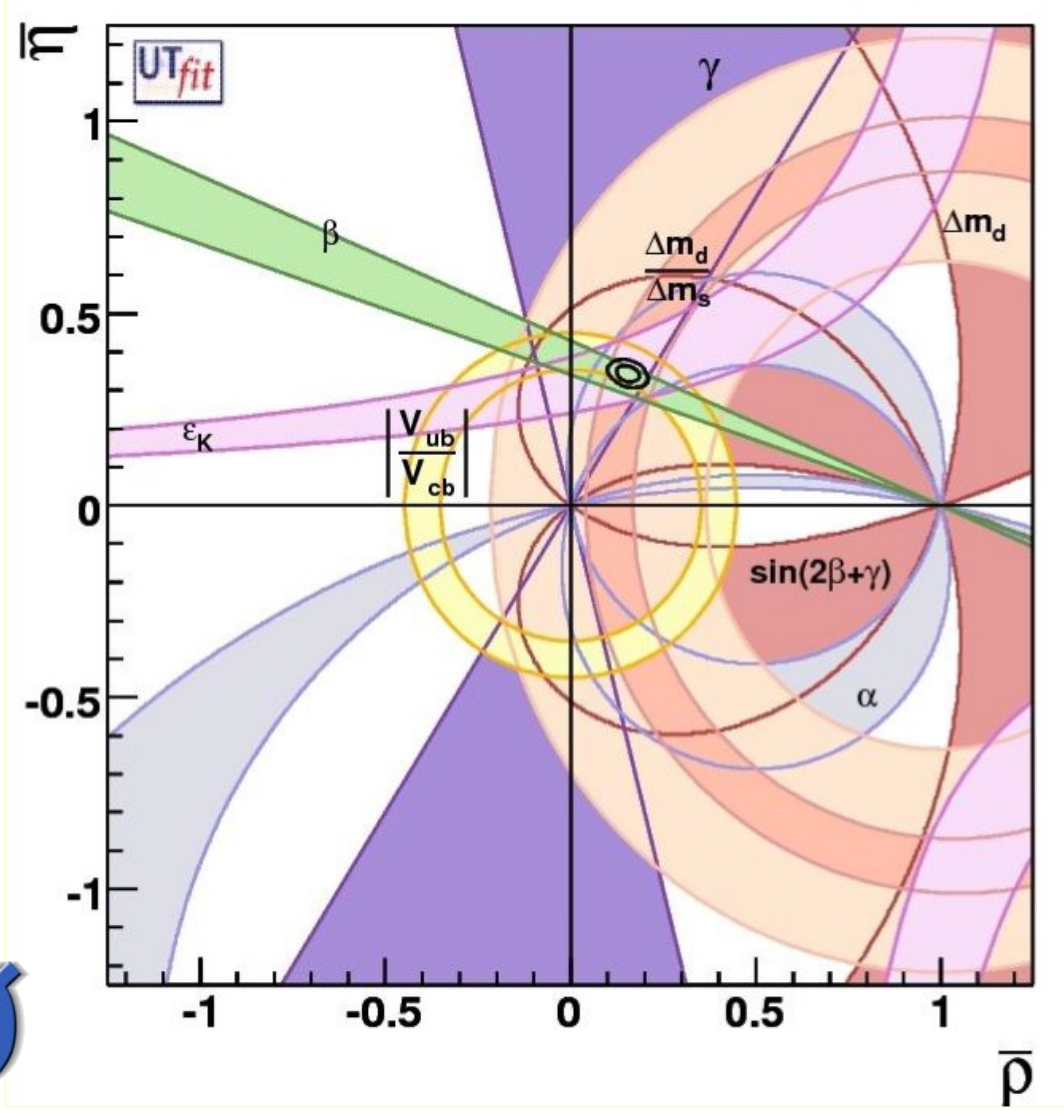
Search for the Higgs boson





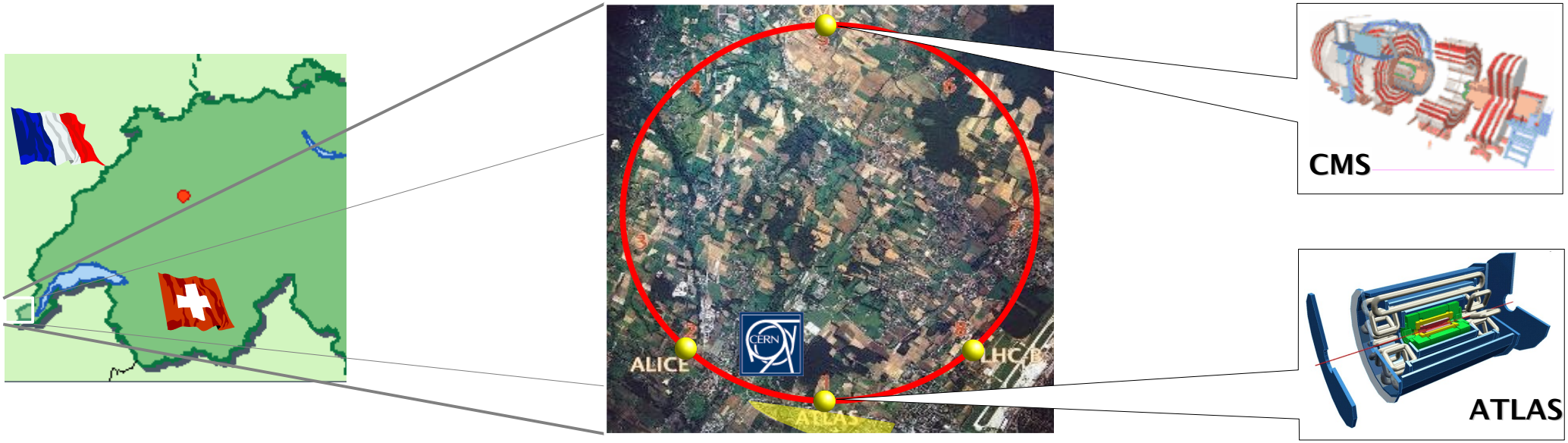


B factories Belle and Babar



e^+e^- colliders PEP (SLAC) and KEK

The Large Hadron Collider (LHC)



The Large Hadron Collider:

- proton–proton collider ⇨ 2 separate beampipes
- 10 fb^{-1} per year
- high energy: $\sqrt{s}$ = 14 TeV
- 40 Mio. collisions per second
- first collisions in 2009
- 4 experiments: ATLAS, CMS, ALICE, LHC–B

2. Quantum Electrodynamics, QED

2. Quantum Electrodynamics, QED

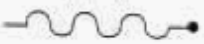
2.3: The Feynman rules of QED

APPENDIX B

The Feynman Rules of QED

(From D. Nachtmann, Elementary Particle Physics, Springer, 1990)

We shall make use of the following correspondence between physical situations, the elements of diagrams, and analytical expressions. Spin indices are not written explicitly.

electron in initial state	incoming electron line
$u(p)$	$e^-(p)$
electron in final state	outgoing electron line
$\bar{u}(p)$	$e^-(p)$
positron in initial state	outgoing electron line
$\bar{v}(q)$	$e^+(q)$
positron in final state	incoming electron line
$v(q)$	$e^+(q)$
photon in initial state	incoming photon line
ε^μ	$\gamma(k)$ 

photon in final state	outgoing photon line
$\varepsilon^{\mu *}$	$\gamma(k)$
virtual photon	internal photon line
$\dfrac{-\mathrm{i}g_{\mu\nu}}{k^2+\mathrm{i}\varepsilon}$	k, μ, ν
virtual electron	internal electron line
$\mathrm{i}\dfrac{\not{p}+m}{p^2-m^2+\mathrm{i}\varepsilon}$	p
elementary process	vertex
$\mathrm{i}e\gamma^\mu$	

The transition amplitude for a particular reaction is ~~given by~~ obtained from the sum of all the diagrams with the appropriate incoming and outgoing lines. Within the diagrams, any number of vertices are allowed. At each vertex, four-momentum must be conserved. This also guarantees overall four-momentum conservation, i.e., the sum of the incoming four-momenta is equal to the sum of the outgoing four-momenta. The four-momenta flow through the diagrams like electrical currents through a network of electrical conductors. Internal currents correspond to loop momenta. Diagrams with no loops are called *tree diagrams*. Examples are the diagrams of Fig. 10.2.

In Fig. B.1 we show an example of a diagram with one loop. The reaction is $\gamma(k_1)+\gamma(k_2)\to\gamma(k_3)+\gamma(k_4)$ and the loop momentum is l.

It is implicitly understood that loop momenta are to be integrated over with a measure given for a single loop by

$$\int \frac{\mathrm{d}l}{(2\pi)^4}. \tag{B.1}$$

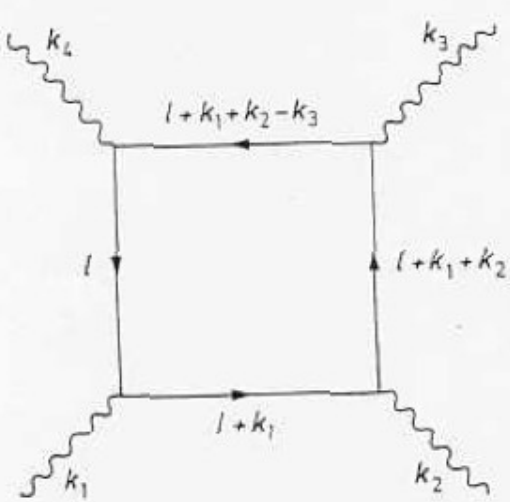


Figure B.1 A diagram for photon–photon scattering with one loop.

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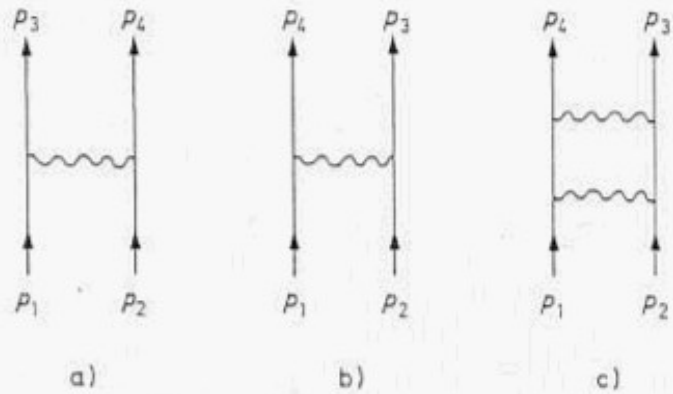


Figure B.2 Examples of diagrams for electron–electron scattering. The diagrams (b) and (c) receive a minus sign relative to (a).

The S-matrix element is obtained from ~~the transition amplitude~~ these rules by an additional multiplication with the δ-function from the energy-momentum conservation:

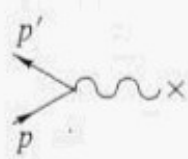
$$(2\pi)^4\delta\left(\sum_f p_f - \sum_i p_i\right), \tag{B.2}$$

where $p_i(p_f)$ are the momenta of the incoming (outgoing) particles.

A few other rules should be noted. For every closed fermion loop a factor of (-1) appears. Diagrams differing from one another only through a permutation of the labels of the external fermion lines receive relative to each other as factors the sign of the corresponding permutation. (An overall phase factor for the complete amplitude remains, of course, arbitrary.) Diagrams which are identical as far as the fermion lines and their labeling are concerned get no extra sign factor relative to each other. Some examples are shown in Fig. B.2.

To the interaction with an external electromagnetic field there corresponds the following analytical expression and vertex:

$$\mathrm{i}e\gamma^\mu \int \mathrm{d}x\, \mathrm{e}^{\mathrm{i}(p'-p)x} A_\mu^{\mathrm{ext}}(x)$$
$$= -\mathrm{i}e\gamma^\mu \frac{1}{(p'-p)^2} \int \mathrm{d}x\, \mathrm{e}^{\mathrm{i}(p'-p)x} j_\mu^{\mathrm{ext}}(x).$$



If such a vertex appears in a diagram, then the δ-function of the energy-momentum conservation (B.2) is to be omitted in the S-matrix element.

For calculating scattering cross-sections for unpolarized particles in the initial (final) state one must average (sum) over the corresponding spin orientations. The spin sums for Dirac spinors are given in (A.22). The spin sums for photons require some further explanation.

Let us consider a reaction in which an unpolarized photon with four-momentum k is incident. The Feynman rules yield as a result the amplitude for a photon with polarization vector ε where the dependence on ε is of the general form

$$\langle \ldots | \mathsf{T} | \ldots \gamma(k, \varepsilon)\rangle = \mathcal{M}_\mu(k)\varepsilon^\mu. \tag{B.3}$$

As discussed in Chap. 7, we can replace the photon state by an equivalent

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state (7.31). The T-matrix element remains unchanged, as do all physical quantities. If c is an arbitrary complex number and

$$\varepsilon' = \varepsilon + ck, \tag{B.4}$$

then it is easy to see that ε and ε' correspond to equivalent photon states:

$$\varepsilon^\mu \mathbf{a}_\mu^\dagger(k)|0\rangle \sim \varepsilon'^\mu \mathbf{a}_\mu^\dagger(k)|0\rangle. \tag{B.5}$$

Thus it must hold that

$$\mathcal{M}_\mu(k)\varepsilon^\mu = \mathcal{M}_\mu(k)\varepsilon'^\mu, \tag{B.6}$$

from which it follows that

$$\mathcal{M}_\mu(k)k^\mu = 0. \tag{B.7}$$

In calculating the cross-section σ for an unpolarized photon we must average over the physical polarization directions of the photon in the initial state:

$$\sigma \propto \sum_{\text{phys.}\,\varepsilon}{}' \, |\mathcal{M}_\mu(k)\varepsilon^\mu|^2. \tag{B.8}$$

To evaluate this sum we make use of the coordinate system defined in (7.24). As physical polarization vectors we can choose $\varepsilon_1 = (0, \boldsymbol{e}_1)$ and $\varepsilon_2 = (0, \boldsymbol{e}_2)$. Equation (B.7) yields

$$\begin{aligned} \mathcal{M}_\mu(k)k^\mu &= k^0(\mathcal{M}_0(k) - \vec{\mathcal{M}}(k)\boldsymbol{e}_3) = 0, \\ \mathcal{M}_0(k) &= \vec{\mathcal{M}}(k)\boldsymbol{e}_3. \end{aligned} \tag{B.9}$$

For the cross-section in (B.8) we then obtain

$$\begin{aligned} \sigma \propto \sum_{\text{phys.}\,\varepsilon}{}' \, |\mathcal{M}_\mu\varepsilon^\mu|^2 &= \tfrac{1}{2}\sum_{i=1}^{2} |\mathcal{M}_\mu\varepsilon_i^\mu|^2 = \tfrac{1}{2}\{|\vec{\mathcal{M}}\boldsymbol{e}_1|^2 + |\vec{\mathcal{M}}\boldsymbol{e}_2|^2\} \\ &= \tfrac{1}{2}\{|\vec{\mathcal{M}}\boldsymbol{e}_1|^2 + |\vec{\mathcal{M}}\boldsymbol{e}_2|^2 + |\vec{\mathcal{M}}\boldsymbol{e}_3|^2 - |\mathcal{M}_0|^2\} \\ &= \tfrac{1}{2}\mathcal{M}_\mu(-g^{\mu\nu})\mathcal{M}_\nu^*. \end{aligned} \tag{B.10}$$

An analogous argument holds for an unpolarized photon in the final state. Thus for calculating cross-sections in QED we can make the replacement

$$\sum_{\text{phys.}\,\varepsilon} \varepsilon^\mu\varepsilon^{\nu *} \to -g^{\mu\nu}. \tag{B.11}$$

A useful formula regarding a type of phase-space integral that frequently occurs in the calculation of cross-sections is

$$\begin{aligned} &\int \mathrm{d}q_1 \int \mathrm{d}q_2\, \delta_+(q_1^2 - m_1^2)\delta_+(q_2^2 - m_2^2)\delta(p - q_1 - q_2) \\ &\quad \cdot \{a + b^\mu(q_1 - q_2)_\mu + c^{\mu\nu}(q_1 - q_2)_\mu(q_1 - q_2)_\nu\} \\ &= \theta(p^0)\theta(p^2 - (m_1 + m_2)^2)\frac{\pi}{2p^2} w(p^2, m_1^2, m_2^2)\left\{a + b^\mu p_\mu \frac{m_1^2 - m_2^2}{p^2}\right. \\ &\quad \left. + c^{\mu\nu}\left[(4p_\mu p_\nu - g_{\mu\nu}p^2)\frac{1}{3(p^2)^2} w^2(p^2, m_1^2, m_2^2) - p_\mu p_\nu \frac{1}{p^2}(p^2 - 2m_1^2 - 2m_2^2)\right]\right\}. \end{aligned} \tag{B.12}$$

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Here p is an external momentum, $m_1 \geq 0$ and $m_2 \geq 0$ are masses, and a, b^μ, $c^{\mu\nu}$ are arbitrary constants. The function w is defined in (2.72) and

$$\delta_+(q^2 - m^2) \equiv \theta(q^0)\delta(q^2 - m^2) = \frac{1}{2q^0}\delta(q_0 - \sqrt{q^2 + m^2}). \qquad \text{(B.13)}$$

In order to prove (B.12) one must first realize that the integral is only nonzero for $p^2 \geq 0$ and $p^0 \geq 0$ and that the individual parts are invariant or covariant under Lorentz transformations. For $p^2 > 0$ the integral is easily calculated in a frame where the spatial part of p is zero. For $p^2 = 0$ the integral gives zero except when $m_1 = m_2 = 0$. In the latter case, we *define* it as the limiting value for $p^2 \to +0$.

APPENDIX A

Dirac Matrices and Spinors

From O.Nachtmann, Elementary Particle Physics, Springer, 1990

Defining relation:

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}. \tag{A.1}$$

Definitions:

$$\not{a} = a^\mu\gamma_\mu, \tag{A.2}$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \tag{A.3}$$

$$\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu). \tag{A.4}$$

Relations:

$$\gamma^\mu\gamma_\mu = 4, \tag{A.5}$$

$$\gamma^\mu\not{a}\gamma_\mu = -2\not{a}, \tag{A.6}$$

$$\gamma^\mu\not{a}\not{b}\gamma_\mu = 4(a, b), \tag{A.7}$$

$$\gamma^\mu\not{a}\not{b}\not{c}\gamma_\mu = -2\not{c}\not{b}\not{a}, \tag{A.8}$$

$$\gamma^\mu\gamma_5 + \gamma_5\gamma^\mu = 0, \tag{A.9}$$

$$\gamma_5\gamma_5 = 1. \tag{A.10}$$

From (A.1) it immediately follows for the trace that

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}. \tag{A.11}$$

The trace of an odd number of γ-matrices vanishes:

$$\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n+1}) = 0. \tag{A.12}$$

To prove this we use the matrix γ_5, for which from (A.9), (A.10) we have

$$\begin{aligned}\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n+1}) &= \mathrm{Tr}(\gamma_5\not{a}_1\gamma_5\gamma_5\not{a}_2\gamma_5\ldots\gamma_5\not{a}_{2n+1}\gamma_5)\\ &= (-1)^{2n+1}\,\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n+1}).\end{aligned} \tag{A.13}$$

A further relationship is

$$\mathrm{Tr}(\not{a}_1\ldots\not{a}_n) = \mathrm{Tr}(\not{a}_n\ldots\not{a}_1) \tag{A.14}$$

for which the proof is similar to that of (A.13), but using the matrix $\mathbf{S}(\mathbf{C})$ (4.124) instead of γ_5.

Relations concerning γ_5:

$$\gamma_\mu\gamma_5 = \frac{\mathrm{i}}{6}\varepsilon_{\mu\nu\rho\sigma}\gamma^\nu\gamma^\rho\gamma^\sigma, \tag{A.15}$$

$$\mathrm{Tr}\,\gamma_5 = \mathrm{Tr}(\gamma^\mu\gamma_5) = \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) = \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma_5) = 0, \tag{A.16}$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = 4\mathrm{i}\varepsilon^{\mu\nu\rho\sigma}. \tag{A.17}$$

The trace of the product of an even number $(2n)$ of γ-matrices can be expressed in terms of traces over $2(n-1)$ γ-matrices:

$$\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n}) = \sum_{k=2}^{2n}(-1)^k(a_1,a_k)\,\mathrm{Tr}(\not{a}_2\ldots\not{a}_{k-1}\not{a}_{k+1}\ldots\not{a}_{2n}). \tag{A.18}$$

To prove this $\not{a}_1$ is moved to the right by repeated anticommutation:

$$\begin{aligned}\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n}) &= \mathrm{Tr}((\{\not{a}_1,\not{a}_2\}-\not{a}_2\not{a}_1)\not{a}_3\ldots\not{a}_{2n})\\ &= 2(a_1,a_2)\,\mathrm{Tr}(\not{a}_3\ldots\not{a}_{2n}) - \mathrm{Tr}(\not{a}_2\not{a}_1\not{a}_3\ldots\not{a}_{2n})\\ &= 2\sum_{k=2}^{2n}(-1)^k(a_1,a_k)\,\mathrm{Tr}(\not{a}_2\ldots\not{a}_{k-1}\not{a}_{k+1}\ldots\not{a}_{2n})\\ &\quad+(-1)^{2n-1}\,\mathrm{Tr}(\not{a}_2\not{a}_3\ldots\not{a}_{2n}\not{a}_1).\end{aligned} \tag{A.19}$$

Since the matrices in a trace can be cyclically permuted, (A.18) follows.

