2 Phenomenology

2.1 Neutrinos in the Standard Model

The Standard Model of particle physics is the basis of our understanding of the world around us today. It has been successful in explaining many different phenomena in particle physics and astronomy.

The theoretical basis of the Standard Model was established by Glashow, Weinberg, and Salam in 1967 (see references [10] and [11]). The fundamental constituents of matter are organized by generation or family. Each of the three generations has two leptons and two quarks, and each of these exists as particle and as anti-particle. The particles that make up the world around us are from the first generation; they are the electron and the electron neutrino, the up and the down quark.

Interactions in the Standard Model are divided into three categories: strong interactions which occur only between quarks are mediated by gluons, electromagnetic interactions which occur only between charged particles are mediated by photons, and weak interactions which occur between all particles are mediated by the W and Z bosons. In composite particles, these interactions occur between the constituents, and in this way, neutral composite particles for example can interact electromagnetically through their dipole moments. The magnetic dipole moment μ of a charged particle is proportional to its spin,

$$\mu_e = g\frac{e}{m}, \tag{2-1}$$

where e is the electric charge and m the mass of the particle [13]. In this equation, the factor g accounts for the anomalous magnetic moment; it is equal to two for electrons in Dirac theory and slightly higher when all Standard Model effects are included [14]. As before,

neutral composite particles such as the neutron also have a magnetic moment, which is a combination of the magnetic moments of the constituent particles [12].

Neutrinos play a special role in the Standard Model in that they are the only particles that have zero mass and only interact via the weak interaction. However, recent experimental evidence indicates that neutrinos are massive as well and several experiments plan to address this specific question.

Since they are massless, neutrinos travel at the speed of light, and a change in the Lorentz frame of reference does not change their helicity[1]; hence they only have one spin state. The neutrino exists only with left-handed chirality (ν_L) and the anti-neutrino exists only with right-handed chirality ($\overline{\nu_R}$). As a result, the total number of spin states is two[2] and the quantum mechanical description of a neutrino is a two-component wave function called Weyl spinor [12]. Since the neutrino is massless, its spin cannot be distinguished from its orbital angular momentum; both are parallel to the momentum vector and Standard Model neutrinos therefore have no magnetic dipole moment [15].

If neutrinos existed in two spin states, the second state should be produced in Z boson decays to two neutrinos. However, the number of neutrinos with a mass smaller than half the Z boson mass has been measured in Z decays to be three [16], which also limits the number of generations in the Standard Model to three. This holds true for all neutrinos that couple to the Z with the usual weak coupling strength.

The current experimental limits for a few neutrino properties are shown in table 2-1 [14].

neutrino	mass	mean life	magnetic moment
ν_e	<15eV	$> 7\times10^{9}$ s/eV	$< 1.8\times10^{-10}\mu_B$
ν_μ	<0.17MeV	> 15.4 s/eV	$< 7.4\times10^{-10}\mu_B$
ν_τ	<18.2 MeV	(no limit)	$< 5.4\times10^{-7}\mu_B$

Table 2-1. Current experimental limits for neutrino properties from reference [14].

1. The direction of the spin of massive particles depends on the frame of reference: A transformation to a frame moving faster than the particle changes the helicity (from left-handed to right-handed for example).

2. Ordinary (massive) spin 1/2 particles have four states: left-handed and right-handed particle and left-handed and right-handed anti-particle.

2.2 Extensions to the Standard Model

There are several indications from experiments looking for neutrino oscillations that the Standard Model description of neutrinos is not complete and that the neutrino mass might not be exactly zero.

Neutrinos interact and are produced in flavor eigenstates. If they are massive, the mass eigenstates (free space wave function) might not be the same as the flavor eigenstates. The time-development of the original state then produces other flavor eigenstates as well. This process is known as neutrino oscillation. If a neutrino beam of one (known) flavor is produced and then observed far away from the production point, the other flavor eigenstates can be observed as well.