For the trace of four γ-matrices we obtain from (A.18)

$$\mathrm{Tr}(\gamma^\lambda\gamma^\mu\gamma^\nu\gamma^\rho) = 4\{g^{\lambda\mu}g^{\nu\rho} + g^{\lambda\rho}g^{\mu\nu} - g^{\lambda\nu}g^{\mu\rho}\}. \tag{A.20}$$

The following are some important relations satisfied by the solutions of the Dirac equation (4.38), (4.41), where $s, r = \pm\frac{1}{2}$ are the spin indices:

$$\begin{aligned}(\not{p}-m)u_s(p) &= 0,\\ (\not{p}+m)v_s(p) &= 0,\end{aligned} \tag{A.21}$$

$$\begin{aligned}\sum_{s=\pm 1/2} u_s(p)\bar{u}_s(p) &= \not{p}+m,\\ \sum_{s=\pm 1/2} v_s(p)\bar{v}_s(p) &= \not{p}-m,\end{aligned} \tag{A.22}$$

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$$\begin{aligned} \bar{u}_r(p)u_s(p) &= 2m\delta_{rs}, \\ \bar{v}_r(p)v_s(p) &= -2m\delta_{rs}, \end{aligned} \tag{A.23}$$

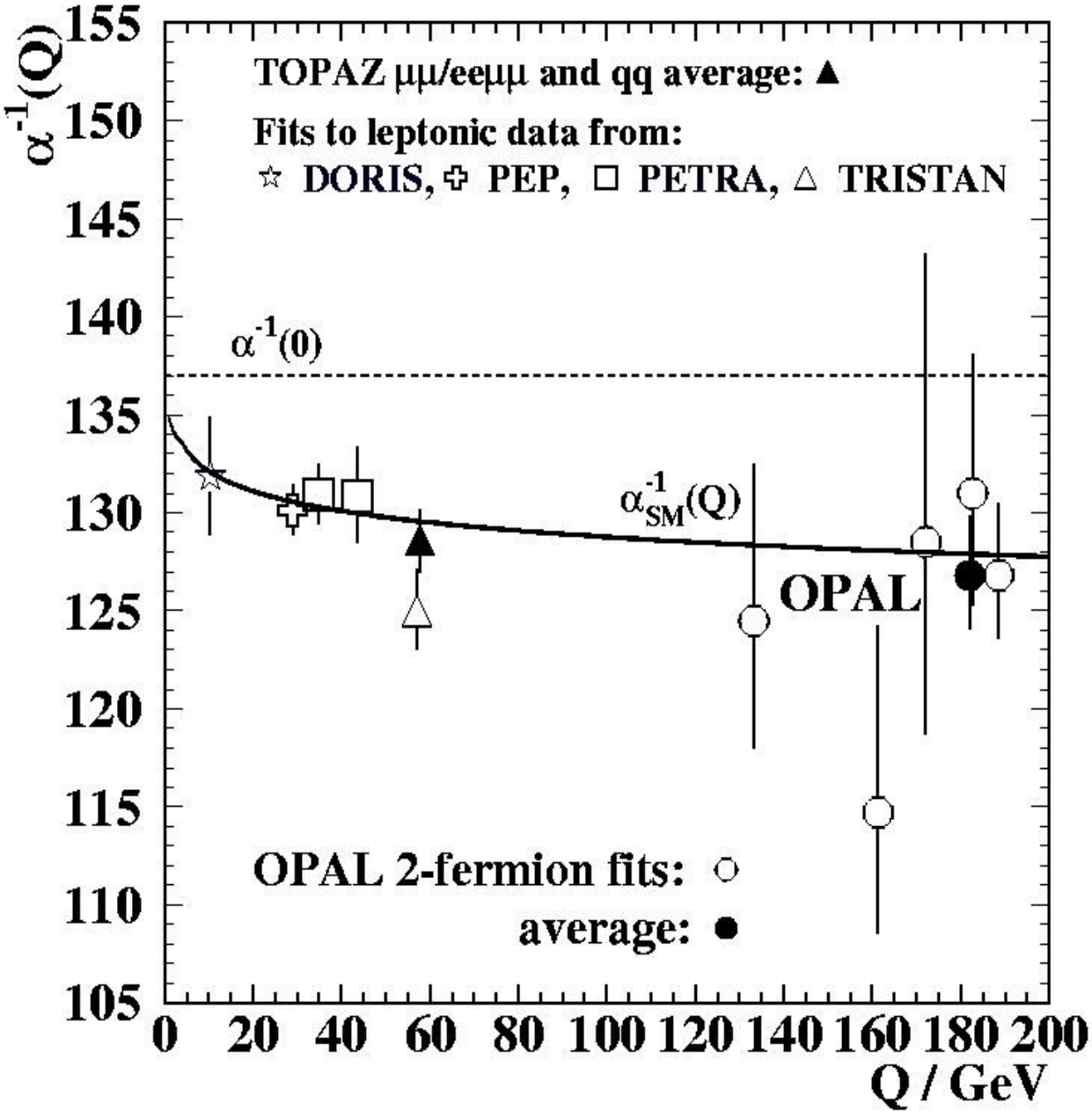
$$\begin{aligned} \bar{u}_r(p)\gamma^\mu u_s(p) &= 2p^\mu\delta_{rs}, \\ \bar{v}_r(p)\gamma^\mu v_s(p) &= 2p^\mu\delta_{rs}, \end{aligned} \tag{A.24}$$

$$\begin{aligned} \bar{u}_r(p')\{2m\gamma^\mu - (p'+p)^\mu - \mathrm{i}\sigma^{\mu\nu}(p'-p)_\nu\}u_s(p) &= 0, \\ \bar{v}_r(p')\{2m\gamma^\mu + (p'+p)^\mu + \mathrm{i}\sigma^{\mu\nu}(p'-p)_\nu\}v_s(p) &= 0. \end{aligned} \tag{A.25}$$

3. Quantum Electrodynamics, QED

3.4: Renormalisation in QED

α^{-1}



α^{-1}_{QED} as function of $\sqrt{s}$ in e^+e^- annihilation

Summary QED Lagrangian

local phase invariance : $\psi(x) \to e^{i\alpha(x)}\psi(x)$ (i)

minimal substitution : $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu(x)$ (ii)

gauge invariance : $A_\mu(x) \longrightarrow A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x)$ (iii)

field strength tensor : $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$

U(1) gauge symmetry

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \bar{\psi}(x)(i\gamma_\mu\partial^\mu - m)\psi(x) + eA_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x)$$

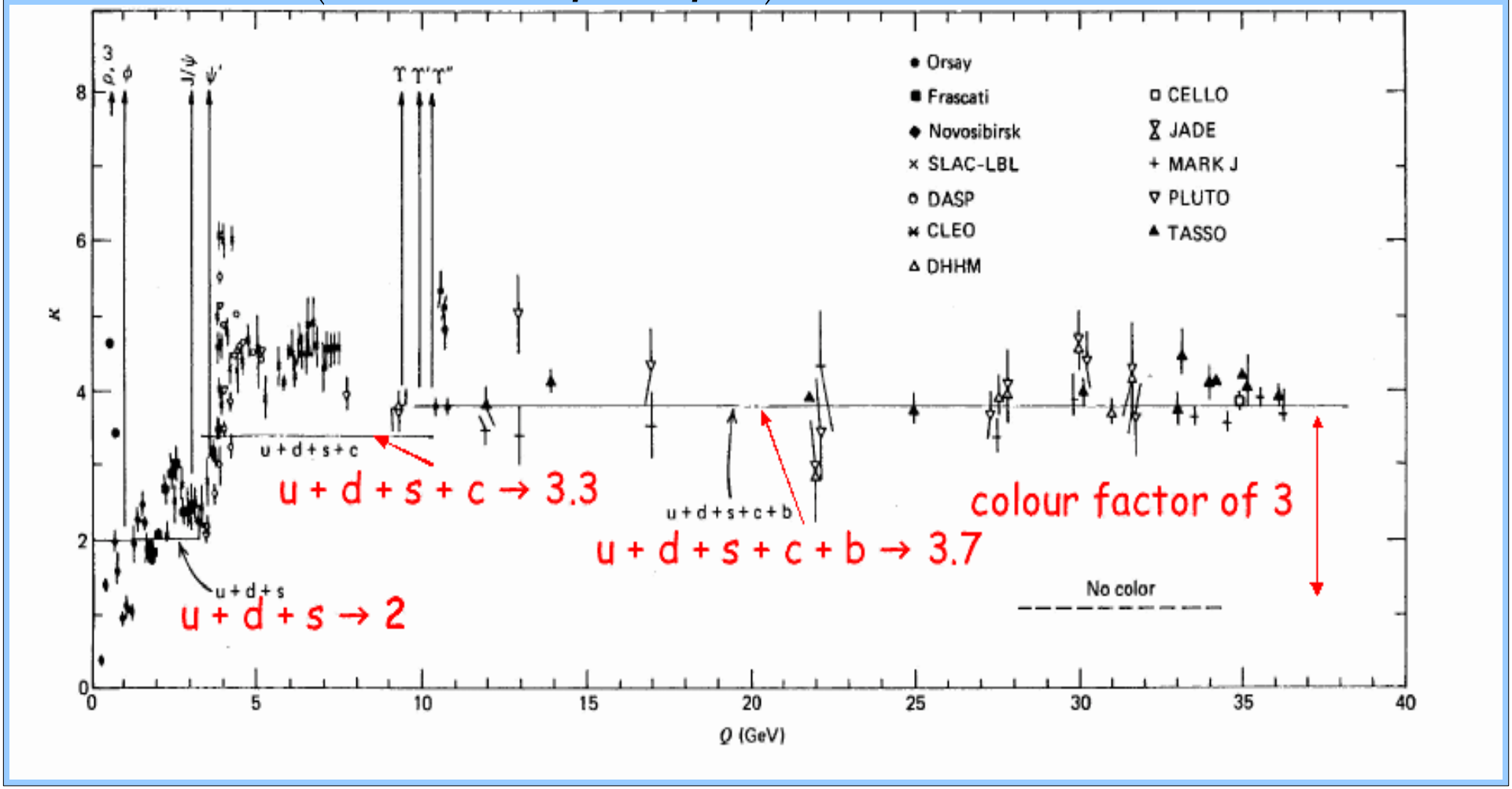
free photon Dirac field interaction

4. Quantum Chromodynamics, QCD

4.1 The Lagrange Density of QCD

Quark Flavours and Colour

$$R = \frac{\sigma(e^+ + e^- \to hadrons)}{\sigma(e^+ + e^- \to \mu^+ + \mu^-)} = \frac{3\sum(quark\ charge)^2}{1^2}$$



The λ_a $(a = 1, \ldots, 8)$ are the Gell-Mann matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_2 = \begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 \end{pmatrix}, \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & \mathrm{i} & 0 \end{pmatrix}, \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{C.44}$$

Relations:

$$\mathrm{Tr}\, \lambda_a = 0, \tag{C.45}$$

$$\mathrm{Tr}(\lambda_a \lambda_b) = 2\delta_{ab}, \tag{C.46}$$

$$[\lambda_a, \lambda_b] = 2\mathrm{i} f_{abc} \lambda_c, \tag{C.47}$$

$$\{\lambda_a, \lambda_b\} = \tfrac{4}{3}\delta_{ab} + 2d_{abc}\lambda_c, \tag{C.48}$$

where f_{abc} is totally antisymmetric, and d_{abc} is totally symmetric with values for the independent components as given in Table C.1.

(From O. Nachtmann, Elementary Particle Physics)

Table 4.1 The independent, nonvanishing components of f_{abc} and d_{abc}.

(a, b, c)	f_{abc}	(a, b, c)	d_{abc}
123	1	118	$1/\sqrt{3}$
147	1/2	146	1/2
156	−1/2	157	1/2
246	1/2	228	$1/\sqrt{3}$
257	1/2	247	−1/2
345	1/2	256	1/2
367	−1/2	338	$1/\sqrt{3}$
458	$\sqrt{3}/2$	344	1/2
678	$\sqrt{3}/2$	355	1/2
		366	−1/2
		377	−1/2
		448	$-1/(2\sqrt{3})$
		558	$-1/(2\sqrt{3})$
		668	$-1/(2\sqrt{3})$
		778	$-1/(2\sqrt{3})$
		888	$-1/\sqrt{3}$

Summary QCD Lagrangian

invariance under lokal gauge transformation in color space : $q^j(x) \to U(x)\, q^j(x) \qquad j = 1, \dots f$ (i)

minimal substitution : $\partial_\lambda \to D_\lambda = \partial_\lambda + i g_s G_\lambda(x)$ (ii)

gauge invariance : $G_\lambda(x) \to G'_\lambda(x) = U(x)\, G_\lambda(x)\, U^+(x) - \frac{i}{g_s} U(x) \partial_\lambda U^+(x)$ (iii)

field strength tensor : $G_{\lambda\rho}(x) = \partial_\lambda G_\rho(x) - \partial_\rho G_\lambda(x) + i g_s [G_\lambda(x), G_\rho(x)]$

SU(3) gauge symmetry

$$\mathcal{L}_{QCD}(x) = \underbrace{-\frac{1}{2} \mathrm{Tr}(G_{\lambda\rho}(x)\, G^{\lambda\rho}(x))}_{\text{free gluon}} + \underbrace{\sum_{j=1}^{f} \bar{q}^j(x) [i\gamma^\lambda \partial_\lambda - m_j]\, q^j(x)}_{\text{Dirac field}}$$

$$\underbrace{- g_s \sum_{j=1}^{f} \bar{q}^j(x)\, \gamma^\lambda\, G_\lambda(x)\, q^j(x)}_{\text{interaction}}$$

Comparison between QED and QCD

	QED	QCD
quantum number	electric charge	colour
fermions	electrons	quarks (colour triplets)
vector bosons	photon (uncharged)	gluon (colour octett)
gauge group	U(1) abelian	SU(3) (non-abelian)
coupling constant	e $\alpha = \frac{e^2}{4\pi}$	g_s $\alpha_s^2 = \frac{g_s^2}{4\pi}$
bound states	positronium	hadrons
free particles (asymptotic states)	electrons, positrons, photons	only hadrons (quarks, gluons are confined)

4. Quantum Chromodynamics, QCD

4.3 The Feynman Rules of QCD

(O. Nachtmann, Elementary Particle Physics)

APPENDIX D

The Feynman Rules of QCD

In this Appendix we present the Feynman rules of QCD. The derivation of these rules from the Lagrange density (19.22) is best achieved using the path-integral formalism. Here we restrict ourselves to giving only the results.

The Feynman diagrams of QCD are constructed from three types of lines: quark lines, gluon lines, and so-called Faddev–Popov ghost lines. Within QCD it is not possible that lines corresponding to different quark flavors are joined together. The ghosts are to be treated like a complex scalar particle with zero mass. Thus ghost and antighost are distinct. The corresponding lines, like the quark lines, are given an arrow. The ghosts transform according to the adjoint representation of the color group SU(3) and thus carry a color index a ($a = 1, \ldots, 8$).

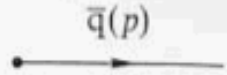
We shall make use of the following scheme to relate physical situations, parts of diagrams, and analytical expressions:

quark in initial state	incoming quark line
$u(p)$	q(p)
quark in final state	outgoing quark line
$\bar{u}(p)$	q(p)

antiquark in initial state

$\bar{v}(p)$

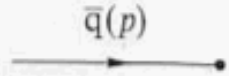
outgoing quark line



antiquark in final state

$v(p)$

incoming quark line



ghost in initial or antighost in final state

1

incoming ghost line

ghost in final or antighost in initial state

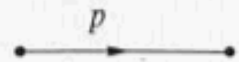
1

outgoing ghost line

virtual quark with flavour j

$$\frac{\mathrm{i}}{\not{p} - m_j + \mathrm{i}\varepsilon}$$

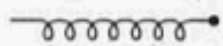
internal quark line



gluon in initial state

ε^{μ}

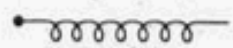
incoming gluon line



gluon in final state

$\varepsilon^{*\mu}$

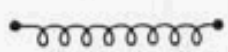
outgoing gluon line



virtual gluon

$$\mathrm{i}\delta^{ab}\left\{\frac{-g_{\mu\nu}}{k^2 + \mathrm{i}\varepsilon} + \frac{(1-\xi)k_\mu k_\nu}{(k^2 + \mathrm{i}\varepsilon)^2}\right\}$$

internal gluon line



virtual ghost

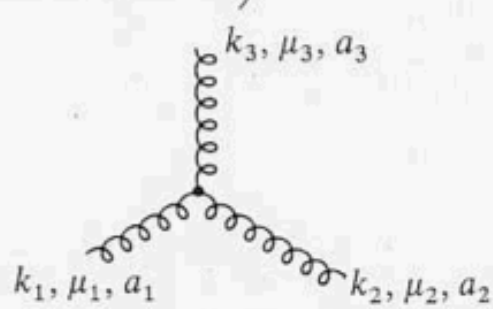
$$\mathrm{i}\frac{\delta^{ab}}{p^2 + \mathrm{i}\varepsilon}$$

internal ghost line

The elementary processes of QCD correspond to the following vertices:

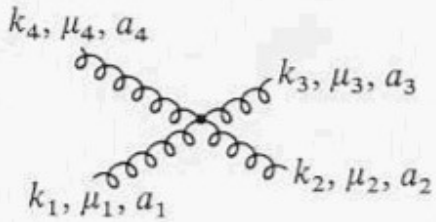
3-gluon vertex $\left(\sum_{j=1}^{3} k_j = 0, \text{ all momenta incoming}\right)$

$$-g_s f_{a_1 a_2 a_3} \cdot \{(k_1 - k_2)_{\mu_3} g_{\mu_1\mu_2} + (k_2 - k_3)_{\mu_1} g_{\mu_2\mu_3} + (k_3 - k_1)_{\mu_2} g_{\mu_3\mu_1}\}$$



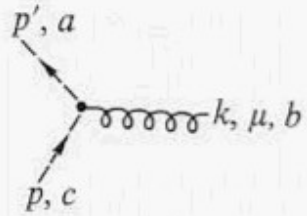
4-gluon vertex $\left(\sum_{j=1}^{4} k_j = 0, \text{ all momenta incoming}\right)$

$$ig_s^2 f_{a_1 a_2 b} f_{a_3 a_4 b} \cdot (g_{\mu_2\mu_3} g_{\mu_1\mu_4} - g_{\mu_1\mu_3} g_{\mu_2\mu_4}) + \text{cyclic perms. of } 1, 2, 3$$



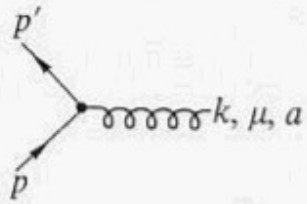
ghost–gluon vertex (p, k incoming; p' outgoing; $p + k = p'$)

$$g_s f_{abc} p'_\mu$$



quark–gluon vertex (p, k incoming; p' outgoing; $p + k = p'$)

$$-ig_s \gamma_\mu \tfrac{1}{2} \lambda_a$$



The parameters that appear in the above Feynman rules are the coupling constant g_s, the quark masses m_j ($j = 1, \ldots, f$), and the arbitrary gauge parameter ξ which must drop out in the expressions obtained for observable quantities. It is usually most convenient to set $\xi = 1$; this is the so-called Feynman gauge. We used this gauge throughout our discussion of QED where an analogous gauge parameter can be introduced.

The rules for loops and for the δ-function of energy-momentum conservation are the same as in QED (Appendix B). For every closed fermion loop and every ghost loop a factor of (-1) must be included. For closed gluon loops one must take the so-called statistical factor into account. This is obtained by counting up all possible contractions of field operators in the perturbation expansion, which, for QCD too, is as given in (9.14). The diagram in Fig. D.1(a), for example, has the statistical factor 1, and that of Fig. D.1(b) has the factor $\frac{1}{2}$. In order to see this we write the three-gluon coupling schematically as

$$\frac{1}{3!} :\mathbf{G}^3(x):,$$

where $\mathbf{G}(x)$ is the gluon field operator. The diagram of Fig. D.1(a) then corresponds to a certain Fourier transform of the vacuum expectation value

$$\langle 0| \frac{1}{3!} :\mathbf{G}^3(x): \mathbf{G}(x_1)\mathbf{G}(x_2)\mathbf{G}(x_3)|0\rangle \propto 1. \tag{D.1}$$

The evaluation is carried out using Wick's theorem from Sect. 8.2. $\mathbf{G}(x_1)$ must be contracted in turn with each of the three operators from $:\mathbf{G}^3(x):$. For $\mathbf{G}(x_2)$

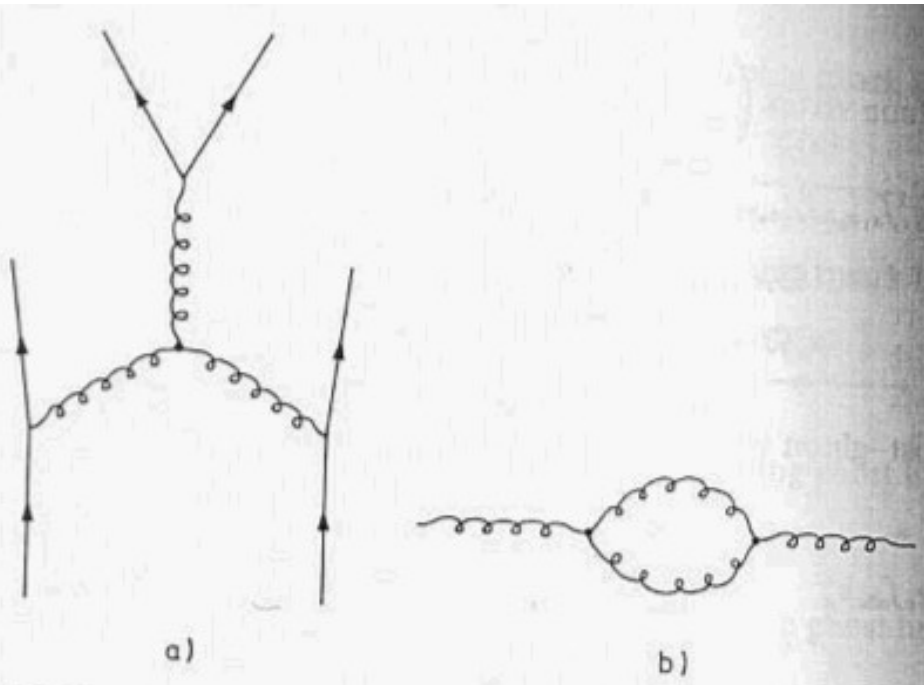


Figure D.1 A diagram with a three-gluon vertex in which no loop occurs, thus giving a statistical factor of 1(a). A self-energy diagram for the gluon propagator. In this a statistical factor of $\frac{1}{2}$ must be taken (b).

only two possibilities then remain, and for $\mathbf{G}(x_3)$ only one. The net result indicated in (D.1) is that the factor 1/3! is cancelled. If we now move on to the diagram of Fig. D.1(b) we find, with the factor 1/2! from the exponential series in (9.14), a corresponding vacuum expectation value of

$$\frac{1}{2!}\langle 0|\frac{1}{3!}:\mathbf{G}^3(x):\frac{1}{3!}:\mathbf{G}^3(y):\mathbf{G}(x_1)\mathbf{G}(x_2)|0\rangle \propto \tfrac{1}{2}. \tag{D.2}$$

This statistical weight of $\frac{1}{2}$ can also be interpreted as phase-space factor for the two gluons in the intermediate state since these are indistinguishable.

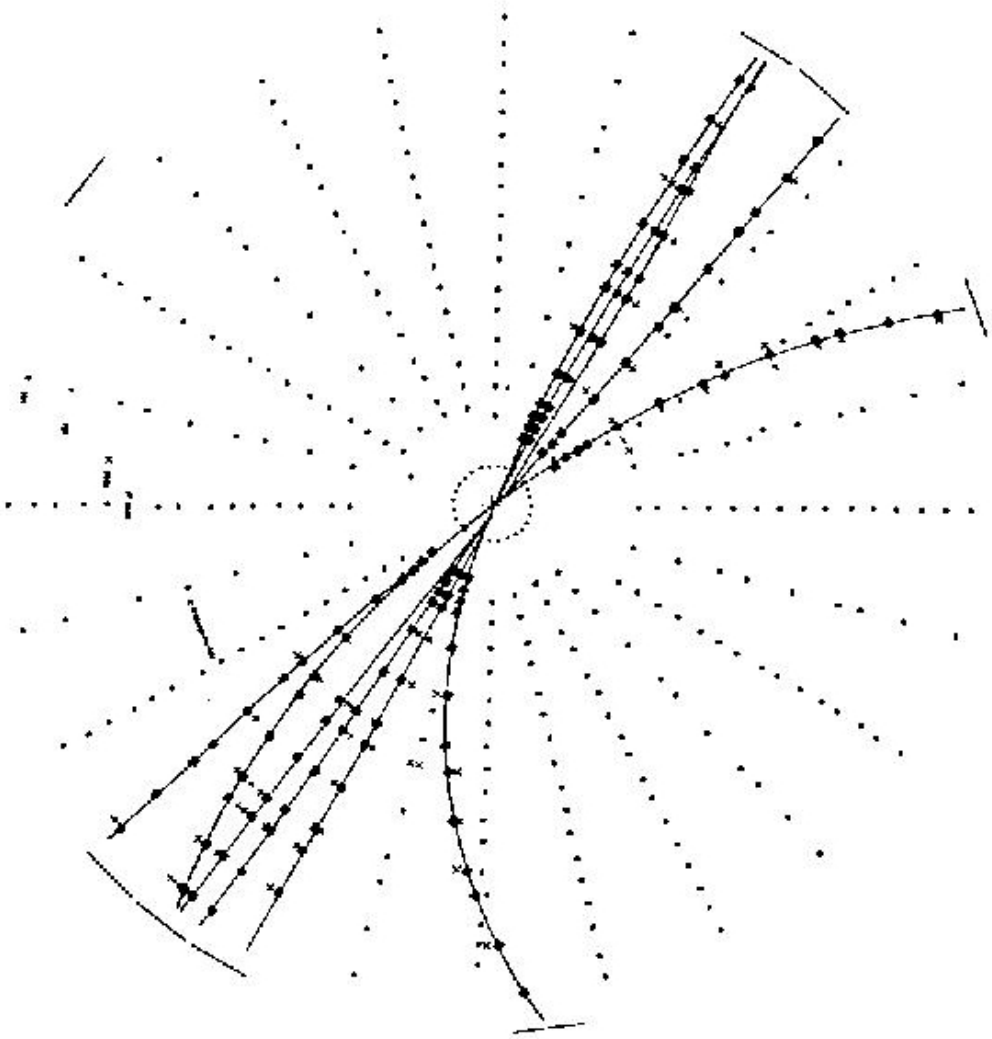
When calculating the polarization sums for gluons in the initial or final states there are likewise a few points that should be noted. It is always possible to do the calculation noncovariantly, i.e., to consider only the transverse polarization degrees of freedom of the gluons (cf. Appendix E). Alternatively, one can work with covariant quantities, i.e., replace the polarization sum of a gluon in the initial or final state by

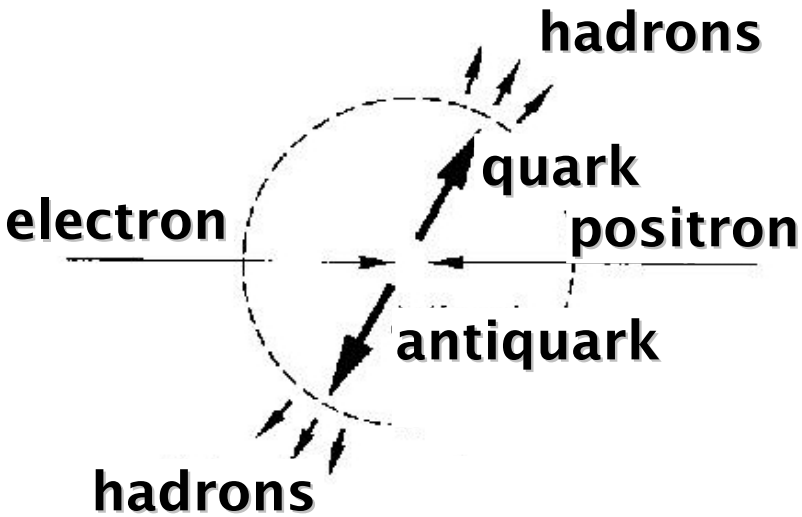
$$\sum_{\text{Spins}} \varepsilon_\mu \varepsilon_\nu^* = -g_{\mu\nu}. \tag{D.3}$$

In this case, however, it is also necessary to take into account processes of the same order with incoming and outgoing ghost lines, treating ghosts and antighosts formally as if they were extra degrees of freedom for the gluons to be considered in their spin sums. (Of course only two external gluon lines can be replaced at a time by ghost lines since an odd number of external ghost lines is impossible, as is an odd number of external fermion lines.) Amplitudes corresponding to diagrams which are identical except for permutations of the labels of external ghost lines receive extra sign factors relative to each other

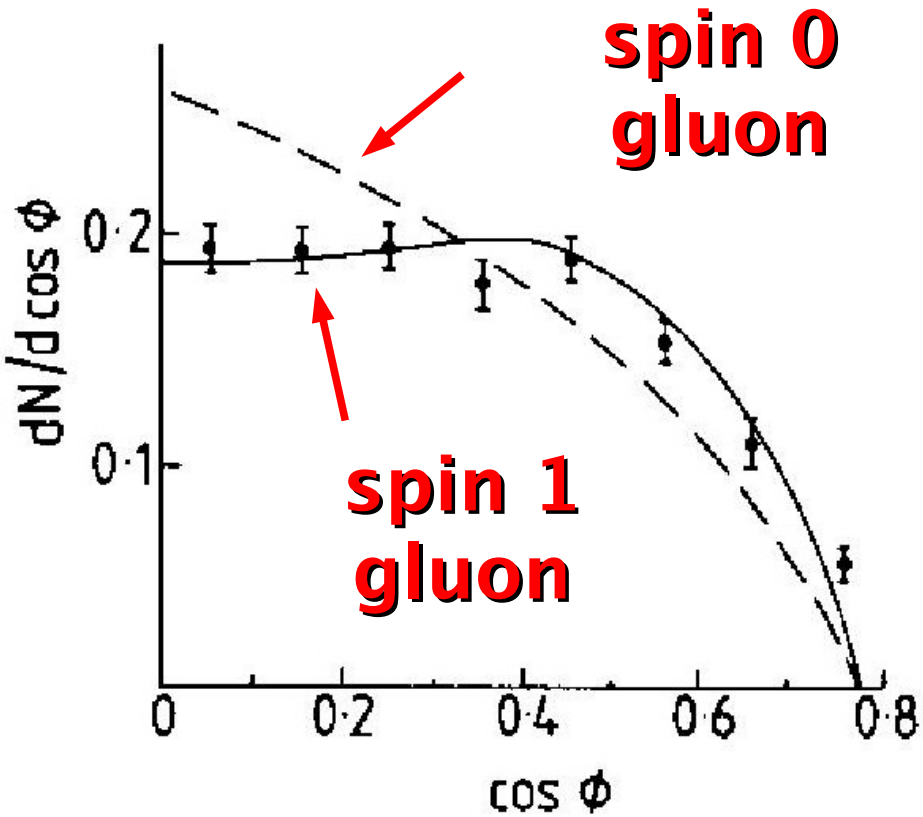
in the same way as for fermions (cf. Appendix B). In calculating transition probabilities one has furthermore to include a factor $(-1)^{n/2}$ where n is the number of external ghost lines in the corresponding diagram. Thus some ghost processes, for instance, the production of a single ghost-antighost pair ($n = 2$), get assigned a negative probability (cf. Cutler 1978).

These rules are sufficient for the calculation of all tree diagrams, i.e., diagrams without loops. For the regularization and renormalization procedures to be applied beyond this level the reader is referred to the available textbooks, e.g., Itzykson (1980).





2 jet event at e^+e^- accelerator PETRA



3 jet event and angular distribution at e^+e^- accelerator PETRA

Feynman rules for QCD. One finds

$$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dT}(e^+e^- \to q\bar{q}G) = \frac{2\alpha_s}{3\pi}\frac{1}{1-T}$$
$$\cdot\left\{\left(\frac{4}{T} - 6 + 6T\right)\ln\frac{2T-1}{1-T} - 3(2-T)(3T-2)\right\} \qquad (\tfrac{2}{3} \le T \le 1). \tag{20.17}$$

This distribution is compared in Fig. 20.13 with data from the TASSO collaboration. The experimental points have been corrected for fragmentation effects. We see that QCD gives a good description of the data. The fitted value of $\alpha_s \cong 0.17$ is compatible with other determinations of this quantity [(20.14), see also Exercise 19.2]. A calculation based on the radiation of "scalar gluons", however, does not yield agreement with experiment.

This and similar investigations can be considered as direct evidence for the existence of gluons with a spin value of 1 as required by theory. What we are still lacking is direct evidence that the gluons also carry color, i.e., that they have the self-coupling represented by the three- and four-gluon vertices of

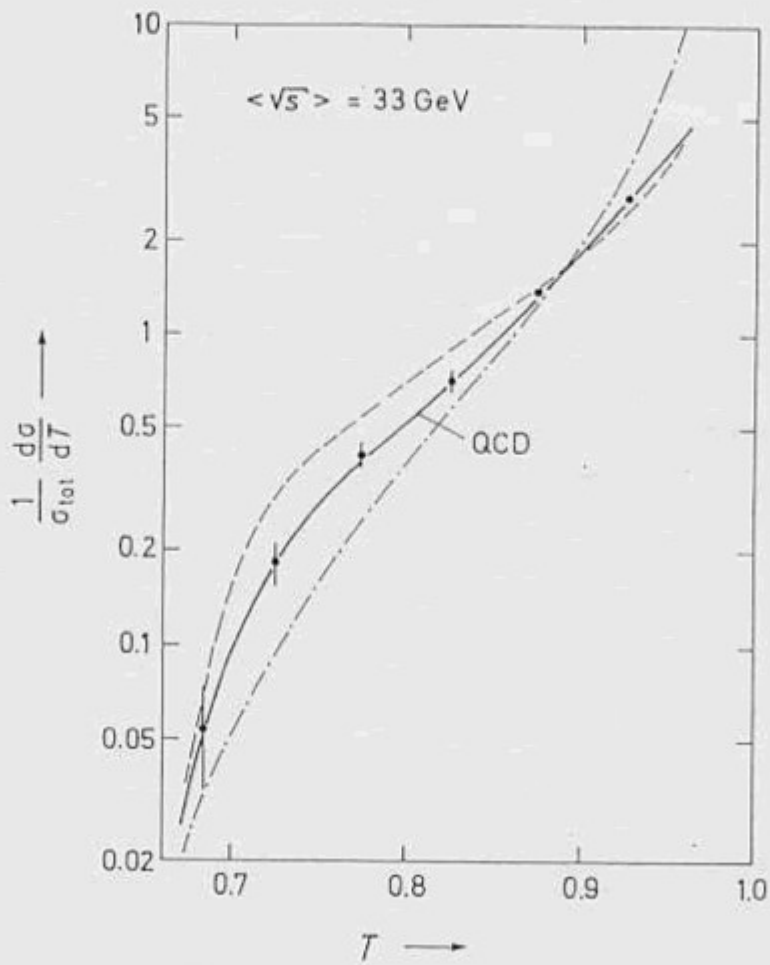


Figure 20.13 The thrust distribution of three-jet events observed in e^+e^- annihilation corrected for the effects of fragmentation. The data are from the TASSO group. The solid line is the first order prediction of QCD for $\alpha_s = 0.17$ (20.17). The dashed line is the result obtained for the emission of scalar gluons, and the dash–dotted line is the result for the so-called "constituent interchange" model (after Wolf 1982).

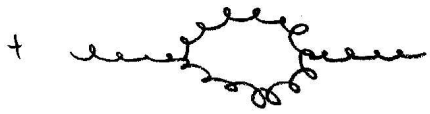
4. Quantum Chromodynamics, QCD

4.4 Asymptotic Freedom

examples:

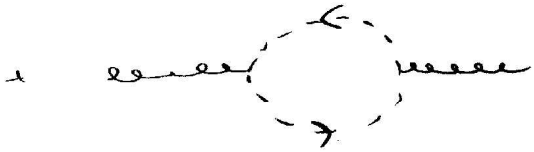
Order

1

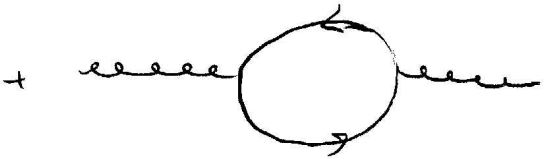


g_s^2

g_s^2



g_s^2



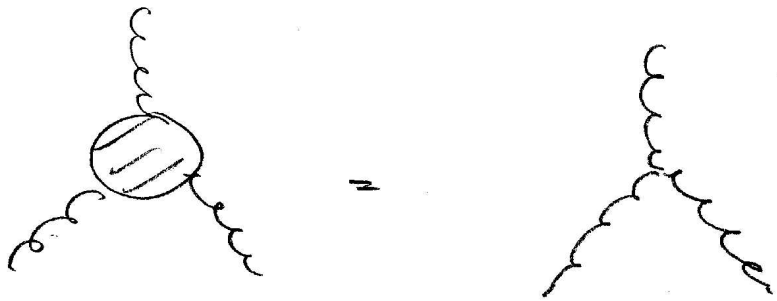
g_s^2

+ ....

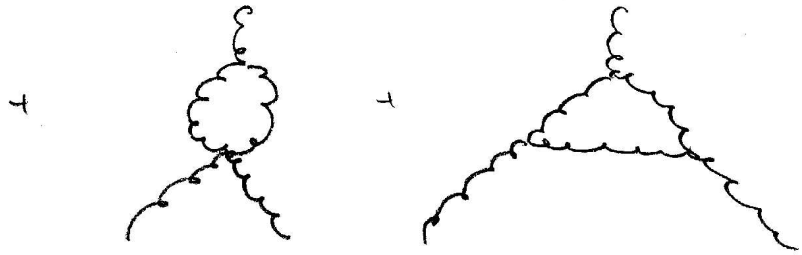
Result:

$$1 + \dots \frac{1}{4-d} g_s^2 + \dots$$

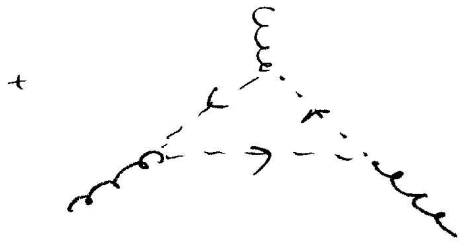
Order $-1+-$



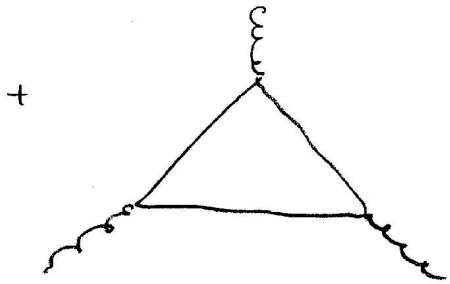
g_s



g_s^3



g_s^3



g_s^3

$+ \quad \ldots$

Result: $g_s + \ldots \frac{1}{4-d} g_s^3 + \ldots$

$$\mu \frac{d}{d\mu} g_s(\mu) = \beta(g_s(\mu)) \qquad (x)$$

$$\beta(g) = -\frac{g}{2}\left(\frac{\beta_0}{2\pi}\alpha_s + \frac{\beta_1}{4\pi^2}\alpha_s^2 + \frac{\beta_2}{64\pi^3}\alpha_s^3 + \dots\right)$$

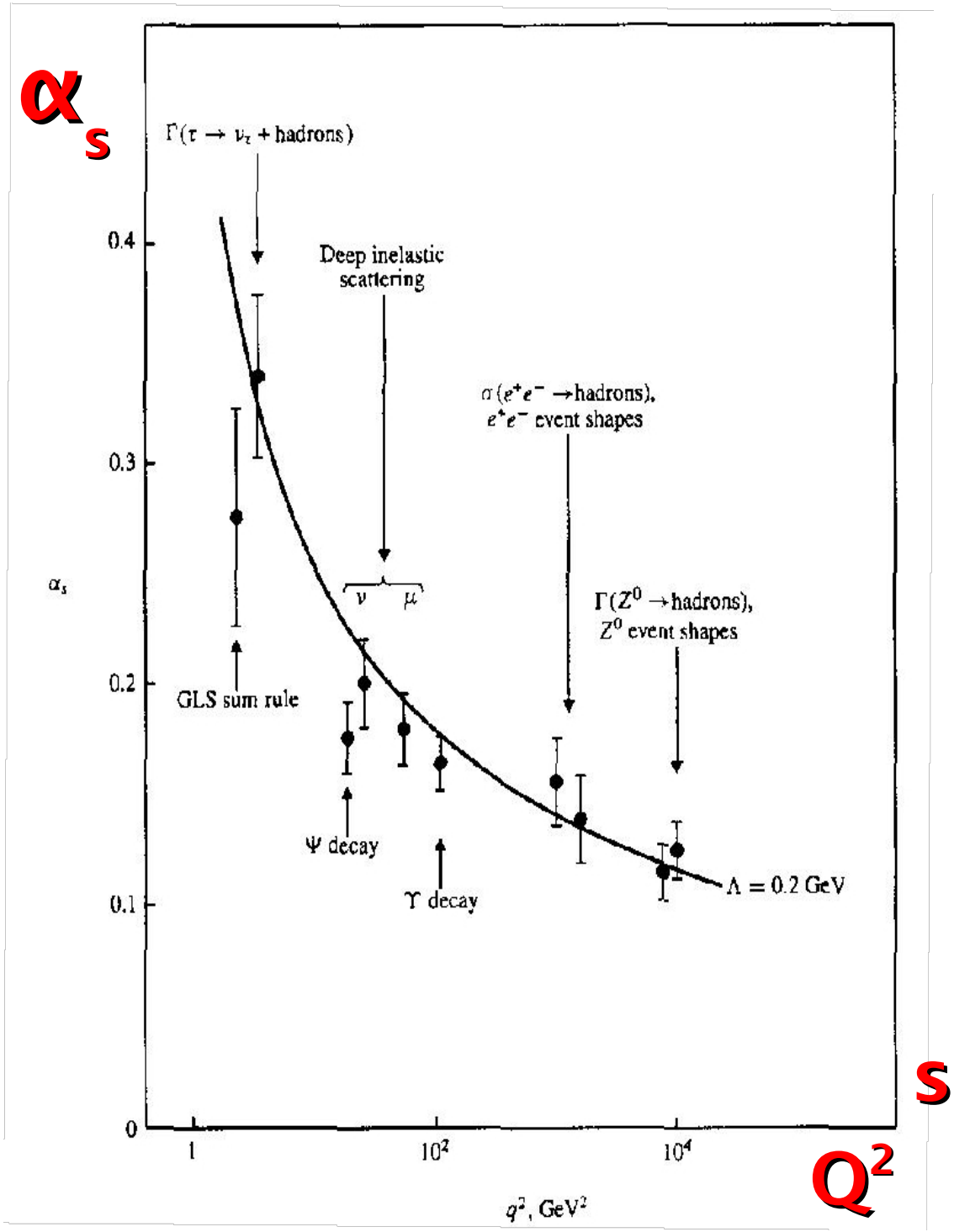
$$\alpha_s = \frac{g^2}{4\pi} \qquad (*)$$

$$\beta_1 = 51 - \frac{19}{3} n_f$$

$$\beta_2 = 2857 - \frac{5033}{9} n_f + \frac{325}{27} n_f^2,$$

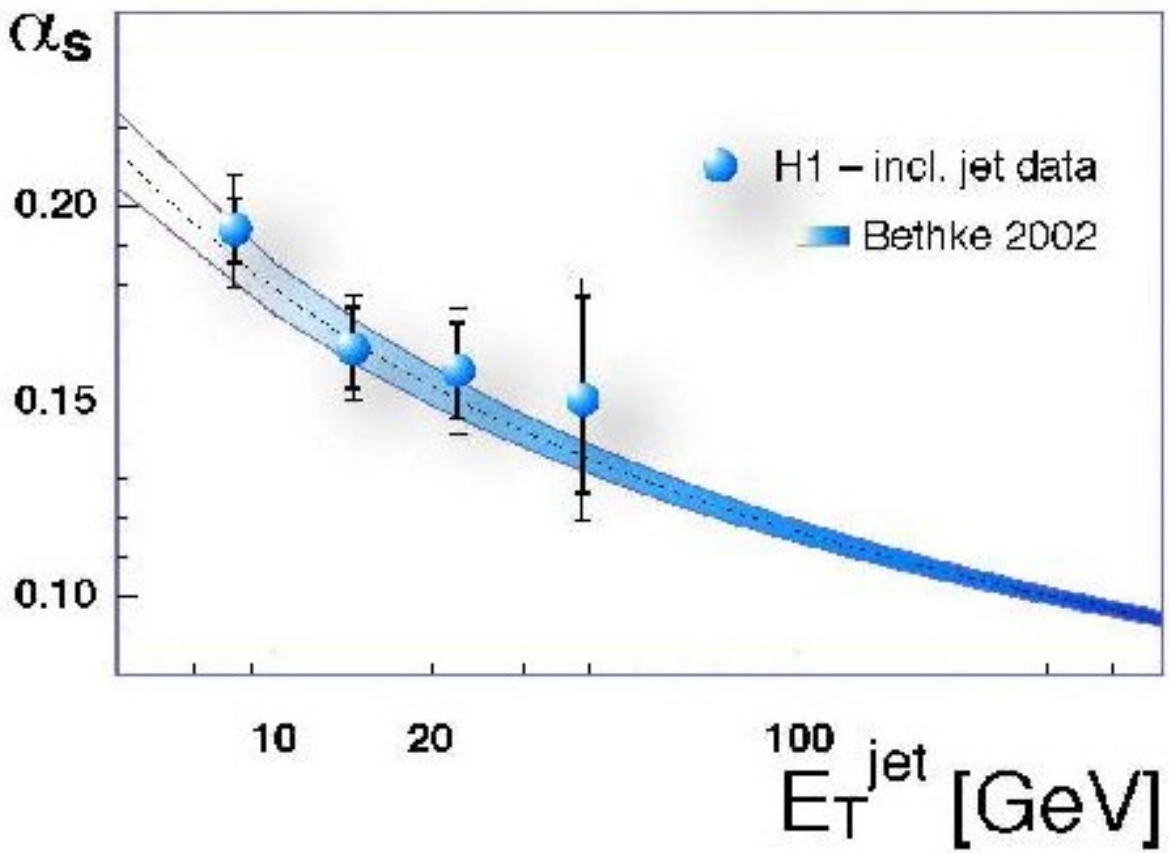
etc.

Also the term β_3 is known by rather lengthy...



strong coupling $\alpha_s(Q^2)$

David Gross, David Politzer and Frank Wilczek awarded the 2004 Nobel Prize in Physics for the discovery of asymptotic freedom



Twenty years ago, David Gross, David Politzer and Frank Wilczek discovered **asymptotic freedom** in the theory of the strong interactions. Measurements published by **H1** in the year 2000 beautifully illustrate this effect: the strong coupling a_s is seen to decrease as the energy at which it is measured, E_T^{jet}, increases.