Neutrino oscillations have already been observed by the LSND collaboration [17]. The experiment detects neutrinos produced in the decay of pions, which is a well understood Standard Model process. The number of electron-neutrino interactions and the number of muon-neutrino interactions observed in a detector 30 meters from the neutrino source do not agree with expectations. The differences can be explained by oscillations from muon neutrinos to electron neutrinos on the path between the neutrino source and the detector. This result has not yet been confirmed by another experiment, instead other experiments that search for the same signal have ruled out most of the predicted parameter space [18].

Cosmic ray experiments observe neutrinos produced when cosmic ray particles hit the upper atmosphere [1]. The interaction produces many pions, which predominantly decay through the channel $\pi^+ \rightarrow \mu^+ \nu_\mu$. The muon itself decays according to $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$. As a result, underground detectors expect about twice as many muon neutrinos as electron neutrinos. Experiments observe as many muon neutrinos as electron neutrinos. The absolute number of observed electron neutrinos is consistent with the predicted flux, whereas the observed number of muon neutrinos is smaller than expected [20].

Neutrinos produced by the fusion process in the solar core have been observed by several experiments ([19], [21], [22], and [23]). The observed energy spectrum of electron neutri-

nos is not consistent with the expected spectrum, but can be explained by neutrino oscillations.

The three oscillation scenarios require three different oscillation channels: the LSND result can only be explained with a large ν_μ–ν_τ mass difference, the atmospheric result can only be explained with a small ν_μ–ν_τ mass difference, and the solar neutrino result can only be explained with a very small ν_e–ν_μ mass difference [21]. Since only two mass differences are independent in a model with three neutrino families, these results are mutually contradicting and at least one requires a different explanation.

The Neutrino Magnetic Moment

One possible explanation of the solar neutrino spectrum is that the electron-neutrino has a magnetic moment of the order of $10^{-11}\mu_B$ [24]. Left-handed neutrinos produced near the core of the sun would experience a spin flip to undetectable right-handed neutrinos as they travel through the magnetic field of the sun and earth-based experiments measure a smaller neutrino flux than expected.

The current upper limits for the neutrino magnetic moment given in table 2-1 were obtained in neutrino-electron scattering experiments that searched for an increase in the neutrino-electron scattering cross section. The limit for the electron-neutrino was obtained in an experiment with neutrinos from a nuclear reactor [25], while the limit for the muon-neutrino was obtained in an experiment with neutrinos from pion decay [26]. The limit for the tau-neutrino was obtained in the CERN BEBC beam dump experiment in an analysis similar to this thesis [8]. It was assumed that the interaction of protons with nucleons produced D_s mesons, which then decayed to tau-neutrinos. Neutrino interactions were recorded in a bubble chamber that provided good track resolution and electron identification. Direct evidence for the interaction of the tau-neutrinos with nuclei was not found, and the tau neutrino flux was based on D_s production and decay parameters measured in other experiments.

Astrophysical arguments lead to more stringent limits for the neutrino magnetic moment, but these depend on the model used and the values assumed for some of the parameters. For this reason only the direct measurements were included in table 2-1.

The nucleosynthesis that occurred at the early age of the universe is responsible for the primordial abundance of ^{4}He. A neutrino magnetic moment of more than about $1.5\times10^{-11}\mu_B$ would modify the process and create an overabundance of helium that is not observed today [27]. This limit should not be exceeded by more than two neutrino flavors. The cooling of helium stars also gives an upper limit. The decay of plasmons in these stars into pairs would accelerate the cooling process. Since no such acceleration has been observed, the largest neutrino magnetic moment has to be smaller than $1.1\times10^{-11}\mu_B$ [28]. The supernova explosion 1987A produced neutrinos, a few of which were observed by underground detectors. The length of the neutrino signal was about 10s, which implies an upper limit for the right-handed neutrino luminosity that in turn corresponds to an upper limit for the magnetic moment of $(0.2-0.8)\times10^{-11}\mu_B$ [29]. This limit applies to all neutrino flavors.