Noble Prize 2004 for running of strong coupling α_s

approximate energy range M (GeV)	number of active quark flavours n_f	$\Lambda^{(n_f)}$ (MeV)
> 200	6	88 ± 11
$10 - 200$	5	208 ± 25
$3 - 10$	4	288 ± 30
< 3	3	326 ± 30

Table 4.1. The number of active quark flavours and $\Lambda^{(n_f)}$ in the $\overline{\text{MS}}$ scheme for various energy ranges

(From: Donnachie, Dosch, Landshoff, Nachtmann: Pomeron Physics and QCD, CUP, 2002)

Quark mass parameters:

$\mu = 2\,\mathrm{GeV}$

$1.5\,\mathrm{MeV} \leq m_u(\mu) \leq 3\,\mathrm{MeV}$

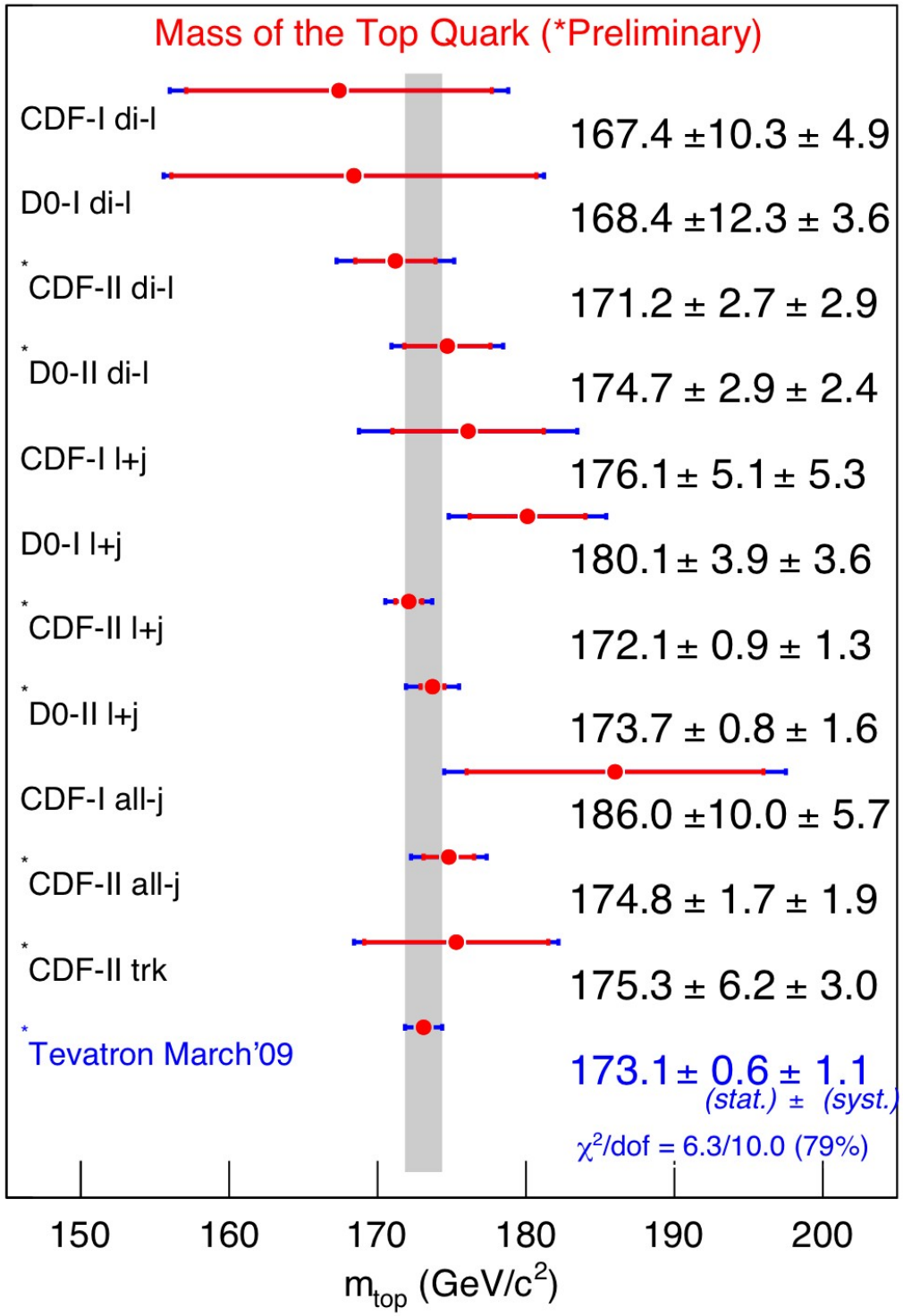
$3.0\,\mathrm{MeV} \leq m_d(\mu) \leq 7\,\mathrm{MeV}$

$70\,\mathrm{MeV} \leq m_s(\mu) \leq 180\,\mathrm{MeV}$

$1\,\mathrm{GeV} \leq m_c(\mu = 1.2\,\mathrm{GeV}) \leq 1.4\,\mathrm{GeV}$

$4\,\mathrm{GeV} \leq m_b(\mu = 4.2\,\mathrm{GeV}) \leq 4.5\,\mathrm{GeV}$

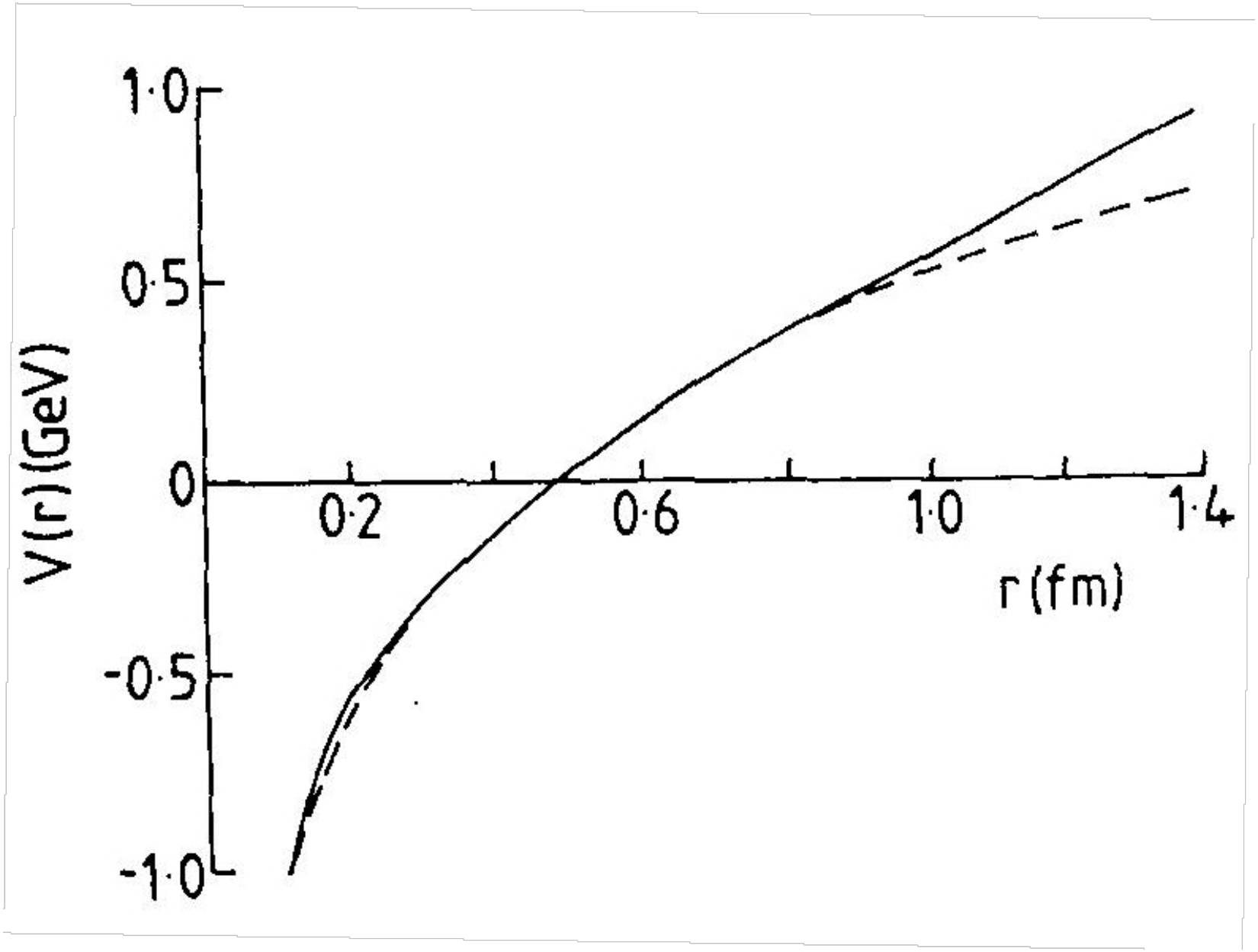
PDG 2004, 2006



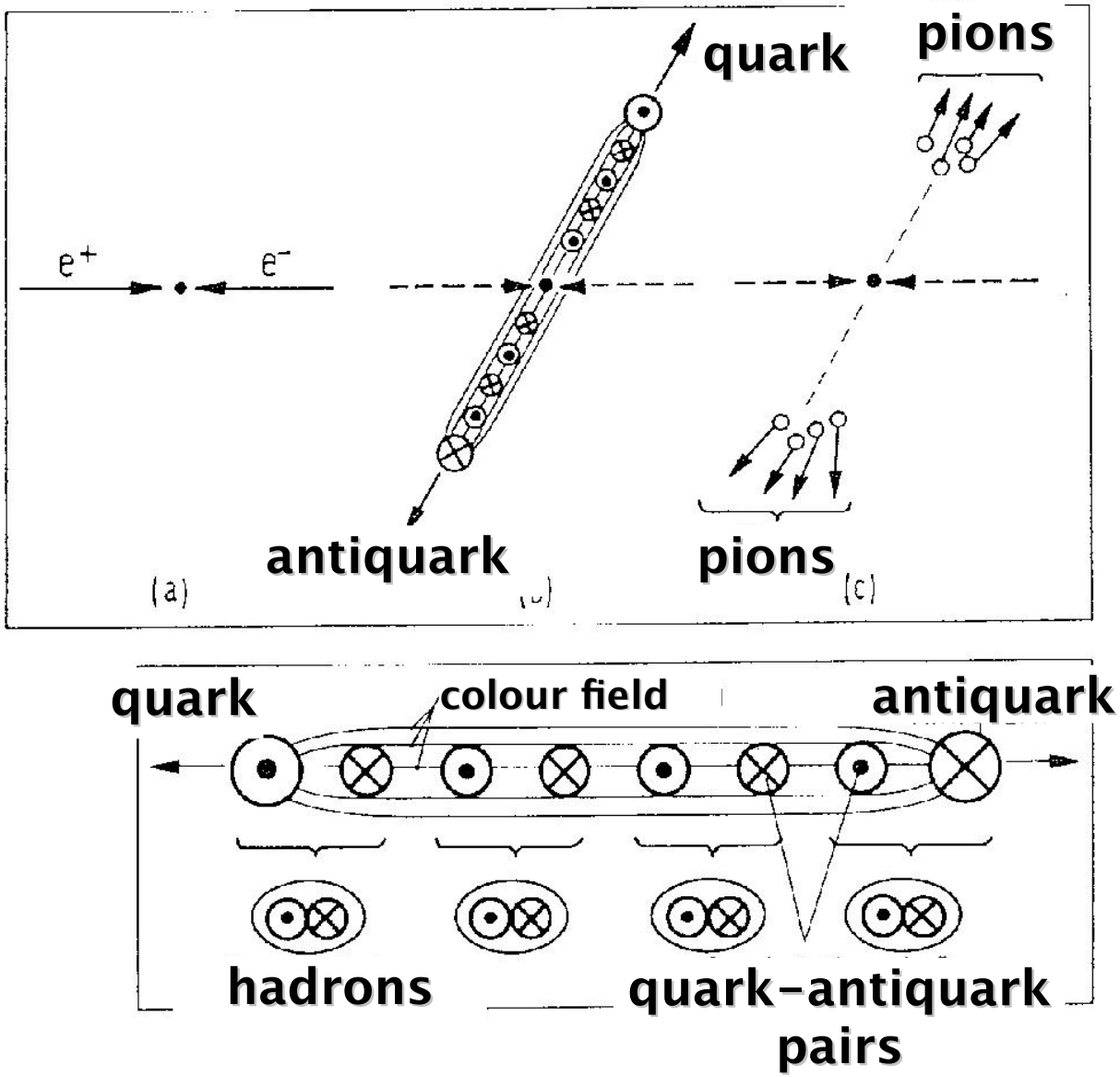
World average (March 2009)

4. Quantum Chromodynamics, QCD

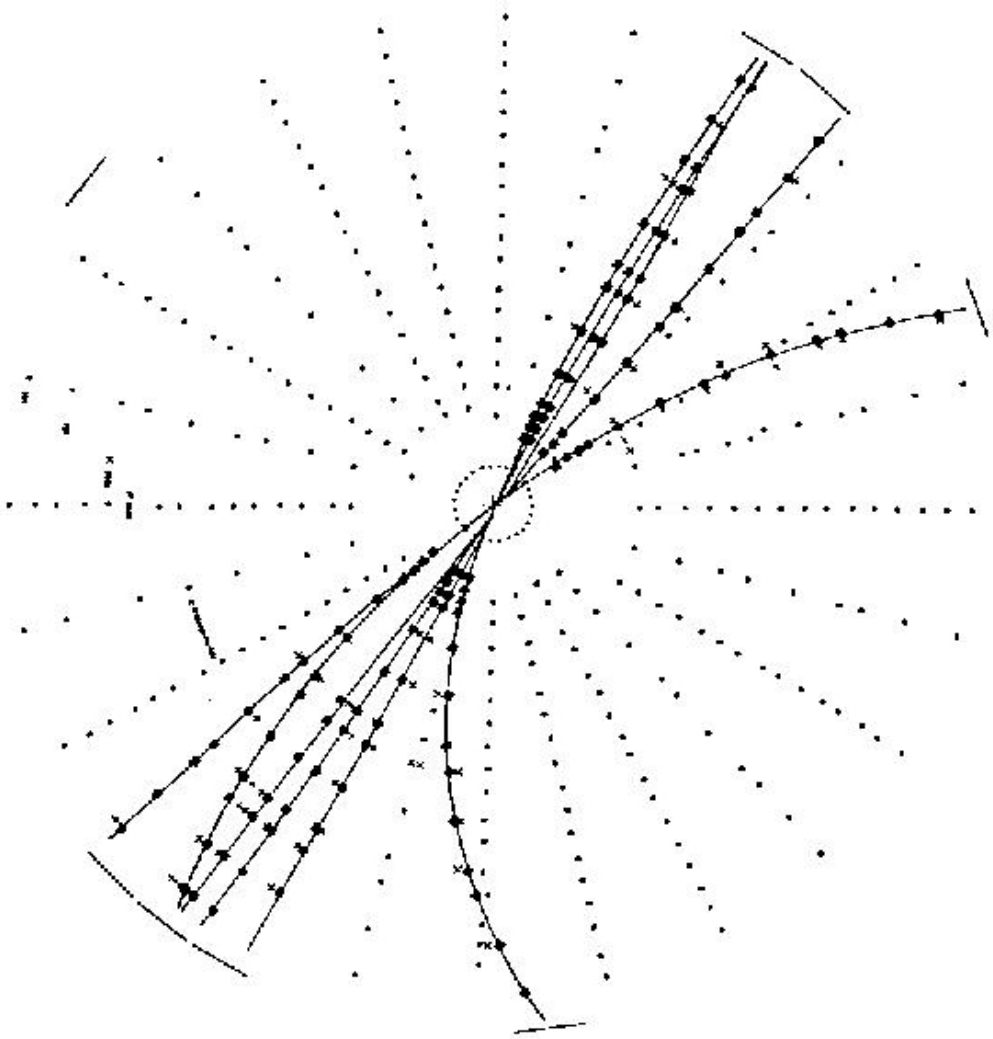
4.5 Confinement

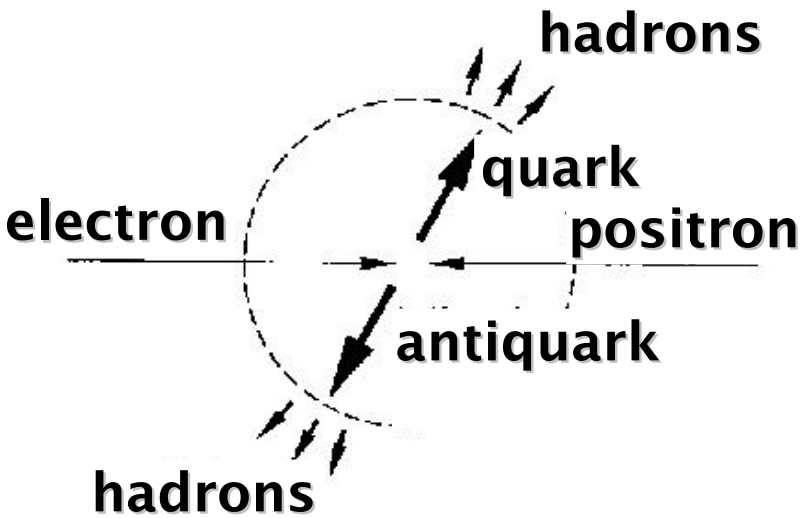


Potential of strong quark interaction from charmonium and bottomium data



Schematic Hadronisation

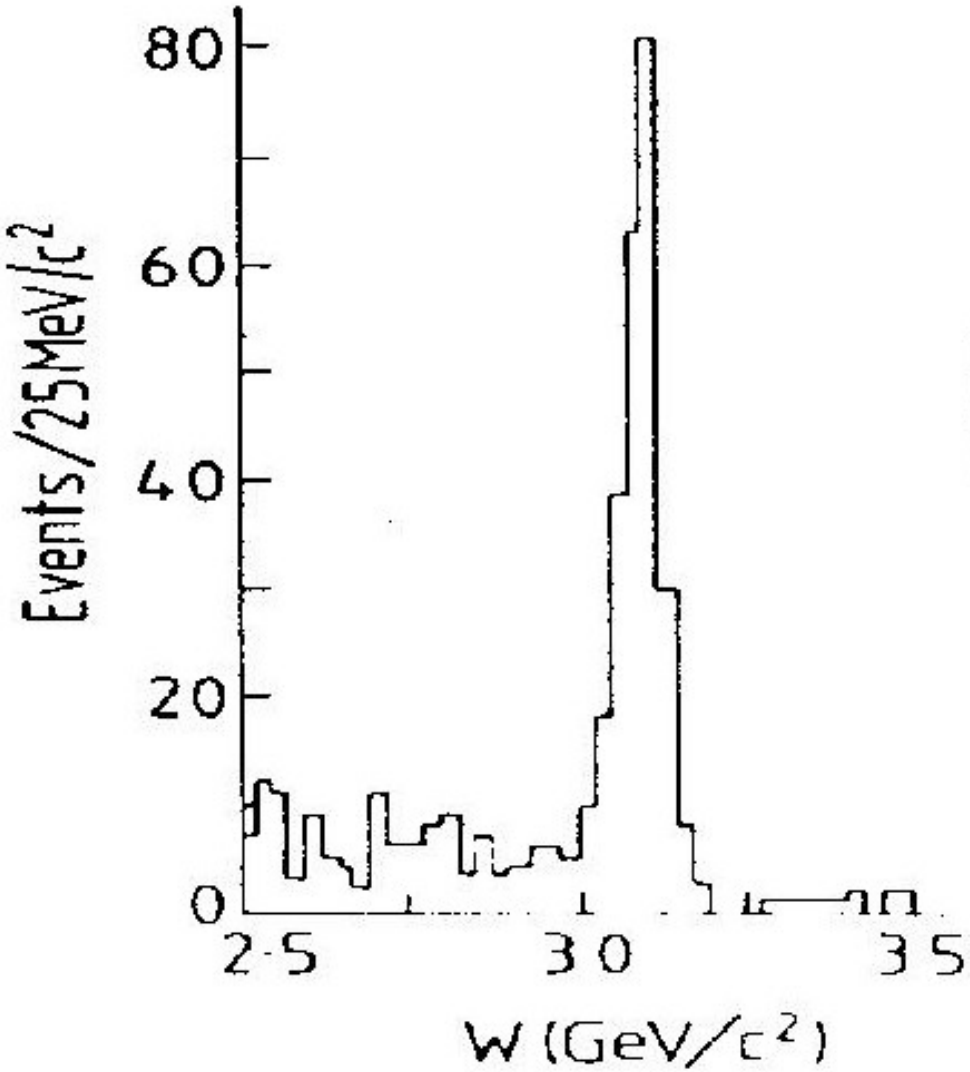


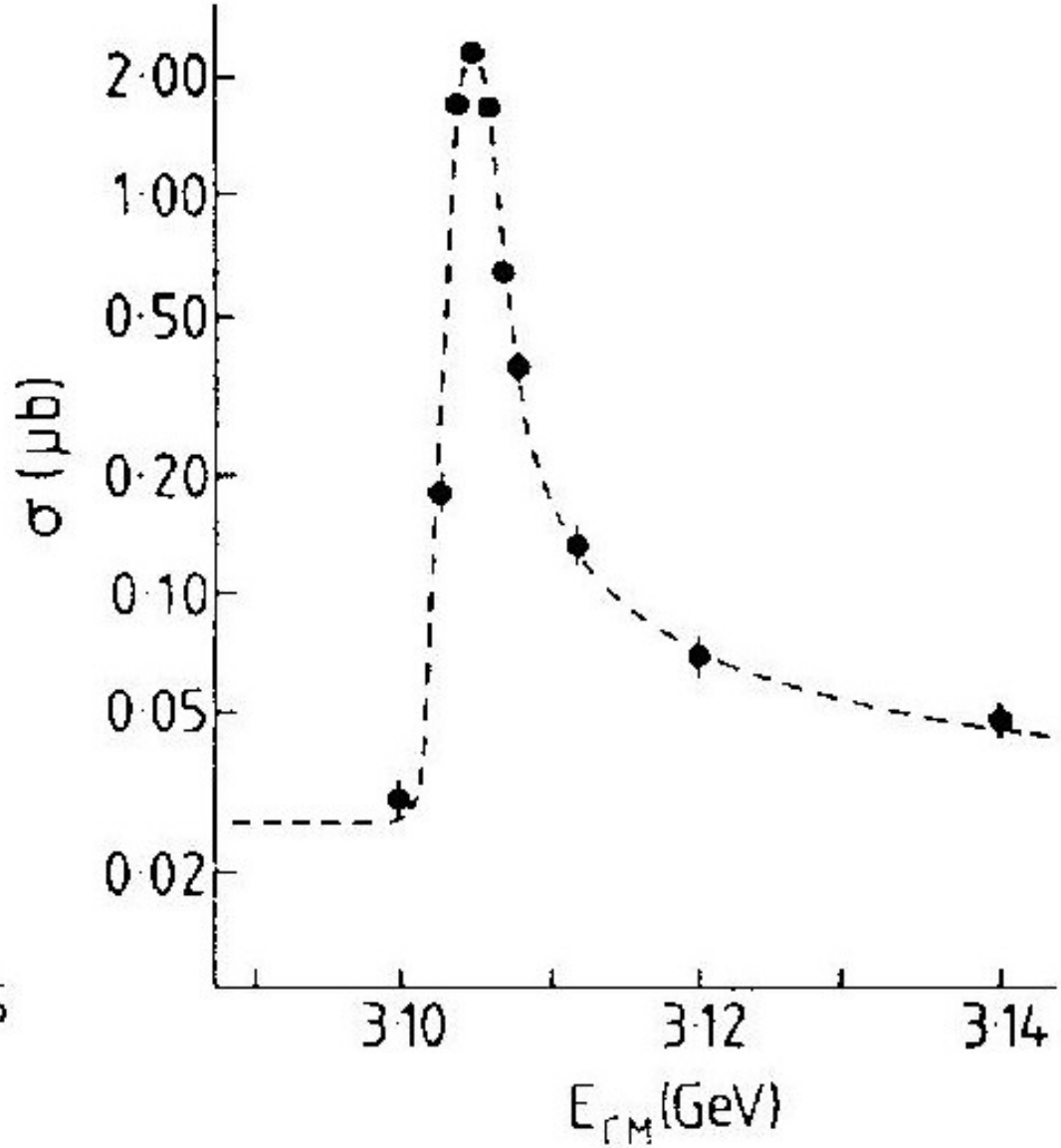


2 jet event at e^+e^- accelerator PETRA

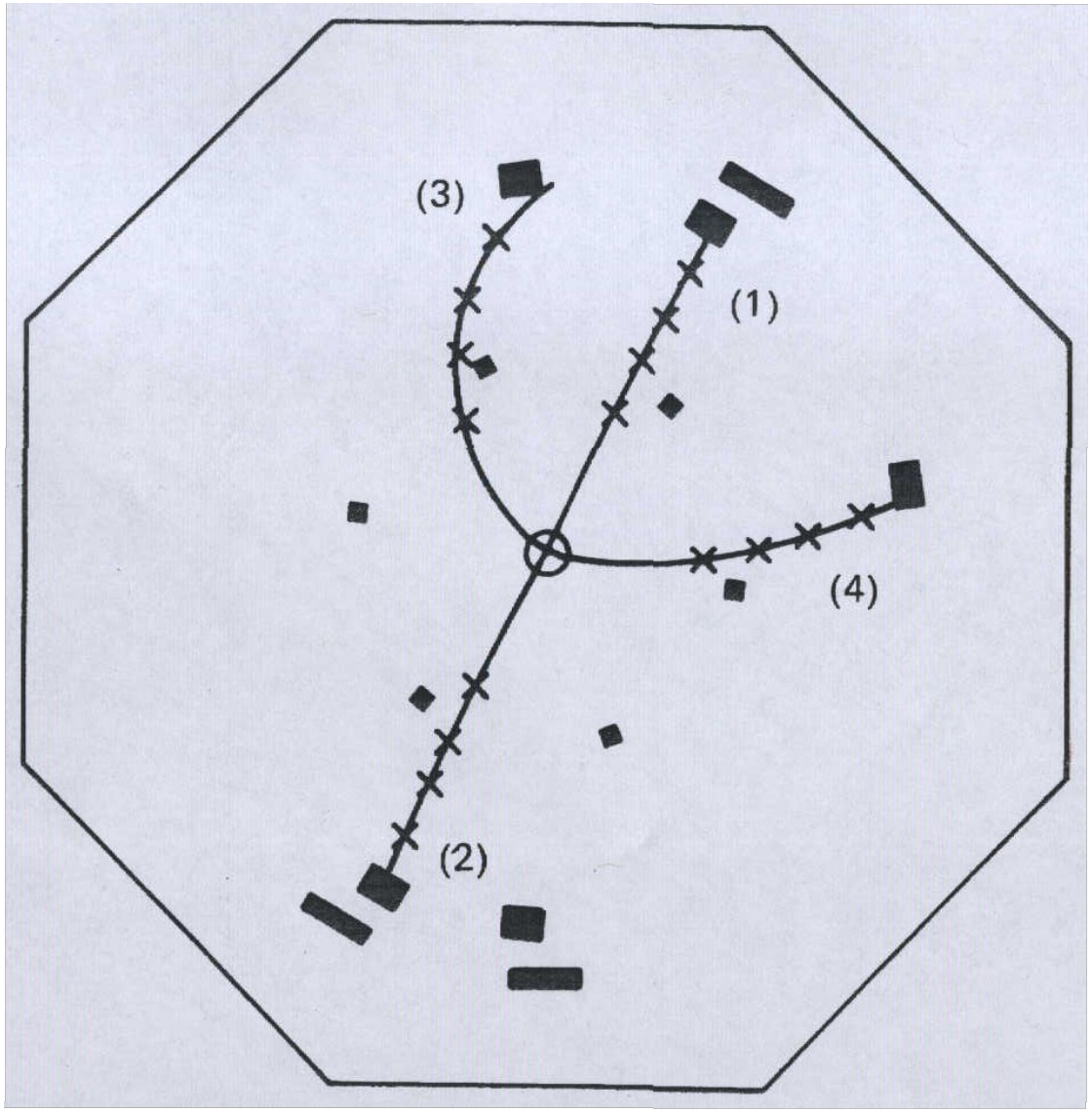
4. Quantum Chromodynamics, QCD

4.6 Quarkonium Physics

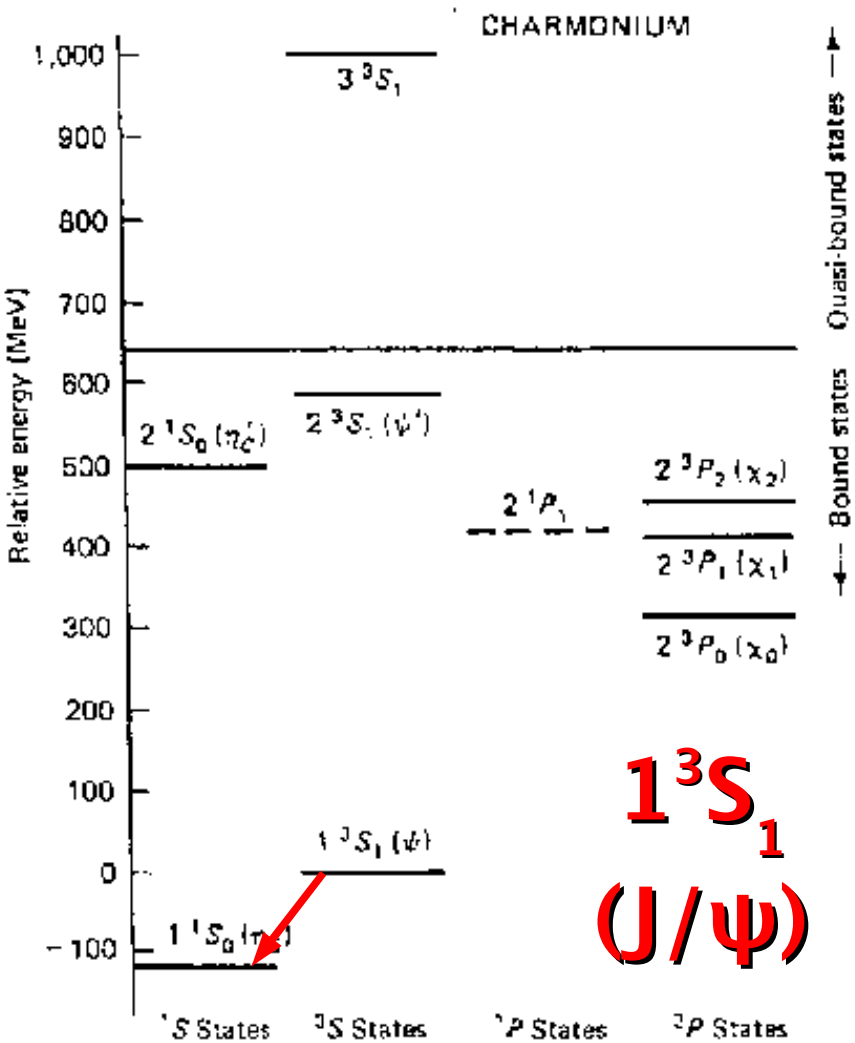




Discovery of J/ψ:
pp- (Brookhaven) and e⁺e-scattering (SLAC)

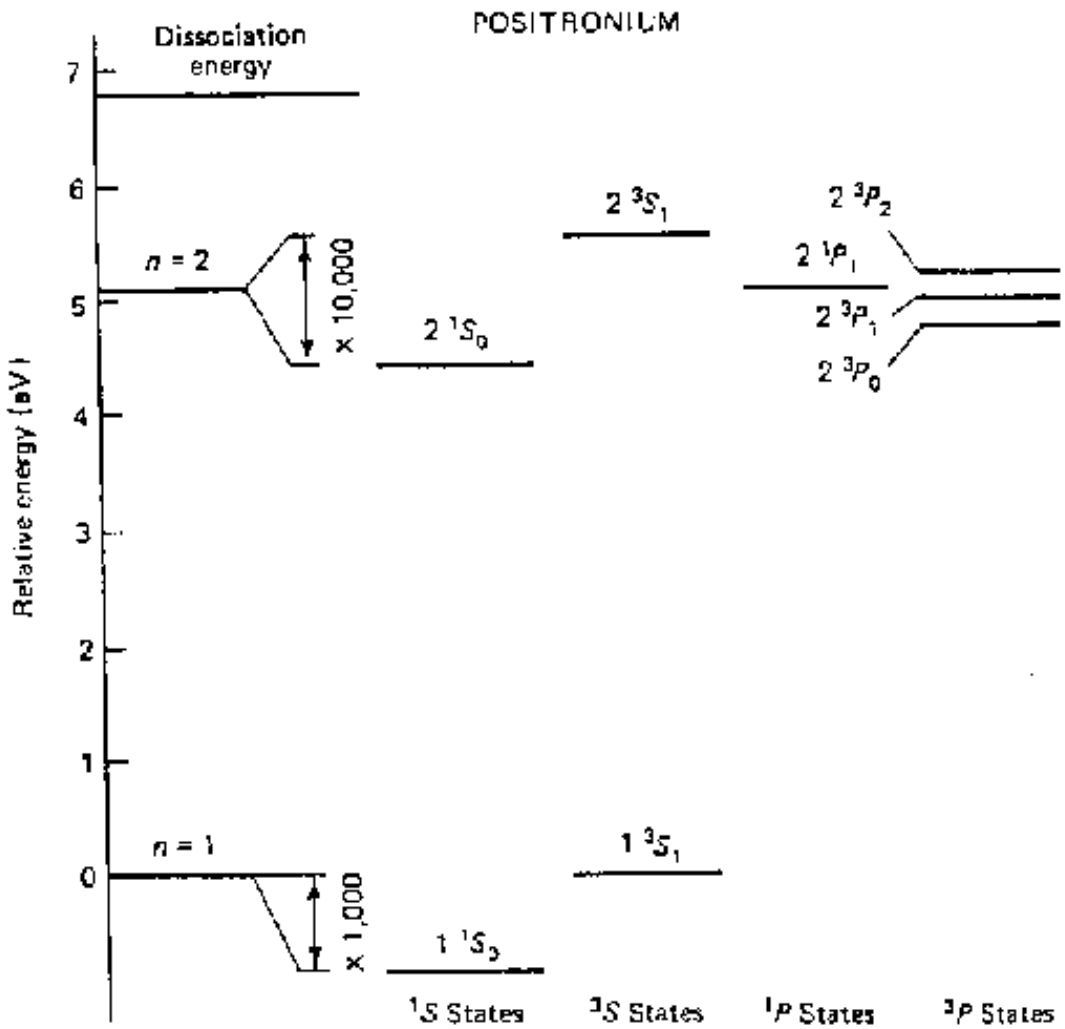


$\psi'(3.7) \rightarrow \psi(3.1)\ \pi^+\pi^- \rightarrow e^+e^-\ \pi^+\pi^-$ in a spark chamber detector



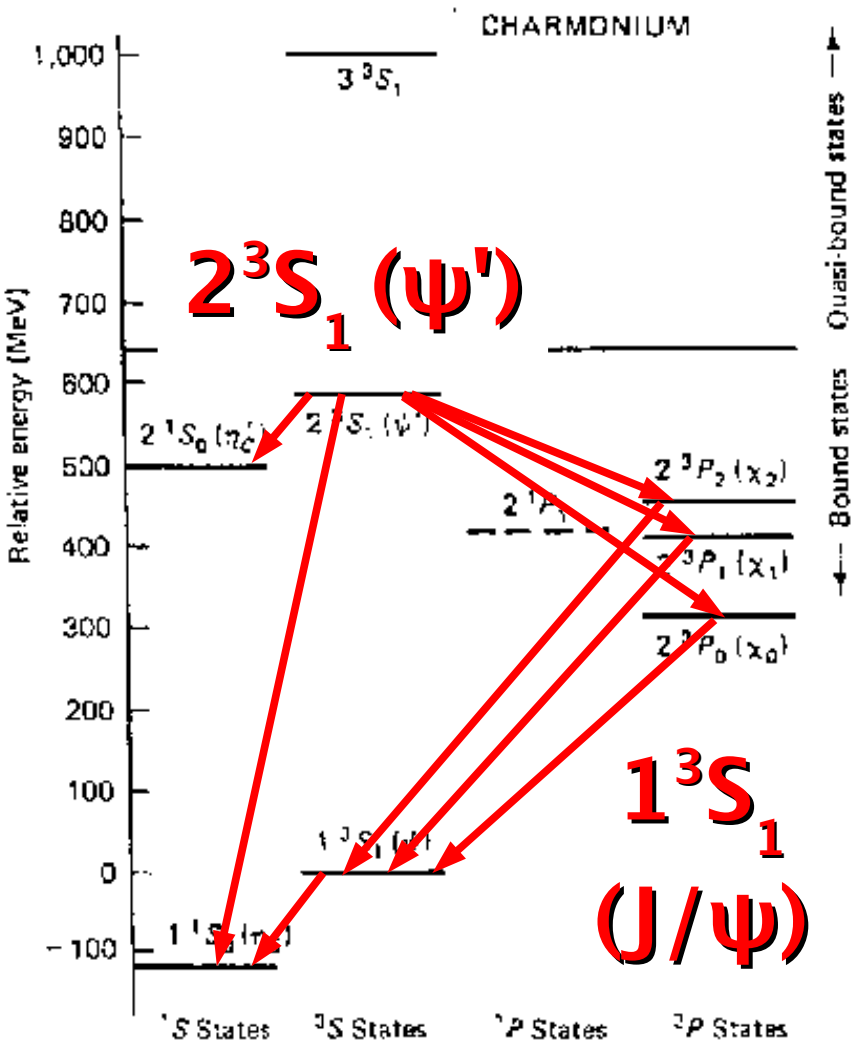
1^3S_1

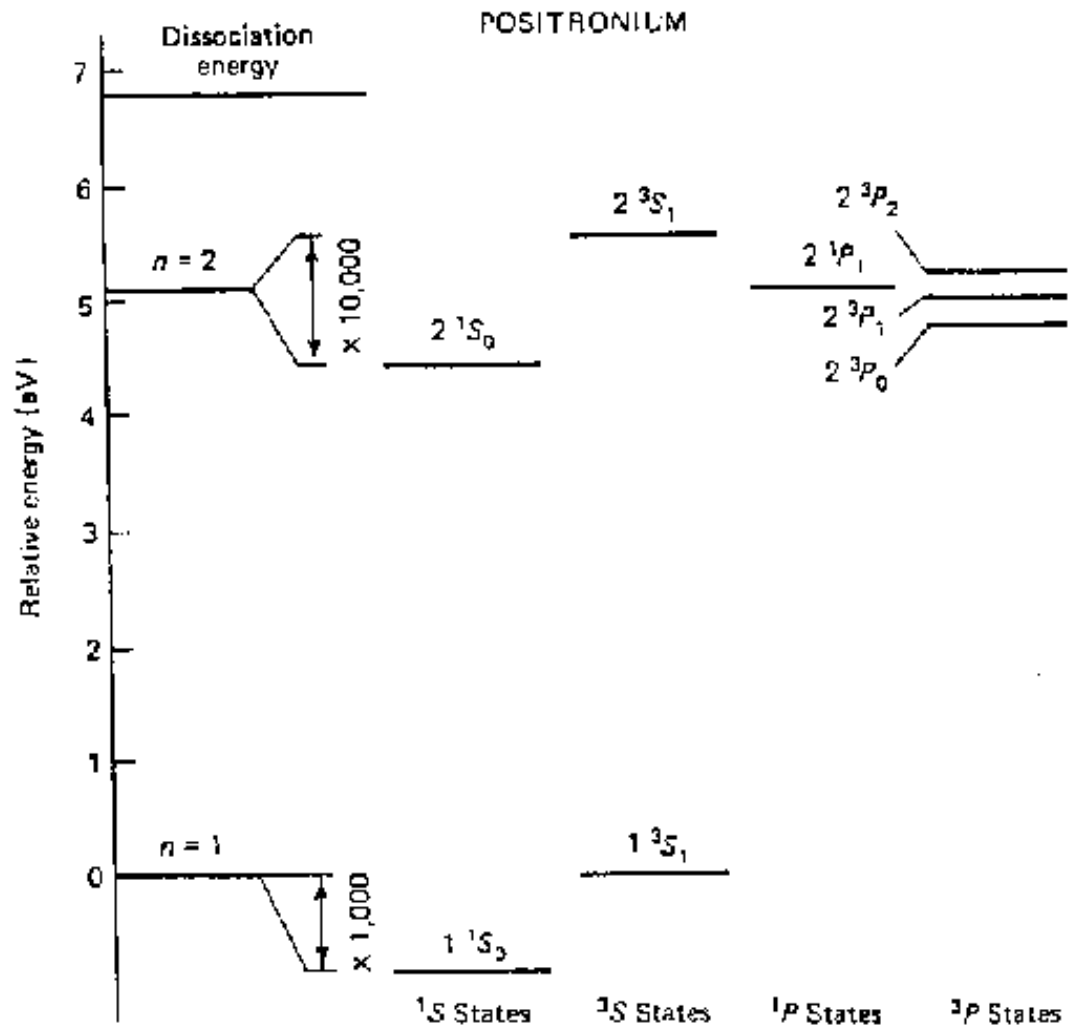
(J/ψ)



$c\bar{c}$

energy levels for charmonium and positronium



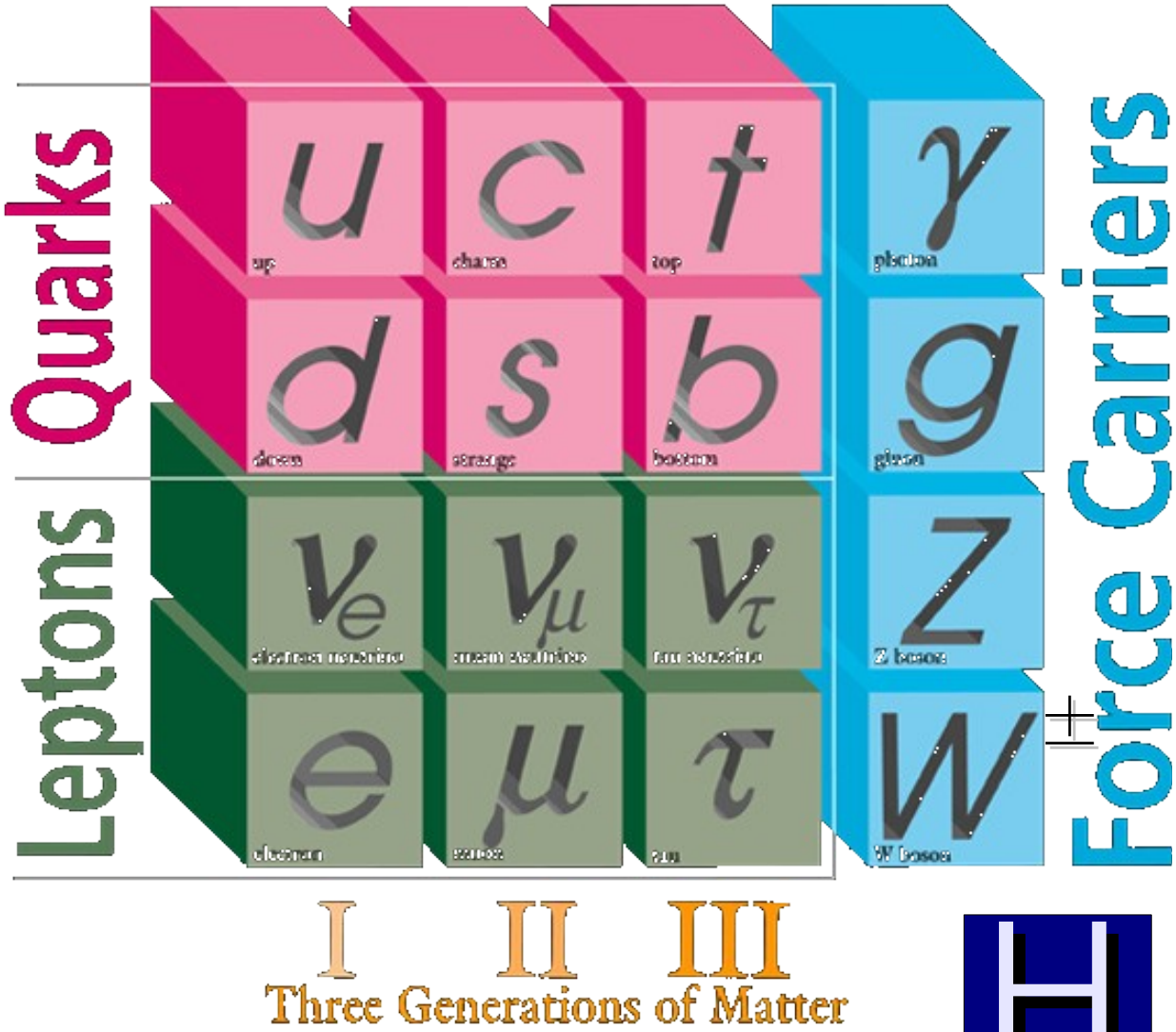


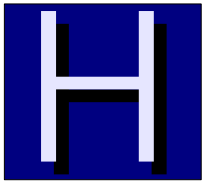
$c\bar{c}$

energy niveaux for charmonium and positronium

4. Quantum Flavour Dynamics, QFD

ELEMENTARY PARTICLES

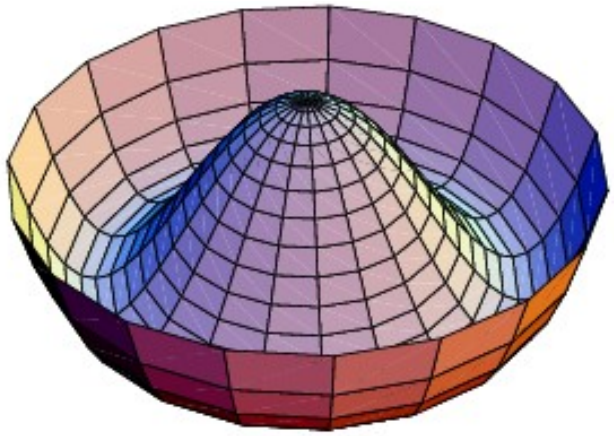




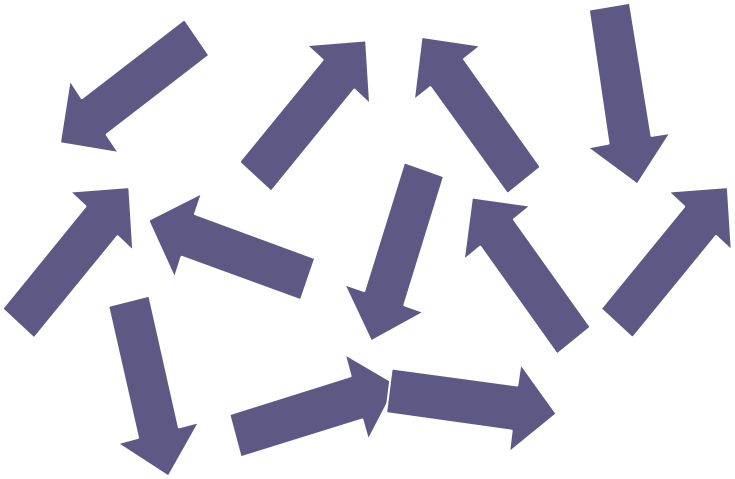
Higgs-boson: (?)
explains masses

Spontaneous Symmetry Breaking (SSB)