Besides these astrophysical considerations, the detection of atmospheric neutrino interactions in underground detectors also leads to a limit [1]. The observation of neutrino oscillations implies that neutrino have mass, see section 2.2 below. If oscillations occur between muon-neutrinos and tau-neutrinos, then the properties of these neutrinos can be investigated in atmospheric neutrino interactions. Since no evidence for neutrino-electron magnetic moment interactions has been found, the upper limit for the tau-neutrino magnetic moment is $1.3\times10^{-7}\mu_B$ [7].

Several experiments have measured a correlation between the flux of solar neutrinos and the number of sun-spots [30]. The flux decreases when there is a lot of sun-spot activity, which could be explained by a ν_e magnetic moment of the order of $(10^{-11}-10^{-12})\mu_B$. The neutrinos would undergo a spin-flip as they pass through the magnetic field associated with those spots, decreasing the number of neutrinos observed at the various detectors [31].

There are several different models that predict a non-zero magnetic moment for the neutrino. The simplest extension to the Standard Model shows how the neutrino magnetic moment scales with the neutrino mass. More complex models introduce additional particles to obtain a large magnetic moment while keeping the neutrino mass small at the same time. Some of the more relevant models are discussed in this section.

If neutrinos have a magnetic moment, then the right-handed chirality state must also exist. If the two spin states are simply combined (ν_R and $\overline{\nu_L}$), neutrinos become Dirac particles with four states [13]. Since the two additional states have never been observed, they can not participate in weak interactions, and they cannot contribute to the width of the Z boson as mentioned above.

The Majorana description of neutrinos does not include the two unobserved neutrino states, and the opposite spin partner of Majorana neutrinos are their anti-neutrinos [20]. Similarly to the Weyl description, a Majorana neutrino has only two states, ν_L and $\overline{\nu_R}$. The difference to the Standard Model is that this neutrino has mass, which means that a Lorentz transformation to a frame of reference moving faster than the neutrino will transform ν_L into $\overline{\nu_R}$. In the Majorana description neutrinos necessarily have no magnetic moment [20].

The Standard Model can be extended in a simple way by adding right-handed neutrinos and left-handed anti-neutrinos [20]. The neutrino magnetic moment in this model is produced when virtual charged bosons and fermions interact with the photon field, which is shown in figure 2-1.

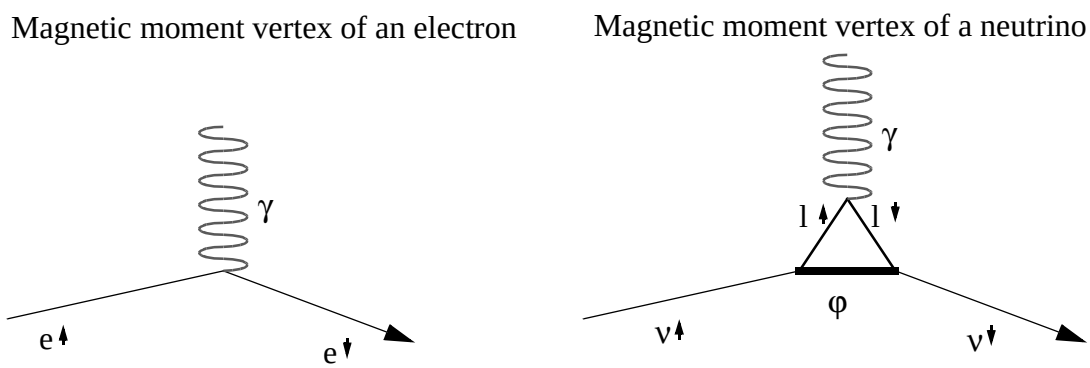

Figure 2-1. Magnetic moment interaction vertex of an electron and a neutrino. Which particles participate in the neutrino loop depends on the model, in this example leptons and gauge bosons φ comprise the loop.

The lowest order contribution to the magnetic moment interaction of a fermion is a simple tree diagram, shown on the left-hand side of figure 2-1. The charged fermion interacts with the photon field and changes its spin. The lowest order contribution to the magnetic moment interaction of a neutrino is a loop diagram, shown on the right-hand side of figure 2-1. The left-handed neutrino produces a pair of virtual particles that can also couple to the

right-handed neutrino. One of the virtual particles then interacts with the photon field and changes its spin as before. Afterwards the two particles re-combine to a right-handed neutrino.