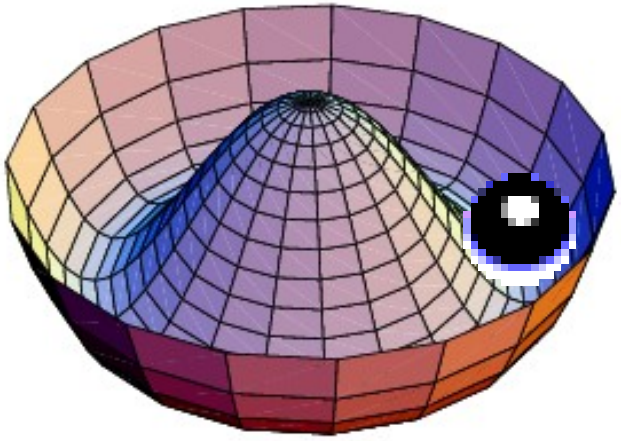
Higgs potential:



unmagnetised material:

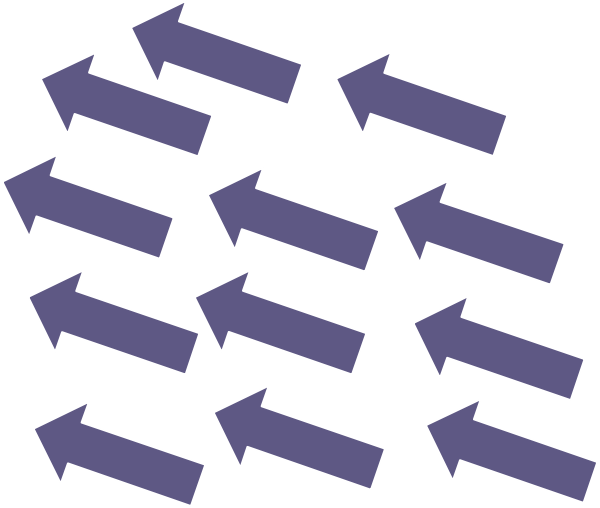


Higgs ground state:



SSB

ferromagnetic material:



$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\left(W_{\lambda\rho}W^{\lambda\rho}\right) - \frac{1}{4}\,B_{\lambda\rho}\,B^{\lambda\rho}$$

$$+\left\{\bar{\nu}_{eL}\,\frac{1}{2}\,i\gamma^{\lambda}\partial_{\lambda}\,\nu_{eL} + h.c.\right\}$$

$$+\left\{\bar{e}\,\frac{i}{2}\,\gamma^{\lambda}\partial_{\lambda}\,e + h.c.\right\}$$

$$+\,W^{+}_{\lambda}\,W^{-\lambda}\,m_W^2\left(1+\frac{H}{v}\right)^2$$

$$+\frac{1}{2}\,Z_{\lambda}\,Z^{\lambda}\,m_Z^2\left(1+\frac{H}{v}\right)^2$$

$$-\,m_e\,\bar{e}e\left(1+\frac{H}{v}\right)$$

$$+\,\frac{1}{2}(\partial_{\lambda}H)(\partial^{\lambda}H)$$

$$-\,\frac{1}{2}\,m_H^2\;H^2\left[1+\frac{H}{v}+\frac{1}{4}\left(\frac{H}{v}\right)^2\right]$$

$$+\,\mathcal{L}_{Int}\,.$$

$$\mathcal{L}_{Int} = -e\left\{A_{\lambda}\,J^{\lambda}_{em} + \frac{1}{\sin\vartheta_W\cos\vartheta_W}\,Z_{\lambda}\,J^{\lambda}_{NC} + \frac{1}{\sqrt{2}\sin\vartheta_W}\left(W^{+}_{\lambda}J^{\lambda}_{CC} + W^{-}_{\lambda}J^{\lambda +}_{CC}\right)\right\}$$

$$J^{\lambda}_{em} = \bar{\psi}\gamma^{\lambda}(T_3+Y)\psi = \bar{\psi}\gamma^{\lambda}Q\psi$$

$$(Q = T_3 + Y),$$

$$J^{\lambda}_{NC} = \bar{\psi}\gamma^{\lambda}T_3\psi - \sin^2\vartheta_W\,J^{\lambda}_{em}$$

$$J^{\lambda}_{CC} = \bar{\psi}\gamma^{\lambda}(T_1+iT_2)\psi\,.$$

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\left(W_{\lambda\rho}W^{\lambda\rho}\right) - \frac{1}{4}\,B_{\lambda\rho}\,B^{\lambda\rho}$$

$$+\left\{\bar{\nu}_{eL}\,\frac{1}{2}\,i\gamma^{\lambda}\partial_{\lambda}\,\nu_{eL} + h.c.\right\}$$

$$+\left\{\bar{e}\,\frac{i}{2}\,\gamma^{\lambda}\partial_{\lambda}\,e + h.c.\right\}$$

$$+\,W^{+}_{\lambda}\,W^{-\lambda}\,m_W^2\left(1+\frac{H}{v}\right)^2$$

$$+\frac{1}{2}\,Z_{\lambda}\,Z^{\lambda}\,m_Z^2\left(1+\frac{H}{v}\right)^2$$

$$-\,m_e\,\bar{e}e\left(1+\frac{H}{v}\right)$$

$$+\,\frac{1}{2}(\partial_{\lambda}H)(\partial^{\lambda}H)$$

$$-\,\frac{1}{2}\,m_H^2\;H^2\left[1+\frac{H}{v}+\frac{1}{4}\left(\frac{H}{v}\right)^2\right]$$

$$+\,\mathcal{L}_{Int}\,.$$

$$Z_{\lambda} = \cos\vartheta_W\,W^3_{\lambda} - \sin\vartheta_W\,B_{\lambda}$$

$$A_{\lambda} = \sin\vartheta_W\,W^3_{\lambda} + \cos\vartheta_W\,B_{\lambda}$$

$$m_W^2 = \frac{g^2v^2}{4} = \frac{e^2v^2}{4\sin^2\vartheta_W}\,,$$

$$m_Z^2 = \frac{(g^2+g'^2)v^2}{4} = \frac{e^2v^2}{4\sin^2\vartheta_W\cos^2\vartheta_W}$$

$$m_e = \kappa_e\,\frac{v}{\sqrt{2}}\,,$$

$$m_H^2 = 2\lambda v^2\,.$$

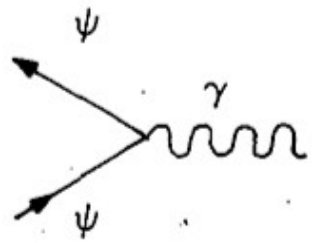
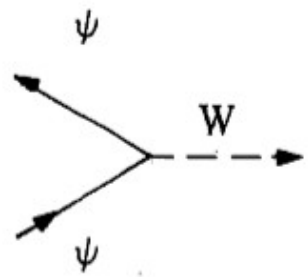
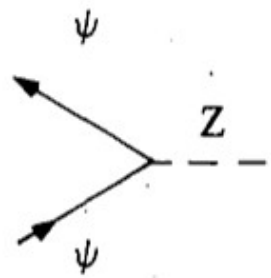
From O.Nachtmann, Elementary Particle Physics, Springer, 1990

			T	T_3	Y	Q
$\begin{pmatrix}\nu_{e_L}\\ e_L\end{pmatrix}$	$\begin{pmatrix}\nu_{\mu_L}\\ \mu_L\end{pmatrix}$	$\begin{pmatrix}\nu_{\tau_L}\\ \tau_L\end{pmatrix}$	1/2	1/2	− 1/2	0
			1/2	− 1/2	− 1/2	− 1
e_R	μ_R	τ_R	0	0	− 1	− 1
$\begin{pmatrix}u_L\\ d'_L\end{pmatrix}$	$\begin{pmatrix}c_L\\ s'_L\end{pmatrix}$	$\begin{pmatrix}t_L\\ b'_L\end{pmatrix}$	1/2	1/2	1/6	2/3
			1/2	− 1/2	1/6	− 1/3
u_R	c_R	t_R	0	0	2/3	2/3
d_R	s_R	b_R	0	0	− 1/3	− 1/3

The flavour quantum numbers of the leptons and quarks.

T : weak isospin, T_3 third component,

Y : hypercharge , Q el. charge

Diagrammteil	analytischer Ausdruck
	$-i e \mathbf{Q} \gamma^{\lambda}$
	$-i \frac{e}{\sqrt{2} \sin \vartheta_W} (\mathbf{T}_1 + i \mathbf{T}_2) \gamma^{\lambda}$
	$-i \frac{e}{\sin \vartheta_W \cos \vartheta_W} (\mathbf{T}_3 - \sin^2 \vartheta_W \mathbf{Q}) \gamma^{\lambda}$

Fermion – gauge boson vertices in QFD

Diagrammteil	analytischer Ausdruck
$\gamma(q)$	$\dfrac{-\mathrm{i}\, g_{\rho\lambda}}{q^2}$
$\mathrm{W}(q)$	$\dfrac{-\mathrm{i}\left(g_{\rho\lambda} - \dfrac{q_\rho q_\lambda}{m_\mathrm{W}^2}\right)}{q^2 - m_\mathrm{W}^2} \xrightarrow[q^2 \ll m_\mathrm{W}^2]{} \dfrac{\mathrm{i}\, g_{\rho\lambda}}{m_\mathrm{W}^2}$
$\mathrm{Z}(q)$	$\dfrac{-\mathrm{i}\left(g_{\rho\lambda} - \dfrac{q_\rho q_\lambda}{m_\mathrm{Z}^2}\right)}{q^2 - m_\mathrm{Z}^2} \xrightarrow[q^2 \ll m_\mathrm{Z}^2]{} \dfrac{\mathrm{i}\, g_{\rho\lambda}}{m_\mathrm{Z}^2}$

Gauge boson propagators in QFD

78

$$\mathcal{L}_{Yuk} = -(\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R)\, C_\ell \begin{pmatrix} \phi^+ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \\ \phi^+ \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \\ \phi^+ \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \end{pmatrix}$$

$$+(\bar{u}_R, \bar{c}_R, \bar{t}_R)\, C_q' \begin{pmatrix} \phi^T \varepsilon \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \\ \phi^T \varepsilon \begin{pmatrix} c_L \\ s'_L \end{pmatrix} \\ \phi^T \varepsilon \begin{pmatrix} t_L \\ b'_L \end{pmatrix} \end{pmatrix}$$

$$-(\bar{d}'_R, \bar{s}'_R, \bar{b}'_R)\, C_q \begin{pmatrix} \phi^+ \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \\ \phi^+ \begin{pmatrix} c_L \\ s'_L \end{pmatrix} \\ \phi^+ \begin{pmatrix} t_L \\ b'_L \end{pmatrix} \end{pmatrix}$$

$$+ h.c. \qquad (5.$$

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Redefinition of fields allows the following standard forms for C_e, C_q', C_q

$$C_e = \begin{pmatrix} c_e & 0 & 0 \\ 0 & c_\mu & 0 \\ 0 & 0 & c_\tau \end{pmatrix}$$

$$C_q' = \begin{pmatrix} c_u & 0 & 0 \\ 0 & c_c & 0 \\ 0 & 0 & c_t \end{pmatrix}$$

$$C_q = V \begin{pmatrix} c_d & 0 & 0 \\ 0 & c_s & 0 \\ 0 & 0 & c_b \end{pmatrix} V^\dagger$$

$$c_e, c_\mu, \ldots, c_b \geq 0$$

V CKM - Matrix (5.100)

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix}$$

$$\equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

$$c_{ij} = \cos \vartheta_{ij} \quad , \quad s_{ij} = \sin \vartheta_{ij}$$

$$0 \leq \vartheta_{ij} \leq \pi/2 \quad , \quad 0 \leq \delta_{13} \leq 2\pi$$

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix} + O(\lambda^4)$$

$$\lambda = s_{12}\,, \quad A = s_{23}/(s_{12})^2\,, \quad \rho - i\eta = \frac{s_{13}\, e^{-i\delta_{13}}}{s_{12}\, s_{23}} \qquad (5.111)$$

We get then for $\mathcal{L}_{Yuk}$

$$\mathcal{L}_{Yuk} = -\Big\{ (\bar{e}, \bar{\mu}, \bar{\tau}) \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$+ (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$$+ (\bar{d}', \bar{s}', \bar{b}') V \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} V^\dagger \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} \Big\}$$

$$\left(1 + \frac{H}{v}\right) \qquad (5.113)$$

Here we set

$$m_e = c_e \frac{v}{\sqrt{2}}, \quad \ldots\ldots, \quad m_b = c_b \frac{v}{\sqrt{2}} . \qquad (5.114)$$

In the unitary gauge:

$$\mathcal{L}_{Yuk} = \Big\{ -(\bar{e}, \bar{\mu}, \bar{\tau}) \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$- (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$$- (\bar{d}, \bar{s}, \bar{b}) \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \Big\}$$

$$\cdot \left(1 + \frac{H}{v}\right)$$

$$m_e = c_e \frac{v}{\sqrt{2}}, \; \ldots\ldots, \; m_b = c_b \frac{v}{\sqrt{2}}$$

(5.116)

$$j^{\lambda}_{NC} = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\, \gamma^\lambda \frac{1}{2} \frac{1-\gamma_5}{2} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$+ (\bar{e}, \bar{\mu}, \bar{\tau})\, \gamma^\lambda \left(-\frac{1}{2} \frac{1-\gamma_5}{2} + \sin^2\vartheta_W \right) \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$+ (\bar{u}, \bar{c}, \bar{t})\, \gamma^\lambda \left(\frac{1}{2} \frac{1-\gamma_5}{2} - \frac{2}{3} \sin^2\vartheta_W \right) \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$$+ (\bar{d}, \bar{s}, \bar{b})\, \gamma^\lambda \left(-\frac{1}{2} \frac{1-\gamma_5}{2} + \frac{1}{3} \sin^2\vartheta_W \right) \begin{pmatrix} d \\ s \\ b \end{pmatrix} .$$

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} = V^{\dagger} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

$$j^{\lambda}_{cc} = (\bar{\nu}_{eL}, \bar{\nu}_{\mu L}, \bar{\nu}_{\tau L})\gamma^{\lambda} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$$

$$+ \underbrace{(\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^{\lambda} \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}}$$

$$= (\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^{\lambda} V \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

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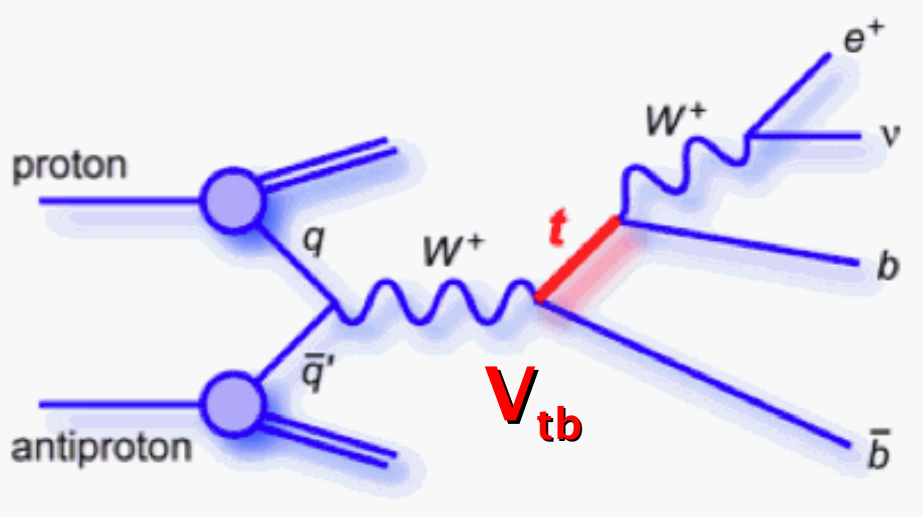
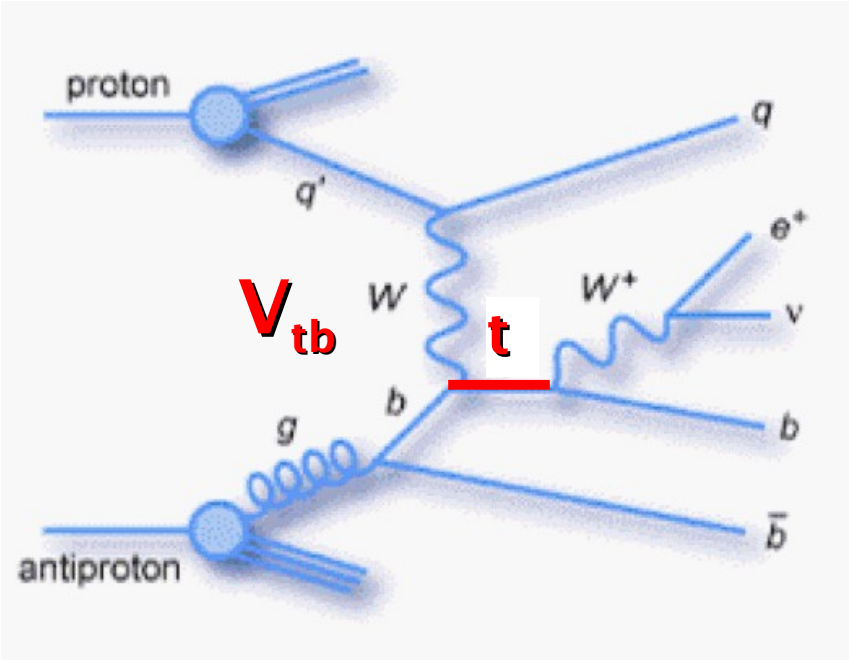
Single Top Quark Production

- **first direct measurement of $|V_{tb}|$**

s-channel:

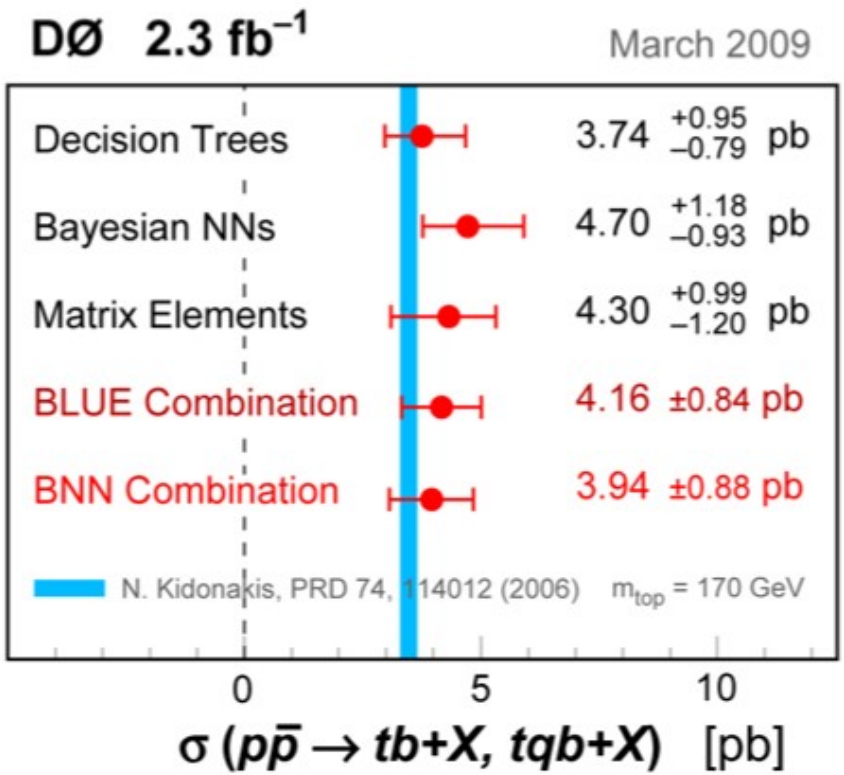


t-channel:

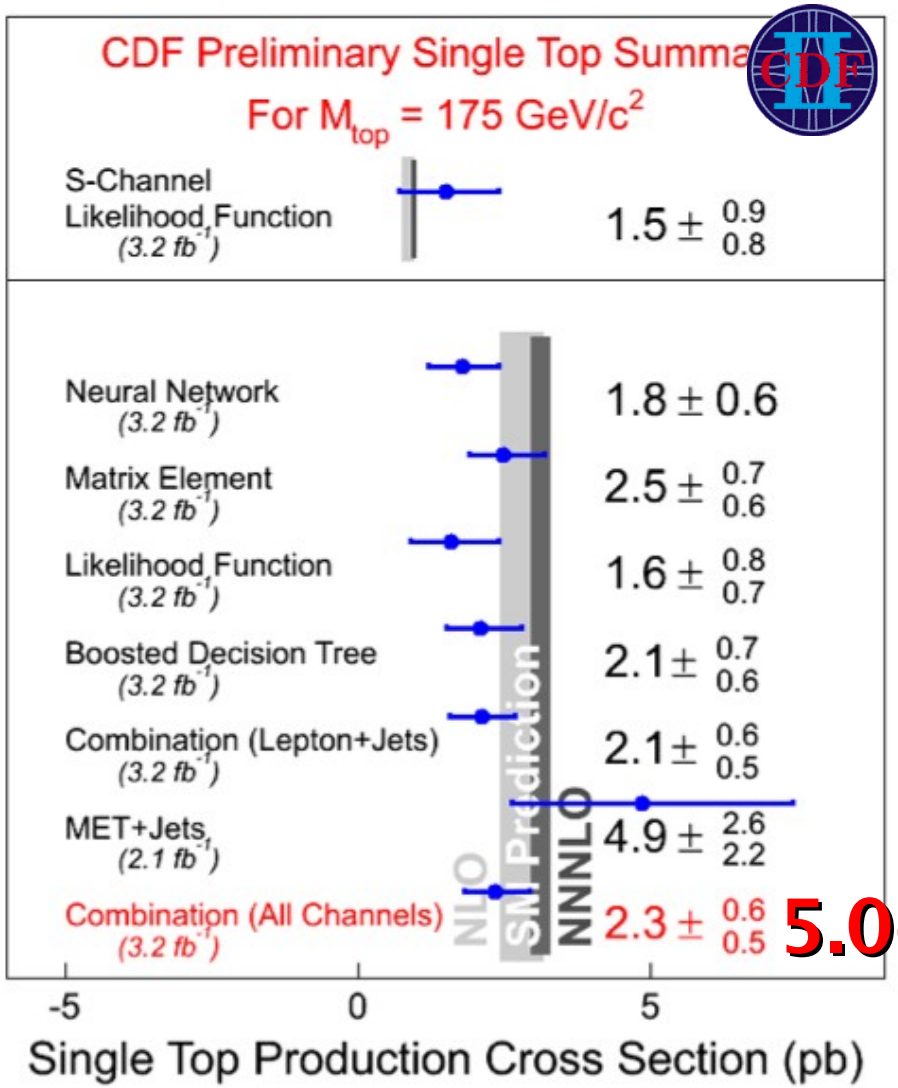


- **jets**
- **isolated lepton**
- **missing E_T**
- **b-jets**

Single Top Quark Observation

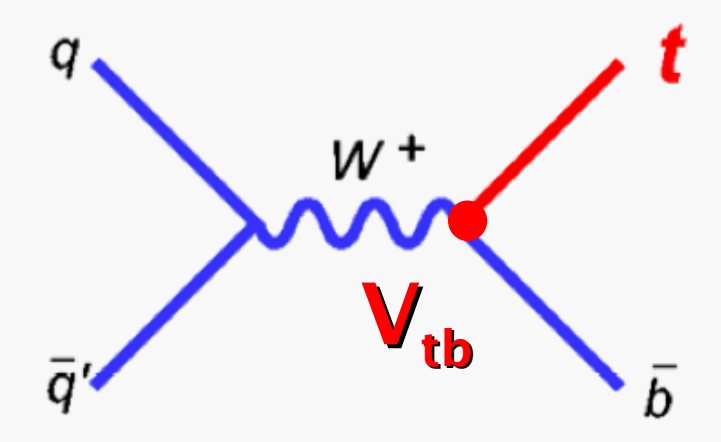


5.0σ



5.0σ

First direct measurement of $|V_{tb}|$



$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM Fitter Group for Beauty 2006