A calculation of the neutrino magnetic moment at the lowest order involves loop diagrams as shown in figure 2-1 with quarks and leptons on the internal lines. The calculation gives a magnetic moment μ_ν that is proportional to the neutrino mass,

$$\mu_\nu = \frac{3eG_F}{8\sqrt{2}\pi} m_\nu , \tag{2-2}$$

where e is the electron charge, G_F the Fermi coupling constant, and m_ν the mass of the neutrino [20], and the numerical value for the neutrino magnetic moment is then

$$\mu_\nu = 3.1 \times 10^{-19} \mu_B \left(\frac{m_\nu}{1\,\mathrm{eV}} \right). \tag{2-3}$$

A tau neutrino mass at the current limit of 18.2MeV would give a magnetic moment of $\mu_\nu = 6 \times 10^{-12} \mu_B$ [14].

Another model that includes Dirac neutrinos is the left-right symmetric model. The left and right-handed neutrino helicities play identical roles at very high energies, while the symmetry is broken at small energies [32]. The predicted neutrino magnetic moment is similar to equation 2-3.

In Grand Unified Theories (GUT) the neutrino magnetic moment is in general not zero [33]; it has been calculated for models that include leptons and quarks in the same sector [34].

Models can also be constructed in which the neutrino mass is small and the magnetic moment nevertheless large. In one such model, the interactions of ν_e and $\overline{N_{eL}}$ (the antiparticle of N_{eR}, the right-handed electron-neutrino) is considered to be symmetric under an $SU(2)_\nu$ symmetry transformation with ν_e and $\overline{N_{eL}}$ forming a doublet [24]. In this case the Dirac mass of ν_e is a triplet under $SU(2)_\nu$ whereas the magnetic moment term is a singlet. As a result, equation 2-3 is invalid and a large magnetic moment is possible.

2.3 Neutrino Interactions

Standard Model neutrinos interact only via the weak interaction, exchanging W bosons in charged-current interactions (CC) or Z bosons in neutral current interactions (NC) with other particles. Examples for the two types of interaction are shown in figure 2-2.

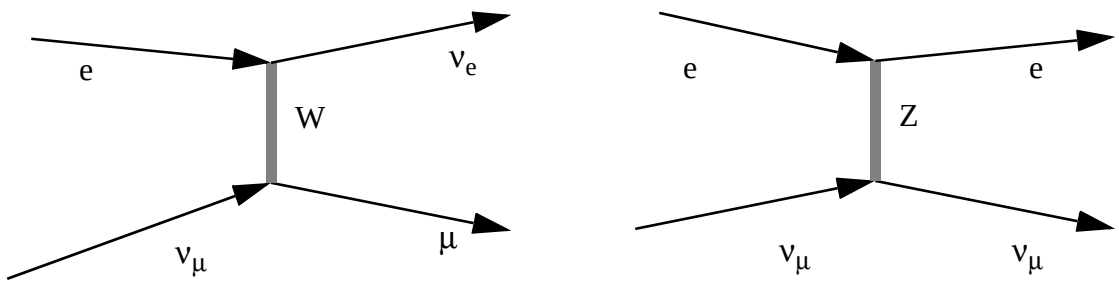

Figure 2-2. Feynman diagram showing two weak interactions of neutrinos (ν) with an electron (e). The left hand side shows a charged-current interaction (exchange of a W boson); the right-hand side shows a neutral-current interaction (exchange of a Z boson).

The differential cross section for a charged-current neutrino-electron interaction is given by

$$\frac{d\sigma^{CC}}{dy} = \frac{G_F^2 s}{\pi}, \tag{2-4}$$

where G_F is the Fermi coupling constant [12]. The fractional energy loss of the neutrino is given by $y = T/E_\nu$, where T is the outgoing electron energy and E_ν is the incoming neutrino energy. The square of the center-of-mass energy s is given in the laboratory frame (target electron at rest) by

$$s = m_e^2 + 2m_e E_\nu, \tag{2-5}$$

where m_e is the electron mass. The charged-current cross section is independent of y.