- before indirect limits: $|V_{tb}| = 0.999127 \pm 0.00026$ (1σ CL)

- assume: $|V_{td}|^2 + |V_{ts}|^2 \ll |V_{tb}|^2$
- assume: pure V–A and CP–conserving Wtb interaction

$$|V_{tb}| = 0.88 \pm 0.07$$

D0 + CDF II

- no assumption on quark families or CKM matrix unitarity

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{2}\,\mathrm{Tr}\left(W_{\lambda\rho}\,W^{\lambda\rho}\right) \\
& -\frac{1}{4}\,B_{\lambda\rho}\,B^{\lambda\rho} \\
& + W^{+}_{\lambda}\,W^{-\lambda}\,m_W^2\left(1+\frac{H}{v}\right)^2 \\
& + \frac{1}{2}\,Z_\lambda\,Z^\lambda\,m_Z^2\left(1+\frac{H}{v}\right)^2 \\
& + \left\{\bar{\psi}\,\frac{i}{2}\,\gamma^\lambda D_\lambda\,\psi + h.c.\right\} \\
& - \bar{\psi}\,M\,\psi\left(1+\frac{H}{v}\right) \\
& + \frac{1}{2}\,\partial_\lambda H\,\partial^\lambda H - \frac{1}{2}\,m_H^2\,H^2\left[1\right. \\
& \qquad \left. + \frac{H}{v} + \frac{1}{4}\left(\frac{H}{v}\right)^2\right]
\end{aligned}
$$

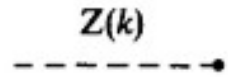
From O.Nachtmann, Elementary Particle Physics, Springer, 1990

The following rules—specific to QFD—represent the additions.

W^- initial state	incoming W line
$\varepsilon(k)$	$W^-(k)$
W^- in final state	outgoing W line
$\varepsilon^*(k)$	$W^-(k)$
W^+ in initial state	outgoing W line
$\varepsilon(k)$	$W^+(k)$
W^+ in final state	incoming W line
$\varepsilon^*(k)$	$W^+(k)$

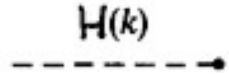
Z in initial (final) state — external Z line

$\varepsilon(k)(\varepsilon^*(k))$



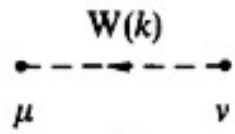
Higgs particle in initial (final) state — external H line

1



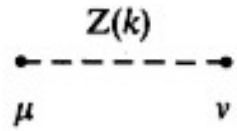
virtual W boson — internal W line

$$\frac{i\left(-g^{\mu\nu}+\frac{k^\mu k^\nu}{m_W^2}\right)}{k^2-m_W^2+i\varepsilon}$$



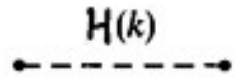
virtual Z boson — internal Z line

$$\frac{i\left(-g^{\mu\nu}+\frac{k^\mu k^\nu}{m_Z^2}\right)}{k^2-m_Z^2+i\varepsilon}$$



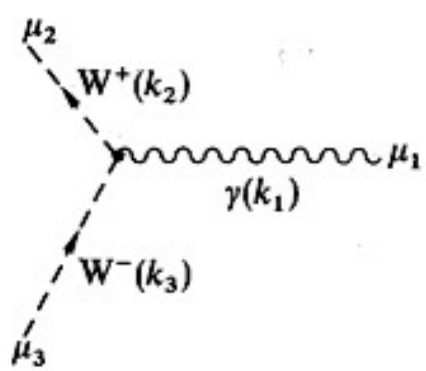
virtual Higgs particle — internal H line

$$\frac{i}{k^2-m^2+i\varepsilon}$$

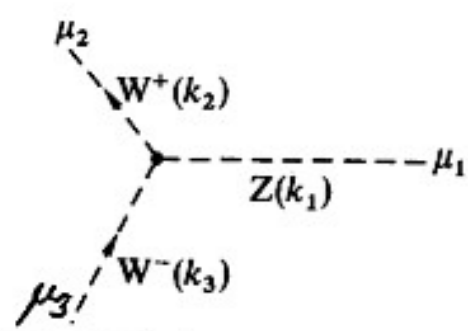


Vertices with 3 vector bosons $\left(\sum_{j=1}^{3} k_j = 0, \text{ all momenta incoming}\right)$:

$$ie\{(k_1-k_2)_{\mu_3}g_{\mu_1\mu_2} + (k_2-k_3)_{\mu_1}g_{\mu_2\mu_3} + (k_3-k_1)_{\mu_2}g_{\mu_3\mu_1}\}$$



$$ie\frac{\cos\vartheta_W}{\sin\vartheta_W}\{\text{as above}\}$$



MANCHESTER 1824
2nd Term 2010 The Standard Model of Particle Physics - Christian Schwanenberger - University of Manchester 103
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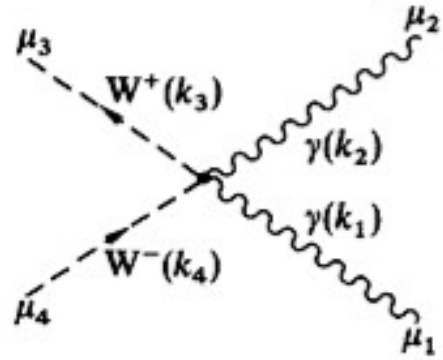
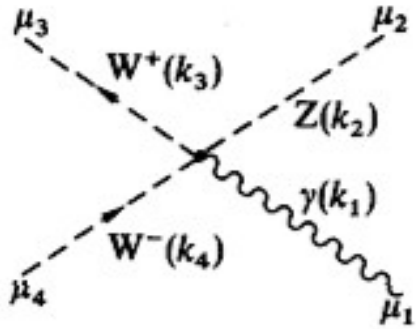
Vertices with 4 vector bosons $\left(\sum_{j=1} k_j = 0, \text{ all momenta incoming}\right)$:

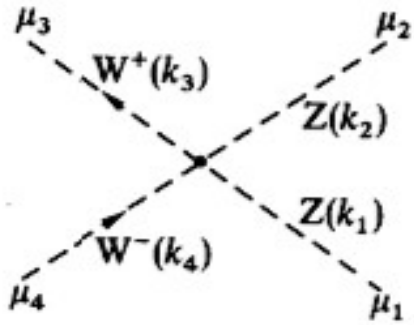
$$ie^2\{g_{\mu_1\mu_3}g_{\mu_2\mu_4} + g_{\mu_1\mu_4}g_{\mu_2\mu_3} - 2g_{\mu_1\mu_2}g_{\mu_3\mu_4}\}$$



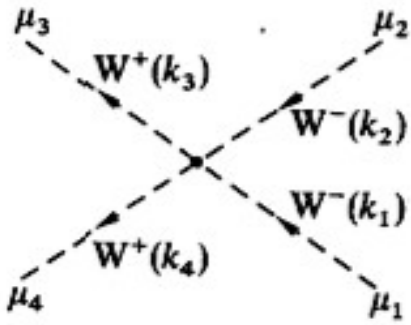
$$ie^2\frac{\cos\vartheta_W}{\sin\vartheta_W}\{\text{as above}\}$$



$$ie^2\frac{\cos^2\vartheta_W}{\sin^2\vartheta_W}\{\text{as above}\}$$

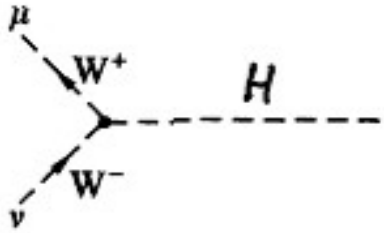


$$-ie^2\frac{1}{\sin^2\vartheta_W}\{\text{as above}\}$$

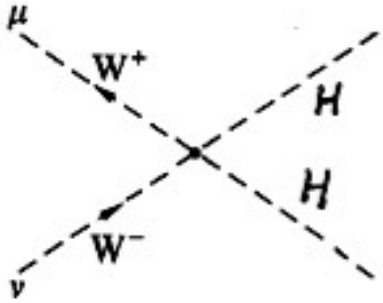


Vertices with Higgs bosons:

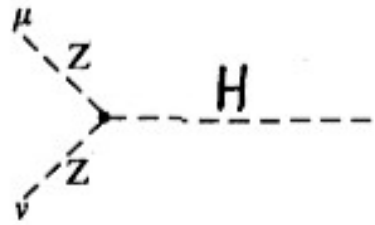
$$ig_{\mu\nu}\frac{2m_W^2}{v}$$



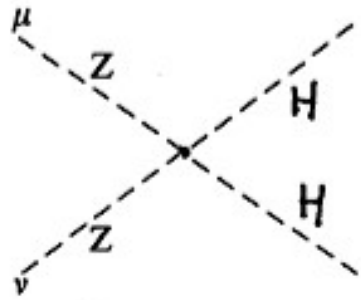
$$ig_{\mu\nu}\frac{2m_W^2}{v^2}$$



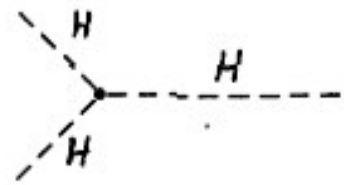
$$ig_{\mu\nu}\frac{2m_Z^2}{v}$$



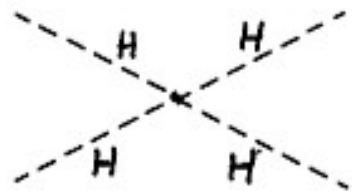
$$ig_{\mu\nu}\frac{2m_Z^2}{v^2}$$



$$-3i\frac{m_H^2}{v}$$



$$-3i\frac{m_H^2}{v^2}$$



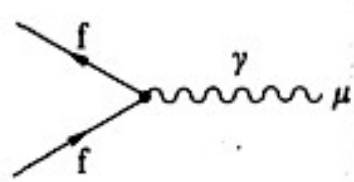
In the following we will number the leptons and quarks of the various families according to the scheme:

$$\nu_1 \equiv \nu_e, \quad \nu_2 \equiv \nu_\mu, \quad \nu_3 \equiv \nu_\tau;$$
$$\ell_1 \equiv e, \quad \ell_2 \equiv \mu, \quad \ell_3 \equiv \tau;$$
$$u_1 \equiv u, \quad u_2 \equiv c, \quad u_3 \equiv t;$$
$$d_1 \equiv d, \quad d_2 \equiv s, \quad d_3 \equiv b.$$

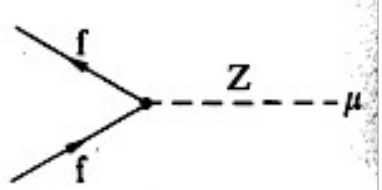
To refer to an arbitrary one of these particles we will use the generic label f; the electric charge is then Q_f, and T_3^f is the eigenvalue obtained by applying the third component of the weak isospin to the *left-handed* part of f. From (22.123), (22.77), (22.112)–(22.114), we then obtain the following vertices:

Fermion–Boson vertices:

$$-ieQ_f\gamma^\mu$$

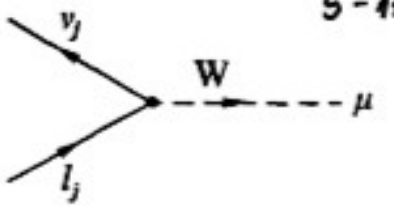


$$-i\frac{e}{\sin\vartheta_W\cos\vartheta_W}\left\{T_3^f\gamma^\mu\frac{1-\gamma_5}{2}-\sin^2\vartheta_W Q_f\gamma^\mu\right\}$$

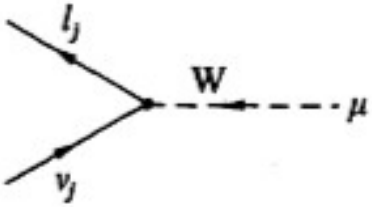


5-11

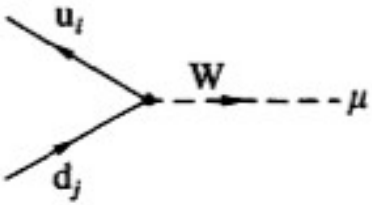
$$-i\frac{e}{\sqrt{2}\sin\vartheta_W}\gamma^\mu\frac{1-\gamma_5}{2}$$



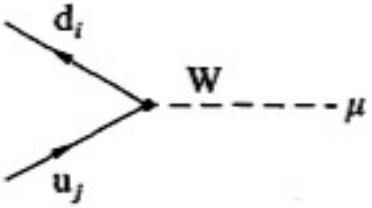
$$-i\frac{e}{\sqrt{2}\sin\vartheta_W}\gamma^\mu\frac{1-\gamma_5}{2}$$



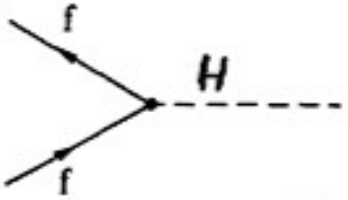
$$-i\frac{e}{\sqrt{2}\sin\vartheta_W}V_{ij}\gamma^\mu\frac{1-\gamma_5}{2}$$



$$-i\frac{e}{\sqrt{2}\sin\vartheta_W}V_{ji}^*\gamma^\mu\frac{1-\gamma_5}{2}$$

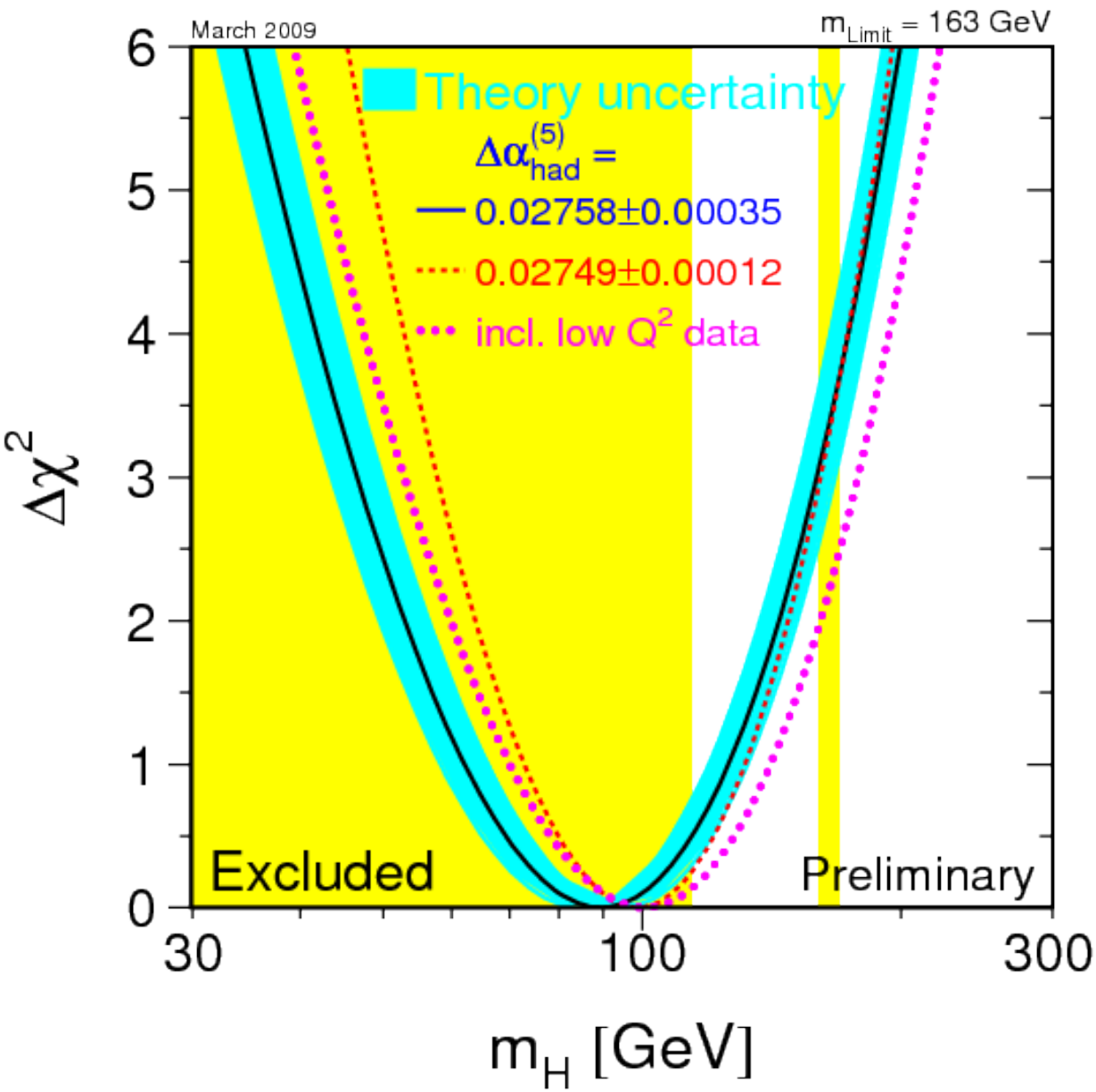


$$-i\frac{m_f}{v}$$

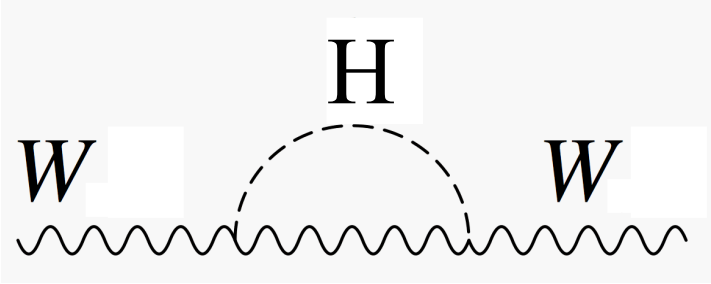


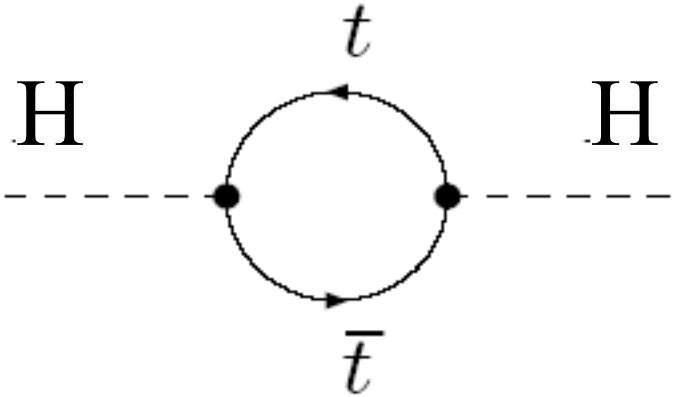
5. Higgs Physics

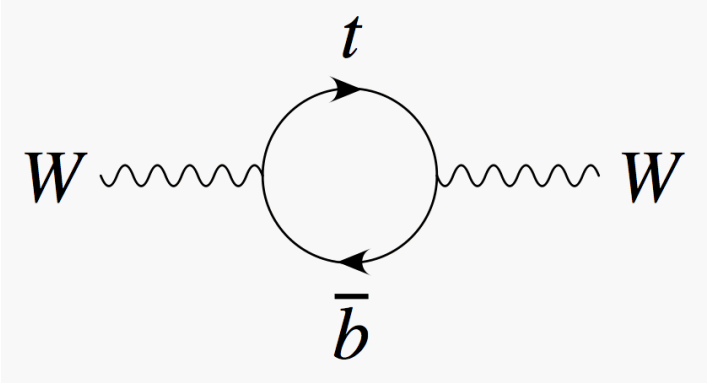
Fits to Electroweak Precision Data



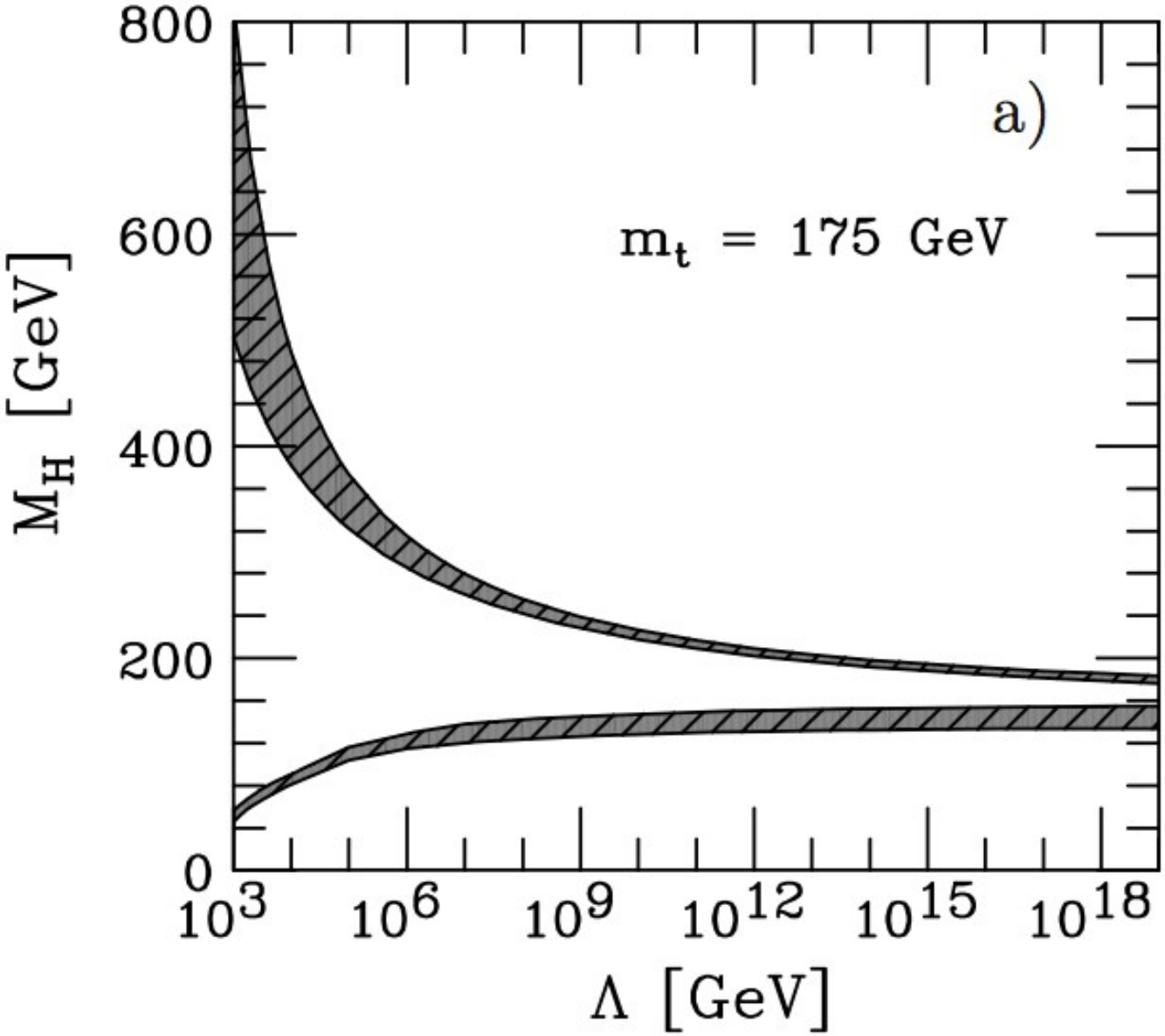
LEP, SLD, Tevatron



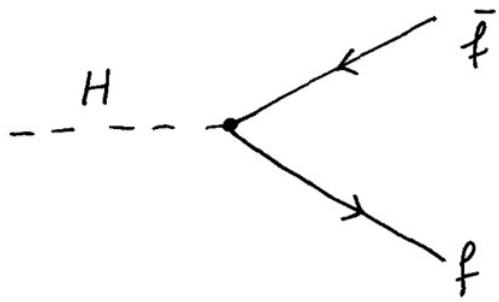




Bounds on SM Higgs Mass



Higgs decays to fermion - antifermion



$$-i\,\frac{m_f}{v}$$

The decay

$$H \to f\bar{f}$$

is possible for $m_H > 2m_f$.