The differential cross section for a neutrino-electron neutral-current interaction is given by

$$\frac{d\sigma^{NC}}{dy} = \frac{G_F^2 m_e E_\nu}{2\pi}[(g_V^{\nu e} + g_A^{\nu e})^2 + (g_V^{\nu e} - g_A^{\nu e})^2(1-y)^2], \tag{2-6}$$

where g_V is the vector coupling, and g_A the axial coupling. The couplings are given by

$$g_V^{\nu e} = -0.5$$
$$g_A^{\nu e} = -0.5 + 2\sin^2\theta_W \quad , \tag{2-7}$$

where θ_W is the Weinberg angle with $\sin^2\theta_W = 0.223$.

The weak cross sections from equations 2-4 and 2-6 are also valid for the interaction between neutrinos and quarks. The neutrino-nucleon cross section is more complicated because it depends on the quark content of the nucleon and is modified by quark-quark interactions. Nevertheless, nucleons are about 2000 times heavier than electrons, and since the weak cross sections are proportional to the target particle mass, most of the events recorded in any neutrino-beam experiment are from neutrino-nucleon interactions.

Neutrino magnetic moment interactions can occur with any charged particle, electron or nucleon, and contrary to weak interactions, the interaction cross section is independent of the mass of the target particle.

The Feynman diagram for the magnetic moment interaction of an electron and a neutrino is shown in figure 2-3. The particles exchange a spin one photon and undergo a spin-flip in the interaction.

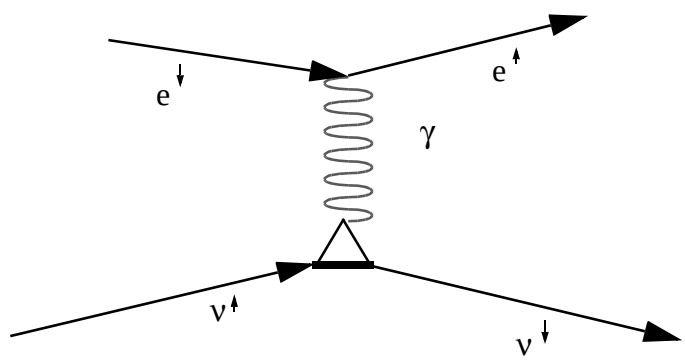

Figure 2-3. Feynman diagram of a neutrino-electron magnetic moment interaction. The arrow next to a particle symbol represents the helicity.

The interaction of the photon with the electron is an electromagnetic Standard Model process shown on the left-hand side of figure 2-1, while the interaction of the photon with the neutrino is shown on the right-hand side of figure 2-1. Since this coupling must include a

right-handed neutrino, the observation of magnetic moment interactions would establish its existence.

The differential cross section for the process shown in figure 2-3 is given in the lab frame by [35]

$$\frac{d\sigma^{\mu}}{dy} = f_{\mu}^{2}\,\frac{\pi\alpha^{2}}{m_{e}^{2}}\left(\frac{1}{y}-1\right), \tag{2-8}$$

where $\alpha \cong 1/137$ is the fine structure constant and $f_{\mu} = \mu_{\nu}/\mu_{B}$ is the size of the neutrino magnetic moment relative to a Bohr magneton. Equation 2-8 is only valid in the high-energy approximation of $y >> (m_{\nu}/E_{\nu})$ and it should not be used near the divergence at $y \rightarrow 0$. However, since the neutrino mass is small, experiments are typically far away from this divergence. The lower limit for *y* is typically determined by the experimental sensitivity to low-energy electrons. Since the differential cross section increases as *1/y*, this limited sensitivity therefore also determines the size of the total cross section and the total number of events that could be observed.

The initial state and final state particles are identical between weak neutral current interactions and magnetic moment interactions, which could give rise to quantum interference effects between the two interactions. However, the magnetic moment process changes the spin of the neutrino and the electron, which means that no such interference can occur.