Its decay rate is

$$\Gamma(H \to f\bar{f}) = \frac{m_H}{8\pi}\left(\frac{m_f}{v}\right)^2 N_c^f \left(1 - \frac{4m_f^2}{m_H^2}\right)^{3/2}$$

$$N_c^f = 1 \quad \text{for } f = \text{lepton},$$

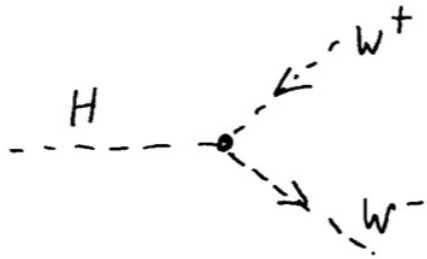
$$N_c^f = 3 \quad \text{for } f = \text{quark}.$$

(6.4) ✓

Higgs decay to W^+W^-

$$H \to W^+W^-$$

is possible for $m_H > 2m_W \cong 161\,\text{GeV}$.



$$i g_{\mu\nu} \frac{2m_W^2}{v}$$

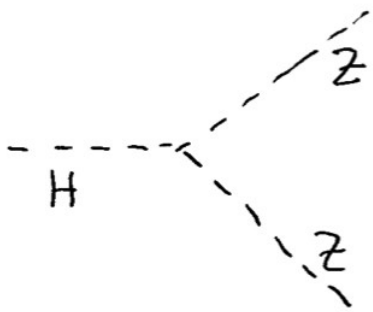
$$\Gamma(H \to W^+W^-) = \frac{m_H}{16\pi}\left(\frac{m_H}{v}\right)^2 \left(1 - \frac{4m_W^2}{m_H^2}\right)^{1/2}\left(1 - 4\frac{m_W^2}{m_H^2} + 12\frac{m_W^4}{m_H^4}\right). \quad (6.5)$$

✓

Higgs decay to ZZ,

$$H \rightarrow ZZ$$

is possible for $m_H > 2m_Z \cong 182\,\text{GeV}$.

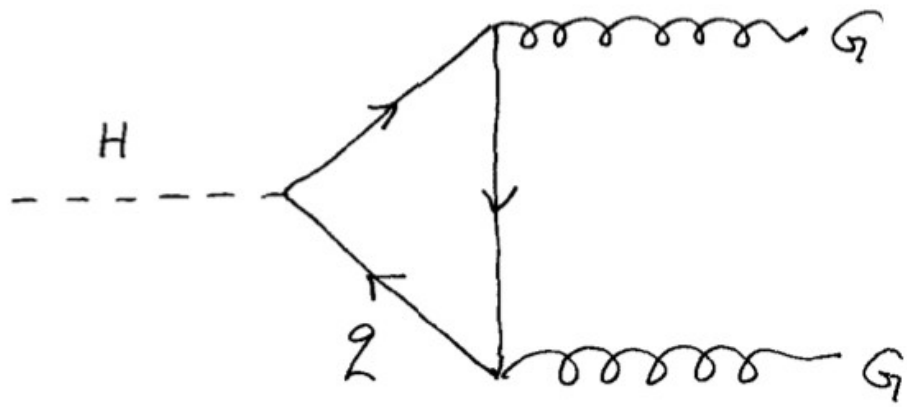


$$i g_{\mu\nu} \frac{2m_Z^2}{v}$$

$$\Gamma(H \rightarrow ZZ) = \frac{m_H}{32\pi}\left(\frac{m_H}{v}\right)^2 \left(1 - \frac{4m_Z^2}{m_H^2}\right)^{1/2}\left(1 - 4\frac{m_Z^2}{m_H^2} + 12\frac{m_Z^4}{m_H^4}\right) \qquad (6.6)$$

✓

decay Higgs → 2 gluons



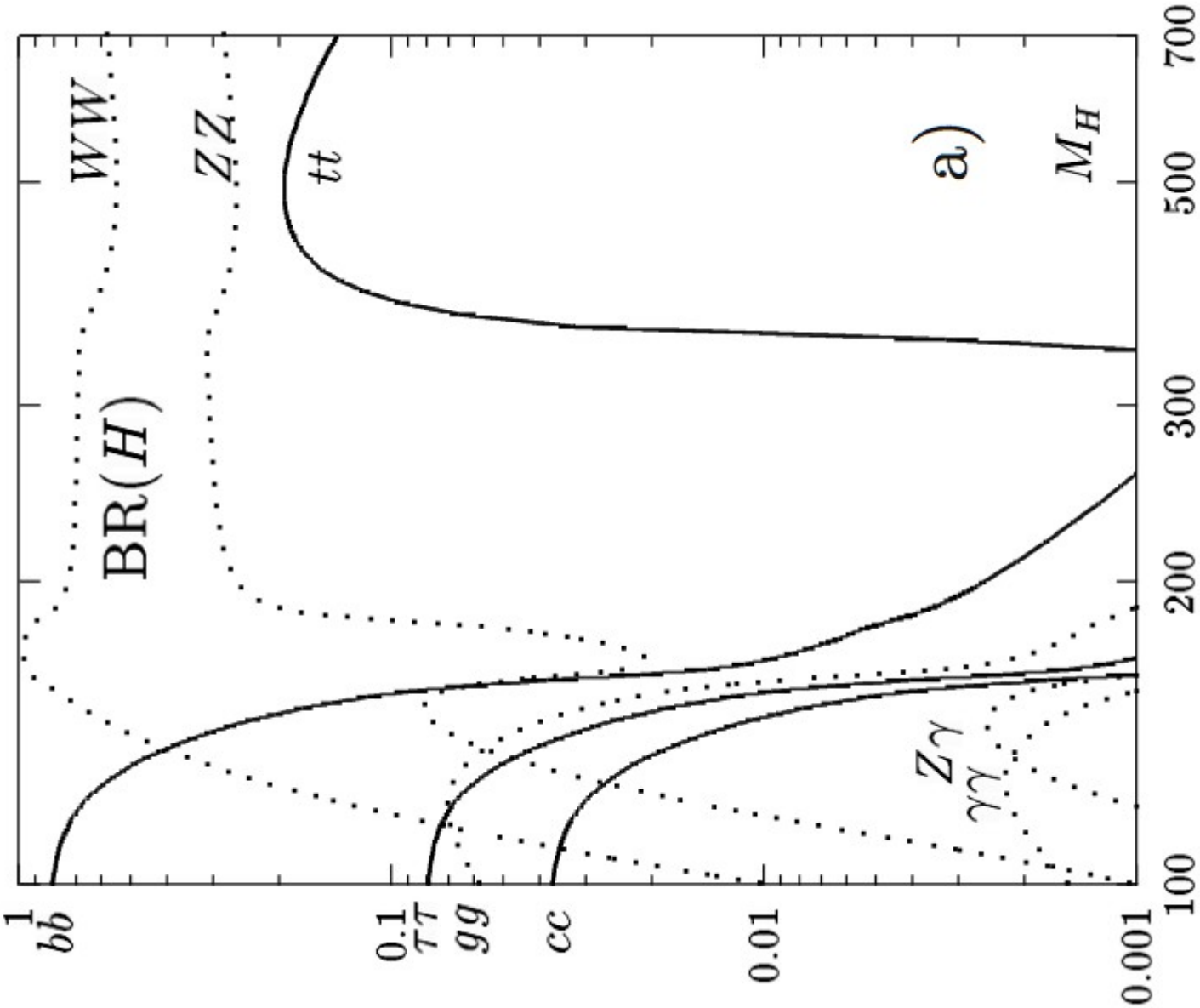
+ crossed diagram

$$\Gamma(H \to GG) = \frac{m_H}{8\pi}\left(\frac{m_H}{v}\right)^2 \left(\frac{\alpha_s}{\pi}\right)^2 \left| \sum_q \left(\frac{m_q}{m_H}\right)^2 J\left(\frac{4m_q^2}{m_H^2}\right)\right|^2 , \qquad (6.13)$$

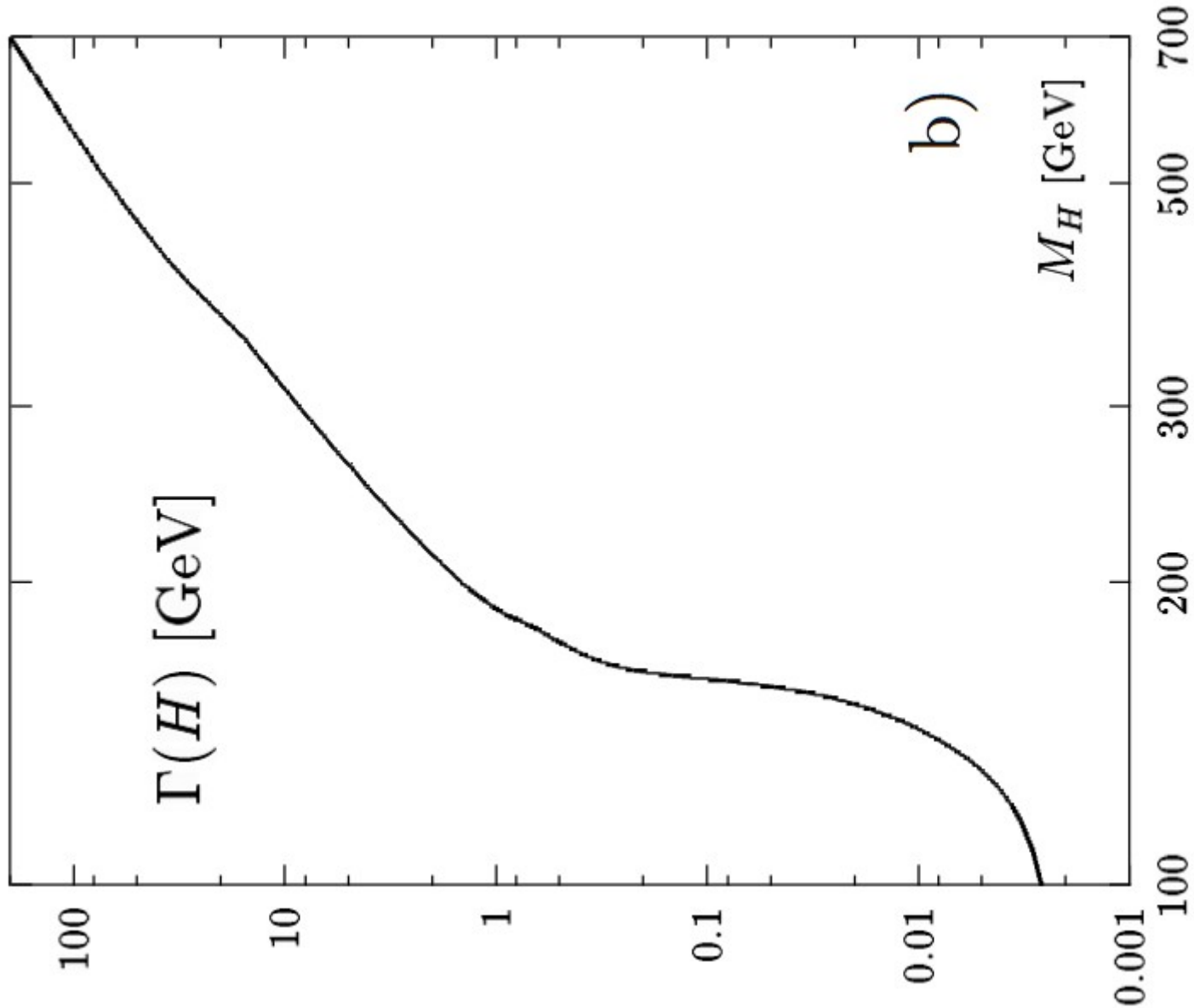
where

$$J(z) = \int_0^1 dv \, \frac{1-v}{z-v-i\varepsilon} \cdot \ln \frac{1+\sqrt{1-v}}{1-\sqrt{1-v}} . \qquad (6.14)$$

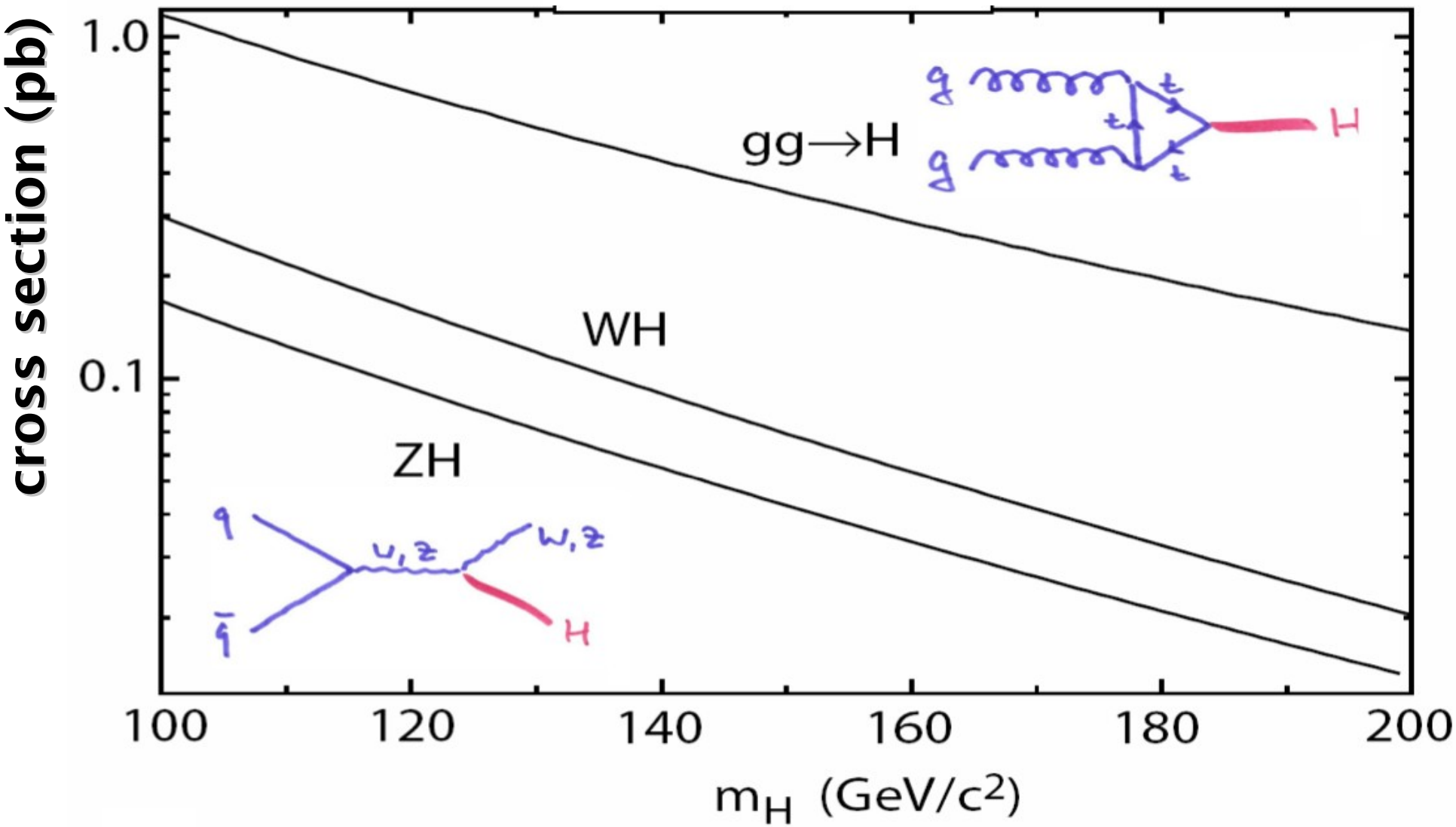
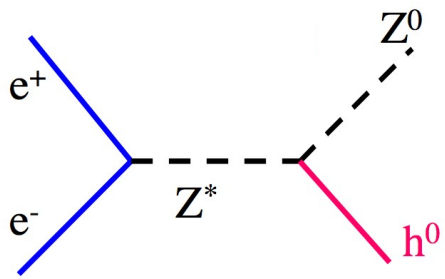
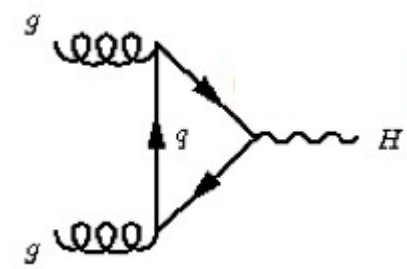
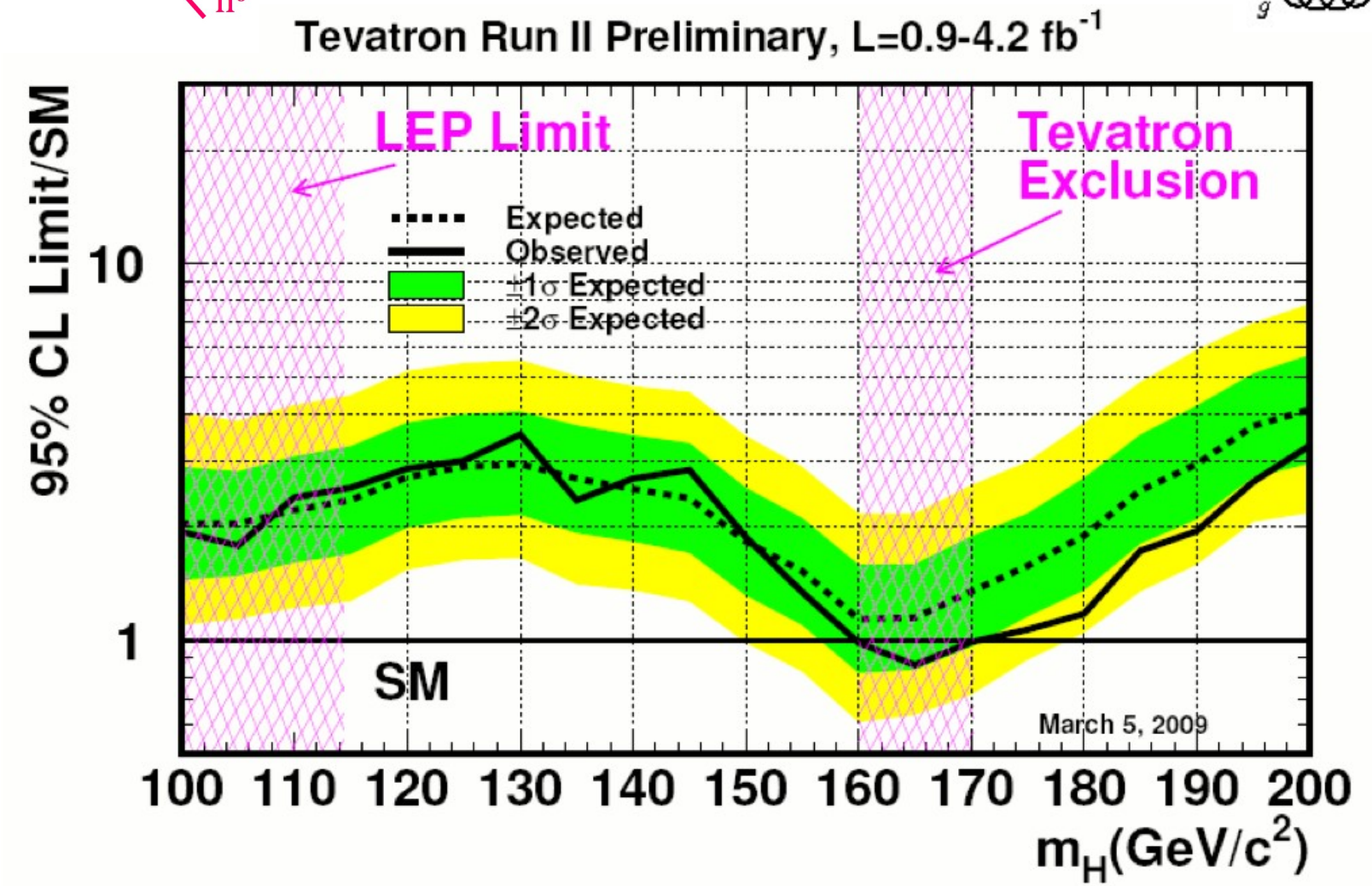
Branching Fractions of SM Higgs Particle



Total Decay Width of SM Higgs Particle



SM Higgs Production at the Tevatron



Search for the Higgs boson