This *y*-dependence can in principle be used to distinguish magnetic moment and neutral-current neutrino-electron scattering processes. Both cross sections are compared in figure 2-4, and the magnetic moment cross section dominates at very small *y*.

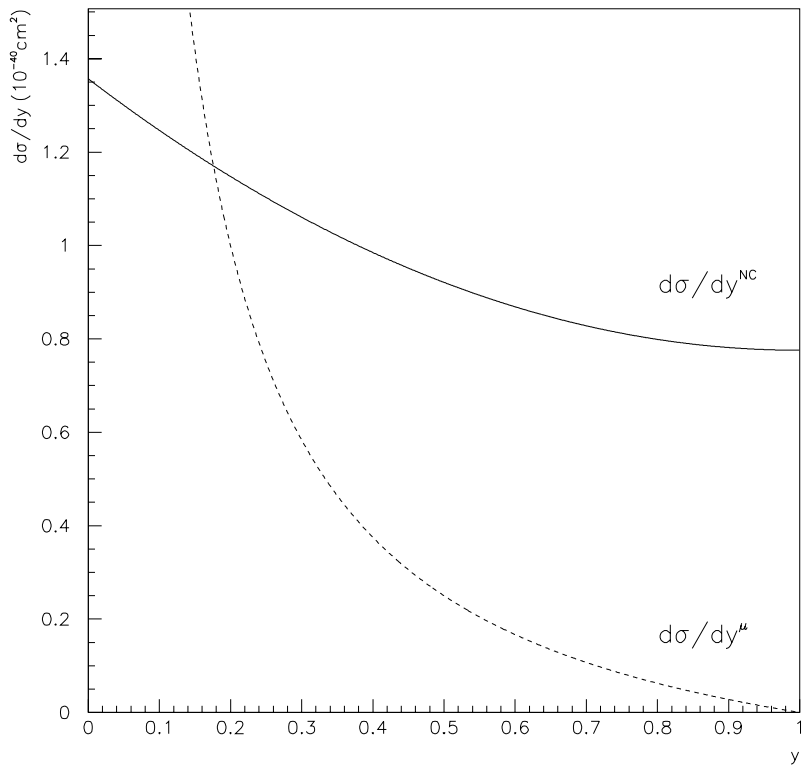

Figure 2-4. Differential cross section for neutrino-electron scattering. The dashed line shows the y-dependence due to a magnetic moment of $10^{-8}\mu_B$. The solid line shows the differential cross section for neutrino-electron neutral-current scattering (ν_μ or ν_τ). A neutrino energy of 50GeV was used to generate this plot.

Thus any search for evidence of a neutrino magnetic moment is concentrated on detecting electrons with small kinetic energies. Due to the small experimental tau neutrino flux it is usually not possible to measure the differential cross. Rather, only the total cross section is measured, which is found for magnetic moment interactions by integrating equation 2-8:

$$\sigma_{tot}^{\mu} = \int_{y_{min}}^{1} \frac{d\sigma^{\mu}}{dy} dy = f_{\mu}^{2} \frac{\pi\alpha^{2}}{m_e^{2}} [y_{min} - \ln y_{min} - 1] \ . \qquad (2\text{-}9)$$

The lower integration limit is given by the low-energy cutoff $y_{min} = T_{min}/E_{\nu}$, where T_{min} is the low-energy experimental detection limit for electrons. Since *y* is smaller than one, the

total cross section depends logarithmically on y_{min}. It also only increases logarithmically with the incoming neutrino energy.

The extensions to the Standard Model mentioned above indicate that neutrinos might possess a magnetic moment, and some of them predict values for the magnetic moment that are just below the current experimental sensitivity [24]. If the solar neutrino problem is indeed due to a ν_e magnetic moment, then the other two neutrino flavors should have magnetic moments of similar magnitude.

Neutrino-electron scattering provides a relatively background-free channel to detect magnetic moment interactions because the weak cross section for this interaction is small. Magnetic moment interactions would increase the number of scattered electrons, predominantly with small momentum. Measuring the number of neutrino-electron scattering events therefore yields an estimate for the tau neutrino magnetic moment.